QUARK DECONFINEMENT IN NEUTRON STAR CORES:
THE EFFECTS OF SPIN-DOWN
JAN E. STAFF & RACHID OUYED
Department of Physics and Astronomy, University of Calgary, 2500 University Drive NW, Calgary, Alberta T2N 1N4, Canada

AND

PRAVANTH JAIKUMAR
Physics Division, Argonne National Laboratory, Argonne, Illinois 60439-4843, USA

ABSTRACT

We study the role of spin-down in driving quark deconfinement in the high density core of isolated neutron stars. Assuming spin-down to be solely due to magnetic braking, we obtain typical timescales to quark deconfinement for neutron stars that are born with Keplerian frequencies. Employing different equations of state (EOS), we determine the minimum and maximum neutron star masses that will allow for deconfinement via spin-down only. We find that the time to reach deconfinement is strongly dependent on the magnetic field and that this time is least for EOS that support the largest minimum mass at zero spin, unless rotational effects on stellar structure are large. For a fiducial critical density of 5ρ0 for the transition to the quark phase (ρ0 = 2.5 × 10^{14} g/cm^3 is the saturation density of nuclear matter), we find that neutron stars lighter than 1.5M⊙ cannot reach a deconfined phase. Depending on the EOS, neutron stars of more than 1.5M⊙ can enter a quark phase only if they are spinning faster than about 3 milliseconds as observed now, whereas larger spin periods imply that they are either already quark stars or will never become one.

Subject headings: neutron stars, spin-down, quark deconfinement

1. INTRODUCTION

Neutron stars, formed in the aftermath of core collapse supernovae, are born rotating with spin periods that can be down in the millisecond range. Coupled to this fast rotation, magnetic flux conservation in the imploding progenitor provides the nascent neutron star with magnetic fields in the range 10^{12}-10^{15} G, as confirmed by a variety of pulsar observations (Lyne & Graham-Smith 1997). A customary first approximation is to model the pulsar as a rotating magnetic dipole which loses angular momentum through electromagnetic radiation. Consequently, the star spins down and the decreased centrifugal force causes the core density to rise. As this occurs, baryonic matter can undergo transitions to exotic phases of strongly interacting matter, finally giving way to a phase in which quarks are deconfined. Such a deconfined phase might explain the most energetic astrophysical phenomena such as gamma-ray bursts and cosmic rays from compact objects (Ouyed, Keränen & Maalampi 2005a; Ouyed, Rapp & Vogt 2005b). A phase transition to quark matter can result in the formation of a quark star, if certain stability criteria are fulfilled (Weber 2005).

Our main objective in this letter is to obtain accurate timescales for spin-down of a neutron star to the deconfinement density. Ultimately, we would like to know the likelihood of a neutron star to undergo a phase conversion to a quark star. To this end, we explore 4 different EOS to determine plausible bounds on masses and spin-periods that are consistent with a phase transition to quark matter in the star’s spin-down lifetime. Aspects of spin-down by magnetic braking and subsequent deconfinement are outlined in section 2 along with the approximations made in this work. In section 3 we elaborate on our computations and the EOS used (see Fig. 1), connecting them to results displayed in Table 1 and Figure 2. Section 4 gathers our principal conclusions based upon an interpretation of results in Table 2 and Figures 3 & 4.

2. SPIN-DOWN AND DECONFINEMENT

It is well-known that the energetics of pulsed emission as well as unpulsed nebular radiation from a neutron star can be generally understood as deriving from its rotational kinetic energy (Manchester & Taylor 1977). While the rotational power can be calculated from measurements of the spin-period (P) and its derivative (P’) in a model-independent way, theoretical calculations linking surface magnetic fields with the spin-down rate (loss of rotational kinetic energy) require a model for the magnetic field configuration. In this paper, we assume a dipolar time-independent magnetic field for the sake of simplicity. Although more complicated configurations with toroidal components are likely in reality, these will not affect our quantitative results which depend more on the absolute strength of the magnetic field rather than its detailed shape. Further, surface magnetic fields are likely to decay over a million years or so, and we do not consider this complication here since its qualitative impact can be easily inferred from our results. We further assume that gravitational radiation is a negligible contributor to spin down, i.e. our neutron stars are born axisymmetric and remain as such, so that the spin-down rate of a neutron star is entirely due to dipolar magnetic field braking as given by (e.g. Deutsch 1955; Manchester & Taylor 1977):

\[ \frac{d\Omega}{dt} = -\frac{2B^2R^6\Omega^3}{3Mc^3}\sin^2\chi \] (1)
where $B$ is the equatorial surface magnetic field, $I$ is the moment of inertia, $R$ the equatorial radius of the star, $\Omega = 2\pi/P$ its instantaneous angular velocity and $c$ the speed of light. In equation (1) above, the inclination angle of the dipole ($\chi$) has been averaged over a sphere representing a population of fixed mass and magnetic field ($\sin^2 \chi = 2/3$). With all quantities expressed in cgs units, equation (1) can be integrated to yield the time for a neutron star to spin down to a particular $\Omega$. The additional (implicit) input that is required to solve for the spin-down time is the EOS, which determines the $R$ and $I$ for a given $\Omega$.

Computing the spin-down time to deconfinement requires precise knowledge of the details of a phase transition to quark matter. Although yet to be determined exactly from first principles, it is expected to occur on general grounds at high baryonic density, whose value depends on the EOS employed (Heiselberg & Pandharipande 2000; Glendenning 2000). If the interface tension between the quark and nuclear phases is too high, a first order phase transition to a pure quark matter core at very high density ($\sim 10\rho_0$) is likely, by which time the star may become gravitationally unstable and collapse to a black hole. Several other phases, such as meson-condensed phases or structured nuclear matter can intervene at lower densities, softening the EOS. The concomitant rapid contraction of the softer matter may lead to non-monotonic behavior of the angular velocity $\Omega$ and cusps in the braking index $\frac{\Omega}{P}$. The transition may then occur smoothly over a range of densities from $(4-8)\rho_0$ with an increasing proportion of quark matter in the mixed phase (Glendenning 1992), and a continuous variation of the pressure and angular velocities. Our working assumptions will be in line with the latter, and we choose a threshold deconfinement density of $5\rho_0$. Obviously, our quantitative results depend on this assumed value, but will remain qualitatively true irrespective of the transition density one may choose. When the central baryonic density reaches the critical density of $\rho_{\text{crit}} = 5\rho_0$, deconfined quark matter will appear in the inner regions of the neutron star. If the strange quark matter hypothesis (Itoh 1970) holds true, it is possible that the entire star converts to a quark star in a violent phenomenon termed the Quark Nova (Ouyed, Dey & Dey 2002, Keränen, Jaikumar & Ouyed 2004). Aspects and consequences of the Quark Nova are discussed in Ouyed et al. (2005a) and Ouyed et al. (2005b).

3. NEUTRON STAR MODELS AND EQUATIONS OF STATE

In order to explore the increase in central density due to spin-down, we create sequences of neutron star models using the RNS code (Stergioulas & Friedman 1995) developed specifically to treat rapidly rotating neutron stars. Models in a particular sequence have the same EOS, constant baryon number, increasing central density and decreasing angular velocity. We assume that the neutron star is born axisymmetric and initially rotates at Keplerian frequencies. For each sequence, the

![Figure 1](image1.png)

**Fig. 1.** The EOS used (see §3).

*RNS code outputs (among other physical quantities) $I$, $\Omega$, and $R$, which feed into equation (1). We emphasize that since the magnetic field does not appear explicitly in the calculations performed by the RNS code, it does not affect the structural evolution of the star or the central density at which the transition to quark matter occurs. In principle, extremely high magnetic fields exceeding $10^{15}$ G can affect neutron star structure (Bocquet et al. 1995). In our simplified approach, as per equation (1), the magnetic field only determines the time to reach the deconfinement density.*

We employ 4 different EOS which are, in increasing order of stiffness: BBB2 (Baldo et al. 1997), C (Bethe & Johnson 1974), APR (Akmal, Pandharipande & Ravenhall 1998) and OBJ (Jaikumar & Ouyed 2006). The pressure versus density curve for each of the above EOS is plotted in Figure 1 for densities ranging from $\rho_0$ to $5\rho_0$. The OBJ EOS is a lot stiffer than the rest at high densities, due to strong repulsion from vector mesons at short distances (details in Jaikumar & Ouyed 2006).

The minimum baryonic mass that is required for a given EOS to support spin-down to deconfinement is found by the RNS code as the sequence that spins down.

![Figure 2](image2.png)

**Fig. 2.** Frequency vs central density for EOS used. All sequences are with constant baryonic mass and for minimum mass configurations (see Table I). The minimum mass configurations are those that can reach a critical density of $5\rho_0$ due to spin down.
to $\rho_{\text{crit}}$ at zero spin. The gravitational mass of such a model at zero spin is what we call minimum mass. Figure 2 displays the spin frequency of neutron stars with minimum mass, as a function of its central density. The curves shown are for a 1.46$M_\odot$ star with EOS BBB2, a 1.53$M_\odot$ star with EOS C, a 1.78$M_\odot$ star with EOS APR, and a 2.8$M_\odot$ star with EOS OBJ (all masses are gravitational masses at zero spin).

Equation (1) implies that it takes infinitely long to spin-down to zero frequency. In practice, we therefore compute the time to reach within 1% of $\rho_{\text{crit}}$, as illustrated in Figure 3. For a minimum mass star, we take this time to be the maximum time for a star with a given EOS to reach deconfinement density. Neutron stars with mass lower than the minimum mass will not reach deconfinement density due to spin-down, and stars with higher masses will reach the critical density sooner, before they have spun down completely. Conversely, a maximum mass can also be found, such that any mass beyond this would already have central densities exceeding $\rho_{\text{crit}}$ at the Kepler frequency so that it is already in a deconfined state and spin-down causes no further change.

The minimum and maximum masses as described above are listed in Table 1 for each EOS. Note that the OBJ EOS, on account of its stiffness, predicts a large minimum mass in order to spin-down to deconfinement. The minimum density at Keplerian frequency required to reach deconfinement density ($\rho_{\text{min,K}}$) is lowest for the OBJ EOS and highest for the APR EOS (these are the two stiffest EOS). This is because the moment of inertia is so much larger for the OBJ EOS as compared to the others, that it more than compensates for the increase in angular momentum with increasing stiffness of the EOS. This also explains the lower spin frequency at a given central density for the OBJ EOS.

**4. RESULTS AND CONCLUSIONS**

The maximum time to reach deconfinement via spin-down can be read off from Figure 3 for each EOS for magnetic fields in the range $10^{12} - 10^{16}$G. For any of the EOS, it takes up to a few hundred years for the star to spin down to 0.99$\rho_{\text{crit}}$ for a $10^{12}$G magnetic field. This time is drastically reduced to about an hour for a $10^{15}$G magnetic field. It is important to emphasize that the curves in Figure 3 correspond to the minimum mass star in each case, i.e. it takes an infinite time for the magnetic field to spin such a star down to $\rho_{\text{crit}}$. In Table 2 we therefore list the time it takes the star to increase its central density to within 1% of the critical density (i.e. 0.99$\rho_{\text{crit}}$) due to spin-down.

Figure 4 shows the period as a function of time for each EOS and for different magnetic field strengths. Just as before, this is for minimum mass stars and we plot the period until the central density is within 1% of $\rho_{\text{crit}}$. It is noteworthy that the central density does not change much after the rotation period slows to about $P_{\text{max}} = 3$ ms (about 6 ms for the OBJ EOS). This is independent of the magnetic field since that only determines the time to deconfinement, not the evolution of period with time. From the above limiting value of the period, and the minimum mass results tabulated in Table 1 for our fiducial critical density of $5\rho_0$, we conclude that only stars with mass greater than about 1.5$M_\odot$ that have a spin period $P > P_{\text{max}}$ can reach a deconfined phase. Conversely, if the star has $P > P_{\text{max}}$ as observed now, it cannot change its central density sufficiently to undergo deconfinement due to spin-down, and must either remain a neutron star or have been born as a quark star.

Although the critical density and consequently $P_{\text{max}}$ is not known precisely, the range of currently observed isolated neutron star masses in conjunction with our study implies that such stars can spin-down to a deconfined phase only if the deconfinement density is not too much above 5$\rho_0$. If the deconfinement density is much larger than this, neutron stars in the currently observed mass range (Stairs 2004) cannot increase their central densities enough via spin-down to reach the threshold, and will remain neutron stars for their lifetimes. This conclusion can only be avoided by assuming that all neutron stars are actually quark stars and are born as such in the aftermath of supernova explosions. Since most realistic quark matter EOS cannot support a maximum mass greater than 1.6$M_\odot$ (see e.g. Dey et al. 1998), it is unlikely that this is true. We therefore believe that the existence of quark stars is predicated upon having a low deconfinement threshold ($\lesssim 5\rho_0$). Once this threshold is pinned down via theoretical studies addressing Quantum Chromodynamics at high baryon density, we will be able to make more definitive statements about the existence of quark stars and related phenomena. Until then, the possibility that some isolated neutron stars are really quark stars cannot be dismissed.

---

**TABLE 1**

<table>
<thead>
<tr>
<th>EOS</th>
<th>$M_{\text{min}}$ ($M_\odot$)</th>
<th>$M_{\text{max}}$ ($M_\odot$)</th>
<th>$\rho_{\text{min,K}}(\rho_0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BBB2</td>
<td>1.46</td>
<td>1.79</td>
<td>3.92</td>
</tr>
<tr>
<td>APR</td>
<td>1.78</td>
<td>2.16</td>
<td>4.12</td>
</tr>
<tr>
<td>C</td>
<td>1.53</td>
<td>1.83</td>
<td>3.88</td>
</tr>
<tr>
<td>OBJ</td>
<td>2.8</td>
<td>3.36</td>
<td>2.52</td>
</tr>
</tbody>
</table>

---

**TABLE 2**

<table>
<thead>
<tr>
<th>EOS</th>
<th>$M_{\text{min}}$ ($M_\odot$)</th>
<th>$10^{15}$ G</th>
<th>$10^{14}$ G</th>
<th>$10^{13}$ G</th>
<th>$10^{12}$ G</th>
</tr>
</thead>
<tbody>
<tr>
<td>BBB2</td>
<td>1.46</td>
<td>96 min</td>
<td>6.7 days</td>
<td>667 days</td>
<td>183 years</td>
</tr>
<tr>
<td>APR</td>
<td>1.78</td>
<td>73 min</td>
<td>5.1 days</td>
<td>505 days</td>
<td>152 years</td>
</tr>
<tr>
<td>C</td>
<td>1.53</td>
<td>89 min</td>
<td>6.2 days</td>
<td>619 days</td>
<td>170 years</td>
</tr>
<tr>
<td>OBJ</td>
<td>2.8</td>
<td>348 min</td>
<td>24 days</td>
<td>2418 days</td>
<td>663 years</td>
</tr>
</tbody>
</table>

---

We thank S. Morsink for help with the RNS code and Ken Nollett for informative discussions. The research of R.O. is supported by an operating grant from the Natural Science and Engineering Research Council of Canada (NSERC) as well as the Alberta Ingenuity Fund (AIF). J.S. acknowledges the hospitality of the University of Alberta and of Argonne National Laboratory where parts of this work were performed. P.J. is supported by the Department of Energy, Office of Nuclear Physics, contract...
for different magnetic fields. The curves terminate when the magnetic field determines only the time to reach deconfinement, and does not enter the hydrostatic equations (see text).

<table>
<thead>
<tr>
<th>Period [ms]</th>
<th>APR</th>
<th>BBB2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>3.0</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>4.0</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>5.0</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>


Fig. 4.— Period vs time for the EOS studied in this paper with $\rho_{\text{crit}} = 5\rho_0$, with the minimum gravitational masses listed in Table I and for different magnetic fields. The curves terminate when $\rho$ is within 1% of $\rho_{\text{crit}}$. Note that for a given EOS all curves reach the same final period since the magnetic field determines only the time to reach deconfinement, and does not enter the hydrostatic equations (see text).

REFERENCES

Deutsch, A. J. 1955, AnAp, 18, 1
Glendenning, N., Compact Stars, 2nd ed. (Springer-Verlag, New York)
Lyne, A. G. & Graham-Smith, F. 1998, Pulsar Astronomy, Cambridge Univ. Press, UK
Manchester, R. N. & Taylor, J. H. 1977, Pulsars, Freeman, San Francisco
Stairs, I. H. 2004, 304, 547