$N = 8$ attractorsSergio Ferrara^{1,*} and Renata Kallosh^{2,†}¹*Physics Department, Theory Unit, CERN, 1211 Geneva 23, Switzerland
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We derive and solve the black hole attractor conditions of $N = 8$ supergravity by finding the critical points of the corresponding black hole potential. This is achieved by a simple generalization of the special geometry symplectic structure to all extended supergravities with $N > 2$. There are two classes of solutions for regular black holes, one for $1/8$ BPS ones and one for the non-BPS. We discuss the solutions of the moduli at the horizon for BPS attractors using $N = 2$ language. An interpretation of some of these results in $N = 2$, STU black hole context helps to clarify the general features of the black hole attractors.

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I. INTRODUCTION

Regular $N = 8$ BPS black holes in four dimensions have a fascinating feature that the entropy formula is known to be given by the square root of the unique Cartan-Cremmer-Julia quartic invariant [1,2] constructed from the fundamental 56 representation (p, q) of the $E_{7(7)}$ group. This follows from U -duality [3], so that

$$\frac{S_{\text{BPS}}}{\pi} = \sqrt{J_4(p, q)}, \quad J_4 > 0 \quad (1.1)$$

For the $N = 2$ BPS extremal black holes the moduli near the horizon are stabilized due to the attractor mechanism [4–7] based on special geometry [8–11]. For $N > 2$ the attractor mechanism was also established starting with [6,7] and was studied in great details in [12–14].

The $N = 2$ BPS as well as non-BPS extremal black holes under certain condition share the common feature that the moduli near the horizon are stabilized due to the attractor mechanism [7,15]. Recently various aspects on the non-BPS black holes were studied, see [16] and references therein.

The purpose of this paper is to establish the black hole attractor conditions of $N = 8$ supergravity which would respect the U -duality of $N = 8$ supergravity and allow the possibility for BPS and non-BPS solutions. This will allow us to develop some new connections between the black holes in string theory/supergravity and quantum information theory (QIT), to continue the recent trend due to [17–19]. It has been argued in [18] on the basis of the connection between the quartic invariant of the $E_{7(7)}$ group and the STU black hole entropy [20] that the critical point of $N = 8$ black hole potential may have a BPS as well as a non-BPS solution.

$$\frac{S_{\text{nonBPS}}}{\pi} = \sqrt{-J_4(p, q)}, \quad J_4 < 0 \quad (1.2)$$

Here we will construct a simple generalization of the tools of $N = 2$ special geometry to all $N > 2$. This will allow us, as an application of the general formalism, to find the attractor equations for any N which are closely related to those in $N = 2$. In particular in $N = 8$ case we will find a set of simple algebraic equations for $N = 8$ BPS and non-BPS black holes. Equations (3.15) and (3.17) will be derived and the basic features of these black holes will be explained. An analysis of the attractor equations for the moduli will be performed by using an $N = 2$ language and properties of $N = 2$ vector multiplets embedded into $N = 8$ supergravity.

II. GENERALIZATION OF $N = 2$ SPECIAL GEOMETRY FOR $N > 2$

Here we present simplified version of flat symplectic bundles which were constructed in [12] as a generalization of $N = 2$ special geometry. Consider any $N \geq 2$, $d = 4$ supergravity interacting with some vector multiplets. In $N = 2$ case the theory is defined by the elegant special geometry [10,11], based on symplectic sections (f, h) ¹:

$$(f, h) \equiv (L^\Lambda, \bar{\mathcal{D}}_{\hat{k}} \bar{L}^\Lambda; M_\Lambda, \bar{\mathcal{D}}_{\hat{k}} \bar{M}_\Lambda) \quad (2.1)$$

Here the “hat” covariant derivative over the moduli $\mathcal{D}_{\hat{k}}$ means the flat derivative in the moduli space. It is related to the “curved” derivative over the moduli as follows: $\mathcal{D}_{\hat{k}} = e_k^{\hat{k}} \mathcal{D}_k$ where the inverse bein $e_k^{\hat{k}}$ is such that the metric of the curved moduli space is $G_{\hat{k}\hat{l}} = e_k^{\hat{k}} e_l^{\hat{l}} \delta_{kl}$ and $\delta_{\hat{k}\hat{l}}$ is the moduli independent flat metric. Here $\mathcal{D}_k \equiv \partial_k + \frac{1}{2} \partial_k K$ where K is the Kähler potential. Note that (f, h) are square complex matrices.

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¹Here, as well as in Refs. [12–14] (f, h) are the complex conjugate of those introduced in [10,11].

The sections (f, h) are related to the real $Sp(2n, \mathbb{R})$ matrix² as follows³

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \Rightarrow \begin{pmatrix} f \\ h \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} A - iB \\ C - iD \end{pmatrix} \quad (2.2)$$

and vice versa

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \sqrt{2} \begin{pmatrix} \text{Re}f & -\text{Im}f \\ \text{Re}h & -\text{Im}h \end{pmatrix} \quad (2.3)$$

The symplectic property of the real $Sp(2n, \mathbb{R})$ matrix requires that

$$S = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad S^t \Omega S = \Omega \quad \Omega = \begin{pmatrix} 0 & -\mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix} \quad (2.4)$$

so that

$$A'C - C'A = 0 \quad B'D - D'B = 0 \quad A'D - C'B = 1 \quad (2.5)$$

For the elements of the section (f, h) this means that

$$i(f^\dagger h - h^\dagger f) = 1, \quad f^t h - h^t f = 0 \quad (2.6)$$

where the superscript $\dagger$ means hermitian conjugation and the superscript t means transpose. We introduce the matrix [10]

$$\mathcal{N} = hf^{-1} \quad (2.7)$$

A more detailed structure of indices is useful. The flat, tangent ones we denote a whereas the vector indices are Λ , so that the section is $(f_a^\Lambda, h_{\Lambda a})$ and

$$f^t h \rightarrow f_a^\Lambda h_{\Lambda b}, \quad f^\dagger h \rightarrow \bar{f}^{\Lambda a} h_{\Lambda b}, \quad \mathcal{N}_{\Lambda\Sigma} = h_{\Lambda a} (f^{-1})_\Sigma^a \quad (2.8)$$

Introducing

$$\mathcal{M}(\mathcal{N}) = \begin{pmatrix} \text{Im}\mathcal{N} + \text{Re}\mathcal{N} \text{Im}\mathcal{N}^{-1} \text{Re}\mathcal{N} & -\text{Re}\mathcal{N} \text{Im}\mathcal{N}^{-1} \\ -\text{Im}\mathcal{N}^{-1} \text{Re}\mathcal{N} & \text{Im}\mathcal{N}^{-1} \end{pmatrix} \quad (2.16)$$

General case, $N \geq 2$

Introduce the new hermitian matrix, $\frac{1}{2}(\mathcal{M} + i\Omega)$ and the covariant symplectic vector $\tilde{V}_a = (\Omega V)_a$ and note that $|\tilde{V}_a\rangle\langle\tilde{V}^a|$ is hermitian. This leads to the following relation

$$\frac{1}{2}(\mathcal{M} + i\Omega) = -(\Omega V)_a (\Omega \tilde{V}^a) \quad (2.17)$$

where

²For $N = 2$, $n = n_v + 1$ where n_v is the number of vector multiplets. For $N > 2$ n refers to the total number of vectors in the theory, i. g. for $N = 8$, $n = 28$.

³This is a standard element of a flat symplectic bundle [9,11].

$$V_a = \begin{pmatrix} f_a^\Lambda \\ h_{\Lambda a} \end{pmatrix} \quad (2.9)$$

we find few simple properties which are the generalizations of the special geometry relations.

$$\langle V_a, V_b \rangle = V_a^t \Omega V_b = -f^t h + h^t f = 0 \quad (2.10)$$

$$\langle \bar{V}^a, V_b \rangle = V^{a\dagger} \Omega V_b = -f^\dagger h + h^\dagger f = i\delta_b^a \quad (2.11)$$

Example of $N = 2$ special geometry

$$V_a \Rightarrow (V, \bar{\mathcal{D}}_{\hat{k}} \bar{V}) \quad (2.12)$$

In $N = 2$ special geometry we have $f_a^\Lambda = (L^\Lambda, \bar{\mathcal{D}}_{\hat{k}} \bar{L}^\Lambda)$ and $h_{\Lambda a} = (M_\Lambda, \bar{\mathcal{D}}_{\hat{k}} \bar{M}_\Lambda = \mathcal{N}_{\Lambda\Sigma} \bar{\mathcal{D}}_{\hat{k}} \bar{L}^\Sigma)$ so that $\mathcal{N}_{\Lambda\Sigma} = h_{\Lambda a} (f^{-1})_\Sigma^a$.

In $N = 2$ special geometry the following relations take place.

$$\langle V, V_{\hat{k}} \rangle = 0, \quad \langle V_{\hat{k}}, \bar{V}_{\hat{k}} \rangle = i\delta_{\hat{k}\hat{k}} \quad (2.13)$$

$$i\langle V, \bar{V} \rangle = 1 \quad \langle V, \bar{V}_{\hat{k}} \rangle = 0 \quad (2.14)$$

If we now take into account that

$$V_a = (V, \bar{V}_{\hat{k}}) = \begin{pmatrix} f \\ h \end{pmatrix} \quad (2.15)$$

we may recover all relations of $N = 2$ special geometry from our generalized relations given in Eqs. (2.10) and (2.11).

Let us also remind here the important matrix of the special geometry, $\mathcal{M}(\mathcal{N})$, [5,11] which plays a significant role in the black hole potential $\mathcal{V}_{\text{BH}} = -\frac{1}{2}Q^t \cdot \mathcal{M}(\mathcal{N}) \cdot Q$ of $N = 2$ supergravity,⁴ where $Q = (p, q)$ is the symplectic charge vector.

$$\begin{aligned} -|\Omega V_a\rangle\langle\Omega \bar{V}^a| &= |\Omega V\rangle\langle\Omega \bar{V}| = \begin{pmatrix} -hh^\dagger & hf^\dagger \\ fh^\dagger & -ff^\dagger \end{pmatrix} \\ &= \begin{pmatrix} -h_{\Lambda a} \bar{h}_{\Sigma}^a & h_{\Lambda a} \bar{f}^{\Sigma a} \\ f_a^\Lambda \bar{h}_{\Sigma}^a & -f_a^\Lambda \bar{f}^{\Sigma a} \end{pmatrix} \end{aligned} \quad (2.18)$$

Note that

$$\frac{1}{2}(\mathcal{M} + i\Omega)V_a = i\Omega V_a \quad (\mathcal{M}V_a = i\Omega V_a) \quad (2.19)$$

By multiplying on Q we find

⁴This invariant was called I_1 in [5].

$$\frac{1}{2}(\mathcal{M} + i\Omega)Q = -(\Omega V)_a \langle Q, \bar{V}^a \rangle \quad (2.20)$$

Contracting this with Q' we get

$$-\frac{1}{2}Q' \cdot \mathcal{M}(\mathcal{N}) \cdot Q = \langle Q, V_a \rangle \langle Q, \bar{V}^a \rangle \quad (2.21)$$

and

$$\begin{aligned} \mathcal{M}Q &= -2 \operatorname{Re}((\Omega V)_a \langle Q, \bar{V}^a \rangle) \\ \Omega Q &= -2 \operatorname{Im}((\Omega V)_a \langle Q, \bar{V}^a \rangle) \end{aligned} \quad (2.22)$$

which is a real and imaginary part of our main new identity (2.20). These equations together with the condition $\mathcal{V}' = 0$ have a general validity and can be used for studies of the attractors and general solutions of stabilization equations in $N \geq 2$ supergravities.⁵

$N = 8$ case

Here we have that the $Sp(56, \mathbb{R})$ matrix

$$\mathcal{S} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

is a coset representative of $\frac{E_{7(7)}}{SU(8)}$. It is a 56-dimensional representation of $E_{7(7)}$ which is real and symplectic (i. e. the antisymmetric product (56×56) contains a singlet). The sections are given in terms of the following elements, $f_{AB}^{\Lambda\Sigma}$, $h_{\Lambda\Sigma,AB}$ and their complex conjugates, $\bar{f}_{AB}^{\Lambda\Sigma}$, $\bar{h}_{\Lambda\Sigma}^{AB}$.

The pair of indices $\Lambda\Sigma$ in $f_{AB}^{\Lambda\Sigma} = -f_{AB}^{\Sigma\Lambda}$ may be taken to run over the 28 of $SL(8, \mathbb{R})$ and in $\mathbf{28}'$ in $h_{\Lambda\Sigma,AB}$.⁶ The pair of indices AB in $f_{AB}^{\Lambda\Sigma} = -f_{BA}^{\Lambda\Sigma}$, run over the 28 of $SU(8)$ both for f and h but in $\mathbf{\bar{28}}$ for the $\bar{f}$ and $\bar{h}$. By writing

$$V_a = \begin{pmatrix} f_{AB}^{\Lambda\Sigma} \\ h_{\Lambda\Sigma,AB} \end{pmatrix} \quad \text{we see that } \Lambda \rightarrow [\Lambda\Sigma] = 1, \dots, 28; \quad a \rightarrow [AB], = 1, \dots, 28 \quad (2.23)$$

The vector coupling matrix is

$$\mathcal{N}_{\Lambda\Sigma, \Gamma\Delta} = h_{\Lambda\Sigma,AB} (f^{-1})_{\Gamma\Delta}^{AB} \quad (2.24)$$

The central charge matrix is

$$Z_{AB} = f_{AB}^{\Lambda\Sigma} e_{\Lambda\Sigma} - h_{\Lambda\Sigma,AB} m^{\Lambda\Sigma} \equiv \langle Q, V_{AB} \rangle \quad (2.25)$$

where the symplectic charge matrix-vector Q for $N = 8$ consists of electric $e_{\Lambda\Sigma}$ and magnetic $m^{\Lambda\Sigma}$ charges forming the fundamental representation of $E_{7(7)}$

$$Q \equiv (m^{\Lambda\Sigma}, e_{\Lambda\Sigma}) \quad (2.26)$$

⁵Note that Eqs. (2.20) and (2.22) are the generalizations of the equations derived in the literature in [5,21–23].

⁶Note that other parametrizations are possible, depending on the particular embedding of the 56 of $E_{7(7)}$ into $Sp(56, \mathbb{R})$, [24–27].

III. $N = 8$ BLACK HOLE POTENTIAL AND ITS CRITICAL POINTS

$N = 8$, $d = 4$ supergravity [2] has the following black hole potential [7,12]

$$\mathcal{V}_{\text{BH}}(\phi, Q) = Z_{AB} Z^{*AB} = \langle Q, V_{AB} \rangle \langle Q, \bar{V}^{AB} \rangle \quad (3.1)$$

$A, B = 1, \dots, 8.$

Here Z_{AB} (and its conjugate Z^{*AB}) is the central charge matrix (and its conjugate).

$$Z_{AB}(\phi, Q) = \langle Q, V_{AB} \rangle = f_{AB}^{\Lambda\Sigma} e_{\Lambda\Sigma} - h_{\Lambda\Sigma,AB} m^{\Lambda\Sigma} \quad (3.2)$$

where Q is charge vector, a fundamental 56 of $E_{7(7)}$ and the bein $f_{AB}^{\Lambda\Sigma}(\phi)$, $h_{\Lambda\Sigma,AB}(\phi)$ is an element of the coset space $\frac{E_{7(7)}}{SU(8)}$ connecting the real 56, to complex 28 of $[AB]$. It depends on 70 real scalars ϕ^i , where the local $SU(8)$ symmetry was used to remove 63 scalars from the 133-dimensional representation of scalars in $E_{7(7)}$. Summation over vector indices AB , $\Lambda\Sigma$ is understood for $A < B$ and $\Lambda < \Sigma$ in Eqs. (3.1) and (3.2).

The covariant derivative of the central charge is defined by the Maurer-Cartan equations for the coset space:

$$\mathcal{D}_i Z_{AB} = \frac{1}{2} P_{i,[ABCD]}(\phi) Z^{*CD}(\phi, Q) \quad (3.3)$$

Here $P_{i,[ABCD]} d\phi^i$ is the 70 $\times$ 70 vielbein of the $\frac{E_{7(7)}}{SU(8)}$ coset space, $i = 1, \dots, 70$ and it is self-dual real

$$P_{i,[ABCD]} = \frac{1}{4!} \epsilon_{ABCDEFGH} (P_{i,[ABCD]})^* \quad (3.4)$$

Here D_i is the $SU(8)$ covariant derivative [12]. Thus the derivative of the black hole potential over 70 moduli is given by the following expression

$$\partial_i \mathcal{V} = \frac{1}{2} (\mathcal{D}_i Z_{AB} Z^{*AB} + Z_{AB} \mathcal{D}_i Z^{*AB}) \quad (3.5)$$

Using Eq. (3.3) we find

$$\partial_i \mathcal{V} = \frac{1}{4} (P_{i,[ABCD]} Z^{*AB} Z^{*CD} + P_i^{[ABCD]} Z_{AB} Z_{CD}) \quad (3.6)$$

and with account of the self-duality condition (3.4) we get

$$\partial_i \mathcal{V} = \frac{1}{4} P_{i,[ABCD]} \left[Z^{*[CD]Z^{*AB}} + \frac{1}{4!} \epsilon^{CDABEFGH} Z_{EF} Z_{GH} \right] \quad (3.7)$$

Now the crucial observation helps to find and extremely simple *algebraic* expression for the critical points of all regular black holes by “flattening its indices”. The 70 $\times$ 70-bein $P_{i,[ABCD]}$ is invertible. This means that we can multiply Eq. (3.7) on $(P_{i,[A'B'C'D']})^{-1}$ and we get a necessary and sufficient condition for the critical points of the black hole potential with regular 70 $\times$ 70-beins:

$$Z^{*[AB}Z^{*CD]} + \frac{1}{4!}\epsilon^{ABCDEFGH}Z_{EF}Z_{GH} = 0 \quad (3.8)$$

To solve these equations we can use the $SU(8)$ -symmetry and work in the canonical basis for the antisymmetric central charge matrix [28] where it has only the nonvanishing complex eigenvalues $z_1 = Z_{12}$, $z_2 = Z_{34}$, $z_3 = Z_{56}$, $z_4 = Z_{78}$. In this basis the attractor equations are

$$\begin{aligned} z_1 z_2 + z^{*3} z^{*4} &= 0 & z_1 z_3 + z^{*2} z^{*4} &= 0 \\ z_2 z_3 + z^{*1} z^{*4} &= 0 \end{aligned} \quad (3.9)$$

The $SU(8)$ symmetry allows to bring all 4 complex eigenvalues to the following normal form [28]

$$z_i = \rho_i e^{i\varphi/4} \quad i = 1, 2, 3, 4. \quad (3.10)$$

so that only 5 real parameters are independent⁷, 4 absolute values ρ_i and an overall phase, φ since the relative phase of each eigenvalue can be changed but not the overall phase. The quartic J_4 invariant in this basis acquires the following form [29]

$$\begin{aligned} J_4 &= [(\rho_1 + \rho_2)^2 - (\rho_3 + \rho_4)^2][(\rho_1 - \rho_2)^2 \\ &\quad - (\rho_3 - \rho_4)^2] + 8\rho_1\rho_2\rho_3\rho_4(\cos\varphi - 1) \end{aligned} \quad (3.11)$$

Without loss of generality we may order the four moduli of eigenvalues as follows

$$\rho_1 \geq \rho_2 \geq \rho_3 \geq \rho_4 \quad (3.12)$$

so that the first term in J_4 is positive, null or negative depending whether $\rho_1 - \rho_2 \geq \rho_3 - \rho_4$ or $\rho_1 - \rho_2 \leq \rho_3 - \rho_4$. The last term is negative or null (it is null if one of the eigenvalues is vanishing or $\varphi = 0$).

As we will see below, in terms of the central charge eigenvalues

$$Z_{AB} = \begin{pmatrix} \rho_1 & 0 & 0 & 0 \\ 0 & \rho_2 & 0 & 0 \\ 0 & 0 & \rho_3 & 0 \\ 0 & 0 & 0 & \rho_4 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} e^{i\varphi/4} \quad (3.13)$$

the 1/8 BPS states attractors correspond to $\rho_2 = \rho_3 = \rho_4 = 0$ and the non-BPS one to $\rho_i = \rho$ and $\varphi = \pi$.

Note that for the non-BPS critical points of the potential the matrix of the second derivative may not be positive definite [15] and a critical point of the potential may not be its minimum, in such case it could be repeller rather than an attractor of the motion. This analysis will be done elsewhere.

$N = 8$ attractor Eqs. (3.9) have 2 solutions for regular black holes

(1)

⁷The black hole solutions with 5 charges have been constructed in [38].

1/8 BPS solution

$$\begin{aligned} z_1 &= \rho_{BPS} e^{i\varphi_1} \neq 0 & z_2 &= z_3 = z_4 = 0 \\ J_4^{BPS} &= \rho_{BPS}^4 > 0 \end{aligned} \quad (3.14)$$

The black hole entropy-area of the BPS black holes with 1/8 of $N = 8$ unbroken supersymmetry is given by

$$\frac{S_{BPS}(Q)}{\pi} = \frac{A_{BPS}(Q)}{4\pi} = \sqrt{J_4^{BPS}(Q)} = \rho_{BPS}^2 \quad (3.15)$$

(2) non-BPS

$$z_i = \rho e^{i\pi/4} \quad J_4^{\text{nonBPS}} = -16\rho_{\text{nonBPS}}^4 \quad (3.16)$$

The black hole entropy-area of the non-BPS black holes all supersymmetries broken is given by

$$\begin{aligned} \frac{S_{\text{nonBPS}}(Q)}{\pi} &= \frac{A_{\text{nonBPS}}(Q)}{4\pi} = \sqrt{-J_4^{\text{nonBPS}}(Q)} \\ &= 4\rho_{\text{nonBPS}}^2 \end{aligned} \quad (3.17)$$

The deep meaning of the extra factor 4 in the non-BPS solution as compared with BPS one will be clear when we will present the $N = 2$ interpretation of the $N = 8$ result.

Various examples of regular BPS and non-BPS $N = 8$ black holes are known in terms of quantized charges. In a series of papers [30,31] multiple examples of BPS and non-BPS extremal black holes in various four-dimensional supergravities have been discovered. In [30] it was shown that the extremal dyonic Reissner-Nordström black hole can be a non-BPS solution of $N = 8$ supergravity. In our case this is $z_i = \rho e^{i\pi/4} = q + ip = \frac{1}{\sqrt{2}}|q|(1 + i)$. The same solution was discovered in [29] in the context of D0-D6 solution dual to KK-monopole with KK-momentum. This solution has been identified in [18] in the setting of QIT as the canonical Greenberger, Horne, Zeilinger (GHZ) state [32], see Fig. 10 in [18]. The contribution to the entropy comes from a single term in J_4 which is always negative, it is equal to $-Tr(xy)^2$ in notation of [18], and therefore it can only be a non-BPS state.

Another class of solutions of $N = 8$ theory were presented in [31] in the context of the no-axion STU model and it was realized there that the flip of the sign of one of the charges relates a 1/8 BPS solution to a non-BPS extremal solution, as was also noticed in more general Calabi-Yau setting in [15,22] and as we have found here in general $N = 8$ setting. The contribution to the entropy comes from a single term in J_4 of the form $-4Pfx$, in notation of [18], which is a product of 4 charges, e. g. x^{12} , x^{34} , x^{56} , x^{78} . For example, if $x^{12}x^{34}x^{56}x^{78} < 0$ and $J_4 > 0$ we have a BPS combination of charges. If the sign of one of these charges is reversed, we find a non-BPS solution. In the context of QIT this state was qualified in [18] as the generalized GHZ class of states, see Fig. 9 there.

IV. GENERALIZED SPECIAL GEOMETRY AT THE ATTRACTOR POINT

The identity (2.20) which generalizes the special geometry formula of $N = 2$ theory can be further simplified at the attractor points. For $1/8$ BPS attractors, using Eq. (3.13) we get for $Z_{12} = \rho_{\text{BPS}} e^{i\varphi_{\text{BPS}}}$.

$$\frac{1}{2}(\mathcal{M} + i\Omega)Q = -(\Omega V)_{AB}Z^{*AB} = -(\Omega V)_{12}Z^{*12} \quad (4.1)$$

If we pick up a particular component of the 56-vector $[(\mathcal{M} + i\Omega)Q]_0$ we get the ratio

$$\frac{(\mathcal{M} + i\Omega)Q}{[(\mathcal{M} + i\Omega)Q]_0} = \frac{(\Omega V)_{12}}{[(\Omega V)_{12}]_0} \quad (4.2)$$

which expresses the scalars in the right hand side as the function of quantized charges Q .

This is in a complete analogy with the $N = 2$ attractors. It was shown in Eqs. (42), (44) of [5] that special geometry requires at $DZ = 0$ that

$$2i\bar{Z}L^\Sigma = p^\Sigma + i\frac{\partial I_1}{\partial q_\Sigma}, \quad 2i\bar{Z}M_\Sigma = q_\Sigma - i\frac{\partial I_1}{\partial p^\Sigma} \quad (4.3)$$

This gives the stabilization eqs. in the form

$$t^\Lambda = \frac{X^\Lambda}{X^0} = \frac{p^\Lambda + i\frac{\partial I_1}{\partial q_\Lambda}}{p^0 + i\frac{\partial I_1}{\partial q_0}}, \quad t_\Lambda = \frac{F_\Lambda}{F_0} = \frac{q_\Lambda - i\frac{\partial I_1}{\partial p^\Lambda}}{q_0 - i\frac{\partial I_1}{\partial p^0}} \quad (4.4)$$

These are the attractor values of special coordinates $t^\Lambda(p, q)$ and their dual, $t_\Lambda(p, q)$, as explicit function of charges for all cases when the entropy formula $S(p, q) = \pi I_1(p, q)$ is known. This form of stabilization equations was used by Bates and Denef in [33] for the $N = 2$ multi-center black hole constituent solutions in cases that the black hole entropy, is known as the function of charges. Here we see that Eqs. (4.1) and (4.2) are the $N = 8$ analogs of $N = 2$ Eqs. (4.3) and (4.4).

There is actually a simple way to recast the Eq. (4.2) in the language of Eqs. (4.3) and (4.4). Before doing this let us first recall that, following the analysis of [13,14] for the $1/8$ BPS, $N = 8$ attractors 30 scalars get fixed and 40 are not fixed. This corresponds to the decomposition of $N = 8$ into $N = 2$ multiplets, resulting from the decomposition $SU(8) \rightarrow SU(2) \times SU(6) \times U(1)$ of the R -symmetry, [14]. Under this splitting one obtains that 70 scalars decompose into 15 complex scalars of 15 vector multiplets and 40 real scalars of 20 (half)-hypermultiplets.

$$\mathbf{70} \Rightarrow (\mathbf{15}, \mathbf{1}) + (\overline{\mathbf{15}}, \mathbf{1}) + (\mathbf{20}, \mathbf{2}) \quad (4.5)$$

under $SU(6) \times SU(2)$. $N = 2$ ensures that 40 scalars are not fixed and 30 are fixed, as proved in [13]. The 15 complex scalars belong to the submanifold $\frac{SO^*(12)}{U(6)}$ of $\frac{E_{7(7)}}{SU(8)}$. Indeed, the vector and hypermultiplet splitting correspond to two different decompositions of $E_{7(7)}$ with

respect to maximal subgroups [34],

$$E_{7(7)} \rightarrow SO^*(12) \times SU(2) \quad (4.6)$$

$$E_{7(7)} \rightarrow E_{6(2)} \times U(1) \quad (4.7)$$

where 15_c parametrize $\frac{SO^*(12)}{U(6)}$ and 40 real scalars parametrize $\frac{E_{6(2)}}{SU(2) \times SU(6)}$. Note that the two factors $SU(2)$, $U(1)$ in the two different $E_{7(7)}$ decompositions simply mean that vector multiplet scalars are $SU(2)$ blinded while the hypermultiplet scalars are $U(1)$ blinded. Of course the two subgroups $SO^*(12)$, $E_{6(2)}$ do not commute, otherwise $\frac{E_{7(7)}}{SU(8)}$ would be a product space. Nevertheless, if we disregard the hyperscalars then

$$\frac{E_{7(7)}}{SU(8)} \Rightarrow \frac{SO^*(12)}{U(6)} \quad (4.8)$$

which is a symmetric special manifold [35]. Special coordinates for this manifold are obtained by its lifting to 5 dimensions to $\frac{SU^*(6)}{Usp(6)}$ which is a 14-dimensional real (very special) manifold [36,37].

The $d = 4$ prepotential of the effective $N = 2$ supergravity is then

$$F(X) = \frac{1}{2^3 \cdot 3!} \epsilon_{\Lambda\Sigma\Gamma\Delta\Pi\Omega} \frac{X^{\Lambda\Sigma} X^{\Gamma\Delta} X^{\Pi\Omega}}{X^0} = (X^0)^2 f(t^{\Lambda\Sigma}) \quad (4.9)$$

$\Lambda, \Sigma = 1, \dots, 6.$

where $t^{\Lambda\Sigma} = -t^{\Sigma\Lambda} = \frac{X^{\Lambda\Sigma}}{X^0}$ are complex coordinates in the 15 of $SU^*(6)$ X^0 is the graviphoton, corresponding to the decomposition of $SO^*(12) \rightarrow SU^*(6) \times SO(1, 1)$ under which the $SO^*(12)$ spinorial 32 representation which represent the $N = 2$ electric and magnetic charges (including graviphoton) decomposes as

$$\mathbf{32} \rightarrow \mathbf{15} + \mathbf{15}' + \mathbf{1} + \mathbf{1}' \quad (4.10)$$

Note that in this formulation only 32 of the 56 charges are retained and the symplectic sections $f^{\Lambda\Sigma}$, $h_{\Lambda\Sigma}$ are identified with the sections $e^{K/2}(X^{\Lambda\Sigma}, \overline{\mathcal{D}}_{\hat{k}} \overline{X}^{\Lambda\Sigma}; F_{\Lambda\Sigma}, \overline{\mathcal{D}}_{\hat{k}} \overline{F}_{\Lambda\Sigma})$ where $F_{\Lambda\Sigma} = \frac{\partial F}{\partial X^{\Lambda\Sigma}}$ and special coordinates are adopted.

Let us see how the 56 of $E_{7(7)}$ decomposes under $SO^*(12) \times SU(2)$: $\mathbf{56} \rightarrow (\mathbf{32}, \mathbf{1}) + (\mathbf{12}, \mathbf{2})$. It is clear that this is a different embedding of 56 into $Sp(56, \mathbb{R})$ with respect to the one in [2,25] where instead $\mathbf{56} \rightarrow \mathbf{28} + \mathbf{28}'$ of $SL(8, \mathbb{R})$, see Sec. II. Here instead the electric -magnetic splitting corresponds to further decomposition $SO^*(12) \rightarrow SU^*(6) \times SO(1, 1)$ leading to the prepotential previously discussed. The 32 electric and magnetic charges correspond to the 15 vector multiplets and the graviphoton. The other 24 remaining charges correspond to the vectors of the remaining 6 gravitino multiplets each of them containing 2 vectors: $6(\frac{3}{2}, 2(1), \frac{1}{2})$. These are the charges which do not appear in the $N = 2$ setting. These charges can be generated by applying to the 32 charges an $SU(8)$ rotation

which is not in $SU(6) \times SU(2) \times U(1)$. Indeed

$$\dim \frac{SU(8)}{SU(6) \times SU(2) \times U(1)} = 24$$

which is precisely the missing charges. To see that this counting is correct we observe that the $N = 2$, BPS black hole has a 32 component charge vector in the coset [36] $\frac{SO^*(12)}{SU(6)}$ which has the signature $(30^-, 1^+)$. The $1/8$, 56 component, $N = 8$ BPS charge vector is in the coset $\frac{E_{7(7)}}{E_{6(2)}}$ with the signature $(30^-, 25^+)$, [36].⁸ We therefore realize that the missing 24 charges are precisely the extra compact directions which correspond to $SU(8)$ rotations. Note that the same reasoning does not apply to the non-BPS black holes since in this case the signature of the coset, $\frac{E_{7(7)}}{E_{6(6)}}$ is $(28^-, 27^+)$.

Having identified the effective $N = 2$ supergravity with the prepotential in Eq. (4.9) describing the $N = 8$ $1/8$ BPS attractors we would need to find the expression for the entropy as the function of quantized charges so that Eqs. (4.2) can be used. One finds that the entropy of the $\frac{SO^*(12)}{U(6)}$ model is the same as in $N = 6$ supergravity where, with a different identification of the central charges we have [12,13] $Z_{AB} \Rightarrow N = 2$ matter charges and $Z \rightarrow N = 2$ central charge, $A, B = 1, \dots, 6$. $P_{i,ABCD} \rightarrow$ is a vielbein of $\frac{SO^*(12)}{U(6)}$. In terms of Z_{AB} , Z the unique $SO^*(12)$ quartic invariant is

$$\frac{S}{\pi} = \frac{1}{2} \sqrt{4I_2 - I_1 + 32I_3 + 4I_4 + 4I_5} \quad (4.11)$$

where $A_A^C \equiv Z_{AB} \bar{Z}^{BC}$ and $I_1 = (\text{Tr} A)^2$, $I_2 = \text{Tr}(A)^2$, $I_3 = \text{Re}(\text{Pf} Z_{AB} \bar{Z})$, $I_4 = \text{Tr} A \bar{Z} \bar{Z}$, $I_5 = \bar{Z} \bar{Z} \bar{Z} \bar{Z}$. Note that at $N = 2$ attractor point $Z_{AB} = 0$ and

$$\frac{S}{\pi} = \sqrt{I_5} = Z \bar{Z} \quad (4.12)$$

as expected. The black hole potential in this model is

$$\mathcal{V} = \sum_{A < B} Z_{AB} \bar{Z}^{AB} + Z \bar{Z} \quad (4.13)$$

The Maurer-Cartan equations [13] are identical to the special geometry relations which imply that $Z_{AB} = 0$, $|Z|^2 = \sqrt{I_4}$ is indeed a BPS attractor point. The I_4 invariant is thus a function of the quantized charges Q_α ($\alpha = 1, \dots, 32$) which form a left spinor of $SO^*(12)$. Since I_4 is moduli independent,

$$I_4(Q_\alpha) = \Gamma^{\alpha\beta\gamma\delta} Q_\alpha Q_\beta Q_\gamma Q_\delta \quad (4.14)$$

where $\Gamma^{\alpha\beta\gamma\delta}$ is a numerical tensor, constructed with the $SO^*(12)$ γ -matrices which is totally symmetric in the

⁸The coset represents this charge vector for the fixed value of I_4 [36].

spinor indices. This accomplishes the solution for $1/8$ BPS attractors.

Non-BPS case

In $N = 8$ theory we can actually find the non-BPS equations from Eq. (3.13) with $z_i^* = \rho_{\text{nonBPS}} e^{-i\pi/4}$

$$\frac{1}{2}(\mathcal{M} + i\Omega)Q = -(\Omega V)_{\text{TRACE}} \rho_{\text{nonBPS}} e^{-i\pi/4} \quad (4.15)$$

where

$$(\Omega V)_{\text{TRACE}} = (\Omega V)_{12} + (\Omega V)_{34} + (\Omega V)_{56} + (\Omega V)_{78} \quad (4.16)$$

Again by dividing on some component of the 56-vector we get

$$\frac{(\mathcal{M} + i\Omega)Q}{[(\mathcal{M} + i\Omega)Q]_0} = \frac{(\Omega V)_{\text{TRACE}}}{[(\Omega V)_{\text{TRACE}}]_0} \quad (4.17)$$

Thus Eqs. (4.2) and (4.17) are defining the scalars of BPS and non-BPS $N = 8$ theory at the black hole horizon. The details of the non-BPS solutions will be considered elsewhere.

V. STU $N = 2$ INTERPRETATION OF $N = 8$ ATTRACTORS

Consider the STU black holes in $N = 2$ supergravity with the prepotential $F = STU$. The corresponding coordinates represent the coset space $(\frac{SU(1,1)}{U(1)})^3$.

The algebraic attractor equations of $N = 8$ theory in Eq. (3.9) can be identified with the corresponding $N = 2$ attractor equations $\partial V_{\text{BH}} = 0$ [7] in the form

$$2(\mathcal{D}_i Z) \bar{Z} + i C_{ijk} G^{j\bar{m}} G^{k\bar{k}} \bar{\mathcal{D}}_{\bar{m}} \bar{Z} \bar{\mathcal{D}}_{\bar{k}} \bar{Z} = 0 \quad (5.1)$$

under the correspondence:

$$z_1 = iZ \quad z_2 = \overline{\mathcal{D}_{\hat{S}} Z} \quad z_3 = \overline{\mathcal{D}_{\hat{T}} Z} \quad z_4 = \overline{\mathcal{D}_{\hat{U}} Z} \quad (5.2)$$

We recover the extremization of the potential in $N = 8$ theory via the special geometry in $N = 2$. The BPS case is

$$\mathcal{D}_{\hat{S}} Z = \mathcal{D}_{\hat{S}} \bar{Z} = \mathcal{D}_{\hat{S}} Z = 0 \quad Z \neq 0 \quad (5.3)$$

which is equivalent to

$$z_2 = z_3 = z_4 = 0 \quad z_1 \neq 0 \quad (5.4)$$

The entropy-area at the BPS attractor points is

$$\frac{S}{\pi} = \frac{A}{4\pi} = |Z|_{\text{BPS}}^2 \quad (5.5)$$

The non-BPS case (for $Z \neq 0$) requires that

$$|Z| = |D_{\hat{S}} Z| = |\mathcal{D}_{\hat{T}} Z| = |\mathcal{D}_{\hat{U}} Z| \quad (5.6)$$

and the entropy-area at the non-BPS attractor point is

$$\frac{S}{\pi} = \frac{A}{4\pi} = |Z|^2 + \sum_{i=1}^3 |\mathcal{D}_i Z|^2 = 4|Z|_{\text{nonBPS}}^2 \quad (5.7)$$

This explains the origin of the factor 4 in the BPS-non-BPS entropy relation of the $N = 8$ attractor (see Eqs. (3.15) and (3.17)).

In conclusion, we have derived the attractor equation by means of an identity (2.20), for generic $N > 2$ supergravity in a form resembling the $N = 2$ attractor equations. This identity (2.20) has to be studied together with the condition for the critical point of the black hole potential, $\mathcal{V}' = 0$ as in $N = 2$ case of new attractors [21–23]. In $N = 8$ supergravity the critical point of the black hole potential $\mathcal{V}' = 0$ is given by an amazing set of algebraic attractor Eqs. (3.8) and (3.9) which clearly produce 2 solutions for regular black holes: one $1/8$ BPS and one non-BPS. The scalars are defined via charges in Eqs. (4.2) and (4.17). Finally, the attractor equations for the fixed scalars at the BPS attractor points can be studied using the prepotential (4.9) for the 15 $N = 2$ vector multiplets embedded into $N = 8$ supergravity.

We have also pointed out some connections between $N = 8$ black holes and quantum information theory in the spirit of the recent proposals in [17–19]. A particular class

of regular non-BPS black holes in $N = 8$ belongs to a canonical GHZ state, some other black holes belong to the generalized GHZ class of states, both in BPS and non-BPS case. It is quite impressive that the extremization of the black hole mass with respect to moduli is related to the process of finding the optimal local distillation protocol of a GHZ state from an arbitrary three-qubit pure state, according to [19].

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