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 $K \ell_3$ DECAY FORM FACTORS AND CURRENT ALGEBRA

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A B S T R A C T

The form factors in $K \ell_3$ decay are discussed in a current algebra approach by writing unsubtracted dispersion relations for the matrix elements of suitable retarded commutators between single particle states and vacuum, at (arbitrary) fixed linear combination of the two momentum transfers. This allows us to include the contributions of all the relevant poles and a consistent solution of the current algebra equations is then possible.

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Recently, new techniques ¹⁾ of applying consistently current algebra method (with pole dominance approximation) have been suggested in order to remedy the failure of current algebra method, in its earlier form, for strong decays ²⁾. The calculations on A_1 decay ^{1),3)} and K_A decay ⁴⁾, admitting that some of the form factors satisfy a subtracted dispersion relation, predict reasonable widths consistent with experimental data.

In this paper we study the $K\ell_3$ decay form factors (including the contribution of kappa meson) following closely a procedure of Brown and West ¹⁾. They calculated the A_1 decay by assuming dispersion relations for vertex functions with an appropriate fixed invariant, so as to include the poles in all the variables. Their approach gives the same results as those obtained by Schnitzer and Weinberg ¹⁾ using Ward identities derived from the current algebras. We will assume in what follows that the form factors are at most once subtracted and the non-constant part is calculated in pole dominance approximation. We illustrate the method by first considering the simple case of $\pi\ell_3$ decay form factor.

$\pi\ell_3$ decay form factor:

We start by considering the matrix element

$$S_\mu = i\sqrt{2k_0V} \int d^4x e^{ip \cdot x} \theta(-x_0) \langle \pi^0(k) | [\partial_\nu A_\nu(0)_2, V_\mu(x)_1] | 0 \rangle \quad (1)$$

$$= (k-q)_\mu F_1(q^2, p^2) + (k+q)_\mu F_2(q^2, p^2)$$

where $p_\mu = k_\mu + q_\mu$ and $k^2 = -m_\pi^2$.

Then

$$p_\mu S_\mu = C - D \quad (2)$$

2.

with

$$C = \sqrt{2k_0 v} \int d^4x e^{ip \cdot x} \delta(x_0) \langle \pi^0(k) | [\partial_\nu A_\nu^{(0)'}{}_2, V_0(x)_1^2] | 0 \rangle \quad (3)$$

and

$$D = \sqrt{2k_0 v} \int d^4x e^{ip \cdot x} \theta(-x_0) \langle \pi^0(k) | [\partial_\nu A_\nu^{(0)'}{}_2, \partial_\mu V_\mu(x)_1^2] | 0 \rangle \quad (4)$$

Here C is independent of p if the equal time commutator is assumed to be a local operator.

We postulate now that $F_{1,2}$ and D ^{*)} satisfy ^{**)} an unsubtracted dispersion relation for fixed μ , where

$$\mu = \alpha q^2 + (1-\alpha)p^2 \quad (5)$$

and α ($0 < \alpha < 1$) is a fixed constant. $F_{1,2}$ and D are evaluated in pole dominant approximation. Equation (2) then leads to the relation :

$$F_\pi m_\pi^2 f(p_1^2) + \frac{G_1 g}{2} \beta(q_1^2) \left[-\left(\frac{1-\alpha}{\alpha}\right) + \frac{q_1^2 + m_\pi^2}{m_\rho^2} \right] = -C \quad (6)$$

^{*)} This is equivalent to writing Feynman graphs for the vertex function. The author is indebted to Professor J.S. Bell for this remark.

^{**)} In the present case D happens to be vanishing if we assume conserved vector current hypothesis for $V_\mu^{(0)'}{}_2$.

where the various form factors and coupling constants are defined as usual by :

$$\begin{aligned}
 \sqrt{4k_0 p_0} V^2 \langle \pi^0(k) | V_\mu^{(0)2} | \pi^+(p) \rangle &= f(q^2) (p+k)_\mu \\
 \sqrt{4k_0 p_0} V^2 \langle \pi^0(k) | \partial_\nu A_\nu^{(0)2} | \pi^+(p) \rangle &= \beta(q^2) e^S \cdot k \\
 \sqrt{2p_0} V \langle 0 | V_\mu^{(0)2} | \pi^+(p) \rangle &= G_S e_\mu^S(p) \\
 \sqrt{2k_0} V \langle 0 | A_\mu^{(0)2} | \pi^-(k) \rangle &= i F_\pi k_\mu
 \end{aligned} \tag{7}$$

and

$$\begin{aligned}
 \mu &= \alpha q_1^2 - (1-\alpha) m_S^2 \\
 &= -\alpha m_\pi^2 + (1-\alpha) p_1^2
 \end{aligned} \tag{8}$$

Equation (6) then implies that

$$\lim_{-\mu \rightarrow \infty} [q_1^2 \beta(q_1^2)]$$

is finite so that $\beta(q^2)$ vanishes for infinite momentum transfer, e.g., $\beta(q^2)$ satisfies unsubtracted dispersion relations. However, the subtraction constant in pion form factor $f(q^2)$ is left unspecified. To obtain information about it we consider :

$$S_{\mu\nu} = i \sqrt{2k_0} V \int d^4x e^{-iq \cdot x} \theta(x_0) \langle \pi^0(k) | [A_\nu^{(x)2}, V_\mu^{(0)2}] | 0 \rangle_{(9)}$$

We easily derive :

$$q_\nu S_{\mu\nu} = i \sqrt{2} F_\pi k_\mu - i S_\mu \tag{10}$$

4.

where we used the current algebra commutation relation :

$$\delta(x_0) [A_0(x)_j^2, V_\mu^{(0)k}] = (\delta_\ell^i A_\mu^{(0)k} - \delta_j^k A_\mu^{(0)i}) \delta^4(x) \quad (11)$$

With the forms of $F_{1,2}$ already derived, we can evaluate the relation (10) at $q_\mu = 0$, $q^2 = 0$, $p^2 = -m_\pi^2$. We obtain the result :

$$f(0) = \sqrt{2} \quad (12)$$

which is expected because of the assumed charge independence and conserved current hypothesis in the simple case discussed. Next we discuss the $K \ell_3$ decay form factors where now the discussion is more involved due to non-conserved vector current and the appearance of two form factors.

$K \ell_3$ decay form factors:

The $K-\pi$ form factor describing $K \ell_3$ decay are defined by the matrix element :

$$\begin{aligned} & \sqrt{4k_0 p_0} V^2 \langle \pi^0(k) | V_\mu^{(0)3} | K^-(p) \rangle \\ &= F_+(q^2) (k+p)_\mu + F_-(q^2) (p-k)_\mu \end{aligned} \quad (13)$$

In the limit of exact $SU(3)$ we have $F_K = F_\pi$, $F_-(q^2) = 0$ and $\sqrt{2} F_+(0) = 1$.

As before, we consider now :

$$\begin{aligned} S_\mu &= i\sqrt{2k_0} V \int d^4x e^{ip \cdot x} \theta(-x_0) \langle \pi^0(k) | [\partial_\nu A_\nu^{(0)3}, V_\mu(x)^3] | 0 \rangle \\ &= (k-q)_\mu F_1 + (k+q)_\mu F_2 \end{aligned} \quad (14)$$

and obtain

$$p_\mu S_\mu = C - D \quad (15)$$

where C and D are defined as in Eqs. (3) and (4), but with index 3 replacing the index 2. The poles involved here are due to K , K^* and an isospinor scalar strangeness carrying meson χ . Proceeding in the same manner as before, we obtain

$$\begin{aligned} & F_\chi [g_+(q_1^2)(m_\pi^2 - m_\chi^2) + g_-(q_1^2)q_1^2] \\ & - F_K m_K^2 [F_+(p_1^2) - (\frac{\alpha}{1-\alpha}) F_-(p_1^2)] \\ & - \frac{G_{K^*}}{2} \gamma(q_2^2) \left[-(\frac{1-\alpha}{\alpha}) + \frac{q_2^2 + m_\pi^2}{m_{K^*}^2} \right] = C \end{aligned} \quad (16)$$

Here $g_\pm$ are the $\chi - \pi$ form factors introduced by

$$\begin{aligned} & \sqrt{4k_0 p_0} V^2 \langle \pi^0(k) | A_\mu(0)_3^1 | \chi^+(p) \rangle \\ & = g_+(q^2) (k+p)_\mu + g_-(q^2) (p-k)_\mu \end{aligned} \quad (17)$$

and

$$\begin{aligned} & \sqrt{4k_0 p_0} V^2 \langle \pi^0(k) | \partial_\mu A_\mu(0)_3^1 | K^{*+}(p) \rangle = \gamma(q^2) e^{K^*} \cdot k \\ & \sqrt{2k_0} V \langle 0 | A_\mu(0)_1^3 | K^-(k) \rangle = i F_K k_\mu \\ & \sqrt{2k_0} V \langle 0 | V_\mu(0)_3^1 | \chi^+(k) \rangle = i F_\chi k_\mu \\ & \sqrt{2p_0} V \langle 0 | V_\mu(0)_3^1 | K^{*+}(p) \rangle = G_{K^*} C_\mu^{K^*}(p) \end{aligned} \quad (18)$$

while

$$\begin{aligned}
 \mu &= \alpha q_1^2 - (1-\alpha) m_\pi^2 \\
 &= \alpha q_2^2 - (1-\alpha) m_K^2 \\
 &= -\alpha m_K^2 + (1-\alpha) p_1^2
 \end{aligned}
 \tag{19}$$

From Eq. (16) we see

$$\lim_{-\mu \rightarrow \infty} \left[F_\pi q_1^2 g_-(q_1^2) - \frac{G_{K^*}}{2m_{K^*}^2} q_2^2 \gamma(q_2^2) \right]
 \tag{20}$$

is a constant independent of α . For Eq. (16) then to be satisfied for arbitrary α we must have $F_-(\omega) = 0$ and $\gamma(\omega) = 0$.

Equation (20) then leads to $g_-(\omega) = 0$. Thus the form factors F_- , g_- and γ satisfy unsubtracted dispersion relations.

To obtain information on the remaining form factors, we consider $S_{\mu\nu}$ defined in Eq. (9) with index 2 replaced by 3, and we proceed as before to obtain the sum rule :

$$F_K F_+(0) - F_\pi g_+(0) = \frac{F_\pi}{\sqrt{2}}
 \tag{21}$$

We can obtain more sum rules by setting K or π on the mass shell and considering retarded products involving the (pionic) current $(A_{\mu 1}^1 - A_{\mu 2}^2)$. The sum rules are :

$$\sqrt{2} F_\pi F_+(0) + F_\pi g(0)/(m_\pi^2 - m_K^2) = F_K
 \tag{22}$$

$$\sqrt{2} F_\pi g_+(0) - F_K g(0)/(m_\pi^2 - m_K^2) = -F_\pi
 \tag{23}$$

where

$$\sqrt{4q_0 p_0} V^2 \langle K^-(q) | \partial_\nu (A_\nu^{(0)1} - A_\nu^{(0)2}) | \pi^-(p) \rangle = i g(k^2) \quad (24)$$

Solving Eqs. (21), (22) and (23)

$$\sqrt{2} F_+(0) = (F_K^2 + F_\pi^2 - F_\kappa^2) / 2 F_K F_\pi \quad (25)$$

$$\sqrt{2} g_+(0) = (F_K^2 - F_\pi^2 - F_\kappa^2) / 2 F_\kappa F_\pi \quad (26)$$

$$g(0) = (m_\kappa^2 - m_K^2) (F_K^2 + F_\kappa^2 - F_\pi^2) / 2 F_K F_\kappa \quad (27)$$

To these we could add the relation

$$F_\kappa^2 \simeq 2 (F_K^2 - F_\pi^2) \quad (28)$$

derived from the Weinberg spectral function sum rules ⁵⁾.

From experiments on $K \ell 3$ decays, one knows $(F_K/F_\pi)(1/\sqrt{2} F_+(0)) \approx 1.28$. Equations (25) and (28) then give ⁶⁾ $(F_K/F_\pi)^2 = 1.17$, $(F_\kappa/F_\pi)^2 \approx 0.34$ and $\sqrt{2} F_+(0) \sim 0.85$. We also obtain

$$\sqrt{2} g_+(0) \simeq -0.25 (F_\kappa/F_\pi) \simeq -0.15 \quad (29)$$

and

$$g(0) \simeq \frac{3}{4} \left(\frac{F_\kappa}{F_K} \right) (m_\kappa^2 - m_K^2) \quad (30)$$

If we assume the very successful hypothesis of partially conserved axial current (PCAC) for pion, Eqs. (24) and (27) then give for the $\pi\pi K$ coupling constant

$$G_{\pi^+\pi^0 K^+} \simeq \frac{3}{4} \left(\frac{F_\pi}{F_K} \right) \frac{(m_\pi^2 - m_K^2)}{\sqrt{2} F_\pi} \quad (31)$$

which is different from the results obtained from PCVC or PCAC assumptions ^{*}) for strangeness carrying currents in Eqs. (13) or (17) respectively. In fact, the $\pi\pi K$ coupling vanishes if $m_\pi = m_K$, as well as in the exact SU(3) symmetry limit.

We can then also calculate $\xi(0)$ for $K \ell_3$ decay and find :

$$\begin{aligned} \xi(0) &= F(0)/F_+(0) \\ &= \left(\frac{m_K^2 - m_\pi^2}{m_\pi^2} \right) \lambda + \frac{(m_\pi^2 - m_K^2)}{m_\pi^2} \frac{(F_K^2 + F_\pi^2 - F_\pi^2)}{2\sqrt{2} F_\pi F_K F_+(0)} \quad (32) \\ &\simeq 11.58 \lambda + 0.28 (1 - m_K^2/m_\pi^2) \end{aligned}$$

where λ is defined, for small momentum transfers, by :

$$F_+(q^2) = F_+(0) \left[1 + \lambda \frac{q^2}{m_\pi^2} \right] \quad (33)$$

Hence an upper limit on the value of ξ is ^{**)}

^{*}) Note that we do not use any such assumption in deriving the above sum rules.

^{**)} If no PCAC assumption is made no upper limit follows and instead we have $G_{\pi^+\pi^0 K^+} \simeq (\xi + 0.27) m_\pi^2 F_+(0)/F_\pi$.

$$\xi(0) < \left(\frac{m_K^2 - m_\pi^2}{m_\pi^2} \right) \lambda + \frac{(F_K^2 + F_\pi^2 - F_\pi^2)}{2\sqrt{2} F_\pi F_K F_+(0)} \quad (34)$$

With the experimental value ⁷⁾ $\lambda \simeq -0.023$, we find $\xi < 0.01$. Thus it excludes the possibility of a large positive value for ξ and favours the negative values found in the experiments measuring the polarization of muon in $K_{\mu 3}$ decay ($\xi_{\text{exp}} \simeq -0.5 \pm 0.3$).

From Eq. (32) we find $\xi \simeq -0.2$ and -0.24 corresponding to $m_\chi \sim 560$ MeV and 520 MeV, respectively. The corresponding values for the $\chi \pi K$ coupling $G_{\chi+\pi^0 K^+}$ are calculated to be 147 MeV and 56 MeV, respectively ^{*)}.

Finally, we remark that the unseen decay mode $K_A \rightarrow \chi \pi$ will be suppressed compared to other modes if g_+ satisfies a subtracted dispersion relation with the subtraction constant close to that given by the right-hand side of Eq. (26).

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*) In a pole model with only pion pole contributing, the former case gives a 10^{-10} sec for the lifetime of χ^+ decaying into $K^+ + 2\pi$. The value is very sensitive to the deviation of χ mass from K mass. For $m_\chi \simeq 615$ MeV the lifetime is $\sim 10^{-13}$ sec.

R E F E R E N C E S

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