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EQUAL TIME COMMUTATOR AND SUPERCONVERGENCE SUM RULE *)

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A B S T R A C T

Applying the infinite momentum technique to the matrix elements of equal time commutators, we obtain superconvergence sum rules for zero-mass-pion nucleon scattering amplitude, which are identical with those derived from dispersion relations, apart from a possible restriction to the trajectory with integer intercept. Using this result, we show that a modification of the equal time commutation relation of pion fields motivated by a simple quark model is not compatible with the Pomeranchuk theorem, and that any other form of modification is also impossible.

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I. INTRODUCTION

An application of the infinite momentum technique ¹⁾ to the matrix element of the equal time commutator of the axial vector current $A^i_\mu(x)$ has led to a successful sum rule [the Adler-Weisberger relation ²⁾], under the assumptions of PCAC ³⁾ and current algebra ⁴⁾. The sum rule is intimately related to the low energy theorem of pion scattering, i.e., relations between the s wave scattering lengths ⁵⁾ which is also in good accord with experimental data ⁶⁾. As a matter of fact, one of the reasons of the success of the infinite momentum technique is that, when applied to the commutator of the axial vector currents, one is led to a sum rule which contains a convergent integral.

A similar method has been applied to other equal time commutators to derive high energy theorems ^{7),8)} or the ratio of the renormalization constants ⁹⁾. In these cases, however, the relevant sum rules contain divergent integrals. In order to avoid this difficulty, various limiting procedures have been used, the validity of which appears to be dubious.

In this article, we discuss similar problems but take into consideration the high energy (Regge) behaviour of the scattering amplitude that appears in the sum rule. In other words, we subtract the Regge amplitude before taking the infinite momentum limit. In Section II, applying this method to the equal time commutator of axial vector current and its divergence (or the pion field), one obtains a superconvergence sum rule ¹⁰⁾⁻¹²⁾ for the symmetric part of the zero mass pion nucleon scattering amplitude. The sum rule thus obtained is shown to be identical to that given by Igi's method, i.e., by taking the infinite energy limit in dispersion relations after subtracting the Regge amplitude. A similar calculation is carried out for equal time commutator of pion fields

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(or of divergences of the axial vector currents), leading to a superconvergence sum rule for the antisymmetric amplitude, which is also identical to that obtained from dispersion relation, (Section III), apart from a possible restriction to the trajectory with integer intercept.

Using the above result, a question of modifying the equal time commutation relation of pion fields is discussed (Section IV).

II. SYMMETRIC AMPLITUDE OF PION-NUCLEON SCATTERING

We start with a relation ¹³⁾

$$\begin{aligned} (2\pi)^4 \delta^4(0) \frac{m}{p_0} 4\pi f^{(+)}(0,0) = \\ = -i f_\pi^{-2} 2\pi \delta(0) \langle p | \left[\int A_0^{(+)}(x, x_0) d^3x, \int \partial_\lambda A_\lambda^{(-)}(y, x_0) d^3y \right] | p \rangle + \\ + i f_\pi^{-2} (2\pi)^4 \delta^4(0) \langle 0 | \left[\int A_0^{(+)}(x, x_0) d^3x, \int \partial_\lambda A_\lambda^{(+)}(y, x_0) d^3y \right] | 0 \rangle, \end{aligned} \quad (1)$$

where $f^{(\pm)}(\nu_L, k_0^2)$ is the (^{symmetric}/_{antisymmetric}) amplitude of a mass k_0 pion nucleon scattering, the lab. energy ν_L is defined by

$$\nu_L = - \frac{pk}{m} = \frac{p_0 k_0}{m} \quad (2)$$

and m, p, μ, k and f_π stand for the nucleon mass, its four-momentum, the pion mass, its four-momentum and the PCAC constant defined by

$$\partial_\lambda A_\lambda^{(\pm)} = f_\pi \mu^2 \phi^{(\pm)}, \quad (3)$$

$\phi^{(\pm)}$ being the pion field. The quantity $f^{(+)}(0,0)$ is understood as ¹³⁾

$$f^{(+)}(0,0) = \lim_{k_0 \rightarrow 0, \frac{k_0^2}{\nu} \rightarrow 0} f^{(+)}(\nu, k_0^2). \quad (4)$$

The states $|p\rangle$ and $|0\rangle$ stand for a proton state and the vacuum, respectively, and the second term of the right-hand side of Eq. (1) represents the subtraction of the disconnected diagram.

The method of Refs. ^{1),2)} enables us to write Eq. (1) as

$$f^{(+)}(0,0) = \frac{\mu^4 |\underline{p}|}{2\pi^2} \int_{\mu + \frac{\mu^2}{2m}}^{\infty} \frac{d\nu}{\sqrt{\underline{p}_0^2 + 2m\nu}} \frac{\sigma^{(+)}(\nu, k_0^2)}{(\mu^2 - k_0^2)^2}, \quad (5)$$

where

$$k_0 = \sqrt{\underline{p}^2 + W^2} - \sqrt{\underline{p}^2 + m^2},$$

$$\nu = \frac{W^2 - m^2}{2m} = \nu_L + \frac{k_0^2}{2m}$$

and

$$\sigma^{(\pm)}(\nu, k_0^2) = \frac{1}{2} \left\{ \sigma_{\pi^- p}(\nu, k_0^2) \pm \sigma_{\pi^+ p}(\nu, k_0^2) \right\},$$

and W is the energy in the c.m.s. In deriving Eq. (5), we insert the intermediate state projections between the two operators of Eq. (1) and sum over them. In contradistinction to the case of the equal time commutator of axial vector current, we do not have a contribution from the one-nucleon state, since

$$\begin{aligned} \langle \underline{p}_n | \int \phi^{(-)}(\underline{x}, x_0) d^3x | \underline{p} \rangle &= (2\pi)^3 \delta(\underline{p} - \underline{p}_n) \langle \underline{p}_n | \phi^{(-)}(0) | \underline{p} \rangle \\ &\propto (2\pi)^3 \delta(\underline{p} - \underline{p}_n) \frac{m}{p_0} \bar{u}(\underline{p}) \gamma_5 u(\underline{p}) = 0. \end{aligned}$$

As for multiparticle intermediate states, the contributions from purely disconnected diagrams of the first term are cancelled by the second term and the remaining, by using unitarity, is brought into the form of Eq. (5), where the singularity of the integral is proved to be the principal value ²⁾, 14).

If one interchanges the integral and the limit $p_0 \rightarrow \infty$ in Eq. (5), one encounters a divergent integral $\int_0^\infty \sigma^{(+)}(\nu, 0) d\nu$:

The high energy behaviour of $\sigma^{(+)}(\nu_L, k_0^2)$ for fixed k_0^2 should be $\nu_L^{\alpha_p(0)-1}$ where $\alpha_p(0)$ is the intercept of the Pomeron, since the Regge trajectory should not depend on the external mass k_0^2 . Therefore one has to subtract the Regge asymptotic form $\sigma_R^{(+)}(\nu_L, k_0^2)$ from $\sigma^{(+)}(\nu_L, k_0^2)$ before taking the infinite momentum limit. That is, instead of Eq. (5), one should deal with the integral

$$\begin{aligned} f^{(+)}(0,0) &= \\ &= \frac{\mu^4 |\underline{p}|}{2\pi^2} \int_\mu^\infty \frac{d\nu}{\sqrt{p_0^2 + 2m\nu}} \frac{1}{(\mu^2 - k_0^2)^2} \left\{ \sigma^{(+)}(\nu_L, k_0^2) \theta\left(\nu - \mu - \frac{\mu^2}{2m}\right) - \right. \\ &\quad \left. - \sigma_R^{(+)}(\nu_L, k_0^2) \right\} + \frac{\mu^4 |\underline{p}|}{2\pi^2} \int_\mu^\infty \frac{d\nu}{\sqrt{p_0^2 + 2m\nu}} \frac{\sigma_R^{(+)}(\nu_L, k_0^2)}{(\mu^2 - k_0^2)^2} \end{aligned} \quad (5')$$

Here θ stands for the step function and $\sigma_R^{(+)}(\nu_L, k_0^2)$ contains the contribution from the Regge trajectories whose intercepts α 's are non-negative, so that one could interchange the above-mentioned limiting procedures in the first term of Eq. (5'). As for the second term, one should integrate first, then take the limit $p_0 \rightarrow \infty$. Since the integrand contains a singular function, there is a possibility of obtaining a sensible result by this procedure ¹⁵⁾. If one chooses, however, the expression for the subtraction term as

$$\sigma_R^{(+)}(\nu_L, k_o^2) = \sum_{0 < \alpha \leq 1} 4\pi \beta_\alpha^{(+)} P_\alpha\left(\frac{\nu_L}{\mu}\right) / \nu_L, \quad (6)$$

as is suggested by Regge theory, the second integral of Eq. (5') leads to a term that behaves like p_o^α , provided that the residue function is independent of k_o^2 . This means that Eq. (6) is not a correct subtraction term to obtain a sensible result by the infinite momentum limit in Eq. (5'). The simplest choice for $\sigma_R^{(+)}(\nu_L, k_o^2)$ to get rid of p_o^α dependence from the second integral of Eq. (5') is to set

$$\sigma_R^{(+)}(\nu_L, k_o^2) = \sigma^{(+)}(\infty, 0) G_1^{(+)}(k_o^2) + \sum_{0 < \alpha < 1} 4\pi \beta_\alpha^{(+)} G_\alpha^{(+)}(k_o^2) P_\alpha\left(\frac{\nu_L}{\mu}\right) / \nu_L \quad (7)$$

where

$$G_\alpha^{(+)}(k_o^2) = 1 + \frac{2-\alpha}{\alpha} \frac{k_o^2}{\mu^2}, \quad 0 < \alpha \leq 1. \quad (8)$$

In Eq. (7) we have separated the contribution of the Pomeron, for convenience. The residue function can still be a function of k_o^2 in such a way that

$$\lim_{k_o^2 \rightarrow 0} \beta_\alpha^{(+)}(k_o^2) = \beta_\alpha^{+}.$$

We assume, however, that such a k_o^2 dependence does not change the essential part of our discussion. This will be justified a posteriori by comparing the result with the superconvergence sum rule which will follow from dispersion relation. The necessity of the factor (8) can be seen from the calculation in the Appendix. In fact, it is just an operation to drop the unwanted p_o^α dependent terms, leaving the constant term unchanged, and thus showing that the procedure is rather stable.

We also assume the absence of the trajectory with $\alpha = 0$, since otherwise it would require the existence of a mass zero particle with spin zero, which has never been observed. This assumption then makes harmless the singularity which appears in the expression of $G_{\alpha}^{(+)}(k_0^2)$. Needless to say that if there is a Regge cut which runs between $1 > \alpha > 0$, the sum in α should be replaced by the corresponding integral. Our discussion will not be changed by this modification.

With these preliminaries and taking the limit $p_0 \rightarrow \infty$ in Eq. (5'), one obtains

$$f^{(+)}(0,0) = \frac{1}{2\pi^2} \int_{\mu}^{\infty} d\nu \left\{ \sigma^{(+)}(\nu,0) \theta\left(\nu - \mu - \frac{\mu^2}{2m}\right) - \sigma_R^{(+)}(\nu,0) \right\} - \frac{\mu}{2\pi^2} \sigma^{(+)}(\infty,0) - \sum_{0 < \alpha < 1} \frac{z \beta_{\alpha}^{(+)}}{\sin \pi \alpha} P_{\alpha}(0). \quad (9)$$

This is identical to the superconvergence sum rule of Igi-type in the case of zero-mass-pion nucleon scattering amplitude. In order to see this in more detail, and for convenience, we repeat his argument in a slightly different form which is closely related to the procedure described above.

One may write the once-subtracted dispersion relation

$$\begin{aligned} \text{Re} f^{(+)}(\nu, k_0^2) = & \text{Re} f^{(+)}(\mu, k_0^2) - \frac{g^2 K(k_0^2)}{4\pi m} \left[\frac{\nu^2}{\nu^2 - \nu_B^2(k_0^2)} - \frac{\mu^2}{\mu^2 - \nu_B^2(k_0^2)} \right] + \\ & + \frac{2(\nu^2 - \mu^2)}{\pi} \int_{\mu + \frac{\mu^2}{2m} - \frac{k_0^2}{2m}}^{\infty} \frac{\nu' d\nu' \text{Im} f^{(+)}(\nu', k_0^2)}{(\nu'^2 - \mu^2)(\nu'^2 - \nu^2)} \end{aligned} \quad (10)$$

where $gK(k_0^2)$ stands for the "pion" nucleon coupling constant,

$$K(\mu^2) = 1, \quad \nu_B(k_0^2) = -\frac{k_0^2}{2m}$$

and

$$\text{Im } f^{(+)}(\nu, k_0^2) = \frac{\sqrt{\nu^2 - k_0^2}}{4\pi} \sigma^{(+)}(\nu, k_0^2). \quad (11)$$

Putting $k_0^2 = 0$ and subtracting the expression for $\nu = 0$, after rearrangement of several terms, one gets the formula :

$$\begin{aligned} & f^{(+)}(0,0) + \frac{\nu^2}{2\pi^2} \int_{\mu}^{\infty} \frac{d\nu'}{\nu'^2 - \nu^2} \left\{ \sigma^{(+)}(\nu,0) \theta\left(\nu - \mu - \frac{\mu^2}{2m}\right) - \sigma_R^{(+)}(\nu,0) \right\} = \\ & = \text{Re } f^{(+)}(\nu,0) - \frac{\nu^2}{2\pi^2} \int_{\mu}^{\infty} \frac{d\nu'}{\nu'^2 - \nu^2} \left\{ \sigma^{(+)}(\nu,0) + \sum_{0 < \alpha < 1} 4\pi \beta_{\alpha}^{(+)} P_{\alpha}\left(\frac{\nu'}{\mu}\right) / \nu' \right\} = \quad (12) \\ & = \text{Re } f^{(+)}(\nu,0) - \frac{1}{2\pi^2} \left\{ \sigma^{(+)}(\nu,0) \frac{\nu}{2} \ln \left| \frac{\nu+\mu}{\nu-\mu} \right| + \right. \\ & \quad \left. + \sum_{0 < \alpha < 1} 4\pi \beta_{\alpha}^{(+)} \frac{\pi}{\sin \pi \alpha} \left(P_{\alpha}(0) - \frac{1}{2} [P_{\alpha}(\nu/\mu) + \bar{P}_{\alpha}(-\nu/\mu)] \right) \right\}, \end{aligned}$$

where $\bar{P}_{\alpha}(-\nu/\mu)$ stands for the principal value

$$\begin{aligned} \bar{P}_{\alpha}(-x) &= \frac{1}{2} \left\{ P_{\alpha}(-x + i\varepsilon) + P_{\alpha}(-x - i\varepsilon) \right\} = \\ &= \cos \pi \alpha P_{\alpha}(x) - \frac{2}{\pi} \sin \pi \alpha Q_{\alpha}(x), \quad x > 1. \end{aligned} \quad (13)$$

One notices that the Born term in the dispersion relation (10) disappears by the required limiting process (4). Since the assumption of the Regge behaviour implies that

$$\lim_{\nu \rightarrow \infty} \left\{ \text{Re} f^{(+)}(\nu, 0) + \sum_{0 < \alpha < 1} \beta_{\alpha}^{(+)} \frac{P_{\alpha}(\frac{\nu}{\mu}) + \bar{P}_{\alpha}(-\frac{\nu}{\mu})}{\sin \pi \alpha} \right\} = 0, \quad (14)$$

the limit $\nu \rightarrow \infty$ in Eq. (12) leads to Eq. (9).

Although one ends up with an identity, one might infer the following two points from the analysis of this section :

- i) the Regge behaviour for the asymptotic form of the scattering amplitude is compatible with the infinite momentum method, and especially
- ii) the k_0^2 dependence of $\sigma_R^{(+)}(\nu_L, k_0^2)$ given by Eqs. (7) and (8) may be justified a posteriori.

Finally, one should remark that the conclusion will not be changed if one includes the trajectory with $\alpha(0) = 0$ which has a vanishing residue, such as in the case of the conspirator^{16), 17)} of the pion in the zero mass approximation. The Regge amplitude due to this trajectory is

$$\begin{aligned} \text{Re} f_{\alpha=0}^{(+)} &= - \lim_{\alpha \rightarrow 0} \left\{ \frac{\beta_{\alpha}^{(+)} (1 + \cos \pi \alpha) P_{\alpha}(\frac{\nu}{\mu})}{\sin \pi \alpha} - \frac{2}{\pi} \beta_{\alpha}^{(+)} Q_{\alpha}(\frac{\nu}{\mu}) \right\} \quad (15) \\ &= - \lim_{\alpha \rightarrow 0} \frac{2 \beta_{\alpha}^{(+)}}{\pi \alpha} = C^{(+)} \end{aligned}$$

and

$$\text{Im } f_{\alpha=0}^{(+)} = \lim_{\alpha \rightarrow 0} \beta_{\alpha}^{(+)} P_{\alpha}\left(\frac{\nu}{\mu}\right) = 0, \quad (16)$$

while the contribution to $\sigma_R^{(+)}(\nu_L, k_0^2)$ may be considered as

$$\lim_{\alpha \rightarrow 0} 4\pi \beta_{\alpha}^{(+)} G_{\alpha}^{(+)}(k_0^2) P_{\alpha}\left(\frac{\nu_L}{\mu}\right) / \nu_L = - \frac{4\pi^2 C^{(+)}}{\nu_L} \frac{k_0^2}{\mu^2} \quad (17)$$

which amounts to adding $C^{(+)}$ to the right-hand side of Eq. (9). Although Eq. (12) is not changed by the addition of this trajectory, one should add $C^{(+)}$ to the right-hand side of Eq. (14), which established the equivalence of the two methods in this case. The argument applies equally to the trajectory of the moving pole, as well as the fixed pole, i.e., $\alpha(t) = 0$, as far as they are handled with the above-described limiting procedure. Incidentally, in the case of a fixed pole with vanishing residue, one may consider the real constant $C^{(+)}$ as a non-Regge behaviour of the scattering amplitude. That is, one may subtract $C^{(+)}$ from the amplitude, then reggeize the remainder as far as the dispersion relation $f^{(+)}(\nu, 0)$ is concerned. In the infinite momentum method, on the other hand, the definition by the limit, as described above, is the only way of getting a consistent result. Therefore, if the fixed pole with vanishing residue cannot be defined as the limiting case $\alpha \rightarrow 0$, which may likely be the case, one would be led to conclude that there should not be such a fixed pole or, equivalently, a real constant $C^{(+)}$ of the non-Regge type in the asymptotic amplitude.

III. ANTISYMMETRIC AMPLITUDE

We now turn our attention to the antisymmetric part of the pion-nucleon scattering amplitude. Application of a similar technique to the preceding section to the matrix element

$$\langle p' | \left[\int \phi^{(+)}(\underline{x}, x_0) d^3x, \int \phi^{(-)}(\underline{y}, x_0) d^3y \right] | p \rangle = 0 \quad (18)$$

leads to

$$\begin{aligned} 0 &= \int_{\mu + \frac{\mu^2}{2m}}^{\infty} \frac{d\nu}{\sqrt{\nu^2 + 2m\nu}} \frac{k_0 \sigma^{(-)}(\nu_L, k_0^2)}{(\mu^2 - k_0^2)^2} \\ &= \int_{\mu}^{\infty} \frac{d\nu}{\sqrt{\nu^2 + 2m\nu}} \frac{k_0}{(\mu^2 - k_0^2)^2} \left\{ \sigma^{(-)}(\nu_L, k_0^2) \theta(\nu - \mu - \frac{\mu^2}{2m}) - \sigma_R^{(-)}(\nu_L, k_0^2) \right\} + \\ &\quad + \int_{\mu}^{\infty} \frac{d\nu}{\sqrt{\nu^2 + 2m\nu}} \frac{k_0 \sigma_R^{(-)}(\nu_L, k_0^2)}{(\mu^2 - k_0^2)^2} \end{aligned} \quad (19)$$

Here $\sigma_R^{(-)}(\nu_L, k_0^2)$ may be chosen either as ¹⁸⁾

$$\sigma_R^{(-)}(\nu_L, k_0^2) = \sum_{-1 \leq \alpha < 1} \frac{4\pi \beta_{\alpha}^{(-)} G_{\alpha}^{(-)}(k_0^2) N_{\alpha} \left(\frac{\nu_L}{\mu}\right)^{\alpha-1}}{\mu} \quad (20a)$$

or as

$$\begin{aligned} \sigma_R^{(-)}(\nu_L, k_0^2) = & \sum_{0 \leq \alpha < 1} 4\pi \beta_\alpha^{(-)} G_\alpha^{(-)}(k_0^2) P_\alpha\left(\frac{\nu_L}{\mu}\right) / \nu_L + \\ & + \sum_{-1 \leq \alpha < 0} \frac{4\pi \beta_\alpha^{(-)} G_\alpha^{(-)}(k_0^2) N_\alpha \left(\frac{\nu_L}{\mu}\right)^{\alpha-1}}{\mu}, \end{aligned} \quad (20b)$$

where

$$G_\alpha^{(-)}(k_0^2) = 1 + \frac{1-\alpha}{1+\alpha} \frac{k_0^2}{\mu^2} \quad (21)$$

and

$$P_\alpha(x) = N_\alpha x^\alpha + N_{-\alpha-1} x^{-\alpha-1} + O(x^{\alpha-2}, x^{-\alpha-3}) \quad (22)$$

with

$$N_\alpha = \frac{2^\alpha \Gamma\left(\frac{1}{2} + \alpha\right)}{\sqrt{\pi} \Gamma(1 + \alpha)} \quad (23)$$

The choice of expression (21) for $G_\alpha^{(-)}(k_0^2)$ is again dictated to guarantee the existence of the proper limit $p_0 \rightarrow \infty$ of Eq. (19). The equality signs at $\alpha = 0$ or -1 of the summation in Eqs. (20) stand for the inclusion of terms which are considered as the limiting case taken from the inside of the boundary, i.e., $\alpha \rightarrow 0+0$ or $\alpha \rightarrow -1+0$. The other terms at $\alpha = -1$, those not considered by the above-mentioned limit, are denoted by $\alpha = -1-0$, and are excluded from the summation of Eqs. (20). Having the vanishing residue factor N_α , all trajectories with intercept -1 do not give a contribution to $\sigma_R^{(-)}(\nu, 0)$ but do contribute to $\text{Re} f_R^{(-)}(\nu, 0)$. In other words, we have

$$\operatorname{Re} f_R^{(-)}(\nu, 0) = \sum_{-1 \leq \alpha < 1} \frac{\beta_\alpha^{(-)} N_\alpha (1 - \cos \pi \alpha) \left(\frac{\nu}{\mu}\right)^\alpha}{\sin \pi \alpha} + \frac{2\beta_{-1-0}^{(-)}}{\pi} \frac{\mu}{\nu}, \quad (24a)$$

and

$$\begin{aligned} \operatorname{Re} f_R^{(-)'}(\nu, 0) = & \sum_{-1 \leq \alpha < 1} \frac{\beta_\alpha^{(-)'} N_\alpha (1 - \cos \pi \alpha) \left(\frac{\nu}{\mu}\right)^\alpha}{\sin \pi \alpha} + \frac{2\beta_{-1-0}^{(-)'}}{\pi} \frac{\mu}{\nu} + \\ & + \frac{\beta_0^{(-)'}}{\pi} \ln \left| \frac{\nu + \mu}{\nu - \mu} \right| + o\left(\frac{1}{\nu}\right) \end{aligned} \quad (24b)$$

The last term of Eq. (24b) derives from the fact that $\lim_{\alpha \rightarrow 0} \overline{P}_\alpha(-x)$ contains the term $Q_0(x) = \frac{1}{2} \ln \frac{x+1}{x-1}$, [see Eq. (13)]. Since the trajectory with $\alpha = -1+0$ contributes to $\sigma_R^{(-)}(\nu, k_0^2)$ as

$$\lim_{\alpha \rightarrow -1} 4\pi \beta_\alpha^{(-)} \frac{1-\alpha}{1+\alpha} \frac{k_0^2}{\mu^3} N_\alpha \left(\frac{\nu}{\mu}\right)^{\alpha-1} = - \frac{8\pi \beta_{-1+0}^{(-)} k_0^2}{\nu^2 \mu}, \quad (25)$$

comparison of Eqs. (20) and (24) leads to relations

$$\beta_{-1-0}^{(-)} = \beta_0^{(-)'} + \beta_{-1-0}^{(-)'} \quad (26)$$

and

$$\beta_\alpha^{(-)} = \beta_\alpha^{(-)'}, \quad -1 \leq \alpha < 1.$$

Choosing Eq. (20a) for the subtraction term, we perform the limit $p_0 \rightarrow \infty$ in Eq. (19) to obtain

$$\int_{\mu}^{\infty} d\nu \left\{ \nu \sigma^{(-)}(\nu, 0) \theta\left(\nu - \mu - \frac{\mu^2}{2m}\right) - \sum_{-1 < \alpha < 1} 4\pi \beta_{\alpha}^{(-)} N_{\alpha} \left(\frac{\nu}{\mu}\right)^{\alpha} \right\} = \quad (27)$$

$$= \sum_{-1 \leq \alpha < 1} \frac{4\pi \beta_{\alpha}^{(-)} \mu N_{\alpha}}{1 + \alpha}$$

where use has been made of the formulae of the Appendix. Note that only the term $\beta_{-1+0}^{(-)}$ appears in the sum rule (27), but not the term $\beta_{-1-0}^{(-)}$. This difference stems from the singularity of $G_{\alpha}^{(-)}(k_0^2)$ at $\alpha = -1$, as stressed earlier.

On the other hand, from the unsubtracted dispersion relation

$$Re f^{(-)}(\nu, k_0^2) = \frac{2\nu f^2 K^2(k_0^2) \left(\frac{k_0^2}{\mu}\right)^2}{\nu^2 - \nu_p^2(k_0^2)} + \frac{2\nu}{\pi} \int_{\mu + \frac{\mu^2}{2m} - \frac{k_0^2}{2m}}^{\infty} \frac{d\nu' \ln f^{(-)}(\nu', k_0^2)}{\nu'^2 - \nu^2} \quad (28)$$

where

$$f^2 = \frac{g^2}{4\pi} \left(\frac{\mu}{2m}\right)^2 = 0.081 \quad (29)$$

one gets

$$\begin{aligned}
 & \frac{\nu^2}{2\pi^2} \int_{\mu}^{\infty} \frac{\nu' d\nu'}{\nu'^2 - \nu^2} \left\{ \sigma^{(-)}(\nu', 0) \theta(\nu' - \mu - \frac{\mu^2}{2m}) - \sigma_R^{(-)}(\nu', 0) \right\} = \\
 & = \nu \left\{ \operatorname{Re} f^{(-)}(\nu, 0) - \sum_{-1 \leq \alpha < 1} \frac{\beta_{\alpha}^{(-)} N_{\alpha} (1 - \cos \pi \alpha) \left(\frac{\nu}{\mu}\right)^{\alpha}}{\sin \pi \alpha} - \sum_{-1 < \alpha < 1} \frac{2\beta_{\alpha}^{(-)} N_{\alpha} \mu}{\pi (1 + \alpha) \nu} \right\} \\
 & + O\left(\frac{1}{\nu^2}\right). \tag{30}
 \end{aligned}$$

Eqs. (24a) and (30) then lead to the sum rule

$$\begin{aligned}
 & \int_{\mu}^{\infty} d\nu \left\{ \nu \sigma^{(-)}(\nu, 0) \theta(\nu - \mu - \frac{\mu^2}{2m}) - \sum_{-1 < \alpha < 1} 4\pi \beta_{\alpha}^{(-)} N_{\alpha} \left(\frac{\nu}{\mu}\right)^{\alpha} \right\} = \\
 & = \sum_{-1 \leq \alpha < 1} \frac{4\pi \beta_{\alpha}^{(-)} \mu N_{\alpha}}{1 + \alpha} - 4\pi \beta_{-1-0}^{(-)} \mu \tag{31}
 \end{aligned}$$

which is compared to Eq. (27) to yield

$$\beta_{-1-0}^{(-)} \mu = \beta_0^{(-)'} \mu + \beta_{-1-0}^{(-)'} \mu = 0. \tag{32}$$

If one used Eq. (20b) for the subtraction term, one would have obtained

$$\int_{\mu}^{\infty} d\nu \left\{ \nu \sigma^{(-)}(\nu, 0) \theta\left(\nu - \mu - \frac{\mu^2}{2m}\right) - \sum_{0 \leq \alpha < 1} 4\pi \beta_{\alpha}^{(-)'} P_{\alpha}\left(\frac{\nu}{\mu}\right) - \sum_{-1 < \alpha < 0} 4\pi \beta_{\alpha}^{(-)'} N_{\alpha} \left(\frac{\nu}{\mu}\right)^{\alpha} \right\} = \quad (27')$$

$$= \sum_{-1 \leq \alpha < 0} \frac{4\pi \beta_{\alpha}^{(-)'} \mu N_{\alpha}}{1 + \alpha} + 4\pi \beta_0^{(-)'} \mu$$

$$= \sum_{-1 \leq \alpha < 0} \frac{4\pi \beta_{\alpha}^{(-)'} \mu N_{\alpha}}{1 + \alpha} - 4\pi \beta_{-1-0}^{(-)'} \mu \quad (31')$$

which lead again to Eq. (32). Equations (27') and (31') can be derived also from Eqs. (27) and (31) by using the relation

$$\int_1^{\infty} (P_{\alpha}(x) - N_{\alpha} x^{\alpha}) dx = \frac{N_{\alpha}}{1 + \alpha}, \quad 1 > \alpha > 0 \quad (33)$$

$$= 0, \quad \alpha = 0.$$

Relation (32) would imply that the real part of the anti-symmetric amplitude should not have a component $1/\nu$, unless it comes from the trajectory with $\alpha \rightarrow -1+0$. This may exclude the existence of a fixed pole at $\alpha(t) = -1$. Of course, it is not clear whether this statement is valid also for physical pion-nucleon

scattering amplitude. The residue function $\beta_{-1-0}^{(-)}$ may well vanish only for zero-mass-pion. Baring this ambiguity, an interesting case would be $\beta_{-1\pm 0}^{(-)} = 0$, so that one has an accessible prediction

$$\beta_0^{(-)'} \mu = 0. \quad (34)$$

Recently, several authors have noticed the necessity of a singularity other than the ρ trajectory from the numerical analysis of superconvergence sum rules ^{10),12)} or from the observations of the polarization of the πp charge exchange scattering ^{17),19),20)} or from a remarkable dipole fit of the electromagnetic form factor of the nucleon ²¹⁾. The second ρ trajectory, the so-called ρ' , has a small intercept, but its precise value is not known. Also the magnitude and the sign of the residue function is not well established ²⁰⁾; they very much depend on the other input information or assumption. If the intercept of ρ' is small, its residue may well be restricted by the condition (34), i.e., the residue function must be small from the continuity reason. Analysis of Ref. 10) shows that the residue function of ρ' at $t=0$ is small indeed. Incidentally, we should note that our conclusion is independent of the existence of any other trajectory or Regge cuts, although we deal with the zero-mass-pion nucleon scattering. The mathematical analysis of the validity of our subtraction method is, however, left to be investigated in more detail ²²⁾.

IV. IMPOSSIBILITY OF MODIFICATION OF EQUAL TIME COMMUTATION RELATIONS OF PION FIELDS

Finally we remark that a modification of the equal time commutator of pion fields, such as

$$[\phi^{(+)}(\underline{x}, x_0), \phi^{(-)}(\underline{y}, x_0)] = C V_0^3(\underline{x}, x_0) \delta(\underline{x} - \underline{y}), \quad (35)$$

where
$$I_3 = \frac{1}{2} \int V_0^3(\underline{x}) d^3x$$

is the third component of the isospin operator, is not compatible with the Pomeranchuk theorem. Equation (35) may be motivated from a simple quark model in which the pion is identified with the pseudoscalar bilinear form of quark fields²³⁾. This is considered as the equal time commutation relation in a simplest example of non-elementary pions. The matrix element of Eq. (35) between one-proton states gives

$$C = \frac{2m}{\pi} \frac{|\underline{p}|}{p_0} \int_{\mu + \frac{\mu^2}{2m}}^{\infty} \frac{d\nu}{\sqrt{p_0^2 + 2m\nu}} \frac{k_0 \sigma^{(-)}(\nu, k_0^2)}{(\mu^2 - k_0^2)^2}, \quad (36)$$

instead of Eq. (19). Using the calculation of the Appendix, and taking the limit $p_0 \rightarrow \infty$ in Eq. (36) associated with the subtraction method described in this article, one obtains

$$C = - \frac{\sigma^{(-)}(\infty, 0)}{\pi \mu^2}, \quad (37)$$

which proves our statement.

$$= -\frac{\pi\mu}{4\sin\pi\alpha}(1-\cos\pi\alpha)\left\{(1-\alpha)N_\alpha y^{1+\alpha} + (2+\alpha)N_{-\alpha-1}y^{-\alpha}\right\} - \quad (\text{A.11})$$

$$- \frac{\mu}{2}M_\alpha y^{-\alpha}(2+\alpha) + O(y^{\alpha-1})$$

and

$$K_\alpha^{(-)} = \frac{\mu^2 p_0^2}{m} \int_{\mu\sqrt{p_0^2+2mV}}^{\infty} \frac{dV}{\sqrt{p_0^2+2mV}} \frac{k_0^3}{(\mu^2 - k_0^2)^2} \frac{P_\alpha\left(\frac{V_L}{\mu}\right)}{V_L}$$

$$= \frac{\mu y}{4} \int_{x_1}^{\infty} \left\{ \frac{y}{(x-y)^2} + \frac{y}{(x+y)^2} - \frac{1}{x+y} + \frac{1}{x-y} \right\} P_\alpha(x) dx \quad (\text{A.12})$$

$$= \frac{\pi\mu}{4\sin\pi\alpha}(1-\cos\pi\alpha)\left\{(1+\alpha)N_\alpha y^{\alpha+1} - \alpha N_{-\alpha-1}y^{-\alpha}\right\} -$$

$$- \frac{\mu M_\alpha}{2} \alpha y^{-\alpha} + O(y^{\alpha-1})$$

Here we have used the asymptotic form of $Q_\alpha(y)$

$$Q_\alpha(y) = M_\alpha y^{-\alpha-1} + O(y^{-\alpha-3}) \quad (\text{A.13})$$

with

$$M_\alpha = \frac{\sqrt{\pi} \Gamma(1+\alpha)}{2^{\alpha+1} \Gamma(\frac{3}{2}+\alpha)} = \frac{1}{(1+2\alpha) N_\alpha}.$$

From Eqs. (A.10)-(A.12), one obtains

$$I_{\alpha}^{(-)} = -\frac{\pi\mu}{2\sin\pi\alpha}(1-\cos\pi\alpha)\frac{1+2\alpha}{1+\alpha}N_{-\alpha-1}y^{-\alpha} - \frac{1+2\alpha}{1+\alpha}\mu M_{\alpha}y^{-\alpha} + O(y^{\alpha-1}), \quad 0 < \alpha < 1. \quad (\text{A.14})$$

For $\alpha = 0$ and 1 , one has

$$I_0^{(+)} = \frac{2}{y^2} J_1^{(+)} = -\mu + O\left(\frac{1}{y^2}\right). \quad (\text{A.15})$$

c. Formulae for power function $N_{\alpha}(y/\mu)^{\alpha}$ are obtained by using the relation

$$\int_0^{\infty} \frac{x^{\alpha}}{x+y} dx = -\frac{\pi}{\sin\pi\alpha} y^{\alpha}, \quad -1 < \operatorname{Re} \alpha < 0, \quad (\text{A.16})$$

and analytic continuation in α . The corresponding quantities are defined by replacing $P_{\alpha}(y/\mu)$ by $N_{\alpha}(y/\mu)$, denoted by

$$\hat{I}_{\alpha}^{(-)} = \hat{J}_{\alpha + \frac{1-\alpha}{1+\alpha}}^{(-)} \hat{K}_{\alpha}^{(-)}.$$

They are calculated easily and read

$$\hat{I}_{\alpha}^{(-)} = -\frac{\mu N_{\alpha}}{1+\alpha} + O\left(\frac{1}{y^2}\right), \quad -1 \leq \alpha < 1. \quad (\text{A.17})$$

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- 14) The proof of Ref. 2) on this point, though plausible, does not seem mathematically rigorous, since it deals with a special limit for an ill-defined quantity such as a product of distributions.
- 15) Such an example is a principal value integral

$$g(y) = P \int_1^{\infty} \frac{y^2 f(x) dx}{x^2 - y^2},$$

where

$$f(x) \xrightarrow{x \rightarrow \infty} C + O\left(\frac{1}{x^{1+\varepsilon}}\right),$$

($\varepsilon > 0$). Obviously one cannot perform the limit $y \rightarrow \infty$ inside the integral. However, arranging the integral as

$$\begin{aligned} g(y) &= y^2 \int_1^{\infty} \frac{f(x) - C}{x^2 - y^2} + C y^2 \int_1^{\infty} \frac{1}{x^2 - y^2} = \\ &= y^2 \int_1^{\infty} \frac{f(x) - C}{x^2 - y^2} + \frac{C y}{2} \ln \left| \frac{1+y}{1-y} \right|, \end{aligned}$$

one obtains the result

$$g(\infty) = - \int_1^{\infty} (f(x) - C) + C.$$

If the kernel $P(1/x^2 - y^2)$ of the integral is replaced by $(1/x^2 + y^2)$, the limit does not exist.

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- 22) Similar problems as in this article have been discussed recently
 in a different way by :
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 S. Fubini and G. Furlan, "Dispersion relation of low energy limit"
 (preprint, 1968).
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- 23) The author is indebted to Professors J.D. Bjorken and W. Weisberger
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