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TEST OF A RELATIVISTIC MANY-CHANNEL RESONANCE FORMULATomas Brunila ^{*)} and Matts Roos

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A relativistic many-channel resonance formula for the ρ meson, proposed recently by Ball and Parkinson, is tested on a compilation of experimental data. It is shown that the formula describes well the behaviour of the phase shift δ_1^1 near the ρ resonance. Ball and Parkinson argue that the $N\bar{N}$, $\pi\omega$ and $K^*\bar{K}$ channels are decisive for the dynamical origin of the ρ meson. We find that the shape of the ρ resonance in the $\pi\pi$ channel is given mainly by the coupling of the ρ meson to the $\pi\pi$ channel. However, the $\pi\pi$ channel alone cannot reproduce the shape satisfactorily, and it is necessary to take one or two additional channels into consideration ($N\bar{N}(+-)$, $\pi\omega$ or $K^*\bar{K}$). We find the ρ meson parameters $M_\rho = (767.6 \pm 1.7)$ MeV, $\Gamma_0 = (152 \pm 18)$ MeV, in agreement with our previous results.

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In two recent papers Roos¹⁾ and Pišút and Roos²⁾ (papers I and II henceforth) studied $\pi\pi$ interactions in the physical region with the aim of extracting information on the ρ meson. The shape of the ρ meson was described²⁾ by a relativistic modification of the resonance scattering formula, built into a phenomenologically modified OPE model. Assuming the $\pi\pi$ scattering in the $T = J = 1$ state to be purely elastic in the ρ region, the dispersion relation for the inverse partial wave was used to parametrize the δ_1^1 phase shift.

The dipion invariant mass distribution in the reaction

$$\pi^- p \rightarrow \pi^- \pi^0 p \quad (1)$$

could then be written

$$\frac{d\sigma}{d\omega} \propto \frac{\omega^2}{q} \sin^2 \delta_1^1 \cdot \frac{1}{D_R} \int_{-T}^{-t_{\min}(\omega)} \exp(D_R t) + \frac{B \cdot q}{D_B} \int_{-T}^{-t_{\min}(\omega)} \exp(D_B t), \quad (2)$$

where, for the purpose of the present study, we have suppressed background interference and other partial waves than the $T = J = 1$ wave. Here ω is the dipion invariant mass, $q = \frac{1}{2}(\omega^2 - 4m_\pi^2)^{1/2}$, $-t_{\min}(\omega)$ is the lower boundary of the Chew-Low plot, $-T$ is the maximum momentum transfer t , and B is an adjustable parameter.

In paper II, Eq. (2) was applied to a large sample of events of reaction (1) with $|t|$ selected to be small, $-t < -T$. The phenomenological parameters D_R and D_B in Eq. (2) were chosen to be $D_R = 9.25 \text{ (GeV/c)}^{-2}$, $D_B = 2.0 \text{ (GeV/c)}^{-2}$. In the energy range $500 \text{ MeV} < \omega < 1100 \text{ MeV}$ the large statistics permitted conclusions to be drawn about various modifications of Eq. (2), such as the presence of s -waves, background interference, energy dependent form factors, CDD poles and barrier penetration factors.

The purpose of the present study is to use the same data compilation and methods of analysis to study a dynamic^{*)}, relativistic many-channel resonance formula, recently proposed by Ball and Parkinson³⁾ (paper BP henceforth). The ρ meson is really an excellent probe for resonance formulae, partly because of its unique $\pi\pi$ decay, partly because it is broad enough to carry a wealth of information about the behaviour of the phase shift far away from the pole.

Ball and Parkinson³⁾ give two apparently different derivations of their resonance formula leading, however, to identical results. In the first approach the ρ meson is assumed to arise through the dynamical interplay of the $\pi\pi$, $\pi\omega$, $K\bar{K}$, $\rho\eta$, $K^*\bar{K}$, $N\bar{N}(++)$ and $N\bar{N}(+-)$ channels. (The indices + and - refer to helicity states.) It is then assumed that the forces can be approximated by one pole on the left hand cut. The pole has the same position in all channels (at $-s_p$) and the residua are factorizable. In such a case, within the framework of the N/D method, one can write

$$N_{ij} = C_i C_j / (s + s_p)$$

provided that D_{ij} is normalized to δ_{ij} at $s = -s_p$. The factorizability assumption appears to be too restrictive; some arguments and criteria for this rough approximation are given in BP. The derivation of the formula is given with all details in BP and need not be reproduced here.

*) The distinction between "dynamic" and "phenomenological" is a question of semantics. By "dynamic" we mean in this study all that pertains to the non-relativistic potential and its relativistic generalizations, such as coupling constants, the structure of left hand singularities in a partial wave, etc.

The second derivation³⁾ is to start with K-matrix formalism: the relativistic BP formula then appears as the simplest generalization of the non-relativistic case, as dictated by the threshold properties, Lorentz invariance and analyticity properties. The derivation is given in Section IV of BP and the non-relativistic pendant is discussed by Dalitz⁴⁾ and Pilkuhn⁵⁾.

The obvious synthesis of the BP formula and Eq. (2) is achieved through the replacement of $\sin^2 \delta_1^1$ by the BP analogue

$$\sin^2 \delta_1^1 \rightarrow \rho_1^2 |T|^2, \quad (3)$$

where T is the $\pi\pi$ scattering amplitude, now in the presence of inelastic channels (even below threshold), normalized in the elastic region to

$$T = e^{i\delta_1^1} \sin \delta_1^1 / \rho_1,$$

and ρ_1 is a kinematical factor. The BP formula gives for T [see Eq. (8) of Ref. 3]:

$$T = \frac{C_1^2}{\tilde{D}(s) - i \sum_i \rho_i(s) \Theta(s - t_i) C_i^2}, \quad (4)$$

where the C_i are coupling constants of the ρ meson to the channels considered, $\Theta(s - t_i)$ is the step function, t_i is the threshold for the i^{th} channel and the $\rho_i(s)$ are the kinematical (phase space) factors. The function $\tilde{D}(s)$ is given by

$$\tilde{D}(s) = s_R - s - \sum_i \frac{C_i^2}{\pi} I_i(s, s_R, \Lambda), \quad (4')$$

where s_R is the resonance energy squared, and Λ is a cut off parameter. The functions $I_i(s, s_R, \Lambda)$ are cumbersome expressions given explicitly in BP. The appearance of the cut off Λ is a common disease of dynamical models.

According to BP, Λ should lie between $200 m_\pi^2$ and $400 m_\pi^2$ and within this region the results are not sensitive to the value of Λ . We have, therefore, kept Λ fixed at the value $\Lambda = 300 m_\pi^2$.

The free parameters in our fit are the mass of the ρ meson, $\omega_R \equiv \sqrt{s_R}$, the background factor B in Eq. (2), and the coupling constants C_i . C_i is simply related to the ρ -meson width Γ_0 by

$$\Gamma_0 = C_i^2 \frac{(s_R - 4m_\pi^2)^{3/2}}{4s_R} . \quad (5)$$

Γ_0 is here the width defined as

$$\frac{d\delta_i'(\omega)}{d\omega} \bigg|_{\omega = \omega_R} = \frac{2}{\Gamma_0} . \quad (6)$$

In BP the C_i are estimated from various sources of information on ρ -meson interactions. Ball and Parkinson then use them as fixed constants in a straightforward calculation of the ρ -resonance shape. One result of the calculation is the relative contributions (intensities) of the different channels at the resonance energy ω_R . In Table I we reproduce this interesting result for later comparison. The constants C_i are given in units of $\hbar = c = m_\pi = 1$.

Table I

Channel i	$\pi\pi$ 1	$\pi\omega$ 2	$K\bar{K}$ 3	$\rho\eta$ 4	$K^*\bar{K}$ 5	$N\bar{N}(++)$ 6	$N\bar{N}(+-)$ 7
C_i rel. strength	0.90 9%	0.47 26%	0.45 1%	0.27 6%	0.47 16%	$0.64(1 + \frac{3.75}{4m_N^2})$ 1%	3.00 42%

Turning now to our analysis, we used the experimental $\pi^- \pi^0$ sample denoted TOT 10⁻ in paper II. In this sample^{†)} the upper momentum transfer is $-T = 10 \text{ m}_\pi^2$, the beam momenta range from 1.6 to 3.2 GeV/c, and in the energy region studied, $540 \text{ MeV} < \omega < 960 \text{ MeV}$, there are 10622 events of reaction (1).

Of course one has to keep in mind that a real fit cannot be expected to be sensitive to all the seven channels in BP. Ball and Parkinson choose so many channels for symmetry arguments, but many of them can be expected to have similar effects on the actual shape of the ρ meson, because their thresholds are distant. Nevertheless we proceed to determine the significance of each of the channels.

The results of our fits are summarized in Table II and discussed below. All coupling constants which have no errors in Table II were

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Table II

Solution no.	$\chi^2/\langle \chi^2 \rangle$	$G_1 (\pi\pi)$	$G_2 (\pi\omega)$	$G_4 (\rho\eta)$	$G_5 (K^*\bar{K})$	$G_7 (N\bar{N}_{+-})$
1	122/43	0.90	0.47	0.27	0.47	3.00
2	68/38	0.96 ± 0.02	0.47	0.27	0.47	3.00
3	29/34	0.99 ± 0.02	$0.90^{+0.06}_{-0.90*}$	$0.1^{+1.2}_{-0.1*}$	$0.0^{+1.8}_{-0.0*}$	0^{+6}_{-0*}
4	30/36	0.99 ± 0.02	$0.7^{+0.1}_{-0.4}$	0	0	1^{+5}_{-1*}
5	30/37	0.99 ± 0.02	0	0	0	6.1 ± 0.2
6	30/37	0.99 ± 0.02	0	0	1.69 ± 0.06	0
7	29/37	0.99 ± 0.02	0.92 ± 0.03	0	0	0
8	85/39	0.95 ± 0.02	0	0	0	0

†) Laboratories which kindly contributed were Bari, Bologna, Colorado, Florence, LRL Berkeley, Orsay, Pennsylvania, Purdue, Saclay, and Wisconsin. Fuller references are given in papers I and II.

*) This error is less than 1 standard deviation.

kept fixed in the corresponding run. The errors are defined as the maximum allowed change in one parameter to produce an increase of 1.0 in χ^2 , when all the other parameters are left free. Note that Eqs. (4) and (4') depend on C_i^2 rather than on C_i . In cases when the best fit value of a C_i is less than one standard deviation away from zero, the negative error is marked with a *.

Solution 1 in Table II represents a fit to the BP choice of coupling constants, with only two free parameters: ω_R and the background factor B. The fit is very poor, thus the C_i values have to be changed.

In solution 2 we leave also C_1 free. It adjusts itself towards a larger ρ width, but the fit is still poor.

Solution 3 is the complete 5-channel fit. All coupling constants C_i are free. We now get an excellent fit; however C_2 , C_4 , C_5 , and C_7 are strongly correlated and the errors come out large. From this fit one can draw the conclusion that several channels have similar influence on the ρ -meson shape, and that they therefore cannot be distinguished from each other in this analysis. We may therefore content ourselves with less than five channels. All the C_i ($i > 1$) are consistent with zero (and consistent with the BP values in Table I), thus it is not clear which channels are sufficient.

In solution 4 we study a 3-channel fit, neglecting channels 4 and 5. The fit is excellent and the values of C_2 and C_7 still agree with the BP values.

Solutions 5, 6, and 7 are 2-channel fits in which one channel is No. 1 and the second channel is 7, 5 or 2, respectively. All these solutions are very good, with a confidence level of about 80%. Since each of the coupling constants is very significantly different from zero, we conclude that two channels are necessary to describe the ρ^- shape. In addition to the

$\pi\pi$ channel one needs either the $\pi\omega$ channel, the $K^*\bar{K}$ channel or the $\bar{N}N(+)$ channel. The χ^2 values for these solutions do not permit any distinction between them.

On the other hand the values of C_2 , C_5 and C_7 now come out quite different from the BP values, and this we may take as an indication that we have reduced the number of channels too far.

Finally we study channel 1 alone in solution 8, which gives a very poor fit. Thus a second channel is necessary.

We conclude that

- i) at least two channels are necessary to give a statistically good fit;
- ii) three channels are necessary and sufficient to reproduce the BP values for C_2 and C_7 ;
- iii) the ρ -meson shape is not sensitive to the presence of further channels.

Using the C_1 value for the best solution 4, and Eq. (5) we find the ρ^- width

$$\Gamma_0 = 152 \pm 18 \text{ MeV.}$$

The ρ^- mass comes out to be

$$M_{\rho^-} = 767.6 \pm 1.7 \text{ MeV.}$$

Both these values are in excellent agreement with the results of the phenomenological analysis in paper II.

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