MESON RESONANCE COUPLINGS IN A FIVE-POINT
VENEZIANO MODEL

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A B S T R A C T

Veneziano's model generalized to the five - point function is applied to the processes $K\bar{K} \rightarrow 3\pi$ and $K\bar{K} \rightarrow K\bar{K}\pi$. Coupling constants of the exchanged resonances are deduced. The results are partial $SU(3)$, $\omega\phi$ and $f f'$ mixing angles, and a universal relation between spin 2^+ and spin 1^- decays. For the processes $K\bar{K} \rightarrow K\bar{K}\pi$, the model is compatible only with G parity conserving couplings, and thus yields pure F coupling for the decay $2^+ \rightarrow 1^- 0^-$.

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1. INTRODUCTION

Veneziano's formula ¹⁾ has recently ²⁾ been generalized to the five-point function for identical, isoscalar, spinless particles. In this note we generalize this result to include isospin and strangeness, and apply it to the processes $K\bar{K} \rightarrow 3\pi$ and $K\bar{K} \rightarrow K\bar{K}\pi$. The Regge trajectories are assumed to be linear and real, and the coupling constants of the exchanged resonances lying on parent trajectories are deduced from the residues at the poles. The main input of the model is the symmetry of the five-point function under cyclic interchange of external particles ("duality"), and the absence of $I = \frac{3}{2}$, 2 and $S = 2$ resonances.

In Section 2, some previous results from the four-point function are reviewed. In Section 3 we summarize our main results. In Sections 4 and 5 the formulae for $K\bar{K} \rightarrow 3\pi$ and $K\bar{K} \rightarrow K\bar{K}\pi$ are discussed. In Section 6 we give some conclusions.

2. $\pi\pi$, πK , KK AND $K\bar{K}$ SCATTERING

We summarize results from previous works ^{3),4)}, relevant to the parent trajectories.

a) The model requires three families of nearly degenerate trajectories :

$$\begin{aligned}\alpha_\rho &= \alpha_\omega = \alpha_f = \alpha_{A_2} \\ \alpha_\phi &= \alpha_{f'} \\ \alpha_{K^*} &= \alpha_{K^{**}}\end{aligned}\tag{1}$$

b) The $\omega\phi$ and ff' mixing angles are given by $\tan^2\theta = \frac{1}{2}$ and one gets the $SU(3)$ results ⁵⁾ :

$$\begin{aligned} g_{\rho K\bar{K}}^2 &= g_{\omega K\bar{K}}^2 = \frac{1}{2} g_{\phi K\bar{K}}^2 \\ g_{A_2 K\bar{K}}^2 &= g_{f K\bar{K}}^2 = \frac{1}{2} g_{f' K\bar{K}}^2 \end{aligned} \quad (2)$$

$$g_{\rho K\bar{K}} g_{\rho \pi\pi} = 2 g_{K^* K\pi}^2 ; \quad g_{f K\bar{K}} g_{f \pi\pi} = 2 g_{K^{**} K\pi}^2$$

where the normalization of the g 's can be read off the couplings :

$$\begin{aligned} g_{\rho \pi\pi} \cdot \frac{1}{2} (\pi_i \vec{\partial}_\mu \pi_j) \rho_K^\mu \varepsilon^{ijk} ; \quad g_{\rho K\bar{K}} \frac{i}{2} (K^+ \vec{\partial}_i \vec{\partial}_\mu K) \rho_i^\mu \\ g_{f \pi\pi} \frac{1}{4} (\pi_i \vec{\partial}_\mu \vec{\partial}_\nu \pi_i) f^{\mu\nu} ; \quad g_{f K\bar{K}} \frac{1}{4} (K^+ \vec{\partial}_\mu \vec{\partial}_\nu K) f^{\mu\nu} \\ \text{etc.} \end{aligned} \quad (3)$$

c) One gets a universal ratio for the spin 1^- and spin 2^+ couplings :

$$\frac{1}{2\alpha'} = \frac{g_{\rho \pi\pi}^2}{g_{f \pi\pi}^2} = \frac{g_{\rho K\bar{K}}^2}{g_{f K\bar{K}}^2} = \frac{g_{\omega K\bar{K}}^2}{g_{A_2 K\bar{K}}^2} = \frac{g_{K^* K\pi}^2}{g_{K^{**} K\pi}^2} = \frac{g_{\phi K\bar{K}}^2}{g_{f' K\bar{K}}^2} \quad (4)$$

where α' is the slope of the trajectories $[\alpha' \approx 1 \text{ (GeV)}^{-2}]$.

Assuming a universal coupling of ρ ($g_{\rho \pi\pi}^2 = 4g_{\rho K\bar{K}}^2$) one gets all SU(3) relations and all couplings are expressed in terms of one.

d) Factorization of ρ couplings implies factorization of f couplings.

3. RESULTS FROM THE FIVE-POINT FUNCTION

The model again requires the three families of degenerate parent trajectories given by (1).

From the discussion in Section 4, we find that $K\bar{K} \rightarrow 3\pi$ is described by a unique five-point function which gives rise to the three double pole graphs of Fig. 1. In particular, the model forbids exchange of the $\emptyset$ trajectory and expresses all couplings at the middle vertex in terms of one of them and of $1^-0^-0^-$ couplings. We get again the same $\omega\emptyset$ and ff' mixing angle and partial SU(3), for example :

$$g_{\phi\pi\pi} = 0 \quad g_{\omega\pi\pi} = 2 g_{K^*K^*\pi} \quad (5)$$

Using (2) and (4), our model gives the same universal ratio (4) between spin 1^- and spin 2^+ couplings :

$$\frac{1}{2\alpha'} = \frac{g_{\omega\pi\pi}^2}{g_{A_2\pi\pi}^2} = \frac{g_{K^*K^*\pi}^2}{g_{K^{**}K^*\pi}^2} = \frac{g_{K^*\rho K}^2}{g_{K^{**}\rho K}^2} \quad (6)$$

We also get

$$\frac{g_{K^*K^*\pi}^2}{g_{K^*\rho K}^2} = \frac{g_{K^{**}K^*\pi}^2}{g_{K^{**}\rho K}^2} = \frac{\sqrt{g_{\rho\pi\pi}}}{\sqrt{2 g_{\rho K\bar{K}}}} \quad (7)$$

This ratio is equal to one if ρ is universally coupled, in which case one gets the SU(3) result :

$$\begin{aligned} & g_{\omega\pi\pi} \cdot \frac{1}{4} \varepsilon_{\lambda\mu\nu\sigma} (\partial^\lambda \omega^\mu) (\partial^\nu \rho_i^\sigma) \pi_i \\ & g_{A_2\pi\pi} \cdot \frac{1}{4} \varepsilon_{\lambda\mu\nu\sigma} [\partial^\lambda (A_2)_i^{\mu\alpha}] (\partial^\nu \rho_j^\sigma) (\partial_\alpha \pi_k) \varepsilon_{ijk} \\ & \text{etc.} \end{aligned} \quad (8)$$

The discussion in Section 5 shows that $K\bar{K} \rightarrow K\bar{K} \pi$ can be described by two five-point functions, the first one giving rise to the three double pole graphs of Fig. 2. The model predicts that G parity is conserved at the middle vertex in graph 2c.

The second five-point function gives rise to the same graphs, except that in 2b, $\emptyset f'$ is replaced by ωf .

We therefore cannot predict $\omega \phi$ mixing in this case. Using, however, our previous result, we shall drop the second term.

Some of the couplings are the same as before, and one verifies that the same relations are obtained. In addition, we get the SU(3) results (amongst others)

$$\frac{1}{2} g_{\phi K^* K}^2 = g_{\rho K^* K}^2 = g_{\omega K^* K}^2 = \frac{1}{4} g_{\omega \rho \pi}^2 \quad (9)$$

and the universal ratio of spin 1^- to spin 2^+ couplings :

$$\frac{1}{2\alpha'} = \frac{g_{K^* \omega K}^2}{g_{K^* \omega K}^2} = \frac{g_{K^* \phi K}^2}{g_{K^* \phi K}^2} = \frac{g_{\omega K^* K}^2}{g_{A_2 K^* K}^2} \quad (10)$$

Using all results, we get the following relations which can be compared to experiment :

$$g_{\omega \rho \pi}^2 = \frac{1}{2\alpha'} g_{A_2 \rho \pi}^2 \quad (11)$$

and

$$\frac{1}{4} g_{A_2 \rho \pi}^2 = g_{K^* K^* \pi}^2 = g_{K^* \rho K}^2 = g_{K^* \omega K}^2 \quad (12)$$

which means that in the $2^+ \rightarrow 1^- 0^-$ decay only the F coupling of the octet part of the 1^- nonet is allowed. This result is usually deduced from SU(3) and C conservation ⁶⁾. Here we remark that in the process $K\bar{K} \rightarrow K\bar{K}\pi$, we have not used C conservation as an input, but find it to be a consequence of the other requirements, namely absence of $I=\frac{3}{2}$ and $S=2$ resonances.

4. $K\bar{K} \rightarrow 3\pi$

The five-point function ²⁾ has poles in all five variables :

$$S_{i,i+1} = (P_i + P_{i+1})^2 \quad i=1 \dots 5, \quad 6 \equiv 1 \quad (13)$$

and corresponding Regge trajectories

$$\alpha_{i,i+1} = \alpha_{i,i+1}^0 + \alpha' S_{i,i+1} \quad (14)$$

We require absence of $I=\frac{3}{2}$, $I=2$ and $S=2$ resonances and the lowest spin of the parent to be 1^- (0^- and 1^+ are obviously forbidden). Hence, the possible parent trajectories are ω , ρ , ϕ and K^* . However, it turns out to be impossible to introduce ϕ without introducing a $I=1$, $J=2^+$ particle on the same trajectory. It is also impossible to have K^* poles in four variables without introducing $I=\frac{3}{2}$ poles. Hence we get the unique answer

$$A(K\bar{K}\pi\pi\pi) = C_1 \sum_{P(3,4,5)} K_2^+ \tau_{i_3} \tau_{i_4} \tau_{i_5} K_1 \times \quad (15)$$

$$\varepsilon_{\mu_1\mu_2\mu_3\mu_4} p_1^{\mu_1} p_2^{\mu_2} p_3^{\mu_3} p_4^{\mu_4} B_5(\alpha_{12}-1, \alpha_{23}^{K^*}-1, \alpha_{34}^{\rho}-1, \alpha_{45}^{\rho}-1, \alpha_{51}^{K^*}-1)$$

The indices $1, \dots, 5$ label the particles in Fig. 1. The sum is over all permutations of the three pions. The kinematical factor is imposed by parity and gives the correct Regge behaviour, as well as satisfying Adler's self-consistency relations for soft mesons. The function B_5 is defined in Ref. 2). It can be written in many particular forms, one of which being :

$$B(\alpha_{12}-1, \alpha_{23}-1, \alpha_{34}-1, \alpha_{45}-1, \alpha_{51}-1) = \int_0^1 du_1 \int_0^1 du_4 u_1^{-\alpha_{12}} (1-u_1)^{-\alpha_{23}} u_4^{-\alpha_{45}} (1-u_4)^{-\alpha_{34}} (1-u_1 u_4)^{-\alpha_{51} + \alpha_{23} + \alpha_{34} - 1} \quad (16)$$

We recall its main properties :

- a) for identical $\alpha_{i,i+1}$ it is cyclically invariant;
- b) it has simple poles for $\alpha_{i,i+1} = 1, 2, 3, \dots$;
- c) it has simultaneous poles in α_{ij} and α_{mn} if i, j, m, n are all different;
- d) it gives the correct single and double Regge behaviour.

Finally, the isospin factor in front satisfies the requirements about the absence of exotic resonances, K being an isospinor and τ the Pauli matrices.

We now show that in the 12 channel the pole $\alpha_{12} = 1$ has isospin zero and the parent related to the pole $\alpha_{12} = 2$ has isospin one, thus requiring a degenerate ω, A_2 trajectory and forbidding ϕ exchange.

From (16), one sees that the residue at the pole $\alpha_{12} = 1$ is obtained by setting $u_1 = 0$

$$\text{Res} [B_5 (\alpha_{12} = 1, \dots)] = \int_0^1 du_4 u_4^{-\alpha_{45}^p} (1-u_4)^{-\alpha_{34}^s} \quad (17)$$

which is just the Veneziano function. It is symmetric under the interchange $3 \leftrightarrow 5$. Using the identity :

$$\tau_{i_3 i_4 i_5} = i \epsilon_{i_3 i_4 i_5} + \delta_{i_3 i_4} \tau_{i_5} + \delta_{i_4 i_5} \tau_{i_3} - \delta_{i_5 i_3} \tau_{i_4} \quad (18)$$

One sees that the $I=0$ wave function for three pions is antisymmetric, the $I=1$ part symmetric under $3 \leftrightarrow 5$. The kinematical factor in (15) being antisymmetric, the symmetrization in (15) eliminates the $I=1$ part.

To get the residue at $\alpha_{12} = 2$, one expands (16) around $u_1 = 0$, keeping terms linear in u_1 . We find

$$\text{Res} [B_5 (\alpha_{12}=2, \dots)] = \int_0^1 du_4 u_4^{-\alpha_{45}^p} (1-u_4)^{-\alpha_{34}^p} \times$$

$$\times \frac{1}{2 - \alpha_{34} - \alpha_{45}} [\alpha_{23} (1 - \alpha_{34}) + \alpha_{15} (1 - \alpha_{45}) + 1 - \alpha_{34} - \alpha_{45} + \alpha_{34} \alpha_{45}] \quad (19)$$

The terms which contribute to the parent with spin 2 are linear in the momentum transfers S_{23} and S_{15} , which, due to momentum conservation, are antisymmetric under $3 \leftrightarrow 5$. Hence, only the $I=1$ part survives symmetrization.

In a similar way, considering channels 34 and 45, one shows that the $\alpha_{34} (\alpha_{45}) = 1$ pole has $I=1$ and the $\alpha_{34} (\alpha_{45}) = 2$ pole contributes to a parent with $J=2$ and $I=0$, thus requiring the degeneracy of ρ and f .

Finally, the poles in channels 23 and 51 give parents with $J=1^-$, 2^+ and $I=\frac{1}{2}$.

Comparing the residues with the couplings (8), one finds the relations of Section 3.

5. $K\bar{K} \rightarrow K\bar{K} \pi$

Because $S=2$ resonances are excluded, K and $\bar{K}$ (all incoming) must alternate in Fig. 2. Absence of $I=\frac{3}{2}$ resonances in the $K\pi$ channels 15 and 45 completely fixes the isospin factor. We find

$$A(K\bar{K}K\bar{K}\pi) = C_2 \sum_{P[(13), (24)]} (k_1^\dagger \tau_{i_5} k_4) (k_3^\dagger k_2) \times$$

$$\epsilon_{\mu_1 \mu_2 \mu_3 \mu_4} p_1^{\mu_1} p_2^{\mu_2} p_3^{\mu_3} p_4^{\mu_4} B_5(\alpha_{12}^f - 1, \alpha_{23}^f - 1, \alpha_{34}^f - 1, \alpha_{45}^{\pi^+} - 1, \alpha_{51}^{\pi^+} - 1) \quad (20)$$

The sum is over the permutations $1 \leftrightarrow 3$, $2 \leftrightarrow 4$, $(1 \leftrightarrow 3) \otimes (2 \leftrightarrow 4)$.

As already mentioned, ϕ could be replaced by ω , but this would not be consistent with the $\phi\omega$ mixing deduced from (15) and the four-point functions.

Using the symmetry properties of (20) and the Fierz identity

$$\begin{aligned} (K_1^+ \tau_{i_5} K_4) (K_3^+ K_2) &= \frac{1}{2} (K_1^+ \tau_{i_5} K_2) (K_3^+ K_4) \\ &+ \frac{1}{2} (K_1^+ K_2) (K_3^+ \tau_{i_5} K_4) - \frac{i}{2} \epsilon_{i_5 j l} (K_1^+ \tau_j K_2) (K_3^+ \tau_l K_4) \end{aligned} \quad (21)$$

one shows that G parity is conserved at the middle vertex in Fig. 2c. Eq. (20) requires

$$\alpha^{\rho} = \alpha^{\omega} = \alpha^f = \alpha^{A_2} ; \alpha^{K^+} = \alpha^{K^{*+}} ; \alpha^{\phi} = \alpha^{f'}$$

6. CONCLUSIONS

We have shown that the four-point and five-point functions give consistent results, which means that factorization works for parents and that duality was correctly incorporated into the five point function (cyclic symmetry). The universal ratio (4), (6), (10) can be generalized: a natural parity meson couples uniquely to 0^-0^- or to 1^-0^- . On the other hand, parents lying on the same trajectory have proportional residues, the factor being in this model the slope of the trajectory raised to the spin difference (apart from normalization factors). The results concerning $SU(3)$ and mixing angles are common to models with duality, parallel trajectories, where exotic resonances are excluded. Our expressions (15) and (20) have of course many more applications, such as decay correlations, high energy behaviour in peripheral graphs, etc.

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a)

b)

c)

FIG. 1

a)

b)

c)

FIG. 2