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ON THE ELEMENTARY PARTICLE APPROACH TO MUON CAPTURE

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A B S T R A C T

In this paper a new formulation of the theory of muon capture, when all intervening particles are treated as elementary particles, is presented. Using the helicity formalism invariant form factors are introduced. The theory is applied to several cases.

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I. INTRODUCTION

In the past few years there has been a wide-spread interest in the theory of muon capture when all intervening particles are treated as elementary particles, as first suggested by Fujii and Yamaguchi ¹⁾, Kim and Primakoff ²⁾. This method has been successfully applied to nuclei of spin 0, $\frac{1}{2}$, 1 and 2 ^{2),3)} and lately to spin $\frac{3}{2}$ ⁴⁾. The hadronic matrix elements of the weak current are, therein, constructed in terms of Dirac spinors, polarization four vectors, tensors and Rarita-Schwinger wave functions.

In this paper a new formulation is given which is valid for nuclei of arbitrary spin. The helicity formalism is used to avoid redundant components in the wave functions and the hadronic matrix elements of the weak currents are written in terms of invariant form factors.

In Section II the expressions for muon capture, beta decay, electromagnetic decay and backward inelastic electron scattering are given in terms of the matrix elements of the hadronic current. In Section III, the general expression for these matrix elements is given, and the implications of CVC, PCAC and G parity conservation are discussed. In Section IV, some applications of the method are presented.

II. GENERAL FORMULAE

We shall consider the following four processes

$$N_2 \rightarrow N_1 + e^- + \bar{\nu}_e \quad (1.a)$$

$$\mu^- + N_1 \rightarrow N_2 + \nu_\mu \quad (1.b)$$

$$N_1^* \rightarrow N_1 + \gamma \quad (1.c)$$

$$e^- + N_1 \rightarrow N_1^* + e^- \quad (1.d)$$

2.

where N_1 and N_2 denote two nuclei and N_1^* is an excited state of N_1 . The first two processes are described by the ordinary current $\times$ current phenomenological Lagrangian density

$$L_w(x) = \frac{G}{\sqrt{2}} J_\mu(x) \ell^\mu(x) + h.c. \quad (2)$$

where $G \simeq 10^{-5} M_p^{-2}$ and $J^\mu(x)$ ($\ell^\mu(x)$) is the hadronic (leptonic) weak current. Reactions (1.c) and (1.d) are, respectively, first order and second order electromagnetic processes governed by the interaction Lagrangian

$$L_{EM}(x) = -e j_\mu(x) A^\mu(x) \quad (3)$$

where $j_\mu(x)$ is the electromagnetic current operator and $A^\mu(x)$ is the photon field.

In Section III, once the invariant form factors have been introduced, we will discuss how to obtain information on the form factors using processes (1.a), (1.c) and (1.d). Here we will limit ourselves to derive the needed formulae.

i) β -decay

Let

$$L_\mu^{(\beta)} \equiv (N_1 | J_\mu^+(0) | N_2) \quad (4)$$

and

$$L_{\mu\nu}^{(\beta)} \equiv \sum \sum L_\mu^{(\beta)} L_\nu^{(\beta)*} \quad (5)$$

where the summation runs over the possible helicity states.

Clearly $L_{\mu\nu}^{(\beta)}$ is a tensor quantity which should be constructed out of the four-momenta p_1, p_2 of the entering nuclei. If the recoil of the final nucleus is neglected, then $p_1 \propto p_2$ and a simple algebraic manipulation lead to

$$L_{\mu\nu}^{(\beta)} = \frac{1}{3} \left[\left(L_{\alpha}^{(\beta)\alpha} - \frac{1}{m_1^2} p_1^\alpha L_{\alpha\gamma}^{(\beta)} p_1^\gamma \right) g_{\mu\nu} - \right. \\ \left. - \left(L_{\alpha}^{(\beta)\alpha} - \frac{4}{m_1^2} p_1^\alpha L_{\alpha\gamma}^{(\beta)} p_1^\gamma \right) \frac{p_{1\mu} p_{1\nu}}{m_1^2} \right] \quad (6)$$

where m_1 and m_2 are the masses of nuclei N_1 and N_2 , respectively. Then one immediately obtains

$$f_{12} \tau_{1/2} = (2\pi)^{-3} \ln 2 (2J_2 + 1) \frac{m_1 m_2}{G^2 m_e^5} \left(\frac{2}{m_1^2} p_1^\mu L_{\mu\nu}^{(\beta)} p_1^\nu - L_{\mu}^{(\beta)\mu} \right)^{-1} \quad (7)$$

where

$$f_{12} = \int_1^{\Delta/m_e} dx \, x \sqrt{x^2 - 1} \left(\frac{\Delta}{m_e} - x \right)^2 F(Z, x\Delta) \quad (8)$$

where $\Delta \equiv m_2 - m_1$ and $F(Z, E)$ is the Fermi correcting factor which takes into account the effect of the Coulomb final state interaction.

ii) μ -capture

Let now

$$L_{\rho}^{(\mu)} \equiv (N_2 | J_{\rho}(0) | N_1) \quad (9)$$

and

$$L_{\rho\sigma}^{(\mu)} \equiv \sum \sum L_{\rho}^{(\mu)} L_{\sigma}^{(\mu)*} \quad (10)$$

By neglecting as before the recoil of the final nucleus, we can write for $L_{\rho\sigma}^{(\mu)}$ an expression similar to (6). Then the partial capture rate (when the hyperfine structure is disregarded) corresponding to (1.b) is given by

4.

$$\Gamma^{(\mu)} = \frac{(2\pi)^4}{2} G^2 \frac{1}{2J_1+1} (Z_1 \alpha m'_\mu)^3 C_1 \frac{\nu_{12}^2}{m_1^2} \left\{ \frac{2}{m_1^2} p_1^\rho L_{\rho\sigma}^{(\mu)} p_1^\sigma - L_{\rho}^{(\mu)\rho} \right\} \quad (11)$$

where m'_μ is the reduced muon mass, C_1 is defined as in Ref. 2) or, equivalently, as 5)

$$C_1 = (Z_{1,eff}/Z_1)^4 \quad (12)$$

and

$$\nu_{12}^2 = E_\nu^2 \left(1 - \frac{m_\mu}{m_1} \right) \quad (13)$$

Equation (11) is correct up to terms of order $(m_\mu/m_1)^2$, Δ/m_1 and higher.

iii) Electromagnetic decay

Process (1.c) allows us to obtain information on the matrix elements of the electromagnetic current at zero squared momentum transfer. If

$$\mathcal{V}^2(0) \equiv \sum \sum (N_1 | j^\mu(0) | N_1^*) (N_1^* | j_\mu(0) | N_1) \quad (14)$$

then the decay width for (1.c) is

$$\Gamma^* = \frac{(2\pi)^6}{2} \alpha \frac{E_\gamma}{m_1^{*2}} \frac{1}{2J_1^*+1} (-\mathcal{V}^2(0)) \quad (15)$$

where m_1^* and J_1^* denote, respectively, the mass and the spin of the nuclear state N_1^* . E_γ stands for the energy of the emitted photon in the N_1^* rest system.

iv) Backward inelastic electron-nucleus scattering

Defining

$$v_{\mu\nu} \equiv \sum \sum (N_i^* | j_{\mu}(0) | N_i) (N_i | j_{\nu}(0) | N_i^*) \quad (16)$$

we can write for $v_{\mu\nu}$ an expression similar to (6) and the differential cross-section for (1.a) at $\theta = 180^\circ$ is therefore

$$\left. \frac{d\sigma}{d\Omega} \right)_{\text{back.}} = \frac{(2\pi)^6}{12} \alpha^2 \frac{1}{E_i^2 m_i^2} \frac{1}{2J_i + 1} \left(\frac{1}{m_i^2} p_{i\mu} v^{\mu\nu} p_{i\nu} - v_{\mu}^{\mu} \right) \quad (17)$$

where E_i denotes the energy of the incoming electron (assumed much bigger than m_e), which is related to the squared momentum transfer, q^2 , between the initial and final nucleus through

$$q^2 \simeq - [2E_i - (m_i^* - m_i)]^2 + (m_i^* - m_i)^2 \quad (18)$$

III. INVARIANT FORM FACTORS

The discussion of the number of independent form factors as well as its behaviour at small momentum transfer can be suitably carried out in the helicity framework. Let $X^{\mu}(x)$ be any four-vector current local operator and let us write

$$\bar{X}^{\mu}(p_2, p_1)_{\lambda_1}^{\lambda_2} \equiv (p_2, j_2, -\lambda_2 | X^{\mu}(0) | p_1, j_1, \lambda_1) \quad (19)$$

To write this in terms of invariant form factors we will use a method analogous to that of de Rafael ⁶⁾. Nevertheless since we are interested in small values of q^2 , we are going to define all quantities in such a way that no singularities arise as $q^2 \rightarrow 0$, which is not the case in Ref. 6).
Let

$$a^\mu \equiv \frac{k^\mu}{\sqrt{k^2}}, \quad k^\mu \equiv p_1^\mu + p_2^\mu, \quad a^0 > 0, \quad a^2 = +1$$

$$e^\mu = [(q \cdot k)^2 - k^2 q^2]^{-1/2} [(k \cdot a) q^\mu - (q \cdot a) k^\mu], \quad e^2 = -1, \quad a \cdot e = 0 \quad (20)$$

where $q^\mu \equiv p_1^\mu - p_2^\mu$. Let $t, n^{(i)}$ ($i=1,2,3$) be a tetrad of reference with the following properties

$$t^2 = +1, \quad n^{(i)} \cdot n^{(j)} = -\delta^{ij}, \quad t \cdot n^{(i)} = 0$$

$$\epsilon^{\mu\nu\rho\sigma} t_\mu n_\nu^{(1)} n_\rho^{(2)} n_\sigma^{(3)} = +1 \quad (21)$$

Furthermore, if c and d are two arbitrary four vectors such that $c^2 = d^2 = +1$, $c^0 > 0$, $d^0 > 0$, then a particular Lorentz transformation which carries c into d is chosen as

$$(\Lambda_{c \rightarrow d})^\nu_\mu = g^\nu_\mu - (1 + c \cdot d)^{-1} \{ d_\mu d^\nu + c_\mu c^\nu + c_\mu d^\nu - [1 + 2(c \cdot d)] d_\mu c^\nu \} \quad (22)$$

Finally, we will introduce the three four vectors

$$f_{(\pm 1)} = \mp \frac{1}{\sqrt{2}} (f^{(1)} \pm i f^{(2)}), \quad f_{(0)} = f^{(3)} \quad (23)$$

where

$$f^{(i)} = \Lambda_{t \rightarrow a} n^{(i)} \quad (24)$$

which are orthogonal to a^μ and satisfy $f^{(i)} \cdot f^{(j)} = -\delta^{ij}$.

Then a straightforward calculation analogous to those carried out in Ref. 6) allows us to write

$$X^\mu(p_2, p_1)_{\lambda_1}^{\lambda_2} = D^{(j_2)} [R(a, u_2)]_{\lambda_2'}^{\lambda_2} D^{(j_1)} [R(u_1, a)]_{\lambda_2}^{\lambda_1'} \cdot \left\{ \sum_{LM} a^\mu \begin{pmatrix} \lambda_2' & j_1 & M \\ j_2 & \lambda_1' & L \end{pmatrix} A^{(L)}(q^2) \sqrt{4\pi} Y_m^{(L)} [\vec{w}(a, e)] + \right.$$

$$\left. + \sum_{\ell m} \sum_{LM} f_{(M-m)}^{\mu*} \begin{pmatrix} \lambda_2' & j_1 & M \\ j_2 & \lambda_1' & L \end{pmatrix} \begin{pmatrix} M-m & L & m \\ 1 & M & \ell \end{pmatrix} F^{(\ell, L)}(q^2) \sqrt{4\pi} Y_m^{(\ell)} [\vec{w}(a, e)] \right\} \quad (25)$$

where $L = |j_1 - j_2|, \dots, j_1 + j_2$, $\ell = |L-1|$, L , $L+1$, and $R(u, a)$ is the $SL(2, \mathbb{C})$ matrix corresponding to the rotation

$$R(u, a) = R^{-1}(a, u) = \Lambda_{t \rightarrow a}^{-1} \Lambda_{u \rightarrow a} \Lambda_{t \rightarrow u} \quad (26)$$

where $u = p/m$. The unitary vector $\vec{\omega}(a, e)$ is defined by the equation $\vec{\omega}(a, e) = -e \cdot \Lambda_{t \rightarrow a} \vec{n}$. Finally, we have used the covariant Wigner $3j$ symbols which are defined in terms of the usual ones through the equation

$$\begin{pmatrix} \lambda_1 & j_2 & \lambda_3 \\ j_1 & \lambda_2 & j_3 \end{pmatrix} = (-1)^{j_1 - \lambda_1 + j_3 - \lambda_3} \begin{pmatrix} j_1 & j_2 & j_3 \\ -\lambda_1 & \lambda_2 & -\lambda_3 \end{pmatrix} \quad (27)$$

It is particularly simple to work in the Breit reference system defined by

$$p_1 \equiv (\sqrt{x^2 + m_1^2}, 0, 0, x) \quad p_2 \equiv (\sqrt{x^2 + m_2^2}, 0, 0, -x) \quad (28)$$

$$x \equiv \sqrt{\frac{\lambda(q^2, m_1^2, m_2^2)}{4(2m_1^2 + 2m_2^2 - q^2)}}$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$. Then equation (25) reduces to

$$\begin{aligned} \Delta^\mu(p_2, p_1)^{\lambda_2}_{\lambda_1} &= \sum_L \sqrt{2L+1} a^\mu \begin{pmatrix} \lambda_2 & j_1 & 0 \\ j_2 & \lambda_1 & L \end{pmatrix} A^{(L)}(q^2) + \\ &+ \sum_L \sum_e \sqrt{(2L+1)(2L+1)} f_{(\lambda_1 - \lambda_2)}^{*\mu} \begin{pmatrix} \lambda_2 & j_1 & \lambda_1 - \lambda_2 \\ j_2 & \lambda_1 & L \end{pmatrix} \begin{pmatrix} \lambda_1 - \lambda_2 & L & 0 \\ 1 & \lambda_1 - \lambda_2 & e \end{pmatrix} F^{(e, L)}(q^2) \end{aligned} \quad (29)$$

where now

$$a \equiv (1, 0, 0, 0), \quad f_{(\pm 1)} = \left(0, \mp \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right), \quad f_{(0)} = (0, 0, 0, 1) \quad (30)$$

To analyze the behaviour of $A^{(L)}(q^2)$ and $F^{(\ell, L)}(q^2)$ for small momentum transfer, we use the same method as Durand et al. ⁷⁾ in their study of the electromagnetic form factors. Then it can be proved that if we define

$$A^{(L)}(q^2) = \nu^L B^{(L)}(q^2), \quad F^{(\ell, L)}(q^2) = \nu^\ell G^{(\ell, L)}(q^2) \quad (31)$$

where

$$\nu = \frac{\sqrt{\lambda(q^2, m_1^2, m_2^2)}}{2 m_1 m_2} \quad (32)$$

then $B^{(L)}(q^2)$ and $G^{(\ell, L)}(q^2)$ are smooth functions of q^2 in that range of momentum transfers we are interested in. Using the techniques of Williams ⁸⁾ it can further be shown that in the region of interest $B^{(L)}(q^2)$ and $G^{(\ell, L)}(q^2)$ practically coincide with the analytic form factors ^{*}).

It is easy to prove that the form factors introduced above are real if X^μ transforms, as we assume, like a four current density operator under time reversal. Similarly, if under the parity operation X^μ is a vector (axial vector) current, then $A^{(L)}(q^2) = 0$ when $(-1)^L = \eta \Pi_1 \Pi_2$ where Π_1 and Π_2 are the intrinsic parities of the particles, say, N_1 and N_2 , entering (19) and $\eta = -1$ (+1). Similarly $F^{(\ell, L)}(q^2) = 0$ if $(-1)^{1+\ell} = \eta \Pi_1 \Pi_2$.

Further limitations on the number of non-zero form factors can be obtained when N_1 and N_2 are members of the same isospin multiplet and X^μ is supposed to be the raising X_{+1}^μ (or lowering X_{-1}^μ) component of an isovector operator, with $X_{-1}^\mu = X_{+1}^{\mu*}$. Then a simple application of the Wigner-Eckart theorem shows that

$$(p_2, j_2; T, T_3+1 | X_{+1}^\mu | 0) (p_1, j_1; T, T_3) = (p_1, j_1; T, T_3+1 | X_{+1}^\mu | 0) (p_2, j_2; T, T_3) \quad (33)$$

which implies

$$A^{(L)}(q^2) = 0 \quad \text{if} \quad (-1)^{L+1} = 1$$

$$F^{(\ell, L)}(q^2) = 0 \quad \text{if} \quad (-1)^L = 1 \quad (34)$$

^{*}) $B^{(L)}(q^2)$ and $G^{(\ell, L)}(q^2)$ are the analytic form factors multiplied by $D^{(j_2)}[\Lambda_{t \rightarrow u_2}^*] D^{(j_1)}[\Lambda_{t \rightarrow u_1}]$, which is practically a constant in the region of interest ($0 \leq -q^2 \leq m_\mu^2$).

Equations (34) are the usual restrictions on the form factors due to G parity conservation by strong interactions.

The matrix elements of the divergence of the current can be calculated from (29) with the result

$$q_\mu \bar{\Sigma}^\mu(p_2, p_1) \lambda_1^{\lambda_2} = - \sum_L \sqrt{2L+1} \begin{pmatrix} \lambda_2 & j_1 & 0 \\ j_2 & \lambda_1 & L \end{pmatrix} [\Delta A^{(L)}(q^2) + \\ + \sum_e \sqrt{2e+1} \begin{pmatrix} 0 & L & 0 \\ 1 & 0 & e \end{pmatrix} m_\nu F^{(e,L)}(q^2)] \quad (35)$$

Then an algebraic manipulation implies that the conservation of the current is equivalent to

$$\Delta A^{(L)}(q^2) + m_\nu \left[\sqrt{\frac{L}{2L+1}} F^{(L-1,L)}(q^2) - \sqrt{\frac{L+1}{2L+1}} F^{(L+1,L)}(q^2) \right] = 0 \quad (36)$$

In particular, thus, the conservation of the current implies that $\nu^{-L} F^{(L-1,L)}(q^2)$ remains finite as $q^2 \rightarrow 0$.

The PCAC hypothesis can be reasonably written, in the spirit of Ref. 2), as ^{*)}

$$\Delta A^{(L)}(q^2) + m_\nu \left[\sqrt{\frac{L}{2L+1}} F^{(L-1,L)}(q^2) - \sqrt{\frac{L+1}{2L+1}} F^{(L+1,L)}(q^2) \right] = \frac{C \nu^L}{q^2 - m_n^2} \quad (37)$$

where, as usual, C is determined in terms of the form factors at zero momentum transfer, i.e.,

$$C = - \frac{m_n^2 m_\nu^L}{\Delta^{L-1}} \left[A^{(L)}(0) + \sqrt{\frac{L}{2L+1}} F^{(L-1,L)}(0) - \sqrt{\frac{L+1}{2L+1}} F^{(L+1,L)}(0) \right] \quad (38)$$

*) If the effect of the nuclear size is taken into account, it seems also plausible that the right-hand side of (37) must be multiplied by the form factor describing the nuclear charge distribution. We would like to thank Professor J.S. Bell for this remark.

The combinations of the hadronic matrix elements which appear in (7) and (11) can be written in terms of the invariant form factors just introduced, taking them at $q^2 = 0$ for process (1.a) and at finite q^2 for process (1.b). A straightforward calculation yields

$$\begin{aligned}
 L^\mu_\mu &= \sum_L [A^{(L)}(q^2)]^2 - \sum_L \sum_e [F^{(e,L)}(q^2)]^2 \\
 \frac{2}{m_1^2} (p_{1\mu} L^{\mu\nu} p_{1\nu}) &= 2 \left(1 + \frac{x^2}{m_1^2}\right) \sum_L [A^{(L)}(q^2)]^2 + \\
 &+ 4 \frac{x}{m_1} \sqrt{1 + \left(\frac{x}{m_1}\right)^2} \sum_L A^{(L)}(q^2) \left[\sqrt{\frac{L+1}{2L+1}} F^{(L+1,L)}(q^2) - \sqrt{\frac{L}{2L+1}} F^{(L-1,L)}(q^2) \right] + \\
 &+ 2 \left(\frac{x}{m_1}\right)^2 \sum_L \left[\sqrt{\frac{L+1}{2L+1}} F^{(L+1,L)}(q^2) - \sqrt{\frac{L}{2L+1}} F^{(L-1,L)}(q^2) \right]^2
 \end{aligned} \tag{39}$$

Since in practical applications $x \ll m_1$ we can write approximately

$$\frac{2}{m_1^2} (p_{1\mu} L^{\mu\nu} p_{1\nu}) - L^\mu_\mu \simeq \sum_L [A^{(L)}(q^2)]^2 + \sum_L \sum_e [F^{(e,L)}(q^2)]^2 \tag{40}$$

Usually it is impossible to handle all the non-zero form factors and since we are interested in values of $(-q^2)$ between 0 and m_μ^2 , which correspond to $\nu \ll 1$ we will keep in the applications only one or two of the leading form factors for each current, i.e., those which behave as the lowest powers of ν for $q^2 \rightarrow 0$, and neglect the remaining ones.

Should no direct measurements of the vector form factors be available, a method which proves useful in some cases is the following one: let N_1^* be an excited state of N_1 with isotopic spin $T_1^* = T_1 \pm 1$ and suppose that N_1^* and N_2 belong to the same isomultiplet. Then processes (1.c) and (1.d)

are mediated by the isovector part of the electromagnetic current and $\gamma_{\mu\nu}$ can be related to the vector contribution to $L_{\mu\nu}$, using the hypotheses that the isovector part of the electromagnetic current and the vector weak current are members of the same isotopic multiplet. If we keep only the leading form factor for the vector weak current then processes (1.c) and (1.d) allow us to compute this form factor at $q^2 = 0$ and at the values of q^2 corresponding to the muon capture process in which we are interested.

Since in process (1.a) only the leading vectorial and axial form factor appear, it can be used to compute the leading axial form factor at $q^2 = 0$. Since the impulse approximation suggests that this form factor behaves like the leading vector form factor as a function of q^2 , we can use this property to compute its value at the q^2 corresponding to muon capture. Finally, the PCAC allows us to compute the second leading axial form factor at the value of q^2 in which we are interested. Then using Eq. (11) the muon capture rate can be calculated.

IV. APPLICATIONS

In this Section we shall apply the method described above to three cases :

Since ${}^3\text{He}$ and ${}^3\text{H}$ are members of the same isomultiplet, the non-zero form factors are :

$$\begin{aligned} \text{Vector form factors} & : A^{(0)}(q^2), F^{(1,1)}(q^2) \\ \text{Axial form factors} & : F^{(0,1)}(q^2), F^{(2,1)}(q^2) \end{aligned} \quad (41)$$

Our form factors can be written in terms of the usual ones as

$$A^{(0)}(q^2) = -m_l \frac{\sqrt{2}}{4\pi^3} F_V(q^2) \quad (42)$$

$$F^{(1,1)}(q^2) = \frac{x}{2\pi^3} \left[F_V(q^2) + \frac{m_l}{M_p} F_M(q^2) \right]$$

From the known values of $F_V(q^2)$ and $F_M(q^2)$, we get $(-q^2 = 1.0319 \times 10^4 \text{ MeV}^2)$

$$\begin{aligned} A^{(0)}(0) &= -0.3202 \cdot 10^{+2} \text{ MeV}, & A^{(0)}(q^2) &= -10.2626 \pm 0.0032 \cdot 10^{+2} \text{ MeV} \\ G^{(1,1)}(0) &= -(0.1232 \pm 0.0012) m_l, & G^{(1,1)}(q^2) &= -(0.1076 \pm 0.0012) m_l \end{aligned} \quad (43)$$

For $q^2=0$ the leading axial form factor is $F^{(0,1)}(q^2)$ and its value at $q^2=0$ can be obtained from the decay ${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$ which has $t_{1/2} = (1.137 \pm 0.020) \cdot 10^3 \text{ sec}$. Therefore

$$[A^{(0)}(0)]^2 + [F^{(0,1)}(0)]^2 = \frac{\ln 2}{t_{1/2}} \frac{m_l m_2}{4\pi^3 G^2 m_e^5} = (0.572 \pm 0.011) \cdot 10^{+4} \text{ MeV}^2 \quad (44)$$

which implies $F^{(0,1)}(0) = (0.685 \pm 0.009) \cdot 10^2 \text{ MeV}$. If we assume that $F^{(0,1)}(q^2)$ behaves as $A^{(0)}(q^2)$ as a function of q^2 we obtain $F^{(0,1)}(q^2) = (0.562 \pm 0.010) \cdot 10^2 \text{ MeV}$. The PCAC hypothesis can be written now as

$$F^{(2,1)}(q^2) = \frac{1}{\sqrt{2}} \left[F^{(0,1)}(q^2) - \frac{m_n^2}{(-q^2) + m_n^2} F^{(0,1)}(0) \right] \quad (45)$$

and hence $F^{(2,1)}(q^2) = (0.090 \pm 0.013)$. Hence we obtain

$$\Gamma^{(P)} = (1300 \pm 100) \text{ sec}^{-1} \quad (46)$$

to be compared with the weighted average value of five experiments ⁹⁾

$$\Gamma^{(P)} = (1468 \pm 50) \text{ sec}^{-1} \quad (47)$$

Other similar calculations have been carried out by Fujii and Yamaguchi ¹⁾, and by Kim and Primakoff ²⁾ and they obtain, respectively, $\Gamma^{(\mu)} \simeq (1430 \pm \pm 110) \text{sec}^{-1}$ and $\Gamma^{(\mu)} = (1510 \pm 40) \text{sec}^{-1}$, in good agreement with our results. We would like to emphasize that all theoretical results quoted in Ref. 2) are too optimistic about the errors involved in the calculation.

In this case the non-zero form factors are

$$\begin{aligned} \text{Vector form factors} & : F^{(1,1)}(q^2) \\ \text{Axial form factors} & : A^{(1)}(q^2), F^{(0,1)}(q^2), F^{(2,1)}(q^2) \end{aligned} \quad (48)$$

Notice that there is only one invariant vector form factor and not two as stated in Ref. 2). The error is due to the fact that these authors do not impose the necessary condition that the current matrix element must be linear in the polarization four vector and besides they make use of an unorthodox Levi-Civita tensor.

All needed information on $F^{(1,1)}(q^2)$ can be obtained from processes (1.c) and (1.d), where N_1^* is the excited state of ${}^{12}\text{C}$ with excitation energy of 15.11 MeV which decays, emitting a γ ray, with $\Gamma^* = (50 \pm 5) \text{eV}$, and belongs to the same isomultiplet as ${}^{12}\text{B}$. Using equations (15) and (17) we obtain $(-q^2 = 0.8223 \cdot 10^4 \text{ MeV}^2)$

$$-v^2(0) = (5.54 \pm 0.55) \text{ MeV}^2 \quad (49)$$

$$-v_{\mu}^{\mu} + \frac{1}{m_1^2} (p_{1\mu} v^{\mu\nu} p_{1\nu}) = (67 \pm 10) \text{ MeV}^2$$

where for the inelastic electron scattering cross-section we have used the data of Ref. ¹⁰⁾. Taking into account the fact that the isovector part of the electromagnetic current and the weak current are members of the same isomultiplet, we obtain

$$\begin{aligned} [F^{(1,1)}(0)]^2 &= (11.08 \pm 0.78) \text{ Mev}^2 \\ [F^{(1,1)}(q^2)]^2 &= (134 \pm 15) \text{ Mev}^2. \end{aligned} \quad (50)$$

which imply $G^{(1,1)}(0) = (0.249 \pm 0.009)m_1$ and $G^{(1,1)}(q^2) = (0.127 \pm 0.007)m_1$.

The leading axial form factor at $q^2 \rightarrow 0$ is $F^{(0,1)}(q^2)$. Its value at $q^2 = 0$ can be obtained from the pure Gamow-Teller β decay process $^{12}\text{B} \rightarrow ^{12}\text{C} + e^- + \bar{\nu}_e$ with $f_{12} \tau_{1/2} = (1.17 \pm 0.03)10^4 \text{ sec}$. Equation (7) implies

$$[F^{(0,1)}(0)]^2 = \frac{3}{8\pi^3} \frac{\ln 2}{f_{12} \tau_{1/2}} \frac{m_1 m_2}{G^2 m_e^5} = (1.321 \pm 0.034)10^4 \text{ Mev}^2. \quad (51)$$

Hence $F^{(0,1)}(0) = (1.150 \pm 0.025)10^2 \text{ MeV}$. If we assume that it depends on q^2 as $G^{(1,1)}(q^2)$ we obtain

$$F^{(0,1)}(q^2) = (0.587 \pm 0.040)10^{+2} \text{ Mev}. \quad (52)$$

We are going to neglect, according to the general discussion, the term $F^{(2,1)}(q^2)$. Then $A^{(1)}(q^2)$ can be computed using PCAC hypothesis which can be written as

$$B^{(1)}(q^2) = \frac{m_1}{\Delta\sqrt{3}} \left[\frac{m_n^2}{-q^2 + m_n^2} F^{(0,1)}(0) - F^{(0,1)}(q^2) \right] \quad (53)$$

Then $B^{(1)}(q^2) = (0.933 \pm 0.191)m_1$. Using equation (11) we obtain

$$\Gamma^{(1)} = (0.92 \pm 0.33)10^4 \text{ sec}^{-1} \quad (54)$$

to be compared with the experimental values $^{11),12)}$

$$\Gamma^{(\mu)} = (0.67 \pm 0.09) 10^4 \text{ sec}^{-1} \quad (55)$$

$$\Gamma^{(\mu)} = \left(\begin{array}{c} 0.675 + 0.030 \\ - 0.075 \end{array} \right) 10^4 \text{ sec}^{-1}$$

and with the theoretical one ²⁾ $\Gamma_{\mu} = (0.66 \pm 0.10) 10^3 \text{ sec}^{-1}$, where the errors are underestimated.

In this case the non-zero form factors are the same as in the last example. ${}^6\text{Li}$ has an excited state at 3.56 MeV which belongs to the same isomultiplet as ${}^6\text{He}$. This state decays with $\Gamma^* = (6.4 \pm 0.6) \text{ eV}$. As in the last example, and using for the inelastic electron scattering the data of Ref. 10), we obtain $G^{(1,1)}(0) = (0.202 \pm 0.007) m_1$ and $G^{(1,1)}(q^2) = (0.109 \pm 0.006) m_1$, $(-q^2 = 1.0217 \cdot 10^4 \text{ MeV}^2)$. The ${}^6\text{He}$ decays, through β emission, into ${}^6\text{Li}$ with $f_{12}^2 \approx \frac{1}{2} = (8.08 \pm 0.30) 10^3 \text{ sec}$. As before we obtain $F^{(0,1)}(0) = (0.400 \pm 0.007) 10^2 \text{ MeV}$ and assuming $F^{(0,1)}(q^2)/F^{(0,1)}(0) \simeq G^{(1,1)}(q^2)/G^{(1,1)}(0)$ we get $F^{(0,1)}(q^2) = (0.216 \pm 0.014) 10^2 \text{ MeV}$. Using the PCAC hypothesis we obtain $B^{(1)}(q^2) = (0.723 \pm 0.251) m_1$ and hence

$$\Gamma^{(\mu)} = (1.08 \pm 0.64) 10^{+3} \text{ sec}^{-1} \quad (56)$$

to be compared with the experimental result ¹³⁾

$$\Gamma^{(\mu)} = \left(\begin{array}{c} 1.60 + 0.33 \\ - 0.12 \end{array} \right) 10^{+3} \text{ sec}^{-1} \quad (57)$$

and the theoretical value ²⁾ $\Gamma^{(\mu)} = (0.98 \pm 0.15) 10^3 \text{ sec}^{-1}$.

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R E F E R E N C E S

- 1) A. Fujii and Y. Yamaguchi, Progr.Theoret.Phys. 31, 107 (1964).
- 2) C.W. Kim and H. Primakoff. Phys.Rev. 139, B1447 (1965).
- 3) C.W. Kim, Phys.Rev. 146, 691 (1966).
- 4) A. Galindo and P. Pascual, An.Real.Soc.Esp.Fis.Quim., (to be published).
- 5) C. Sens, Phys.Rev. 113, 679 (1959).
- 6) E. de Rafael, Thesis, Université de Paris (1966).
- 7) L. Durand III, P.C. DeCelles and R.B. Marr, Phys.Rev. 126, 1882 (1962).
- 8) D.N. Williams, UCRL 11113 (1963).
- 9) R. Pascual and P. Pascual, Nuovo Cimento 44B, 434 (1966).
- 10) J. Goldemberg, W.C. Barber, F.H. Lewis and J.D. Walecka, Phys.Rev. 134, B1022 (1964).
- 11) G.T. Reynolds, D.B. Scarl, R.A. Swanson, J.R. Waters and R.A. Zdanis, Phys.Rev. 129, 1790 (1963).
- 12) E.J. Maier, R.M. Edelstein and R.T. Siegel, Phys.Rev. 133, B663 (1964).
- 13) J.P. Deutsch, L. Grenacs, P. Igo-Kemenes, P. Lipnik and P.C. Macq, Phys.Letters 26B, 315 (1968).