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A NEW EXAMPLE OF A CROSSING SYMMETRIC REGGE BEHAVED AMPLITUDE

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A B S T R A C T

An example is shown of a Regge behaved, crossing symmetric amplitude, with linearly rising trajectories, such that it has the right branch points in s and t , and the residue of a pole, say in t , is a polynomial in s . The example does not show duality, it is rather like a sum of direct and crossed channel resonances.

There has been recently a great interest in the construction of examples of crossing symmetric scattering amplitudes showing Regge behaviour with linearly rising trajectories ¹⁾⁻⁴⁾. We want to present here a new example which, improving the other attempts known to us, satisfies the following requirements.

The crossing symmetric amplitude $A(s,t) = A(t,s)$ (later on we will also consider crossing symmetry with respect to the third variable u) has to be such that:

- i) for $|s| \rightarrow \infty$ at fixed t it goes as $(-s)^{\alpha(t)}$, no matter the value of $\text{Re} \alpha(t)$, which in our case of linearly rising and decreasing trajectory can be made arbitrarily high positive or negative. This, together with dispersion relations, will ensure the automatic fulfilment of the finite energy sum rules;
- ii) it must have a cut in s from $s=s_0$ to $+\infty$, and a cut in t from $t=t_0$ to $+\infty$, the position of the branch point in s being independent on t and vice versa. Poles must appear only in the second sheet, apart from bound states;
- iii) the poles in the second sheet, say in t , near the real axis (resonances) must have a residue which is a polynomial in s . This ensures that at the value of t in question only a finite number of partial waves will resonate, and this is what we would expect from a theory of the Regge type;
- iv) the partial waves, say for the t channel, have a good asymptotic behaviour in t . This means that the asymptotic behaviour of our amplitude for $t, s \rightarrow \infty$ at fixed $\cos \theta$ must not be worse than a power behaviour.

2.

All previous models fail in one of the points i) to iv).

For instance, the model by Khuri ¹⁾ is a function which has a complex branch point in t for s complex and it does not satisfy ii). The model by Veneziano ²⁾ either satisfies i) and ii) or iii), but it cannot work for all of them. Indeed the function $\frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$ has no singularities in $\alpha(s)$ and $\alpha(t)$ except for poles, so that if $\alpha(s)$ is a polynomial in s it cannot have a Regge behaviour with the proper signature for $\alpha(s) \rightarrow \infty$ along the real axis, the proper signature requiring a cut. If $\alpha(s)$ has an imaginary part for s real positive, the function can have a Regge behaviour in s , but then the residue at the pole in t is a polynomial in $\alpha(s)$ and cannot be a polynomial in s . In such a case the formula by Veneziano shows poles at all the partial waves for a value of t (not only the daughter poles).

The same criticism, based on point iii), applies to the models by Atkinson and Dietz ³⁾, and by Kupsh ⁴⁾. In order to construct our example let us first study the function defined by the integral representation

$$G(s, t) = \frac{1}{2\pi i} \int_{\gamma} dn \frac{1}{n - \alpha(t)} \frac{(s_0 - s)^n}{\Gamma(n+1) \sin \pi n} \frac{1}{\Gamma(n-c) \sin \pi(n-c)} \frac{1}{\Gamma(n-d) \sin \pi(n-d)} \quad (1)$$

where c and d are real positive non-integer numbers. The integral is done along the contour γ shown in the Figure. It consists of a part parallel to the imaginary axis at the left of the poles of the integrand at $n=0, 1, 2, \dots$ and $n=c+1, c+2, \dots$ and $n=d+1, d+2, \dots$ plus a path round the moving pole at $n=\alpha(t)$, if eventually it is to the right. Therefore we have a contribution from the moving pole of the form

$$\frac{1}{\Gamma(\alpha(t)+1)} \frac{(s_0 - s)^{\alpha(t)}}{\sin \pi \alpha(t)} \frac{1}{\Gamma(\alpha(t)-c) \sin \pi(\alpha(t)-c)} \frac{1}{\Gamma(\alpha(t)-d) \sin \pi(\alpha(t)-d)} \quad (2)$$

plus the contribution from a "background" integral of the form of (1), integrated along a parallel to the imaginary axis. For $n \rightarrow \text{Re } n \pm i\infty$, at fixed $\text{Re } n$, the integrand in (1) behaves as $e^{-\pi/2|n|}$, so that the integral converges for every $\text{Re } n$.

It is immediately seen that $G(s, t)$ has a branch in s at s_0 , and we choose the cut from s_0 to $+\infty$. As we can perform parallel displacements on the line of integration of the background, it is immediately apparent that $G(s, t)$ has a Regge behaviour in s : $G(s, t)|_{|s| \rightarrow \infty} \sim (-s)^{\alpha(t)}$.

From (2) it is seen that, apart from the cut of $\alpha(t)$, $G(s, t)$ has poles in t for $\alpha(t)$ positive integer, whose residue is a polynomial in s . This corresponds in general to a family of parallel daughters. $G(s, t)$ has also poles for $\alpha(t) = c+m$ and $\alpha(t) = d+m$, whose residue is not a polynomial in s . We can choose however c and d very large in such a way that these poles are reached for large $\text{Re } t$ and also, presumably, $\text{Im } t$, so that they will not have to be identified with resonances. Anyhow, if we do not like these extra poles, we can eliminate them by multiplying $G(s, t)$ by a function of t as it will be shown later.

Let us now consider the behaviour of $G(s, t)$ for $|t| \rightarrow \infty$. If $|\alpha(t)| \rightarrow \infty$ in such a way that $\text{Re } \alpha < N$ we can always choose the line of integration so as to leave to the left the moving pole, eventually picking a contribution from some poles at $n = \ell$, $n = c+m$, $n = d+m$ (ℓ, m positive integers). It is clear that the contribution from these poles will decrease as $|\alpha(t)| \rightarrow \infty$ like $1/(|\alpha(t)|)$, whereas the integral along the line will behave at most as a constant. If $\text{Re } \alpha(t) \rightarrow +\infty$ we are forced to take into account the contribution from the moving pole of the form (2), and it is clear that for $\alpha(t)$ going to infinity linearly with t this term goes to zero faster than the exponential. Let us note also that if s and t go to infinity together, being fixed s/t , the term (2) again goes to zero whereas the line integral behaves surely not worse than a power in s .

4.

In order to eliminate the extra poles at $\alpha(t) = c+m$, $\alpha(t) = d+m$ we multiply $G(s, t)$ by $g(t, c)g(t, d)$ where

$$g(t, c) = \frac{1}{\Gamma(c+1-\alpha(t)) e^{-\alpha(t)} e^{(\alpha(t)-c-\frac{1}{2}) \log b(t_0-t)}} \quad (3)$$

and b is determined by the asymptotic behaviour of $\alpha(t)$:

$$\alpha(t) \xrightarrow{|t| \rightarrow \infty} bt + h + O((\log t)^L)$$

$g(t, c)$ has a cut in t from t_0 to $+\infty$, is zero for $\alpha(t) = c+1, c+2, \dots$ and behaves at most like $e(\log t)^L$ when $|t| \rightarrow \infty$.

We then construct a function $f(t)$ having a cut from t_0 to $+\infty$ and such as to kill this asymptotic behaviour, for instance

$$f(t) = \exp - [\log(t_0 - t)]^{2L}$$

or

$$f(t) = \exp - (t_0 - t)^{1/4}$$

Then we consider

$$F(s, t) = f(t) g(t, c) g(t, d) G(s, t) \quad (4)$$

This function has a cut in s from s_0 to $+\infty$, a cut in t from t_0 to $+\infty$, poles in t in the second sheet whose residue is a polynomial in s , it behaves as $(-s)^{\alpha(t)}$ for $s \rightarrow \infty$ at fixed t , it goes to zero faster than power when $t \rightarrow \infty$ at fixed s or at fixed t/s (fixed $\cos \vartheta$). The asymptotic properties in t are indeed determined by $f(t)$. At this point the job is done. We construct

$$B(s, t) = F(s, t) + F(t, s) \quad (5)$$

and then

$$A(s, t, u) = B(s, t) + B(t, u) + B(s, u) \quad (6)$$

[Note that for $s \rightarrow \infty$ at fixed t $B(s, u)$ goes to zero faster than power, as explained before.] The function A satisfies the properties i) to iv). As a first comment we note that even if the representation (1) may recall the Khuri representation ⁵⁾, it is not an integral over Khuri amplitudes. Indeed we cannot expand a function in powers of $(s_0 - s)$ if it has a branch at $s = s_0$. We want also to point out that even if the integrand of Eq. (1) shows fixed singularities at $n = c+1, c+2, \dots$, the Khuri amplitude (the coefficient in the s^r development) shows only moving poles at $r = \alpha(t), \alpha(t)-1, \dots$

Second comment. The way in which the finite energy sum rules are satisfied does not show, by the very construction of the amplitude [see also Ref. 4)], any duality of the Dolen-Horn-Schmid ⁶⁾ type. Rather it resembles the interference model, or better the model proposed by Alessandrini, Amati and Squires ⁷⁾ to which we refer for questions concerning double counting. Namely $B(s, t)$ is composed of two parts [see Eq. (4)]: I) $F(s, t)$ which has resonances in the t channel and is smooth in the s channel giving the Regge behaviour in s , and II) $F(t, s)$ which has resonances in the s channel but goes to zero for $s \rightarrow \infty$ faster than any power and then it is strictly superconvergent (all the moment) and does not contribute anything to the Regge behaviour.

As a third comment the finite energy sum rules do not give any bootstrap-like condition on the parameters of our amplitude.

We want also to note that it is less complicated, and in fact can be done rather easily, to construct models satisfying i) to iv) if the trajectory $\alpha(t)$ goes to a finite limit for $\text{Re } t \rightarrow +\infty$ and/or $\text{Re } t \rightarrow -\infty$. These models are again easily constructed if we do not demand duality of the Dolen-Horn-Schmid type.

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R E F E R E N C E S

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Fig. I