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DUALITY IN STRONG INTERACTION PHYSICS

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1. INTRODUCTION

The purpose of this series of lectures is not to introduce a theory, but rather an approach to strong interaction physics. This approach appears at present as fruitful and exciting and many papers exploring the consequences of duality have recently been published or are being circulated in preprint form. Nevertheless, I should mention to begin with that the whole matter is still in a controversial form. There is not yet agreement on how far duality can be used, since no precise definition for it is yet available. As a consequence, results presented here as definitive success are still bitterly challenged there. I will therefore not even try to start by a tentative definition which will emerge only as we proceed. In any case, after Horn's series of lectures ¹⁾ on the use of finite energy sum rules, a very good idea of what it should be has already been gathered.

I shall start by listing a few facts which, refusing to call for mere coincidences, we will try to correlate in the most simple way, thus building a duality approach to strong interaction physics where Regge behaviour and resonances are interlocked.

This will lead us to the prominent features of a simple bootstrap scheme based on rising Regge trajectories. This is not a full bootstrap programme. Nevertheless it is a simple way of reaching some general properties of two-body collision amplitudes which should, to a very good approximation, result from a more refined but still apparently too complicated approach, and this, independently of the details of so far poorly known inelastic mechanisms. We will then introduce the Veneziano model as providing a tentative answer to the solution looked for which meets many of the required properties. We will then see how it yields in fact very simple expressions for meson-meson scattering where current algebra constraints together with broken SU_3 relations can be indeed most simply incorporated. We will then close on a few remarks about its generalization to many particle production and reactions among particles with spin.

2. RISING TRAJECTORIES

The known particles show striking regularities when set on a Chew-Frautschi diagram. Several particles can now be safely associated with the same Regge trajectory. This is in particular the case for the three baryon trajectories which are drawn on Fig. 1, and which respectively include the Δ ,

the Υ_1^* and the $\Lambda^{2),3)}$. All three trajectories are amenable to a straight line approximation with a common slope α' . Another striking fact is the possibility of superposing on each of these trajectories another trajectory with opposite signature. Such coincident trajectories have in most cases different residue functions but their superposition will not be considered as a coincidence. As a result this will be associated with the absence of either direct or exchange forces in the channel where the corresponding resonances occur, a property which will in turn have to be interpreted in the realm of unitarity and crossing symmetry ³⁾.

This degeneracy between trajectories of opposite signatures can be used to help drawing meson trajectories since only in the case of the ρ we already have two particles present: the $\rho(1^-)$ and the $g(3^-)$. The $f(2^+)$ indeed falls on the line joining the ρ and g points as if we had degenerate trajectories with opposite signatures. This assumption is supported by the similarity of the two trajectories determined for negative t values from the high energy scattering data. As a matter of fact, Regge analysis would favour slightly different values of the ρ and f (or P') trajectories in the scattering region. Nevertheless we will here refuse to consider their close similarity as a mere coincidence, or consider their degeneracy as a good approximation. The ω and the A_2 ⁴⁾ have masses which are respectively very close to the masses of the ρ and the f . Drawing Regge trajectories through the corresponding points with the slope of the ρ trajectory, a procedure supported by the Regge analysis of high energy data, we get four almost coincident trajectories with isospin $I=0$ and 1 and, for each isospin, the two signatures. Here again we probably oversimplify the picture since the masses involved show but an approximate equality and high energy Regge fits would prefer slightly different intercepts and behaviour. However, it is tempting to see here again more than a coincidence and to try to interpret such an apparent redundancy of parameters. We also note at this point that the equality in masses of the ω and ρ on the one hand, and of the f and the A_2 on the other hand, follow from "ideal" nonet structure with a common mixing angle such that:

$$\tan^2 \theta_V = \tan^2 \theta_T = \frac{1}{2} \quad (1)$$

where V and T , respectively, stand for vector and tensor.

In this mixing scheme, we expect that the two other members of each nonet are equally spaced in mass squared. This is fairly well met by the facts. Furthermore, on the Chew-Frautschi diagram, the straight lines joining the $K^*890(1^-)$ to the $K^*1420(2^+)$ and the $\varphi(1^-)$ to the $f'(2^+)$, as two

coincident Regge trajectories with opposite signatures, have within a good approximation a common slope equal to the ρ slope and are equally spaced. This is displayed on Fig. 2.

This leads us to consider as an important fact the degeneracy of meson Regge trajectories of opposite signatures. We have so far considered only the natural parity trajectories and leave the others for later discussion. The natural parity trajectories are indeed the only relevant ones in meson-baryon scattering where at present these ideas can be submitted to experimental tests.

A third fact which will also not be considered as a coincidence is the equality between the slopes of the meson and baryon trajectories. All slopes are very similar, of the order of 1 GeV^{-2} , and may - within a good approximation - be considered as identical. This strongly suggests the possibility of a bootstrap between the different trajectories, the baryon resulting from the presence of forces associated with meson (and baryon) exchange in the crossed channels, and vice versa, self-consistency then imposing a common slope for all trajectories.

One must stress that such a bootstrap based on linearly rising trajectories may at present appear as too bold a goal. It is very likely that as energy increases, new structures will appear and that rising trajectories extrapolated from the yet sketchy data which we have (Figs. 1 and 2) should be considered as only a simple approximation ⁶⁾. Nevertheless, the basic philosophy underlying the bootstrap approach is that nearby singularities are enough to build themselves out in a self-consistent way, the effect of distant and unknown singularities being lumped together into an assumed simple asymptotic behaviour. We may therefore try to understand the low energy baryons lying on linear Regge trajectories in terms of the low energy mesons also lying on linear Regge trajectories and vice versa. We consider that the observed equality between the baryon and meson slopes is a strong hint of doing so. The observed degeneracy in quantum numbers is a further hint. Any resonance being associated with a particular set of internal quantum numbers, this calls for equal (or opposite) forces due to different sets of quantum numbers in the crossed channels. Baryons with different isospin would otherwise in general show degeneracy in mass. This is not met by the facts and imposes that some forces (and therefore here some meson trajectories) should show degeneracy in internal quantum numbers.

Our aim is therefore to build a bootstrap approach in which linear Regge trajectories in one channel result from the exchange of linear Regge trajectories in the crossed channels, thus linking together the direct channel

resonance and crossed channel Regge pole descriptions of collision processes ⁷⁾. Whether Regge trajectories continue to rise indefinitely or eventually fall is not relevant here since our bootstrap approach postulates the existence of an approximate self-consistent solution involving only nearby singularities in all channels.

3. FINITE ENERGY SUM RULE BOOTSTRAP

The present basic difficulty with strong interaction physics is the impossibility to yet safely separate out a non-trivial but still simple process which, once understood, would give much insight for more complicated reactions. All processes are connected through unitarity and we cannot hope to understand in detail two-body collisions, about which we already have an impressive amount of data, without tackling at the same time more complicated production processes about which we still know very little.

In order to try to get anyway some understanding of two-body processes which we could readily test against experiment, we are forced to make guesses, writing down tentative expressions which incorporate many properties and carry but a very few adjustable parameters. In so doing, we hope that information yielded by unitarity and not yet exploited by lack of appropriate means, will eventually fix the parameters but not completely upset the present tentative form tried for collision amplitudes. This is in particular the case with a Regge amplitude used in the analysis of high s , low t results

$$F(s, t) \simeq \sum_i \beta_i(t) \frac{\eta_i - e^{-i\pi\alpha_i(t)}}{\sin \pi \alpha_i(t)} \left(s/s_0\right)^{\alpha_i(t)} \quad (2)$$

where $\eta = \pm 1$ stands for signature ∓ 1 . It is a priori not incompatible with unitarity, when a one-particle exchange expression :

$$F(s, t) \simeq \frac{B(t)}{t - M^2} P_J(\cos \theta_t)$$

with $\cos \theta_t = ((u-s)/(4m^2-t))$, written for equal mass spinless scattering, would be.

Relation (2), however, does not explicitly satisfy unitarity and our hope in using it is that unitarity will eventually help fixing β and α

but not fully upset (2) as a result of calling for dominating Regge cuts as opposed to dominant Regge poles. As far as we know, at present this hope is met by the facts ⁸⁾.

In the present approach we will also have to make a guess in order to write a tentative expression for two-body reaction amplitudes but we will now try to incorporate Regge behaviour in all channels and this together with the presence of resonances, the resonances in one channel building up the Regge trajectories in the crossed channels and vice versa.

It is obvious that a finite set of resonances ⁹⁾, which we may write in a zero width approximation as

$$F(s, \cos \theta) \simeq \sum_i \frac{B_i}{s - M_i^2} P_{J_i}(\cos \theta) \quad (3)$$

will not give a Regge asymptotic behaviour, but rather a fixed pole behaviour at high s and that on the other hand a Regge amplitude such as (2) will not give s channel poles as usually associated with resonances. Imposing unitarity shifts the real axis poles, as they appear in the K matrix, say, to the unphysical sheets. Only an infinite set of s channel resonances can yield an asymptotic Regge behaviour at large s and only an infinite set of Regge trajectories may lead to a second sheet pole. These two approximations to two-body amplitudes, the first one (s channel resonances) being fruitful at low energies (< 2 GeV, say) and the second one (Regge poles) being fruitful at high energies (> 5 GeV, say) are far from equivalent when stopped short after a few terms, as legitimate in order to analyze either low or high energy experiments. Nevertheless, it has been realized that the two descriptions have a large overlap so that many features readily interpreted in terms of Regge pole exchange can be fruitfully analyzed as co-operative effects of direct channel resonances and conversely that properties most easily associated to the dominance of resonances can be interpreted as due to relations among Regge trajectories.

Relations (2) and (3) may then be considered at the same time as two possible ways to describe a reaction amplitude in the sense of asymptotic series. One is thus led to introduce a duality between the resonance and Regge pole description of hadron collisions ¹⁰⁾. Both expansions, when stopped short after a few terms correspond to two incomplete but complementary ways of describing a particular process.

As a consequence, even at relatively low energies, the sum of all resonance contributions corresponding to a particular set of internal quantum numbers in the crossed channel should be well approximated in a semi-local sense ¹¹⁾ by the extrapolation of the Regge amplitude, with the same quantum numbers, determined from the analysis of high energy data. To the extent that such a property is met by the facts at energies where the amplitude is still very well known in terms of phase shift or multipole expansions, so that it can be safely continued to unphysical t values, the Regge trajectories can be determined from the low and intermediate energy data in terms of finite energy sum rules ¹⁾ (FESR) :

$$\int_0^N \nu^n J_m\{F(\nu, t)\} d\nu = \sum_i \beta_i(t) \frac{N^{\alpha_i(t)+1+n}}{\alpha_i(t)+1+n} \quad (4)$$

where $\nu = (s-u)/2m$.

Continuing to timelike t values, one eventually reaches right signature integer (or half-integer) points on the trajectory, the crossed channel particles being thus determined in terms of the direct channel ones ¹²⁾. The finite energy sum rules result from a combination of analyticity, which leads to the possible existence of superconvergence relations, Regge asymptotic behaviour, which warrants writing such superconvergence relations and l'Hopital's rule which, after a simple regrouping of terms ⁸⁾, directly leads to (4), as an asymptotic relation for large N . Provided that N can be taken low enough, this leads to a simple and powerful bootstrap scheme which has already met with several successes ¹⁰⁾.

The main point of (4) is its linear character so that one may separate out the exchange of the quantum numbers of the vacuum, and in particular the problems associated with the complicated structure of the Pomeranchuk singularity, from the contributions of other well established simple Regge trajectories. However, finite energy sum rules may lead to a bootstrap only to the extent that continuation to timelike t values is possible, which we shall do in an approximation with zero width resonances and real Regge trajectories. This means that we will thus have to neglect constraints imposed by unitarity on the ground that crossing symmetry is really important enough to yield a solution. This departs from standard N/D bootstrap, when one tries to keep both unitarity and crossing symmetry, but very poorly indeed however when it comes down to actual calculations.

As written explicitly in (4), the direct channel resonances "build up" the Regge trajectory in the crossed channel.

4. REGGE POLES AND ARGAND LOOPS

Conversely, Regge behaviour implies the presence of direct channel resonances or at least phase variations in partial wave amplitudes which could be attributed to the presence of resonances. Relation (2) imposes a fixed t structure to the differential cross-section up to $\log(s/s_0)$ terms which we will first neglect. In a semi-classical picture this corresponds to a fixed impact parameter and therefore to an increase of "effective" angular momentum with energy given by

$$\langle J \rangle \sim \sqrt{s} \quad (5)$$

The phase of F , as given by (2), is further fixed with t when each partial wave is associated with a particular θ behaviour. Since θ and t are related by :

$$t \sim \frac{s}{2} (1 - \cos \theta) \quad (6)$$

a phase fixed with t for all s implies phase variations in all partial waves. As can be readily checked in (2), each partial wave will increase and then decrease with energy, the corresponding maximum being reached for higher s values, the higher the angular momentum J ¹³⁾. Furthermore, with standard Regge parameters such as those of the ρ trajectory, the real part increases first and is then followed by the imaginary part. An anti-clockwise loop is thus eventually obtained on an Argand diagram, a feature which is generally associated with the presence of a resonance¹⁴⁾.

In order to analyze this in a more quantitative way, we can consider the partial wave projection of the phase factor in (2)¹⁵⁾. It brings the most conspicuous t variation at relatively low energy, when $\log(s/s_0) \ll \pi$. Taking for simplicity a linear trajectory of slope α' , this gives :

$$\int e^{-i\pi\alpha(t)} P_J(\cos \theta) d\cos \theta = e^{-i\pi(\alpha(0) - 2q^2\alpha')} i^J d_J(-2q^2\alpha') \quad (7)$$

where we have introduced the spherical Bessel function :

$$d_J(z) = \int e^{iz \cos \theta} P_J(\cos \theta) d\cos \theta$$

For each J value, the phase of the partial wave amplitude increases with s ($q^2 \sim s/4$) and eventually reaches $\pi/2$ for $s = s_J$. For another partial wave, $J' = J + \Delta J$, the same phase will be reached but for a different value of s , $s_{J'}$, with $-$ as readily seen from (7) :

$$s_{J'} = s_J + \frac{\Delta J}{\alpha'} \quad (8)$$

One thus obtains from the Regge amplitude a phase behaviour for the partial wave amplitudes as if a series of direct channel resonances were present, these resonances being further spaced in energy as if they were on a rising trajectory with the same slope α' as the exchanged Regge trajectory.

This analysis is only very rough. Nevertheless, it carries the basic properties involved and is supported by a more detailed one taking all factors into account, but which has then to be carried out on a computer¹⁵⁾. The Argand loops thus obtained do not yet correspond to direct channel resonances. The amplitude written as in relation (2), with a finite sum or Regge poles, does not have poles in s as normally associated with resonances¹⁶⁾. A good approximation on the real axis (as an asymptotic series), it probably becomes very bad when continued to an unphysical sheet.

We will nevertheless assume that a partial wave phase increasing through $\pi/2$ (or $\pi/2 + n\pi$) obtained from a Regge amplitude indicates the presence of a direct channel resonance. The phase behaviour is taken as a common feature of (2) and (3). Only a more detailed Regge description of the amplitude, in terms of poles and cuts, would yield second sheet poles in s ⁸⁾. The Regge poles alone are however enough to locate these s poles. As a result exchange of linear Regge trajectories implies linear direct channel trajectories and vice versa, all trajectories having the same slope. Moreover, the presence (or absence) of resonances in one channel is to be associated with relations among the exchanged trajectories which summarize the forces involved. This is the full implication of the duality between the Regge pole and direct channel resonance descriptions of collision amplitudes as we will understand it here.

It should be stressed though that, if we assume that the conspicuous overlap between the two descriptions is meaningful, one should take seriously into account only big effects which should remain in a more refined analysis and disregard small ones which are most likely associated with the inaccurate character of either (2) or (3). In particular we will consider a Regge trajectory as definitively built of direct channel resonances only to the extent that different moments in (4) yield compatible Regge parameters. Conversely, we will take into account an Argand loop only if it does not lead to an

excessively small inelasticity parameter. A tiny loop is very unstable against variation of the Regge parameters which we know but imperfectly, whereas a large and neat one is not. The recently much debated question of ancestors, Refs. 13), 17), is considered to offer a typical example of such spurious effects.

We are therefore aiming at a tentative expression for two-body collision amplitudes with direct and crossed channel resonance poles equally spaced in s and t and asymptotic Regge behaviour at high s and high t , the two properties being fully interlocked.

Relations (8) and (5) are not incompatible. Their simultaneous presence tells us that several lower rising trajectories have to be included in such a way that the contributions from these lower trajectories become so important as s increases that the "effective" $\langle J \rangle$ value is definitely smaller than the highest resonance angular momentum.

5. DUALITY AND POMERANCHUK SINGULARITY

The direct channel resonances being closely associated with exchanged Regge trajectories, it is necessary to leave aside the leading vacuum singularity with effective pole intercept 1 at $t=0$. Since the positivity of cross-sections requires that no trajectory with internal quantum numbers other than those of the vacuum can be degenerate with the leading vacuum singularity, direct channel resonances, if associated with Pomeron exchange, would show isospin degeneracy when this is not observed. The Pomeron should then not appreciably contribute to direct channel resonances when all other trajectories may do. This is indeed the case, following the previous analysis of partial wave amplitudes, if it is approximated as an almost flat Regge trajectory with $\alpha_P(0) = 1$; $\alpha'_P \ll 1 \text{ GeV}^{-2}$.

The corresponding contribution to (2) is pure imaginary for all t values and its partial wave components will not have a phase varying with energy as obtained in (7). At least with $\alpha'_P \ll 1 \text{ GeV}^{-2}$, its effect cannot be related to the observed resonance pattern.

Excluding the Pomeron contribution from the bootstrap scope, one is therefore led to a scheme where only the resonances in one channel generate resonances in crossed channel, all resonances lying on approximately linear Regge trajectories. Appreciable forces are therefore to be considered only

when resonances exist. They do not result from unitarity alone, whether resonances are present or not, as they do in ordinary N/D bootstrap. As a result, either direct or exchange forces may be absent from many channels where Pomeron exchange could have been a priori expected to generate these forces, even in the absence of resonances.

This separation of the Pomeron contribution, as possible in the linear bootstrap scheme offered by FESR, has several direct consequences¹⁸⁾. Resonances being associated to the exchange of well-established trajectories such as the ρ or the f^0 (P') trajectories with $\alpha(0) \sim \frac{1}{2}$ and $\alpha' \sim 1 \text{ GeV}^{-2}$, the absence of resonances calls for exchange degeneracy between crossed channel trajectories. Two trajectories with opposite signatures have to coincide with the same α but further with opposite residues such that their sum would be purely real for all t values in the scattering region. Following (2) and (7), no resonances result. Conversely, as a result of the absence of resonances in a particular channel, one type of forces (direct or exchange) is missing and, as stressed already, resonances with both signature have to be present, thus leading to exchange degenerate trajectories.

If the standard Regge amplitude is real, the total cross-section is proportional to the Pomeron contribution alone and should be constant already at relatively low energies instead of showing the standard fractional power Regge decrease¹⁹⁾. Such properties are well met by the facts. As an example the K^+p total cross-section, a reaction with no direct channel resonance observed, is almost constant as compared to the decreasing K^-p total cross-section, a reaction where conspicuous resonances show up. Absence of resonances in K^+p also calls for exchange degeneracy between the ρ and A_2 trajectories when both trajectories are indeed found to coincide (Fig. 2). Broken SU_3 nonet structure for exchanged trajectories requires the standard mixing angle $\tan^2 \theta = \frac{1}{2}$ ²⁰⁾, so that the singled out φ decouples.

In this scheme the Pomeron appears as the over-all effect of all the inelastic channels coupled to the particular elastic channel studied. A bootstrap solution is first obtained from the resonances alone in all channels. The cumulative effects obtained through unitarity from all production reactions are then globally added in each elastic channel as a Pomeron contribution. As a result, the Pomeron should be considered only at relatively high energy when many channels get open. It should not be present below conspicuous inelastic thresholds, below 1 GeV, say, in $\pi\pi$ scattering.

6. EXOTIC STATES AND EXCHANGE DEGENERACY

The bootstrap scheme which is thus being built up is then based upon resonances as generating the only important forces. It lies, of course, on the assumed dominance of relatively nearby singularities which are then associated to rising Regge trajectories. In the framework of such a nearby singularity approach, one may then fully exclude from the scheme all types of possible resonance states which happen not to be found experimentally, even though they might well show up at higher energies. It is well known that all the presently unambiguous resonance states can be classified according to SU_3 octets and singlets (for mesons) and decuplet octets and singlets (for baryons). In effect, all states which can be built out of a quark and an antiquark (for mesons) or three quarks (for baryons)²¹⁾. This excludes in particular resonances with isospin or strangeness 2. Such states which cannot appear in the standard quark model are called "exotic" states and one then takes as a basic property that exotic channels do not generate appreciable forces. As a result, exchange degeneracy is expected in both crossed channels whenever an exotic channel is present.

As a particular example of exchange degeneracy, we will consider here K^+ nucleon scattering, following a recent paper by Schmid²⁾. Both the K^+p and K^+n channels are exotic ones and, as a matter of fact, no unambiguous resonances exist in these channels. The presence of an exotic channel implies then, according to our rules, exchange degeneracy in both crossed channels (Fig. 3) : $K\bar{K} \rightarrow N\bar{N}$ (channel 2) and $\bar{K}N \rightarrow \bar{K}N$ (channel 3). In channel 2 one finds the ρ , ω , A_2 and f trajectories, the φ and f' being only very weakly coupled to the NN system. These four Regge trajectories should show exchange degeneracy and since it must work for both K^+p and K^+n scattering, the isospin 1 and 0 trajectories have to be separately exchange degenerate

$$\begin{aligned} \alpha_\rho &= \alpha_{A_2} & \beta_\rho &= -\beta_{A_2} \\ \alpha_\omega &= \alpha_f & \beta_\omega &= -\beta_f \end{aligned} \quad (9)$$

When going from K^+N scattering to K^-N scattering, the ρ and ω couplings (odd under C) change sign, whereas the f and A_2 couplings (even under C) stay the same. Accordingly the Regge amplitude :

$$F(s,t) \simeq \sum_i \beta_i(t) \frac{\eta_i - e^{-i\pi\alpha_i(t)}}{\sin \pi\alpha_i(t)} \left(\frac{s}{s_0}\right)^{\alpha_i(t)}$$

which excludes the Pomeron, purely real for K^+N scattering becomes complex and Argand loops now occur in each partial wave, thus indicating the actual hyperon resonances. The Regge amplitude for charge exchange :

$$K^0 + p \rightarrow K^+ + n$$

is, however, purely real with no Pomeron present. Figure 2 shows how the exchanged trajectories indeed appear to coincide. The absence of forces in the exotic $\pi^+\pi^+$ channel in $\pi\pi$ scattering further implies exchange degeneracy between the ρ and f trajectories so that the four trajectories coincide.

A similar degeneracy should further hold for channel 3. Indeed, as shown on Fig. 2, two exchange degenerate Y_0^* and Y_1^* trajectories can be drawn through known resonances, and the observed couplings are compatible with the equality of the residue functions ²⁾. It should be stressed, though, that this leaves aside several hyperon resonances which we cannot associate with exchange degenerate trajectories as one a priori should. Such as the $Y_0^*(1405, \frac{1}{2}^-)$; the $Y_0^*(1690, \frac{3}{2}^-)$, the $Y_0^*(1830, \frac{5}{2}^-)$; and the $Y_0^*(1660, \frac{1}{2}^-)$ on the one hand and the $\Sigma(1190, \frac{1}{2}^+)$ and $Y_1^*(1910, \frac{5}{2}^+)$ on the other. This leads us to two important points about FESR bootstrap. We are dealing with one particular set of channels connected through crossing and, if we draw conclusions from the absence of forces in the exotic K^+N channel, they have to affect only those states conspicuously coupled to the $\bar{K}N$ (channel 3) or $N\bar{N}$ (and $K\bar{K}$) systems (channel 2). We should not draw any conclusion for states relatively weakly coupled to those particular channels. This is indeed the case for all the resonances left apart which we already listed ²²⁾ except for the $Y_0^*(1405, \frac{1}{2}^-)$ and $Y_0^*(1660, \frac{1}{2}^-)$ states. As a first rule, we should then use duality arguments only among strongly coupled states and not trust them a priori with channels about which we know almost nothing (say, A_1 exchange in $\pi\pi$ elastic scattering). The two leftover Y_0^* states have a very low spin for their relatively high mass. They therefore contribute but a negligible amount in "building up" the crossed channel Regge trajectories which, as already stressed, get their major contribution from those partial wave amplitudes which are prominent at high unphysical values of $\cos\theta_s$:

$$\cos \theta_s = 1 + \frac{t}{2q^2} \quad (t \sim m_\rho^2)$$

They are the resonating peripheral partial waves. The low partial waves are of almost no effect on the leading Regge trajectories. Exchange degeneracy therefore tells us almost nothing about the low partial waves. Our bootstrap

scheme relates peripheral partial waves to leading trajectories and leaves aside all secondary effects. One cannot use it a priori to make refined prediction such as polarization and we cannot conclude for instance that exchange degeneracy implies the absence of polarization effects in KN charge exchange ²³⁾. At most, can we say that the polarization due to interference between the high (real) and low (complex) partial waves should a priori show little structure.

A similar remark holds for NN charge exchange where the absence of dibaryon resonance also implies exchange degeneracy and therefore real peripheral waves.

In standard N/D bootstrap exchange degeneracy does not occur so generally as a result of the presence of coupled channels where both direct and exchange forces are present. In this simplified approach we obtain but a first guess of the whole amplitude which does not incorporate such effects. This, however, clearly shows some general constraints of the bootstrap solution which are to be at least approximately satisfied by the exact solutions and which can readily be reached.

To sum up, we aim at building a model where narrow resonances would be the only relevant objects, in scattering. This leads to many obvious oversimplifications but allows us to deal with two-body scattering apart. We guess directly at a solution which would satisfy crossing symmetry if not unitarity. We hope that this simple bootstrap approach, even though very approximate, will already teach us many things about resonance location and properties. The equality of slopes of all leading trajectories and present evidence for exchange degeneracy are strong hints that this is a fruitful track.

7. GENERALIZED INTERFERENCE MODELS

The duality model presented here is, of course, not the only way of getting resonances and Regge behaviour in all channels. As shown by Van Hove ²⁵⁾ an infinite sum of resonances can result in an asymptotic Regge behaviour. The scattering amplitude written as

$$F(s, t) = \sum^{\pm} \frac{\ell(J, t)}{M_J^2 - t} P_J(\cos \theta_t)$$

where the sum runs over even (+) or odd (-) non-negative integer values of J , can be rewritten in terms of a Sommerfeld-Watson transformation. Its high s behaviour is then dictated by the leading pole in J (the location of which depends on t) of the continued partial wave amplitude : $b(J,t) \times (M^2(J)-t)^{-1}$.

If the t channel resonances are associated with a rising Regge trajectory, $\alpha(M^2(t)) = J$, $M^2(\alpha(t)) = t$, the partial wave amplitude will have a pole at $J = \alpha(t)$ which will correspond to Regge behaviour $(-s)^{\alpha(t)}$ at high s . The contribution of an infinite set of t channel resonances can then be resummed as a t channel Regge pole.

It is possible to build explicit examples of amplitudes which have poles in t and behave as $(-s)^{\alpha(t)}$ as s goes to infinity. Such functions may be further constructed as to have cuts in s and t as imposed by unitarity in such a way that the poles are on the unphysical sheet. The residues of the pole in t are polynomial in s ²⁶⁾. If it is further imposed that the function goes to zero faster than a power when t goes to infinity, as shown in Ref. ²⁶⁾, a sum of two such functions, where s and t are interchanged, will have resonance poles and Regge behaviour in both channels, even though no relation will exist between the resonances in one channel and the Regge poles in the crossed channel. Such an amplitude does not show duality and allows full dissociation between prominent resonances, such as the Δ , and asymptotic Regge behaviour ²⁷⁾. This provides a generalized interference model. It is, however, much different from standard interference model adding somewhat casually resonance contributions to Regge pole contributions since the constraints imposed on either pieces of the amplitude lead in practice to the presence of a strong Regge background interfering with the resonance pole contributions. The residue function would therefore be in general very different from what one would assume considering standard Breit-Wigner formulae to describe the prominent resonances ²⁷⁾.

In these lectures we will consider that resonances and Regge behaviour are fully interlocked. As already stressed, the subject should however be considered as still controversial, both the dualist and the generalized interference approaches corresponding probably to too radical choices.

8. DUALITY DIAGRAMS

Isolating a particular set of two-body channels related through crossing, duality will not always act in such a way that the observed resonances in one channel can be considered as generated from the observed resonances in both crossed channels. Cancellations may occur in such a way that only one channel is relevant even if both do not carry exotic resonances. Consider as an example $K \pi$ scattering (Fig. 4a). Only isospin $\frac{1}{2}$ K^* resonances are to be considered in the s and u channels. They cannot mutually sustain themselves since a force with isospin $\frac{1}{2}$ in one channel will obviously generate at the same time effects in the isospin $\frac{1}{2}$ and $\frac{3}{2}$ states in the crossed channel. Selection of the isospin $\frac{1}{2}$ state can only result, in a bootstrap scheme based on nearby, and therefore non-exotic, singularities, from a particular relation between the $I=0$ and $I=1$ t channel forces obtained through ρ f exchange degeneracy. Conversely $I=\frac{1}{2}$ forces in the s and u channels will create both $I=0$ and 1 effects in the t channel.

Duality will therefore manifest itself between the s and t , u and t but not s and u channels. Such a property can be simply exhibited on duality diagrams where the SU_3 properties or quark content of each particles are clearly written out. Contraction between the SU_3 indices of the external particles, as obtained in terms of exchanged vector or tensor mesons in this case (or their corresponding Regge trajectories), is graphically represented as quark exchange (Figs. 4b and 4c, obtained from each other using $s \leftrightarrow u$ exchange). The coupling of a Regge trajectory to the external meson corresponds to

$$\begin{aligned} \text{Tr} (M [V, M]) & \quad \text{F coupling, vector meson} \\ \text{Tr} (M \{T, M\}) & \quad \text{D coupling, tensor meson} \end{aligned} \tag{10}$$

The pseudoscalar mesons here considered are represented by 3×3 matrices and the curly bracket stands for the anticommutator. We exclude a coupling of the type :

$$\text{Tr}(T) \text{Tr}(MM) \tag{11}$$

the absence of which can be seen to correspond to the vanishing of $\pi \pi$ coupling once the f' and f are defined using the standard 35° nonet mixing angle; it is already forbidden by C invariance for vector mesons ²⁸⁾.

With (10), and not (11), as possible couplings, the SU_3 indices of the two incoming particles are always connected through the exchanged meson indices. As a result, all graphs are connected as in Figs. 4b and 4c. Graphs like 4d, which would be associated with (11), are excluded. All possible diagrams being connected graphs, they can be "planar", as in 4b, or "not planar" as in 4c. The rule is then very simple^{29),30)}. Duality is effective among channels associated with a planar graph. There is s, t duality (4b). The s channel K^* resonances in the s channel "build up" an imaginary part for high s which is to be associated with non exchange degenerate Regge exchange in the t channel. On the other hand, there is no $s-u$ duality (4c). The $K^+ \pi^+$ amplitude should be purely real at high s but for the Pomeron contribution. In other words, two channels connected by duality should have non-exotic quantum numbers and this is a fortiori the case when visualized in terms of quark-antiquark pairs exchanged in both channels, as in 4b, since a coupling like (11) is excluded. The rule is readily extended to meson-baryon scattering, each external baryon being represented in terms of three quarks. All graphs are again connected provided that the ψ and f' are not coupled to the $N\bar{N}$ system, these two states being again defined together with the f and f' through standard nonet mixing : $\tan^2 \theta = \frac{1}{2}$ ³¹⁾.

We have here merely stated the rule for the presence or absence of duality relation between two channels. Its derivation based on standard SU_3 and Regge pole analysis is given in Ref. 30). Applications to several interesting cases and extension to many particle production are described in Ref. 29).

Coming back to $\bar{K}$ nucleon scattering as a further example we have s, t duality (Fig. 5a) and we would have u, t duality with K instead of $\bar{K}$. We do not have s, t duality for the reaction $K^- p \rightarrow \Sigma^+ \pi^-$ (Fig. 5b), but u, t and s, u duality (Fig. 5c). This is particularly interesting. The resonances in the $K^- p$ channel should not build up the exchanged K^* trajectory. The t channel resonance having isospin $\frac{1}{2}$, this would have implied degeneracy between the $I=0$ and $I=1$, Y^* trajectories in order to exclude isospin $\frac{3}{2}$ resonances. This would go against experimental information (Fig. 1). On the other hand, the K^* trajectory is built from the N^* resonances in the u channel ($u-t$ duality). This implies exchange degeneracy between decuplet and octet states and is indeed compatible with present information (Fig. 1). The $K^- p \rightarrow \Sigma^+ \pi^-$ amplitude should be purely real at high s ^{30),31)} and small t values (no $s-t$ duality). As already stressed, the rules obtained from duality diagrams imply more than SU_3 symmetry and the absence of exotic states. The decoupling of one of the nonet members from particles without strange quark here assumed will be necessary to incorporate standard broken SU_3 [SU_3 couplings but nonet (or octet) mass splitting] in the FESR bootstrap scheme. We will therefore use them to limit the number of possible terms in the amplitude.

9. VENEZIANO AMPLITUDE

After discussing the general ideas underlying a tentative linear bootstrap solution, we now apply them to the very simple and most interesting model proposed by Veneziano ³²⁾, which offers solutions exhibiting duality properties. We first consider for simplicity an amplitude with only s and t channel resonances ($\pi^+\pi^- \rightarrow \pi^+\pi^-$, say). The amplitude should show explicitly a string of resonances (poles) in both the s and t channels located on the linearly rising ρ and f Regge trajectories. The absence of force (resonance) in the $\pi^+\pi^+$ u channel implies exchange degeneracy of these two trajectories and resonances in both the s and t channels should then be spaced by one unit of spin, instead of two. The amplitude should also have s - t crossing symmetry, and asymptotic Regge behaviour in both channels. This is indeed the case for

$$V_{xy}(s,t) = -\lambda \frac{\Gamma(1-\alpha_x(s))\Gamma(1-\alpha_y(t))}{\Gamma(1-\alpha_x(s)-\alpha_y(t))} \quad (12)$$

where x and y label the trajectories exchanged in the s and t channel. Here both correspond to the ρ - f degenerate trajectories. We are working in a zero width approximation and the Regge trajectories are real and linear

$$\alpha(s) = \alpha(0) + \alpha' s \quad \alpha' > 0 \quad (13)$$

Relation (12) shows poles in s and t associated with a zero or negative integer value of the gamma function arguments, that is, with $\alpha(0) \sim \frac{1}{2}$, for timelike s and t values. It has further no double pole. This is important since one should be able to associate each pole term in (12) to particle exchange. It means that the residue of each pole should be a polynomial in the crossed variable. When rewritten in terms of Legendre polynomials, it will correspond to a sum of parent and daughter contributions. A double pole would correspond to an infinite series expansion in the crossed variable leading to arbitrary high spin values (ancestors) which cannot be tolerated. The denominator in (12) indeed cancels all double poles and the residue of each pole is the ratio of two gamma functions which is readily rewritten as a polynomial in $\alpha(t)$ (for a pole in s), which, together with (13), gives a polynomial in t . The pole structure is illustrated in Fig. 6. For $\alpha(s) = J \geq 1$, we have together with the leading particle exchanged with spin J a series of daughters with spins $J-1, J-2, \dots, 0$. Relation (12) implies therefore an infinite series of equally spaced daughter trajectories (Fig. 7).

For large values of the argument the ratio of two gamma functions is given by :

$$\frac{\Gamma(z+a)}{\Gamma(z+b)} = z^{a-b} \left(1 + \frac{2}{2z} (a-b)(a+b-1) + O(z^{-2}) \right) \quad (14)$$

for $-\pi < \arg z < \pi$. Used in (12), with the appropriate limit procedure, this leads to asymptotic Regge behaviour :

$$\begin{aligned} V_{xy}(s,t) &\sim \Gamma(1-\alpha_y(t)) (-\alpha'_s)^{\alpha_y(t)} & s \rightarrow \infty \\ V_{xy}(s,t) &\sim \Gamma(1-\alpha_x(s)) (-\alpha'_t)^{\alpha_x(s)} & t \rightarrow \infty \end{aligned} \quad (15)$$

V , as defined in (12), can be rewritten in terms of the Euler beta function

$$\begin{aligned} B(x,y) &= \int_0^1 u^{x-1} (1-u)^{y-1} du = \frac{\Gamma(x) \Gamma(y)}{\Gamma(x+y)} \\ V_{xy}(s,t) &= (1-\alpha_x(s)-\alpha_y(t)) B(1-\alpha_x(s), 1-\alpha_y(t)) \end{aligned} \quad (16)$$

Relation (12), although extremely simple, cumulates many required properties for the solution we are guessing at. It has resonances and Regge behaviour and they are deeply interlocked as imposed by duality. Its basic drawback is of course its inconsistency with unitarity. In order to impose an analytic structure at least compatible with unitarity, we could depart from linear trajectories and introduce a branch point at the lowest threshold :

$$\begin{aligned} \alpha(t) &= \alpha(0) + \alpha'_0 t + \frac{t^2}{\pi} \int_{4m^2}^{\infty} \frac{\text{Im } \alpha'(t')}{t'^2 (t'-t-i\varepsilon)} dt' \\ \text{Im } \alpha(t) &\neq 0 \quad \text{for } t > 4m^2 \end{aligned} \quad (17)$$

This preserves crossing symmetry but introduces ancestors since the residue at one pole will no longer be a polynomial in the crossed variable. We could also impose unitarity in one channel, using the Baker-Blankenbecler method ²⁴⁾ or Padé approximants. This would, however, spoil crossing symmetry. Trying

to guess a tentative solution for two-body scattering without using any definite information from many particle production amplitudes, we should not worry too much at not reaching an a priori perfect answer. We will use (12) on the ground that crossing symmetry is the main property and that it should be kept exactly, hoping that a bootstrap solution, built with zero width resonances, is not too far apart, as far as the general resonance pattern is concerned, from a solution which would take into account unitarity constraints which we cannot yet all incorporate. Relation (12) is built on ρ (f) exchange and neglects the Pomeron contribution. As already mentioned, Pomeron exchange should be considered only at high energies where inelasticity becomes important. Relation (12) is then suitable for a "low" energy approximation to the $\pi\pi$ scattering amplitude. Nearby singularities are included as linear trajectories. The effect of distant singularities is lumped in the imposed Regge behaviour.

A term like :

$$V'_{xy}(s,t) = -\lambda' \frac{\Gamma(m-\alpha_x(s))\Gamma(n-\alpha_y(t))}{\Gamma(m+n+p-\alpha_x(s)-\alpha_y(t))} \quad (18)$$

has the same basic properties as (12). The first poles appear at $\alpha_x(s) = m$ and $\alpha_y(t) = n$ instead of 1 (we will take $m, n \geq 1$) and the asymptotic behaviours correspond, respectively to

$$s^{\alpha_y(t)-n-p} \quad \text{and} \quad t^{\alpha_x(s)-m-p}$$

with $n+p (m+p) > 0$, this corresponds to daughter behaviour. Poles in s (t) will have polynomial residue in t (s) provided that $p \leq 0$. We are therefore at liberty to add any such terms, called "satellite", to the initially considered solution (12), without modifying any of the desired properties we require. This is an important ambiguity in the bootstrap solution and such terms can be used to modify it at will at the daughter level without changing the leading trajectory contributions. Satellites can be added to eliminate unwanted daughter contributions and in particular odd daughters³³⁾, if we want the solution to show trajectories spaced by two units of spin as corresponding to the minimum number of trajectories. Satellites will not affect the leading high energy behaviour directly obtained from (12).

In order to determine a signature factor, we consider an amplitude with specific s-u symmetry [t channel, $I=0(1)$, $K\pi$ scattering, say, as discussed in detail later]

$$\begin{aligned} \bar{F}^I &= -\lambda^I (V_{xy}(t,s) + (-1)^I V_{xy}(t,u)) \\ &= -\lambda^I \left\{ \frac{\Gamma(1-\alpha_x(t))\Gamma(1-\alpha_y(s))}{\Gamma(1-\alpha_x(t)-\alpha_y(s))} + (-1)^I \frac{\Gamma(1-\alpha_x(t))\Gamma(1-\alpha_y(u))}{\Gamma(1-\alpha_x(t)-\alpha_y(u))} \right\} \end{aligned} \quad (19)$$

We use the relation :

$$\Gamma(z)\Gamma(1-z) = \pi (\sin \pi z)^{-1} \quad (20)$$

to rewrite (19) as

$$\begin{aligned} F^I &= -\frac{\pi \lambda^I}{\Gamma(\alpha_x(t)) \sin \pi \alpha_x(t)} \left\{ (-1)^I \frac{\Gamma(1-\alpha_y(u))}{\Gamma(1-\alpha_x(t)-\alpha_y(u))} + \right. \\ &\quad \left. + \frac{\sin \pi (\alpha_x(t)+\alpha_y(s))}{\sin \pi \alpha_y(s)} \frac{\Gamma(\alpha_x(t)+\alpha_y(s))}{\Gamma(\alpha_y(s))} \right\} \\ &\sim -\frac{\pi \lambda^I}{\Gamma(\alpha_x(t))} \left\{ \frac{\cos \pi \alpha_x(t) + (-1)^I}{\sin \pi \alpha_x(t)} \left(\frac{s}{s_0}\right)^{\alpha_x(t)} + \cot \pi \alpha_y(s) \left(\frac{s}{s_0}\right)^{\alpha_x(t)} \right\} \end{aligned} \quad (21)$$

when $|s| \rightarrow \infty$, $\arg s \rightarrow 0^+$.

The first term corresponds to the properly behaved real part with a signature factor associated with s-u symmetry. The second term corresponds to the absorptive part. It has a succession of poles on the real axis. At this state one may smooth them out, letting the imaginary part of α_y increase with s as it normally should. One then has $\cot \pi \alpha_y(s) \rightarrow -i$ as $s \rightarrow \infty$ and obtains from (21) the standard Regge behaviour :

$$F^I \sim \beta^I(t) \frac{(-1)^I + e^{-i\pi \alpha_x(t)}}{\sin \pi \alpha_x(t)} \left(\frac{s}{s_0}\right)^{\alpha_x(t)} \quad (22)$$

Two important properties appear in this analysis. First s_0 , as it is introduced in (21), is indeed α^{-1} , the energy scale is therefore 1 GeV, as indicated through standard Regge pole analysis. Second, the residue function

includes a factor $1/\Gamma(\alpha_x(t))$, which vanishes for zero or negative integer values of $\alpha_x(t)$. This corresponds to nonsense choosing with Gell-Mann mechanism ³⁴⁾, when writing a Veneziano term for each invariant amplitude in order to extend (22) to reactions with particles with spin (π nucleon scattering ³⁵⁾, for instance). Both the helicity flip and non-flip amplitude vanish when $\alpha(t) = 0$.

10. RESONANCE WIDTH IN THE VENEZIANO MODEL

In the neighbourhood of an s channel pole $\alpha(s) \sim J$ one may write $\Gamma(1-\alpha(s))$ as :

$$\Gamma(1-\alpha(s)) \sim \frac{(-1)^J}{(J-1)! \alpha'(s-M_J^2)}$$

with $\alpha(M_J^2) = J$. As a result, the pole residue in (12) is given by :

$$\lambda \frac{(2qq')^J}{(J-1)!} \alpha'^{J-1} \cos^J \theta + O(\cos^{J-1} \theta) \quad (23)$$

with $t = -2qq'(1-\cos\theta)$, q and q' standing for the initial and final centre-of-mass momenta.

The constant λ thus determines all pole residues on the rising trajectory and the coefficient of the leading power in $\cos\theta$ is (in the spinless case) enough to determine the coupling constant and then the width of the leading trajectory resonances. This result is not affected by satellite terms.

Writing the $\rho\pi\pi$ coupling as : $G_\rho \varepsilon \cdot (q_1 + q_2)$, and therefore the ρ pole contribution in $\pi\pi$ scattering as : $((4qq')/(M^2-s))G^2$ we get from (12) and (23) :

$$G_\rho = \frac{\lambda}{2}$$

and

$$\Gamma_\rho = \frac{1}{12 m_\rho^2} \frac{\lambda}{8\pi} (m_\rho^2 - 4m_\pi^2)^{3/2} \quad (24)$$

a width of 100 MeV corresponds to $\lambda = 50$. The ratio of the ρ and f elastic widths is readily obtained from (23). One finds :

$$\Gamma_\rho = 1.2 \Gamma_f$$

The main point is that the range parameter which allows us to connect decay widths of resonances of different spins and therefore with decay coupling of different dimensions is the square root of the universal slope $\alpha' = 0.9 \text{ GeV}^{-2}$. Such a relation among the leading trajectory widths could be, for instance, checked through a detailed study of the $p\bar{p} \rightarrow \pi\pi$ reaction at low and intermediate energies.

The daughter width associated with a single term can be obtained from the expansion

$$B(x, y) = \sum_{j=0}^{\infty} C_j \binom{y-1}{j} (x+j)^{-1}$$

or

$$V_{xy}(s, t) = \sum_{j=0}^{\infty} \binom{\alpha_y(t) + j + 1}{j} \frac{1 - \alpha_x(s) - \alpha_y(t)}{j + 1 - \alpha_x(s)} \quad (25)$$

The ratio of the ρ and Σ^0 width is then found to be 2/9, the Σ^0 being the first ρ daughter. This implies the existence of a wide (450 MeV) s wave resonance at the mass of the ρ ³⁶). At the same time, a P wave daughter is expected at the mass of the f, with a similar elastic width (9/10 ratio). This conclusion now relies, however, upon the absence of satellite terms. This ρ' resonance could, however, be already highly inelastic and therefore not spoil the isospin determination of the f which is known to be peripherally produced as the ρ' would also be.

As seen from (25), the daughter couplings will in general increase with the daughter order and, as the energy increases, daughters should then take a more and more important role in elastic scattering. The "effective" angular momentum will therefore increase much less than linearly with s (as $\sqrt{s}$ as a first approximation) to give a fixed forward peak up to logarithmic terms. The parent still plays a prominent part, however, when going to high unphysical t value and in particular when reaching the crossed channel trajectories.

11. PION-PION SCATTERING

We already wrote an amplitude for $\pi^+\pi^-$ scattering (12), taking into account the absence of force in the u channel. There is only one

trajectory involved : the exchange degenerate ρ f trajectory. We exclude the f' , assumed to have no coupling to $\pi\pi$ as a result of standard nonet mixing and a particular ratio of octet and singlet couplings. Had we kept the f' and considered it as separated from the f , we would have needed a new ρ' trajectory exchange degenerate with the f' . A broken SU_3 nonet structure therefore imposes the decoupling of the f' ²⁰⁾. Considering

$$V_{\rho\rho}(s,t) = -\lambda \frac{\Gamma(1-\alpha_\rho(s))\Gamma(1-\alpha_\rho(t))}{\Gamma(1-\alpha_\rho(s)-\alpha_\rho(t))}$$

as the simplest tentative amplitude for $\pi^+\pi^-$ scattering, as opposed to more complicated terms involving satellites, the three s channel amplitudes can be written, imposing obvious symmetry relations, as :

$$\begin{aligned} S_0 &= \beta (V_{\rho\rho}(s,t) + V_{\rho\rho}(s,u)) + \gamma V_{\rho\rho}(t,u) \\ S_1 &= \alpha (V_{\rho\rho}(s,t) - V_{\rho\rho}(s,u)) \\ S_2 &= V_{\rho\rho}(t,u) \end{aligned} \quad (26)$$

The isospin crossing matrix between the s and t channel being

$$M_{ts} = \frac{1}{6} \begin{pmatrix} 2 & 6 & 10 \\ 2 & 3 & -5 \\ 2 & -3 & 1 \end{pmatrix}$$

with $T_i = (M_{ts})_{ij} S_j$.

One may exclude t poles in T_2 , forbidden as exotic resonances, provided that

$$\gamma = -\frac{1}{2} \quad \beta = \frac{3}{2}\alpha$$

We should further impose that the pole at $\alpha(t)=1$ in the T_0 amplitude vanishes. This gives $\alpha = 1$. All $\pi\pi$ amplitudes can then be written in terms of one single constant λ ³⁷⁾. We choose to write :

$$\begin{aligned} A(\pi^+\pi^-\rightarrow\pi^+\pi^-) &= -\lambda V_{\rho\rho}(s,t) \\ A(\pi^+\pi^0\rightarrow\pi^+\pi^0) &= -\frac{\lambda}{2} (V_{\rho\rho}(u,t) + V_{\rho\rho}(s,t) - V_{\rho\rho}(s,u)) \\ A(\pi^0\pi^0\rightarrow\pi^0\pi^0) &= -\frac{\lambda}{2} (V_{\rho\rho}(s,t) + V_{\rho\rho}(t,u) + V_{\rho\rho}(u,s)) \end{aligned} \quad (27)$$

An extremely important point about (12) is that the amplitude vanishes when $\alpha_\rho(s) + \alpha_\rho(t) = 1$, or with linear trajectories when $s+t = 2\mu^2$ with:

$$\alpha_\rho(\mu^2) = \frac{1}{2}$$

This coincides with the Adler condition ³⁸⁾ which requires the $\pi\pi$ amplitude to vanish when $s=t=u=m_\pi^2$, provided that $\mu^2 = m_\pi^2$. Such a condition is compatible with what is known about the ρ trajectory ³⁷⁾ and it is therefore tempting to write :

$$\begin{aligned} \alpha_\rho(t) &= \frac{1}{2} + \alpha'(t - m_\pi^2) \\ \alpha' &= \frac{1}{2(m_\rho^2 - m_\pi^2)} = 0.88 \text{ GeV}^{-2} \end{aligned} \quad (28)$$

thus having the Adler relation satisfied by (26) without having to call for satellite terms or a vanishing of λ with one of the external masses. Relation (26) can then be used to continue the $\pi\pi$ amplitude off the mass shell, considering s, t and u as independent variables. Its linear terms can further be identified with the Weinberg amplitude ³⁹⁾, since both amplitudes vanish at $s=t=u=m_\pi^2$ ^{40), 41)}. The Weinberg amplitude reads :

$$F_{\beta\delta, \alpha\gamma} = -\frac{1}{f_\pi^2} \left\{ \delta_{\alpha\gamma} \delta_{\beta\delta} (s - m_\pi^2) + \delta_{\alpha\beta} \delta_{\gamma\delta} (t - m_\pi^2) + \delta_{\alpha\delta} \delta_{\beta\gamma} (u - m_\pi^2) \right\}$$

with

$$\begin{aligned} S_0 &= -\frac{1}{f_\pi^2} (3s + t + u - 5m_\pi^2) \\ S_1 &= -\frac{1}{f_\pi^2} (t - u) \\ S_2 &= -\frac{1}{f_\pi^2} (t + u - 2m_\pi^2) \end{aligned} \quad (29)$$

Writing a linear approximation to (12) around $s=t=u=m_\pi^2$ one gets $-\lambda \pi \alpha'(s+t-2m_\pi^2)$ and identifying (26) and (29) this gives :

$$\lambda = (\pi f_\pi^2 \alpha')^{-1} \simeq 50 \quad (30)$$

thus yielding a ρ width of the order of 100 MeV, together with the Weinberg scattering lengths ³⁹⁾.

The structure of (29) follows from the Adler relation, current algebra and the absence of $I=2$ ∇ term. This last assumption is translated in our approach by the absence of force in the $I=2$ state which is due to the

absence of resonance in this channel. We can therefore use current algebra through the low energy theorem; it provides to fix the value of λ . We already found :

$$G_e^2 = \frac{\lambda}{2} = \frac{m_e^2 - m_\pi^2}{\pi f_\pi^2} \quad (31)$$

Following Sakurai, one may also assume ρ dominance and write the T_1 amplitude at $t=0$ obtained from relations (29) as if it were dominated by ρ exchange. One obtains the KSFR ^{*)} relation :

$$G_e^2 = \frac{m_e^2}{2 f_\pi^2} \quad (32)$$

Relations (31) and (32) differ by a factor $2/\pi$, the ρ width obtained from the Veneziano formula being smaller than the one obtained from the KSFR relation. This reduction is needed in order to meet experiment.

The Veneziano amplitude is then very well suited to describe $\pi\pi$ scattering from low energy up to the first resonances. It can be checked that, as written, it has no ghost before deep daughter levels ⁴²⁾ and, as a result, barely meets Martin's bound based on positivity ⁴³⁾. The first fixed t derivative with respect to s at $s=m^2$ is matched with the current algebra value. The second derivative of the amplitude, which can be calculated in terms of an unsubtracted dispersion relation involving the Pomeron contribution cannot, however, be identified with the value obtained from the Veneziano term. Nevertheless, the error made appears to be small.

The consistent picture for $\pi\pi$ s wave scattering thus obtained is based on small scattering length and large effective range (as imposed by the unphysical zero near threshold) as opposed to the large scattering length small effective range approximation initially tried.

As emphasized by Lovelace ³⁷⁾, the Veneziano amplitude can be used to describe three pion final state interaction and relation (27) is still valid (with different λ) when any of the pions is replaced by an isospin 1 0^- state. When comparing to experiment, it is however now necessary to include an imaginary part in α to replace physical region poles by resonances. One may, in particular, describe K and η decay assuming that graphs, as drawn on Fig. 8, are dominant. To the extent that one limits oneself to linear terms, the results are, however, identical to those which would result from the use of the Weinberg amplitude with the same dynamical assumptions. The

^{*)} Kawarabayashi, Suzuki, Fayyazudin and Riazuddin.

Veneziano amplitude has more to say when phase space is big enough so that at least two resonances on the same trajectory can have important final state effects. This is the case of $\bar{p}n$ annihilation at rest ³⁷⁾ where the Σ^0 and ρ' effects are correlated.

12. NONET STRUCTURE IN THE VENEZIANO AMPLITUDE

The $K\pi$ scattering amplitudes can be considered along the same lines ⁴³⁾. From obvious symmetry reasons, one writes the $\pi\pi \rightarrow K\bar{K}$, t channel, amplitudes as :

$$\begin{aligned} T_0 &= \alpha' (V_{\rho K^*}(t,s) + V_{\rho K^*}(t,u)) \\ T_1 &= \beta' (V_{\rho K^*}(t,s) - V_{\rho K^*}(t,u)) \end{aligned} \quad (33)$$

The (ρf) trajectory is exchanged in the t channel and the $(K^*(890), K^{**}(1420))$ exchange degenerate trajectory is exchanged in both the s and u channels. Exchange degeneracy is imposed by the absence of resonance in the exotic $I=\frac{3}{2}$ channel. The s channel amplitudes are readily obtained in terms of the crossing matrix

$$M_{st} = \frac{1}{2} \begin{pmatrix} \sqrt{\frac{2}{3}} & 2 \\ \sqrt{\frac{2}{3}} & -1 \end{pmatrix}$$

and the absence of $I=\frac{3}{2}$ s pole imposes $\alpha' = \sqrt{\frac{3}{2}} \beta'$. It also forbids the presence of a $V_{K^*K^*}(s,u)$ term in T_0 which we already argued away. We get

$$S_{\frac{1}{2}} = \frac{\beta'}{2} (3 V_{\rho K^*}(t,s) - V_{\rho K^*}(t,u)) \quad (34)$$

The K^* pole residue in $s_{\frac{1}{2}}$ is proportional to $(3\beta')/2$, while the pole residue in T_1 is proportional to $2\beta'$. It then follows that

$$G_{K^*K\pi}^2 = \frac{3}{4} G_{\rho\pi\pi} G_{\rho K\bar{K}} \quad (35)$$

We also relate the K^{**} pole residue in $s_{\frac{1}{2}}$ to the f residue in T_0 .

$$G_{K^*K\pi} = \frac{\sqrt{3}}{2\sqrt{2}} G_{f\pi\pi} G_{fK\bar{K}} \quad (36)$$

If we further relate the $\rho\pi\pi$ and $\rho K\bar{K}$ coupling through SU_3 : $G_{\rho\pi\pi} = 2G_{\rho K\bar{K}}$, (35) corresponds to the relation we could have directly written using SU_3 :

$$G_{K^*K\pi}^2 = \frac{3}{8} G_{\rho\pi\pi}^2 \quad (37)$$

while (36) corresponds to SU_3 coupling with standard nonet mixing. This follows from the fact that we have assumed the f' to decouple from $\pi\pi$ in order to be able to maintain the ff' mass difference without introducing a ρ' . We imposed symmetry between an $s=0$, $I=1$ and an $s=\frac{1}{2}$, $I=\frac{1}{2}$ object and it is therefore not too surprising that we obtain group structure constants of the simplest unitary group where these objects may be members of the same irreducible representation. It should be remarked that SU_3 breaking appears even at the first daughter level since only the parent trajectory couplings does not involve the external masses.

Normalizing (33) together with (26), with SU_3 symmetry, we get $\beta' = \frac{1}{2}$. The denominators in (33) will become infinite when the pion four momentum vanishes (Adler condition) if :

$$\alpha_{\rho}(m_{\pi}^2) + \alpha_{K^*}(m_K^2) = 1 \quad (38)$$

We will then impose $\alpha_{K^*}(m_K^2) = \frac{1}{2}$ in agreement with experiment (Fig. 2), so that (33) automatically satisfies the Adler condition. With common slope for the ρ and K^* trajectories, this implies the well satisfied relation

$$m_{K^*}^2 - m_{\rho}^2 = m_{K^*}^2 = m_{\rho}^2 = m_K^2 - m_{\pi}^2 \quad (39)$$

which is readily understood in terms of quark content [broken $SU_6 \times O_3$ symmetry ²¹⁾].

The soft kaon limit corresponds to $s = m_{\pi}^2$, $t = m_K^2$ and, with linear and parallel trajectories, it also corresponds to a vanishing amplitude.

The K^* and the ρ trajectories with both isospins can therefore sustain themselves through duality. The K^* and the f' could not and the f' must therefore decouple from the $\pi\pi$ channel if we require a difference in mass between f and f' and as a result different trajectories.

$\pi\eta$ scattering immediately raises a problem since the A_2 and f trajectories which can be, respectively, exchanged in the $\pi\eta$ and $\pi\pi \rightarrow \eta\eta$ channels have already been found to be degenerate. The Adler condition cannot be satisfied unless a satellite term is added since $\alpha_e(m_\eta^2) \neq \frac{1}{2}$ ⁴¹⁾. It would, however, readily hold with "ideal" nonet mixing for the pseudoscalar mesons, the π and the η having the same mass (no strange quark), thus departing from an $SU_6 \times O_3$ classification. The X would then decouple from the A_2 and f trajectories since it would collect all the strange quarks. There seems to be, however, a very big difference in mixing schemes between the vector (and tensor) mesons and the pseudoscalar ones, which has to be considered as an important fact, the pseudoscalar mesons having an almost perfect octet structure. As a result, a solution tailored according to the mass pattern of the vector mesons cannot readily match all pseudoscalar meson channels and at least a satellite term has to be introduced, the coupling of which translates the difference between the vector and pseudoscalar meson mixing schemes. It should further be remarked that with the splitting of the A_2 also found in the $K\bar{K}$ channel ⁴⁴⁾ one cannot yet make any definite statement about exchange degeneracy involving the f and the A_2 trajectories when used in such reactions.

We finally consider KK or $K\bar{K}$ scattering ⁴³⁾. The degenerate ρ, f, ω and A_2 trajectories can be exchanged in the s and u channels ($K\bar{K}$). These trajectories will be globally labelled according to the ρ . The $\varphi(f')$ exchange degenerate trajectory can also be exchanged and is assumed to differ from the ρ (Fig. 2). No force (or resonance) is present in the t channel and this imposes exchange degeneracy for the crossed channel trajectories. For obvious symmetry reasons, one writes :

$$\begin{aligned} T_0 &= \alpha'' (V_{\rho\varphi}(u,s) - V_{\rho\varphi}(s,u)) \\ T_1 &= \beta'' (V_{\rho\varphi}(u,s) + V_{\rho\varphi}(s,u)) \end{aligned} \quad (40)$$

A non-vanishing T_0 imposes φ coupling. We have already excluded an a priori possible $V_{\varphi\varphi}(s,u)$ term in T_1 , since the isolated $I=0$ φ trajectory cannot sustain itself. We also exclude a possible $V_{\rho\rho}(s,u)$ term since we take $\rho\omega fA_2$ mass degeneracy and forbid $f'^2\pi$ coupling. Such assumptions warrant the use of duality diagrams and a $V_{\rho\rho}(s,u)$ term cannot be obtained from a planar graph. [The $\lambda\bar{\lambda}$ quark pair has to go one way.]

The Adler condition will be readily satisfied for zero momentum kaon ($s=t=u=m_K^2$) if :

$$\alpha_p(m_K^2) + \alpha_\varphi(m_K^2) = 1 \quad (41)$$

This implies :

$$\alpha'(m_\varphi^2 + m_\pi^2 - 2m_K^2) = \frac{1}{2} \quad (42)$$

with

$$\alpha' = (2(m_p^2 - m_\pi^2))^{-1}$$

which is well satisfied. The straight line φ trajectory is therefore written as :

$$\alpha_\varphi(s) = \frac{1}{2} + \alpha'(s - 2m_K^2 + m_\pi^2) \quad (43)$$

with

$$\alpha_{K^*}(s) = \frac{1}{2} + \alpha'(s - m_K^2)$$

$$\alpha_p(s) = \frac{1}{2} + \alpha'(s - m_\pi^2)$$

Neglecting the pion mass as is legitimate here since $m_\omega^2 - m_p^2 > m_\pi^2$, we obtain three equally spaced trajectories, as readily expected in our "ideal" nonet scheme (Fig. 2).

The s channel amplitudes are obtained from the crossing matrix :

$$M_{st} = \frac{1}{2} \begin{pmatrix} -1 & 3 \\ 1 & 1 \end{pmatrix}$$

One imposes that there is no φ pole in the s_1 amplitude and get as a result $\beta'' = -\alpha''$. It follows that

$$\begin{aligned} S_0 &= \beta'' (2 V_{p\varphi}(u, s) + V_{p\varphi}(s, u)) \\ S_1 &= \beta'' V_{p\varphi}(s, u) \end{aligned} \quad (44)$$

One can then again relate the leading trajectory pole residues. The p and ω residues (s channel poles) are obviously equal, whereas the φ residue is equal to twice the ω residue (s channel poles in s_0). Similar relations hold for the tensor mesons. This reads :

$$\begin{aligned} g_{\rho K\bar{K}}^2 &= g_{\omega K\bar{K}}^2 = \frac{1}{2} g_{\phi K\bar{K}}^2 \\ g_{A_2 K\bar{K}}^2 &= g_{f K\bar{K}}^2 = \frac{1}{2} g_{f' K\bar{K}}^2 \end{aligned} \quad (45)$$

These relations of course result from our excluding $V_{\rho\rho}$ and $V_{\phi\phi}$ terms in (40) as imposed by the nonet scheme. Relations (45) are SU_3 results with standard mixing angle $\tan^2\theta = \frac{1}{2}$ and with no $f'2\pi$ coupling.

Using SU_3 we can again normalize the $K\bar{K}$ amplitudes to (26). We get $\beta'' = -\frac{1}{2}$.

For $\pi\pi$, πK and KK scattering, where octet or nonet structure is irrelevant, one can then easily incorporate the broken SU_3 nonet scheme for the vector and tensor mesons generating the forces and resulting as resonances. In all three cases the simplest terms readily satisfy the constraints of PCAC and current algebra as the result of relations between the vector and pseudo-scalar masses :

$$2(m_\rho^2 - m_\pi^2) = 2(m_{K^*}^2 - m_K^2) = \kappa'^{-1}$$

together with the relation :

$$m_\phi^2 - m_{K^*}^2 = m_{K^*}^2 - m_\rho^2$$

13. GENERALIZATION OF THE VENEZIANO MODEL

These relations among masses may not be considered as a coincidence. One may attribute a dynamical "raison d'être" to the simplest Veneziano amplitude, thus written without satellite, and therefore gets as a result of the Adler condition, relations among trajectory intercepts which can be translated into mass formulae ⁴⁵⁾ :

one may attach a great importance to the fact that the simplest Veneziano amplitude written for $\pi\pi$ (and πK) scattering satisfies the Adler condition. As emphasized by Lovelace ³⁷⁾, this might be connected with chiral symmetry. Considered as a dynamical hypothesis ⁴⁵⁾, it leads to mass formulae. This cannot be true in general since there are several counter

examples such as $\bar{p}n(0^-) \rightarrow 3\pi$, where the simplest amplitude differs from the one written for $\pi\pi$ scattering by only a numerical factor ³⁷⁾, and where the Adler condition is not satisfied. It might nevertheless hold in many cases in connection with a mass pattern which it is very interesting to compare against experiment. For this purpose one imposes that each term of the amplitude, written for the reaction :

$$\pi + A \rightarrow B + C$$

namely :

$$V_{xy}(s,t) = \frac{\Gamma(m-\alpha_x(s))\Gamma(n-\alpha_y(t))}{\Gamma(m+n+p-\alpha_x(s)-\alpha_y(t))} \quad (46)$$

p , integer ≤ 0 , should vanish when the momentum of the pion goes to zero ($s=m_A^2$; $t=m_B^2$; $u=m_C^2$) m and n are the lowest resonance spins which can contribute to this particular term.

It follows that

$$\alpha_x(m_A^2) + \alpha_y(m_B^2) = m+n+p+N = m+n+N_{AB}$$

where N is either zero or a positive integer.

The same condition imposed for the amplitudes where $B=A$ ($y=x$) gives in either case

$$\begin{aligned} \alpha_x(m_A^2) &= m + \frac{N_{AA}}{2} \\ \text{or} \quad \alpha_y(m_B^2) &= n + \frac{N_{BB}}{2} \end{aligned} \quad (47)$$

If the required property holds for some A and B particles

$$N_{AA} + N_{BB} = 2N_{AB} \quad (48)$$

and both N_{AA} and N_{BB} must therefore be either even or odd integer. Since we already know that for $\pi\pi$ scattering, $N_{\pi\pi}$ is odd, the conclusion is that N_{AA} is odd whatever A is ⁴⁵⁾.

Particle A with spin s_A being itself on a linear Regge trajectory

$$m_A^2 = \frac{s_A - \alpha_A(0)}{\alpha'_A} \quad (49)$$

and

$$\alpha_X(0) + \frac{\alpha'_X}{\alpha'_A} (s_A - \alpha_A(0)) = m + \frac{N_{AA}}{2} \quad (50)$$

With identical slopes ⁴⁶⁾, and s_A and m being obviously both integers or half-integers, one concludes that the intercepts of the A and X trajectories, associated to particles which can decay into each other by pion emission, differ by an odd integer. This result holds, provided that the A and X trajectories have opposite normality, defined as $P(-1)^J$ for mesons and $P(-1)^{J-\frac{1}{2}}$ for baryons ⁴⁵⁾. The Adler condition is otherwise satisfied by definition and no conclusion follows.

The ρ and π ; ρ and A_1 ; Δ and N ; Y_1^* and Σ intercepts should then be spaced by half an integer (in effect $\frac{1}{2}$). This is very well satisfied. With the same slope for all trajectories, relations among intercepts can be rewritten as relations among masses. With $\alpha' = 1/(2(m_\rho^2 - m_\pi^2))$, the previously mentioned relations give for instance :

$$\begin{aligned} m_{A_1}^2 &= 2m_\rho^2 - m_\pi^2 \\ m_\Delta^2 &= m_N^2 + m_\rho^2 - m_\pi^2 \end{aligned} \quad (51)$$

They are both very nicely satisfied.

Generalizing these results to too many reactions, one however meets the already mentioned difficulty that the mass differences between the internal and external particles do not obey the same rules (nonet or octet structures of mesons). For instance one cannot satisfy, at the same time, the relations $m_\rho^2 = 3m_\pi^2 - 2m_\pi^2$ and $m_{A_2}^2 = 3m_\rho^2 + m_\eta^2 - 3m_\pi^2$ with a standard nonet of tensor mesons and an almost pure octet of pseudoscalar mesons. At the same time we do not, a priori, know whether to write $m_{Y_1^*}^2 = m_\Sigma^2 + m_\rho^2 - m_\pi^2$ or $m_\Lambda^2 + m_\rho^2 - m_\pi^2$, a clash between decuplet and octet mass splittings. Applying duality arguments according to coupling strength, we choose the second relation. The correct result is half-way in between. There is not yet any general rule to tell when to impose the Adler condition on each of the leading Veneziano terms. In other words, we do not yet know whether the simplicity of the Veneziano amplitude comes from the mass pattern, or the other way round.

If satellites are needed, the Adler condition could be imposed in a different way. We know that the simplest Veneziano term has all desired properties for $\pi\pi$ scattering, but in other cases it is no longer true. Such complications readily occur when one tries to extend the model to reactions

between particles with spin. In order to do so, one may write Veneziano expressions for each of the invariant amplitudes which satisfy the Mandelstam representation and which are introduced to describe the reaction.

Since these invariant amplitudes do not have the same asymptotic behaviour either for high s (u) or t values, each Veneziano term has to be tailored so as to meet the required properties. One may also use regularized helicity amplitudes where each Regge trajectory contributes asymptotically as $s^{\alpha(t)-\Lambda}$, where Λ is the corresponding helicity flip. A standard term such as :

$$V_{xy}(s,t) = \frac{\Gamma(m-\alpha_x(s))\Gamma(n-\alpha_y(t))}{\Gamma(m+n+p-\alpha_x(s)-\alpha_y(t))}$$

will have $s^{\alpha_y(t)-n-p}$ or $t^{\alpha_x(s)-m-p}$ asymptotic behaviours which are respectively obtained when s or t go to infinity. However, if the same trajectory is exchanged in both channels, the same asymptotic behaviour will automatically follow when the amplitude might have to correspond to different helicity flips in the two channels and therefore to different asymptotic behaviours. In order to maintain the required power in s or t , the standard Veneziano amplitude initially written down will then have to be multiplied by a polynomial in s or in t . For example :

$$(t-t_0) \frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))}$$

will have $s^{\alpha(t)-1}$ and $t^{\alpha(s)}$ asymptotic behaviours. The trajectories being linear, it is readily written as the sum of two Veneziano terms. One has lost, however, the great simplicity initially met with $\pi\pi$ scattering.

Attempts have been made to describe as simply as possible πN and KN scattering^{35),47)}, writing Veneziano terms for the two invariant amplitudes. As is well known, linear trajectories lead to a difficult question with parity doubling.

We will rather discuss in some detail $\rho\pi$ scattering which brings into focus many interesting questions⁴⁸⁾. The amplitude is usually written as :

$$T_{\mu\nu} = A p_\mu p_\nu + B(p_\mu q_\nu + p_\nu q_\mu) + C_1 q_\mu q_\nu + C_2 g_{\mu\nu} \quad (52)$$

where $P = \frac{1}{2}(p+p')$ and $Q = \frac{1}{2}(q+q')$; p and q (p' and q') stand for the pion and vector meson momenta in the initial (final) state. The four invariant amplitudes A , B , C_1 and C_2 are known to satisfy the Mandelstam representation. They are analytic functions of the variables $s = (p+q)^2$ and $t = (p'-p)^2$. The amplitudes are found to respectively have $s^{\alpha-2}$, $s^{\alpha-1}$, s^{α} and s^{α} asymptotic behaviour in s at fixed t , where α is the leading Regge trajectory, but the same t^{α} behaviour at large t and fixed s .

The absence of force in the exotic $\rho^+\pi^+$ and $\rho^+\rho^+ \rightarrow \pi^+\pi^+$ channels imposes exchange degeneracy in both channels. In the t channel ($\pi\pi \rightarrow \rho\rho$) we have the ρ, f exchange degenerate trajectories. In the s (or u) channel ($\pi\rho \rightarrow \pi\rho$), we have the ω, A_2 exchange degenerate trajectories on the one hand and, a priori, the π and the A_1 trajectories on the other. It is tempting to use πA_1 exchange degeneracy, from which the already obtained mass relation

$$m_{A_1}^2 = 2m_\rho^2 - m_\pi^2$$

readily follows. This, however, does not work. The π and the A_1 having the same isospin, their contribution at building up the crossed channel Regge trajectories, will not vanish in the isospin 2 state, if it is present in the isospin 0 and 1 states. We furthermore cannot get cancellations between the ω, A_2 and the π, A_1 contributions, since the trajectories are half a unit apart. $\pi\rho$ duality therefore imposes that an unnatural parity trajectory with negative signature and isospin 0 should be exchange degenerate with the pion trajectory⁴⁹⁾, and by the same token coincide with the A_1 trajectory, since we can consider the coincidence of the π and A_1 trajectories as an "experimental" fact. A 1^+ meson with isospin zero and G parity -1 should then be degenerate (or almost degenerate) in mass with the A_1 . Since most of the evidence for the A_1 comes from diffraction dissociation experiments, such an isospin 0 state might have yet escaped unambiguous detection. In a similar way, one needs an isospin 0 trajectory exchange degenerate with the A_1 , the first resonance of which would have spin 2^- .

In other words, if a forward peak is expected in the $\rho^+\pi^- \rightarrow \rho^+\pi^-$ reaction as a result of ρ if t channel exchange, it should change into a backward peak for the reaction $\rho^-\pi^+ \rightarrow \rho^+\pi^-$ where no resonance occurs in the t channel. Charge conjugation applied on the initial state does not, however, modify the π, A_1 contribution in the s channel (same isospin) as it does for the ω, A_2 contribution (opposite isospin). We therefore cannot reach the same answer both ways (in terms of t or s channel exchanges) as implied through duality.

The C_2 amplitude can be written as :

$$C_2(s,t) = \beta_4 \frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(s)-\alpha(t))} + \gamma_4 \frac{\Gamma(1-\alpha_1(s))\Gamma(1-\alpha(t))}{\Gamma(1-\alpha_1(s)-\alpha(t))} \quad (53)$$

(we consider $\rho^+ \pi^-$ scattering and therefore do not have u channel resonances). α represents the ρ (or f, ω, A_2) trajectory and α_1 the π trajectory (or the three other trajectories which should coincide with it). These two terms will be referred to as the $\rho\omega$ and $\rho\pi$ contributions. Since the pion pole does not contribute to C_2 , the first resonance here included on α_1 has spin 1. C_2 thus written has the correct poles and asymptotic behaviours. C_2 is singular at $\alpha(s) = 1$. This corresponds to the ω pole. The pole residue is easily found to be

$$\lim_{s \rightarrow m_\omega^2} (s - m_\omega^2) C_2(s,t) = -\beta_4 \left(\frac{\alpha(t)}{\alpha'} + t \right) \quad (54)$$

This corresponds to a superposition of vector and scalar exchanges, but since nothing yet fixes the parity, we could also have a superposition of axial, vector and pseudoscalar exchanges. However, pseudoscalar exchange is excluded for C_2 and scalar exchange is forbidden by parity conservation. If ω exchange corresponds to vector exchange (and not to a mixture of vector and pseudovector, which would cancel the spin zero part), (54) would lead to some forbidden spin zero exchange unless an extra satellite term is added to (53) in order to cancel it⁵⁰). This term affects (53) only at the first daughter level. It does not change the asymptotic behaviour and reads

$$\beta'_4 \frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))}$$

One finds : $\beta'_4 = \beta_4/2$, considering that $\alpha' m_\pi^2 \ll 1$ and neglecting the $\rho\omega$ mass difference. The absence of satellite would impose the excluded condition $\alpha_\rho(0) \simeq 0$ ⁵⁰). Eliminating parity doublets therefore imposes satellites. Further terms have similarly to be added in order to eliminate unwanted terms at $\alpha(s) = 2, 3, \dots$. They will act at higher daughter levels. Veneziano amplitudes can therefore still be written and they relate in an unambiguous way the width of the resonances on the leading trajectories ($\omega, A_2, \dots$). The simplicity of the $\pi\pi$ amplitudes is however lost !

The A amplitude in (52) behaves as $s^{\alpha(t)-2}$ at high s . We could write tentatively a Veneziano amplitude with such a behaviour for both the

ω, ρ and the π, ρ contributions, namely :

$$A(s,t) = \beta_1 \frac{\Gamma(1-\alpha(s))\Gamma(2-\alpha(t))}{\Gamma(3-\alpha(s)-\alpha(t))} + \gamma_1 \frac{\Gamma(-\alpha_1(s))\Gamma(2-\alpha(t))}{\Gamma(2-\alpha_1(s)-\alpha(t))}$$

This is, however, not correct since either term leading separately to a superconvergent amplitude, a cancellation of s channel poles would be required. This is not possible since all residues, if correctly given by the Veneziano amplitude, have to have the same sign. One should therefore impose superconvergence only to the combination of the ω, ρ and π, ρ contributions, writing up to extra necessary satellite terms :

$$A(s,t) = \beta_1 \left\{ \frac{\Gamma(1-\alpha(s))\Gamma(2-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))} + (s-s_1) \alpha' \frac{\Gamma(-\alpha_1(s))\Gamma(2-\alpha(t))}{\Gamma(2-\alpha_1(s)-\alpha(t))} \right\} \quad (55)$$

This expression has the required poles in s , no pole at $\alpha(t)=1$ and the correct asymptotic behaviours. The residues at the ω and π poles can be calculated, when they are, respectively, known to be $((G_{\omega e\pi}^2/8)(t-2m_e^2))$ and $G_{e\pi\pi}^2$.

One finds

$$(s_1 - m_\pi^2) = \frac{8 G_{e\pi\pi}^2}{G_{\omega e\pi}^2}$$

when from saturation of superconvergence sum rules with the ω and π contributions alone, one gets ⁵¹⁾

$$G_{e\pi\pi}^2 = \frac{1}{4} (m_e^2 G_{\omega e\pi}^2) \quad (56)$$

Both results agree with $s_1 \sim 2m_e^2 \sim \alpha'^{-1}$. Using (55) we should fix s_1 taking the experimental value for $((G_{e\pi\pi}^2)/(G_{\omega e\pi}^2))$. As already discussed we expect both an $I=1$ and an $I=0$ contribution at the A_1 mass, and β_1 and s_1 , thus determined, do not fix one of the $A_1 e\pi$ coupling constants.

A generalization of the Veneziano formula to the N point function has been worked out ^{52),53)}. This is in effect obtained in the case of identical spin zero mesons. The Euler beta function :

$$B(x,y) = \int_0^1 u^x (1-u)^y du \quad (57)$$

used in the standard, four-point function, Veneziano amplitude, has poles in x and y at negative integer values but no double poles. As seen on (57) a pole appears in x (or y) when u goes to zero (or to 1). The aim is therefore to generalize (57) with as many independent variables as necessary to describe a production amplitude (5 for the five-point function, 9 for the six-point function,...), in such a way that certain multiple poles occur, but only some of them. As an example, the graph of Fig. 9 shows multiple poles in the variable s_{12} , s_{123} , and s_{56} but a double pole in the variable s_{12} and s_{23} is not allowed. We refer to Refs. 52) and 53) for a detailed discussion of the construction proposed, which we do not attempt to sketch here. The amplitude is written as a multiple integral over as many variables as allowed coincident poles, while the integrand contains as many terms, all varying between 0 and 1 raised to the powers $(-1-\alpha(s_i))$, as there are different independent subenergy squared s_i . These terms are not independent and are related so that they cannot all vanish and be equal to 1 at the same time.

The four-point function :

$$B_4(x_1, x_2) = \int_0^1 du_1 u_1^{x_1} u_2^{x_2}$$

with $u_2=1-u_1$, is thus generalized to a five-point function 52) as

$$B_5(x_1, x_2, x_3, x_4, x_5) = \int_0^1 du_1 \int_0^1 du_4 \frac{1}{u_5} u_1^{x_1} u_2^{x_2} u_3^{x_3} u_4^{x_4} u_5^{x_5}$$

with $u_1=1-u_5u_2$, $u_2=1-u_1u_3$,.... The six-point functions correspond to nine independent subenergies, but only three possible coincident poles (Fig. 9). As shown by Chan 53) the procedure can be extended to an arbitrary number of particles. The amplitude thus written has poles and Regge behaviour in all sub-channels, as required by duality. A very important point is that the residue of spin zero poles are found to be product of Veneziano amplitudes corresponding to lower multiplicity. Residues at a pole corresponding to integer values of $\alpha(s_i)$ give dual amplitude for reactions with particles with spin. As already mentioned, important complications occur in order to avoid parity doubling.

These constructions are very important since they allow to treat on a similar footing two-body collisions and multiparticle reactions, for which we need a common framework before unitarity can be fully used.

14. CONCLUSION

An attempt has been made to show some prominent facets of the approach to hadron physics followed under the name of duality. It is difficult to describe a method which claims at being somewhat general and nevertheless is at the same time bound to be only approximate as a result of too drastic steps made in dealing with unitarity. The resonance and Regge pole approximations to two-body amplitudes, which have been reconciled with powerful use of their overlap, still appear as different as they were initially. A generalized interference model compatible with the unitarity constraints, if not yet using them, is probably the next step. It should, however, be constructed in a dual way, as well as in a crossing symmetric way, interlocking the resonance recurrence in one channel with the asymptotic Regge behaviour in the same channel. Present checks of exchange degeneracy are enough to provide a very strong support to duality as thus imposed. One of the key points is, of course, the hitherto unobserved rising daughter trajectories. An immediate consequence is the postulated existence of a string of 1^- mesons separated in mass squared by 1.15 GeV^2 . They should show up in e^+e^- collisions with the coming generation of storage rings. As already stressed, these resonances should be highly inelastic and could be searched for in multipion states, as well as in two-particle channels.

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$$\alpha(t) \simeq \alpha(s) + \alpha' t + \frac{t^2}{\pi} \int_s^\infty \frac{\text{Im } \alpha(t')}{t'^2(t'-t-i\epsilon)} dt'$$

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FIGURE CAPTIONS

Figure 1 : Three baryon trajectories (Δ, Y_1^*, Λ) and their exchange degenerate partners. The negative signature trajectory which coincides with the Δ trajectory corresponds to iso-spin $\frac{1}{2}$. The underlined masses correspond to well-established spins and parities.

Figure 2 : Regge trajectories for the vector and tensor mesons. The black (white) dots correspond to positive (negative) parity.

Figure 3 : Mandelstam diagram for $KN \rightarrow KN$ scattering. Absence of resonance in one channel (KN) corresponds to exchange degenerate resonances in both crossed channels.

Figure 4 : Duality diagrams for $K \pi$ scattering.

Figure 5 : Duality diagrams for $K^- p \rightarrow K^- p$ and $K^- p \rightarrow \Sigma^+ \pi^-$ scattering.

Figure 6 : s and t channel poles (solid lines). The dashed lines correspond to zeros (pole of the denominator) which eliminate double poles.

Figure 7 : Leading trajectory (two degenerate trajectories with opposite signatures) and daughter sequence.

Figure 8 : K and η decay in the one-particle exchange approximation.

Figure 9 : A particular term involved in the six-point function.

FIG.1

FIG.2

FIG.3

FIG. 4

FIG.5

FIG.6

FIG.7

FIG. 8

FIG. 9