An effective vacuum refractive index from gravity and the present ether-drift experiments

M. Consoli and E. Costanzo

Istituto Nazionale di Fisica Nucleare, Sezione di Catania
Dipartimento di Fisica e Astronomia dell’ Università di Catania
Via Santa Sofia 64, 95123 Catania, Italy

Abstract

Re-analyzing the data published by the Berlin and Düsseldorf ether-drift experiments, we have found a clean non-zero daily average for the amplitude of the signal. The two experimental values, $A_0 \sim (10.5 \pm 1.3) \cdot 10^{-16}$ and $A_0 \sim (12.1 \pm 2.2) \cdot 10^{-16}$ respectively, are entirely consistent with the theoretical prediction $(9.7 \pm 3.5) \cdot 10^{-16}$ that is obtained once the Robertson-Mansouri-Sexl anisotropy parameter is expressed in terms of $N_{\text{vacuum}}$, the effective vacuum refractive index that one would get, for an apparatus placed on the Earth’s surface, in a flat-space picture of gravity.
1. Introduction

The present generation of ether-drift experiments, combining the possibility of active rotations of the apparatus with the use of cryogenic optical resonators, is currently pushing the relative accuracy of the measured frequency shifts to the level $O(10^{-16})$. As we shall try to illustrate, this level of accuracy could be crucial to determine basic properties of the vacuum such as its space-time structure.

To this end, we’ll present a re-analysis of the observations reported in Refs. [1, 2] for the anisotropy of the speed of light in the vacuum. This re-analysis leads to two conclusions: i) both experiments exhibit a non-zero daily average for the amplitude of the signal ii) the magnitude of this average amplitude is entirely consistent with the theoretical Robertson - Mansouri - Sexl (RMS) [3, 4] anisotropy parameter

$$\left|(1/2 - \beta + \delta)_{\text{th}}\right| \sim 3(N_{\text{vacuum}} - 1) \sim 42 \cdot 10^{-10}$$  \hspace{1cm} (1)

that one would get [5, 6, 7] in terms of $N_{\text{vacuum}}$, the effective vacuum refractive index that arises in a flat-space picture of gravity.

The plane of the paper is as follows. In Sect.2, we shall first illustrate the basic formalism and report the experimental data of Refs. [1, 2]. Then, in Sect.3, we shall use these data to deduce the daily average amplitude of the signal for the two experiments. Further, in Sect.4 we shall compare these experimental values with the theoretical prediction that one would get, if there is a preferred frame, in a flat-space description of gravity. Finally, in Sect.5, we shall present our summary and conclusions.

2. Basic formalism and experimental data

The experimental data reported in Ref. [1] refer to 15 short-period observations, performed from December 2004 to April 2005, while the observations of Ref. [2] refer to a single short-period observation, taken around February 8th 2005. The starting point for our analysis is the expression for the relative frequency shift of two optical resonators at a given time $t$. For the Berlin experiment [1], this can be expressed as

$$\frac{\Delta \nu(t)}{\nu_0} = S(t) \sin 2\omega_{\text{rot}} t + C(t) \cos 2\omega_{\text{rot}} t$$  \hspace{1cm} (2)

where $\omega_{\text{rot}}$ is the rotation frequency of one resonator with respect to the other which is kept fixed in the laboratory and oriented north-south. The Fourier expansions of $S(t)$ and $C(t)$
are predicted to be

\[ S(t) = S_0 + S_{s1} \sin \tau + S_{c1} \cos \tau + S_{s2} \sin(2\tau) + S_{c2} \cos(2\tau) \]  

\[ C(t) = C_0 + C_{s1} \sin \tau + C_{c1} \cos \tau + C_{s2} \sin(2\tau) + C_{c2} \cos(2\tau) \]  

where \( \tau = \omega_{\text{sid}} t \) is the sidereal time of the observation in degrees and \( \omega_{\text{sid}} \sim \frac{2\pi}{23h56'} \). Introducing the colatitude of the laboratory \( \chi \), and the unknown average velocity, right ascension and declination of the cosmic motion with respect to a hypothetical preferred frame (respectively \( V, \alpha \) and \( \gamma \)), one finds the expressions reported in Table I of Ref. [1]

\[ C_0 = -\frac{K \sin^2 \chi}{8} (3 \cos 2\gamma - 1) \]  

\[ C_{s1} = \frac{1}{4} K \sin 2\gamma \sin \alpha \sin 2\chi \]  

\[ C_{c1} = \frac{1}{4} K \sin 2\gamma \cos \alpha \sin 2\chi \]  

\[ C_{s2} = \frac{1}{4} K \cos^2 \gamma \sin 2\alpha (1 + \cos^2 \chi) \]  

\[ C_{c2} = \frac{1}{4} K \cos^2 \gamma \cos 2\alpha (1 + \cos^2 \chi) \]  

where

\[ K = (1/2 - \beta + \delta) \frac{V^2}{c^2} \]  

and \((1/2 - \beta + \delta)\) indicates the RMS anisotropy parameter. The corresponding \( S \)--quantities are also given by \((S_0 = 0)\)

\[ S_{s1} = -\frac{C_{c1}}{\cos \chi} \]  

\[ S_{c1} = \frac{C_{s1}}{\cos \chi} \]  

\[ S_{s2} = -\frac{2 \cos \chi}{1 + \cos^2 \chi} C_{c2} \]  

\[ S_{c2} = \frac{2 \cos \chi}{1 + \cos^2 \chi} C_{s2} \]  

For the Düsseldorf experiment of Ref.[2], one should just re-nominate the two sets

\[(C_0, C_{s1}, C_{c1}, C_{s2}, C_{c2}) \rightarrow (C_0, C_1, C_2, C_3, C_4)\]  

\[(S_0, S_{s1}, S_{c1}, S_{s2}, S_{c2}) \rightarrow (B_0, B_1, B_2, B_3, B_4)\]  

and introduce an overall factor of two for the frequency shift since, in this case, two orthogonal cavities are maintained in a state of active rotation.

As suggested by the same authors, it is safer to concentrate on the observed time modulation of the signal, i.e. on the quantities \( C_{s1}, C_{c1}, C_{s2}, C_{c2} \) and on their \( S \)-counterparts. In fact, the constant components \( C_0 \) and \( S_0 = B_0 \) are likely affected by spurious systematic effects such as thermal drift. The experimental C-coefficients are reported in Table 1 for Ref.[2] and in Table 2 for Ref.[1] (these latter numerical values have been extracted from Fig.3 Ref.[1]).
Table 1: The experimental $C$–coefficients as reported in Ref.[2].

<table>
<thead>
<tr>
<th>$C_s[\times10^{-16}]$</th>
<th>$C_c[\times10^{-16}]$</th>
<th>$C_s[\times10^{-16}]$</th>
<th>$C_c[\times10^{-16}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3.0 \pm 2.0$</td>
<td>$11.0 \pm 2.5$</td>
<td>$1.0 \pm 2.5$</td>
<td>$0.1 \pm 2.5$</td>
</tr>
</tbody>
</table>

Table 2: The experimental $C$–coefficients as extracted from Fig.3 of Ref.[1].

<table>
<thead>
<tr>
<th>$C_s[\times10^{-16}]$</th>
<th>$C_c[\times10^{-16}]$</th>
<th>$C_s[\times10^{-16}]$</th>
<th>$C_c[\times10^{-16}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2.7 \pm 4.5$</td>
<td>$5.3 \pm 4.8$</td>
<td>$-3.2 \pm 4.7$</td>
<td>$1.2 \pm 4.2$</td>
</tr>
<tr>
<td>$-18.6 \pm 6.5$</td>
<td>$8.9 \pm 6.4$</td>
<td>$-11.4 \pm 6.5$</td>
<td>$-5.0 \pm 6.4$</td>
</tr>
<tr>
<td>$-0.7 \pm 3.9$</td>
<td>$5.3 \pm 3.6$</td>
<td>$5.0 \pm 3.5$</td>
<td>$1.6 \pm 3.8$</td>
</tr>
<tr>
<td>$6.1 \pm 4.6$</td>
<td>$0.0 \pm 4.8$</td>
<td>$-8.1 \pm 4.8$</td>
<td>$-4.0 \pm 4.6$</td>
</tr>
<tr>
<td>$2.0 \pm 8.6$</td>
<td>$1.3 \pm 7.7$</td>
<td>$16.1 \pm 8.0$</td>
<td>$-3.3 \pm 7.2$</td>
</tr>
<tr>
<td>$3.0 \pm 5.8$</td>
<td>$4.6 \pm 5.9$</td>
<td>$8.6 \pm 5.9$</td>
<td>$-6.9 \pm 5.9$</td>
</tr>
<tr>
<td>$0.0 \pm 5.4$</td>
<td>$-9.5 \pm 5.7$</td>
<td>$-5.5 \pm 5.6$</td>
<td>$-3.5 \pm 5.4$</td>
</tr>
<tr>
<td>$-1.1 \pm 8.1$</td>
<td>$11.0 \pm 7.9$</td>
<td>$0.9 \pm 8.3$</td>
<td>$18.6 \pm 7.9$</td>
</tr>
<tr>
<td>$8.6 \pm 6.5$</td>
<td>$2.7 \pm 6.7$</td>
<td>$4.3 \pm 6.5$</td>
<td>$-12.4 \pm 6.4$</td>
</tr>
<tr>
<td>$-4.8 \pm 4.8$</td>
<td>$-5.1 \pm 4.8$</td>
<td>$3.8 \pm 4.7$</td>
<td>$-5.2 \pm 4.7$</td>
</tr>
<tr>
<td>$5.7 \pm 3.2$</td>
<td>$3.0 \pm 3.4$</td>
<td>$-6.3 \pm 3.2$</td>
<td>$0.0 \pm 3.5$</td>
</tr>
<tr>
<td>$4.8 \pm 8.0$</td>
<td>$0.0 \pm 7.0$</td>
<td>$0.0 \pm 7.6$</td>
<td>$1.5 \pm 7.7$</td>
</tr>
<tr>
<td>$3.0 \pm 4.3$</td>
<td>$-5.9 \pm 4.3$</td>
<td>$-2.1 \pm 4.4$</td>
<td>$14.1 \pm 4.3$</td>
</tr>
<tr>
<td>$-4.5 \pm 4.4$</td>
<td>$-2.3 \pm 4.5$</td>
<td>$4.1 \pm 4.3$</td>
<td>$3.2 \pm 4.3$</td>
</tr>
<tr>
<td>$0.0 \pm 3.6$</td>
<td>$4.6 \pm 3.4$</td>
<td>$0.6 \pm 3.2$</td>
<td>$4.9 \pm 3.3$</td>
</tr>
</tbody>
</table>
Table 3: The experimental $S$—coefficients as extracted from Fig. 3 of Ref.[1].

<table>
<thead>
<tr>
<th>$S_{11}[\times 10^{-16}]$</th>
<th>$S_{c1}[\times 10^{-16}]$</th>
<th>$S_{x2}[\times 10^{-16}]$</th>
<th>$S_{c2}[\times 10^{-16}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$11.2 \pm 4.7$</td>
<td>$11.9 \pm 4.9$</td>
<td>$1.8 \pm 4.9$</td>
<td>$0.8 \pm 4.5$</td>
</tr>
<tr>
<td>$1.8 \pm 6.5$</td>
<td>$-4.3 \pm 6.5$</td>
<td>$6.4 \pm 6.4$</td>
<td>$1.8 \pm 6.4$</td>
</tr>
<tr>
<td>$-3.3 \pm 3.8$</td>
<td>$2.9 \pm 3.8$</td>
<td>$-5.9 \pm 3.8$</td>
<td>$4.6 \pm 4.0$</td>
</tr>
<tr>
<td>$12.7 \pm 5.1$</td>
<td>$14.3 \pm 5.5$</td>
<td>$-1.9 \pm 5.3$</td>
<td>$-3.3 \pm 5.1$</td>
</tr>
<tr>
<td>$4.7 \pm 8.4$</td>
<td>$-6.9 \pm 7.3$</td>
<td>$-1.8 \pm 8.0$</td>
<td>$-7.8 \pm 7.0$</td>
</tr>
<tr>
<td>$5.2 \pm 5.8$</td>
<td>$-3.0 \pm 5.9$</td>
<td>$7.1 \pm 5.9$</td>
<td>$-5.9 \pm 5.8$</td>
</tr>
<tr>
<td>$11.1 \pm 5.3$</td>
<td>$-13.4 \pm 5.4$</td>
<td>$-4.5 \pm 5.5$</td>
<td>$-9.8 \pm 5.5$</td>
</tr>
<tr>
<td>$-12.1 \pm 8.9$</td>
<td>$0.0 \pm 8.8$</td>
<td>$-3.1 \pm 9.0$</td>
<td>$1.4 \pm 8.9$</td>
</tr>
<tr>
<td>$-4.8 \pm 6.3$</td>
<td>$6.5 \pm 6.4$</td>
<td>$-8.1 \pm 6.3$</td>
<td>$3.5 \pm 6.5$</td>
</tr>
<tr>
<td>$9.8 \pm 5.0$</td>
<td>$4.8 \pm 5.0$</td>
<td>$1.9 \pm 5.0$</td>
<td>$-9.2 \pm 4.8$</td>
</tr>
<tr>
<td>$0.0 \pm 3.2$</td>
<td>$-3.9 \pm 3.6$</td>
<td>$1.0 \pm 3.1$</td>
<td>$-2.2 \pm 3.4$</td>
</tr>
<tr>
<td>$-12.7 \pm 7.7$</td>
<td>$8.5 \pm 6.8$</td>
<td>$-8.3 \pm 7.2$</td>
<td>$-7.1 \pm 7.4$</td>
</tr>
<tr>
<td>$-7.9 \pm 4.7$</td>
<td>$-4.3 \pm 4.8$</td>
<td>$-1.9 \pm 4.8$</td>
<td>$-6.2 \pm 4.7$</td>
</tr>
<tr>
<td>$16.1 \pm 4.9$</td>
<td>$12.0 \pm 5.2$</td>
<td>$2.9 \pm 4.9$</td>
<td>$-9.6 \pm 4.8$</td>
</tr>
<tr>
<td>$13.9 \pm 3.9$</td>
<td>$-7.0 \pm 3.4$</td>
<td>$-3.3 \pm 3.5$</td>
<td>$3.0 \pm 3.6$</td>
</tr>
</tbody>
</table>

The relevant numbers for the S-coefficients of Ref.[1] are reported in our Table 3. The S-coefficients of Ref.[2] were constrained, in the fits to the data, to their theoretical predictions in Eqs.(9) and (10). Thus their values will be deduced from Table 1 using these relations.

3. The daily average amplitude of the signal

For our analysis, we shall re-write Eq.(2) as follows

$$\frac{\Delta \nu(t)}{\nu_0} = A(t) \cos(2\omega_{rot}t - 2\theta_0(t))$$ (13)

with

$$C(t) = A(t) \cos 2\theta_0(t) \quad S(t) = A(t) \sin 2\theta_0(t)$$ (14)

$\theta_0(t)$ representing the instantaneous direction of a hypothetical ether-drift effect in the plane of the interferometer.
Within the RMS model, the amplitude of the signal (a positive-definite quantity) can be expressed in terms of \(v(t)\), the magnitude of the projection of the cosmic Earth’s velocity in the plane of the interferometer as

\[
A(t) = \frac{1}{2}|(1/2 - \beta + \delta)|\frac{v^2(t)}{c^2},
\]

To compute \(v(t)\), we shall use the expressions given by Nassau and Morse [8]. These are valid for short-period observations, as those performed in Refs.[1, 2], where the kinematical parameters of the cosmic velocity \(V\) are not appreciably modified by the Earth’s orbital motion around the Sun. In this case, by introducing the latitude of the laboratory \(\phi\), the right ascension \(\alpha\) and the declination \(\gamma\) associated to \(V\), the magnitude of the Earth’s velocity in the plane of the interferometer is defined by the two equations [8]

\[
\cos z(t) = \sin \gamma \sin \phi + \cos \gamma \cos \phi \cos(\tau - \alpha) \tag{16}
\]

and

\[
v(t) = V \sin z(t), \tag{17}
\]

\(z = z(t)\) being the zenithal distance of \(V\).

Replacing Eq. (17) into Eq. (15) and adopting a notation of the type in Eqs.(3)-(4), we obtain

\[
A(t) = A_0 + A_1 \sin \tau + A_2 \cos \tau + A_3 \sin(2\tau) + A_4 \cos(2\tau) \tag{18}
\]

where \((\chi = 90^\circ - \phi)\)

\[
A_0 = \frac{1}{2}|K| \left(1 - \sin^2 \gamma \cos^2 \chi - \frac{1}{2} \cos^2 \gamma \sin^2 \chi \right) \tag{19}
\]

\[
A_1 = -\frac{1}{4}|K| \sin 2\gamma \sin \alpha \sin 2\chi \quad A_2 = -\frac{1}{4}|K| \sin 2\gamma \cos \alpha \sin 2\chi \tag{20}
\]

\[
A_3 = -\frac{1}{4}|K| \cos^2 \gamma \sin 2\alpha \sin^2 \chi \quad A_4 = -\frac{1}{4}|K| \cos^2 \gamma \cos 2\alpha \sin^2 \chi \tag{21}
\]

Since \(A_0\) was not explicitly given by the authors of Ref.[1, 2], we shall now deduce its value from their published data that indeed have been obtained with experimental sessions extending over integer multiples of 24 hours in length [1]. The daily averaging of the signal (here denoted by \(\langle .. \rangle\)), when used in Eq. (18) produces the relation

\[
\langle A^2(t) \rangle = A_0^2 + \frac{1}{2}(A_1^2 + A_2^2 + A_3^2 + A_4^2) \tag{22}
\]
On the other hand, using Eqs. (3), (4) and (14), one also obtains

\[ \langle A^2(t) \rangle = C_0^2 + S_0^2 + \frac{1}{2}(C_{11}^2 + S_{11}^2 + C_{22}^2 + S_{22}^2) \]  

(23)

where we have introduced the combinations

\[ C_{11} \equiv \sqrt{C_{s1}^2 + C_{c1}^2} \quad C_{22} \equiv \sqrt{C_{s2}^2 + C_{c2}^2} \]
\[ S_{11} \equiv \sqrt{S_{s1}^2 + S_{c1}^2} \quad S_{22} \equiv \sqrt{S_{s2}^2 + S_{c2}^2} \]

(24)

As one can check, replacing the expressions (19)-(21), Eq. (22) gives exactly the same result that one would obtain replacing the values for the C- and S- coefficients in Eq. (23). Therefore, one can combine the two relations and get

\[ A_0^2(1 + r) = C_0^2 + S_0^2 + \frac{1}{2}(C_{11}^2 + S_{11}^2 + C_{22}^2 + S_{22}^2) \]

(26)

with

\[ r \equiv \frac{1}{2A_0^2}(A_1^2 + A_2^2 + A_3^2 + A_4^2) \]

(27)

To evaluate \( A_0 \) we shall proceed as follows. On the one hand, we shall compute the ratio \( r = r(\gamma, \chi) \) using the theoretical expressions Eqs. (19)-(21). This gives

\[ 0 \leq r \leq 0.40 \]  

(28)

for the latitude of the two laboratories in the full range \( 0 \leq |\gamma| \leq \pi/2 \). On the other hand, we shall adopt the point of view of the authors of Refs. [1, 2] that, even when large non-zero values of \( C_0 \) and \( S_0 \) are obtained (compare with the value \( C_0 = (-59.0 \pm 3.4 \pm 3.0) \cdot 10^{-16} \) of Ref. [2] and with the large scatter of the data reported in Fig. 3 of Ref. [1]), tend to consider these individual determinations as spurious effects. This means to set in Eq. (26)

\[ C_0 = \langle A(t) \cos 2\theta_0(t) \rangle \sim 0 \]
\[ S_0 = \langle A(t) \sin 2\theta_0(t) \rangle \sim 0 \]

(29)

(30)

The resulting average daily amplitude, determined in terms of \( C_{11}, S_{11}, C_{22} \) and \( S_{22} \) alone, provides, in any case, a lower bound to its true experimental value. The data for the various coefficients are reported in our Tables 4 and 5 together with the quantity

\[ Q = \sqrt{\frac{1}{2}(C_{11}^2 + S_{11}^2 + C_{22}^2 + S_{22}^2)} \sim A_0 \sqrt{1 + r} \]

(31)

from which, taking into account the numerical range of \( r \) in Eq. (28), we finally get

\[ A_0 \sim (0.92 \pm 0.08)Q \]

(32)

For a more precise determination of \( Q \) for the experiment of Ref. [1], we observe that the values reported in Table 5 exhibit a good degree of statistical consistency.
symmetrical errors.

relations.

cients defined in Eqs.(24)-(25) and the resulting $Q$ from Eq.(31). For simplicity, we report symmetrical errors. The values for the S-coefficients, constrained in the fits to the data to their theoretical predictions in Eqs.(9) and (10), have been deduced from Table 1 using these relations.

<table>
<thead>
<tr>
<th>$C_{11} \times 10^{-16}$</th>
<th>$C_{22} \times 10^{-16}$</th>
<th>$S_{11} \times 10^{-16}$</th>
<th>$S_{22} \times 10^{-16}$</th>
<th>$Q \times 10^{-16}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.4 ± 2.5</td>
<td>1.0 ± 2.5</td>
<td>14.7 ± 3.2</td>
<td>1.0 ± 2.5</td>
<td>13.2 ± 2.1</td>
</tr>
</tbody>
</table>

Table 5: The experimental values of Ref.[1] for the combinations of $C$– and $S$– coefficients defined in Eqs. (24)-(25) and the resulting $Q$ from Eq.(31). For simplicity, we report symmetrical errors.

<table>
<thead>
<tr>
<th>$C_{11} \times 10^{-16}$</th>
<th>$C_{22} \times 10^{-16}$</th>
<th>$S_{11} \times 10^{-16}$</th>
<th>$S_{22} \times 10^{-16}$</th>
<th>$Q \times 10^{-16}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.9 ± 4.7</td>
<td>3.5 ± 4.6</td>
<td>16.3 ± 4.8</td>
<td>2.0 ± 4.9</td>
<td>12.6 ± 3.5</td>
</tr>
<tr>
<td>20.6 ± 6.4</td>
<td>12.5 ± 6.5</td>
<td>4.6 ± 6.5</td>
<td>6.6 ± 6.4</td>
<td>17.8 ± 4.7</td>
</tr>
<tr>
<td>5.3 ± 3.6</td>
<td>5.3 ± 3.6</td>
<td>4.4 ± 3.8</td>
<td>7.5 ± 3.8</td>
<td>8.1 ± 2.8</td>
</tr>
<tr>
<td>6.1 ± 4.6</td>
<td>9.0 ± 4.8</td>
<td>19.1 ± 5.3</td>
<td>3.8 ± 5.1</td>
<td>15.7 ± 3.8</td>
</tr>
<tr>
<td>2.4 ± 8.4</td>
<td>16.5 ± 8.0</td>
<td>8.4 ± 7.7</td>
<td>8.0 ± 7.1</td>
<td>14.2 ± 6.1</td>
</tr>
<tr>
<td>5.5 ± 5.9</td>
<td>11.0 ± 5.9</td>
<td>6.0 ± 5.9</td>
<td>9.2 ± 5.9</td>
<td>11.6 ± 4.5</td>
</tr>
<tr>
<td>9.5 ± 5.7</td>
<td>6.5 ± 5.5</td>
<td>17.4 ± 5.4</td>
<td>10.7 ± 5.5</td>
<td>16.6 ± 4.0</td>
</tr>
<tr>
<td>11.0 ± 7.9</td>
<td>18.7 ± 7.9</td>
<td>12.1 ± 8.9</td>
<td>3.4 ± 9.0</td>
<td>17.7 ± 6.2</td>
</tr>
<tr>
<td>9.1 ± 6.5</td>
<td>13.1 ± 6.4</td>
<td>8.1 ± 6.4</td>
<td>8.8 ± 6.4</td>
<td>14.1 ± 4.8</td>
</tr>
<tr>
<td>7.0 ± 4.8</td>
<td>6.5 ± 4.7</td>
<td>10.9 ± 5.0</td>
<td>9.4 ± 4.8</td>
<td>12.2 ± 3.7</td>
</tr>
<tr>
<td>6.4 ± 3.1</td>
<td>6.3 ± 3.2</td>
<td>3.9 ± 3.6</td>
<td>2.4 ± 3.4</td>
<td>7.0 ± 2.4</td>
</tr>
<tr>
<td>4.8 ± 8.0</td>
<td>1.5 ± 7.7</td>
<td>15.3 ± 7.4</td>
<td>10.9 ± 7.3</td>
<td>13.7 ± 5.8</td>
</tr>
<tr>
<td>6.6 ± 4.3</td>
<td>14.3 ± 4.3</td>
<td>9.0 ± 4.7</td>
<td>6.5 ± 4.7</td>
<td>13.6 ± 3.3</td>
</tr>
<tr>
<td>5.1 ± 4.5</td>
<td>5.2 ± 4.3</td>
<td>20.0 ± 5.0</td>
<td>10.0 ± 4.8</td>
<td>16.6 ± 3.6</td>
</tr>
<tr>
<td>4.6 ± 3.4</td>
<td>5.0 ± 3.3</td>
<td>15.6 ± 3.8</td>
<td>4.4 ± 3.5</td>
<td>12.4 ± 2.7</td>
</tr>
</tbody>
</table>
This can be checked through the chi-square of the weighted averages over the 15 observation periods

\begin{align*}
C_{11} &= (6.7 \pm 1.2) \cdot 10^{-16} \quad C_{22} = (7.6 \pm 1.2) \cdot 10^{-16} \\
S_{11} &= (11.0 \pm 1.3) \cdot 10^{-16} \quad S_{22} = (6.3 \pm 1.3) \cdot 10^{-16}
\end{align*}

(33)

which is always of order unity. Using Eqs. (31) and (32) these values give an average \( A_0 \) for the 15 observation periods of Ref. [1]

\[ A_0 \sim (10.5 \pm 1.3) \cdot 10^{-16} \]

(35)

in good agreement with the value

\[ A_0 \sim (12.1 \pm 2.2) \cdot 10^{-16} \]

(36)

of Ref. [2].

4. An effective refractive index for the vacuum

In this section, we shall point out that the two experimental values in Eqs. (35) and (36) are well consistent with the theoretical prediction

\[ A_0^{th} \sim \frac{1}{2} |1/2 - \beta + \delta|_v \frac{v^2}{c^2} \sim (9.7 \pm 3.5) \cdot 10^{-16} \]

(37)

of Refs. [6, 7]. This was obtained, in connection with the RMS parameter [5] \( 1/2 - \beta + \delta|_v \sim 42 \cdot 10^{-10} \), after inserting the average cosmic velocity (projected in the plane of the interferometer) \( v = (204 \pm 36) \text{ km/s} \) that derives from a re-analysis [6, 7] of the classical ether-drift experiments. Due to this rather large theoretical uncertainty, the different locations of the various laboratories and any other kinematical property of the cosmic motion can be neglected in a first approximation.

For a proper comparison, we also remind that in Refs. [6, 7], the frequency shift was parameterized as

\[ \frac{\Delta \nu(\theta)}{\nu_0} = |1/2 - \beta + \delta|_v \frac{v^2}{c^2} \cos 2\theta \]

(38)

This relation is appropriate for a symmetrical apparatus with two rotating orthogonal lasers, as in the Düsseldorf experiment [2], and gives an average amplitude

\[ 2A_0 \sim (19 \pm 7) \cdot 10^{-16} \]

(39)
The theoretical prediction for the RMS parameter was obtained starting from the formal analogy that one can establish between General Relativity and a flat-space description with re-defined masses, space-time units and an effective vacuum refractive index. This alternative approach, see for instance Wilson [9], Gordon [10], Rosen [11], Dicke [12], Puthoff [13] and even Einstein himself [14], before his formulation of a metric theory of gravity, in spite of the deep conceptual differences, produces an equivalent description of the phenomena in a weak gravitational field.

The substantial phenomenological equivalence of the two approaches was well summarized by Atkinson as follows [15]: "It is possible, on the one hand, to postulate that the velocity of light is a universal constant, to define natural clocks and measuring rods as the standards by which space and time are to be judged and then to discover from measurement that space-time is really non-Euclidean. Alternatively, one can define space as Euclidean and time as the same everywhere, and discover (from exactly the same measurements) how the velocity of light and natural clocks, rods and particle inertias really behave in the neighborhood of large masses."

This formal equivalence, which is preserved by the weak-field classical tests, is interesting in itself and deserves to be explored. In fact, "...it is not unreasonable to wonder whether it may not be better to give up the geometric approach to gravitation for the sake of obtaining a more uniform treatment for all the various fields of force that are found in nature" [11].

For a quantitative test, one can start from the Equivalence Principle [14]. According to it, for an observer placed in a freely falling frame, local Lorentz invariance is valid. Therefore, given two space-time events that differ by \((dx, dy, dz, dt)\), and the space-time metric

\[ ds^2 = c^2 dt^2 - (dx^2 + dy^2 + dz^2) \]  

(40)

one gets from \( ds^2 = 0 \) the same speed of light that one would get in the absence of any gravitational effect.

For an observer placed on the Earth’s surface, for which the only gravitational field with respect to which the laboratory is not in free fall is that of the Earth, both General Relativity and the flat-space approach predict the weak-field, isotropic form of the metric

\[ ds^2 = c^2 dt^2 g_{44} - g_{11}(dx^2 + dy^2 + dz^2) = c^2 d\tau^2 - dl^2 \]  

(41)

where \( g_{44} = (1 - \frac{2GM}{c^2 R}) \), \( g_{11} = (1 + \frac{2GM}{c^2 R}) \), \( G \) being Newton’s constant and \( M \) and \( R \) the Earth’s mass and radius. Here \( d\tau \) and \( dl \) denote respectively the elements of "proper" time.
and "proper" length in terms of which, in General Relativity, one would again deduce from 
\[ ds^2 = 0 \] the same universal value \( \frac{dl}{d\tau} = c \).

However, in the flat-space approach the condition \( ds^2 = 0 \) is interpreted in terms of an 
effective refractive index for the vacuum 
\[ N_{\text{vacuum}} - 1 \sim \frac{2GM}{c^2R} \sim 14 \cdot 10^{-10} \] (42)
as if Euclidean space would be filled by a very rarefied medium. Is it possible to distinguish 
experimentally between the two different interpretations?

To this end, let us recall that a moving dielectric medium acts on light as an effective grav-
itational field [10, 16] and that, propagating in the "gravitational medium", light can be seen 
isotropic by only one inertial frame [17], say \( \Sigma \). Thus the following question naturally arises:
according to the ether-drift experiments, does \( \Sigma \) coincide with the Earth’s frame or with the 
hypothetical preferred frame of Lorentzian relativity? In the former case, corresponding to 
no anisotropy of the two-way speed of light in the vacuum, the equivalence between General 
Relativity and the gravitational-medium picture would persist. In the latter case, using 
Lorentz transformations, one predicts an anisotropy governed by the RMS parameter [5, 6, 7]
\[ |1/2 - \beta + \delta|_{\text{th}} \sim 3(N_{\text{vacuum}} - 1) \sim 42 \cdot 10^{-10} \] (43)
whose observation would uniquely single out the flat-space scenario. More precisely, one 
would be driven to conclude that the isotropic form of the metric Eq.(41) does not hold for 
an observer placed on the Earth’s surface and applies to some other frame (whose physical 
interpretation, within standard General Relativity, is not obvious). For this reason, the 
present ether-drift experiments, with their \( O(10^{-16}) \) accuracy, represent precision probes of 
the vacuum and of its space-time structure.

5. Summary and conclusions

In this paper, we have presented a re-analysis of two ether-drift experiments [1, 2] that,
employing rotating cryogenic optical resonators, attempt to establish the isotropy of the 
speed of light in the vacuum to a level of accuracy \( O(10^{-16}) \). For our re-analysis, we started 
by re-writing Eq.(2) as 
\[ \frac{\Delta \nu(t)}{\nu_0} = A(t) \cos(2\omega_{\text{rot}}t - 2\theta_0(t)) \] (44)
and assuming, as the authors of Refs.[1, 2], that experimental results providing large non-
zero values for either \( \langle C(t) \rangle = C_0 \) or \( \langle S(t) \rangle = S_0 \) in Eqs.[3] and [4] should be interpreted
as spurious effects (e.g. due to thermal drift, non-uniformity of the rotating cavity speed, misalignment of the cavity rotation axis,...).

With this assumption, the daily average for the amplitude of the signal \( A_0 = \langle A(t) \rangle \) can be expressed as

\[
A_0 \sim (0.92 \pm 0.08)Q
\]

where

\[
Q = \sqrt{\frac{1}{2}(C_{11}^2 + S_{11}^2 + C_{22}^2 + S_{22}^2)}
\]

is given in terms of the coefficients \( C_{11}, C_{22}, S_{11}, S_{22} \) defined in Eqs. (24)-(25). They represent the simplest rotationally invariant combinations one can form with the elementary coefficients \( C_{s1}, C_{cl}, C_{s2}, C_{c2} \) and with their S-counterparts. As stressed by the authors of Refs. [1, 2], these coefficients, that reflect the time modulation of the signal, should be much less affected by spurious effects than \( C_0 \) and \( S_0 \). Therefore, computing \( A_0 \) in this way should be completely safe. In any case, comparing with the full result in Eq. (26), our method provides a lower bound for the true experimental value of \( A_0 \).

Now, the two resulting experimental determinations in Eqs. (35) and (36), namely

\[
A_0 \sim (10.5 \pm 1.3) \cdot 10^{-16} \quad \text{and} \quad A_0 \sim (12.1 \pm 2.2) \cdot 10^{-16}
\]

are in good agreement with each other and with the theoretical prediction \((9.7 \pm 3.5) \cdot 10^{-16}\) of Refs. [5, 6, 7] that is obtained, in a flat-space description of gravity, in the presence of a preferred reference frame. As far as we can see, this non-trivial level of consistency means that a non-zero anisotropy of the speed of light in the vacuum has actually been measured in these experiments with values of the RMS anisotropy parameter that are one order of magnitude larger than the presently quoted ones.

For instance, in Ref. [1] the set \((V \sim 370 \text{ km/s}, \alpha \sim 168^\circ, \gamma \sim -6^\circ)\), corresponding to parameters obtained from a dipole fit to the COBE data, was assumed from the very beginning in the analysis of the data. In this case, fixing \( V \sim 370 \text{ km/s} \) and replacing the value of the RMS parameter from Ref. [1] \( |(1/2 - \beta + \delta)| \sim (2 \pm 2) \cdot 10^{-10} \) in Eq. (5), one would expect \(|K| \sim (3 \pm 3) \cdot 10^{-16} \) and \( C_{11} = (0.15 \pm 0.15) \cdot 10^{-16}, S_{11} = (0.20 \pm 0.20) \cdot 10^{-16}, C_{22} = (1.2 \pm 1.2) \cdot 10^{-16}, S_{22} = (6.3 \pm 6.3) \cdot 10^{-16} \).

These expectations should be compared with the actual experimental values reported in Table 5 and with their weighted averages

\[
C_{11} = (6.7 \pm 1.2) \cdot 10^{-16} \quad C_{22} = (7.6 \pm 1.2) \cdot 10^{-16}
\]

\[
S_{11} = (11.0 \pm 1.3) \cdot 10^{-16} \quad S_{22} = (6.3 \pm 1.3) \cdot 10^{-16}
\]
For this reason, in our opinion, the very small RMS parameter of Ref. [1] (and of Ref. [2]) rather than reflecting the smallness of the signal, originates from accidental cancellations among the various entries. These might be due to several reasons. For instance, to a wrong input choice for the kinematical parameters \((V, \alpha, \gamma)\) used in the fits or to the procedure used to fix the relative phases for the various parameter pairs (see note [13] of Ref. [1]). These phases are essential to obtain consistent values for the right ascension \(\alpha\) and the sign of \(\gamma\). In any case, even a substantial level of phase error among different experimental sessions, that can produce vanishing inter-session averages for \(C_{s1}, C_{c1}, C_{s2}, C_{c2}\) and their S-counterparts, will not affect the rotationally invariant combinations \(C_{11}, C_{22}, S_{11}, S_{22}\) and our determination of \(A_0\).

To conclude, motivated by the fundamental nature of the questions concerning the vacuum and its space-time structure, we have undertaken a careful re-analysis of the data that leads to the observed values of \(A_0\) in Eqs. (35) and (36). Since these results are entirely consistent with the theoretical prediction Eq. (37), we are driven to conclude that the data support both the existence of a preferred frame and a flat-space description of gravity. At the same time, the novelty of this conclusion emphasizes the importance of comparing different approaches and points of view to achieve a full understanding of the underlying physical problem.
References


