Entropy considerations in constraining the mSUGRA parameter space

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Abstract. We explore the use of two criteria to constrain the allowed parameter space in mSUGRA models. Both criteria are based in the calculation of the present density of neutralinos as dark matter in the Universe. The first one is the usual “abundance” criterion which is used to calculate the relic density after the “freeze-out” era. To compute the relic density we used the numerical public code micrOMEGAs. The second criterion applies the microcanonical definition of entropy to a weakly interacting and self-gravitating gas evaluating then the change in the entropy per particle of this gas between the “freeze-out” era and present day virialized structures. An “entropy-consistency” criterion emerges by comparing theoretical and empirical estimates of this entropy. The main objective of our work is to determine for which regions of the parameter space in the mSUGRA model are both criteria consistent with the 2σ bounds according to WMAP for the relic density:

\[ 0.0945 \leq \Omega_{CDM} h^2 \leq 0.1287 \]

As a first result, we found that for \( A_0 = 0 \), \( \text{sgn} \mu = + \), small values of \( \tan \beta \) are not favored; only for \( \tan \beta \approx 50 \) are both criteria significantly consistent.

Keywords: dark matter, supersymmetry

INTRODUCTION

One of the most accepted candidates to be the major component of dark matter (DM) is the neutralino as an LSP (Lightest Supersymmetric Particle). Supersymmetric models with R-parity conservations predict this type of particles (for an excellent introduction to Supersymmetry see [1]). This type of models have several parameters that can be constrained in its values using observational constraints of the actual density of DM, according with WMAP: \( 0.0945 \leq \Omega_{CDM} h^2 \leq 0.1287 \) [2, 3]. In particular for mSUGRA models this has been done using the standard approach [2, 4] which is based in the Boltzmann equation considering that after the “freeze-out” era, neutralinos cease to annihilate

keeping its number constant. In such an approach, the relic density of neutralinos is approximately: \( \Omega_\chi \approx \frac{1}{\langle \sigma v \rangle} \), where \( \langle \sigma v \rangle \) is the thermally averaged cross section times the relative velocity of the LSP annihilation pair. Within the mSUGRA model five parameters \((m_0, m_{1/2}, A_0, \tan \beta \text{ and the sign of } \mu)\) are needed to specify the supersymmetric spectrum of particles and the final relic density. We will use the numerical code micrOMEGAs \([5]\) to compute the relic density following the past scheme which will be called the “abundance criterion” (AC).

Just after “freeze-out”, we can consider neutralinos then as forming a Maxwell-Boltzmann (MB) gas in thermal equilibrium with other components of the primordial cosmic structures. In the present time, such a gas is almost collisionless and either constitutes galactic halos and larger structures or it is in the process of its formation. In this context, we can conceive two equilibrium states for the neutralino gas, the decoupling (or “freeze-out”) epoch and its present state as a virialized system. Computing the entropy per particle for each one of this states we can use an “entropy consistency” criterion (EC) using theoretical and empirical estimates for this entropy to obtain the relic density of neutralinos \(\Omega_\chi\).

Our objective is then to use AC and EC criteria, to obtain constraints for the parameters of the mSUGRA model by demanding that both criteria must be consistent within them and within the observational constraints required by WMAP.

**ABUNDANCE CRITERION**

Relic abundance of some stable species \(\chi\) is defined as \(\Omega_\chi = \frac{\rho_\chi}{\rho_{\text{crit}}}\), where \(\rho_\chi = m_\chi n_\chi\) is the relic’s mass density \((n_\chi\) is the number density), \(\rho_{\text{crit}}\) is the critical density of the Universe (see \([6]\) for a review on the standard method to compute the relic density). The time evolution of \(n_\chi\) is given by the Boltzmann equation:

\[
\frac{dn_\chi}{dt} = -3Hn_\chi - \langle \sigma v \rangle (n_\chi^2 - (n_{\chi}^{\text{eq}})^2)
\]  

(1)

where \(H\) is the Hubble expansion rate, \(\langle \sigma v \rangle\) is the thermally averaged cross section times the relative velocity of the LSP annihilation pair and \(n_{\chi}^{\text{eq}}\) is the number density that species would have in thermal equilibrium. In the early Universe, the neutralinos \(\chi\) were initially in thermal equilibrium, \(n_\chi = n_{\chi}^{\text{eq}}\). As the Universe expanded, their typical interaction rate started to diminish an the process of annihilation froze out. Since then, the number density of neutralinos has remained basically constant.

There are several ways to solve equation (1), one of the more used is based on the “freeze-out” approximation (see for example \([7]\)). However in order to have more precision, we will use the exact solution to Boltzmann equation using the public numerical code micrOMEGAs 1.3.6 \([5]\) which calculates the relic density of the LSP in the Minimal Supersymmetric Standard Model (MSSM). We will take and mSUGRA model and its five parameters \((m_0, m_{1/2}, A_0, \tan \beta \text{ and the sign of } \mu)\) as input parameters for micrOMEGAs and use Suspect \([8]\), which comes as an interface to micrOMEGAs, to calculate the supersymmetric spectrum of masses of particles. Details about how we used micrOMEGAs for making the calculation will be described in a future paper that is currently in preparation \([9]\).
Using micrOMEGAs, we can obtain the relic density for any region of the parameter space to discriminate regions that are consistent with the WMAP constraints in this abundance criterion.

**ENTROPY CONSISTENCY CRITERION**

Since the usual MB statistics that can be formally applied to the neutralino gas at the "freeze-out" era can not be used to describe present day neutralinos subject to a long range gravitational interaction making up non-extensive systems, it is necessary to use the appropriate approach that follows from the microcanonical ensemble in the "mean field" approximation which yields an entropy definition that is well defined for a self-gravitating gas in an intermediate state. Such an approach is valid at both the initial ("freeze-out" era, \(f\)) and final (virialized halo structures, \(h\)) states that we wish to compare. Under these conditions, the change in the entropy per particle \(s\) between these two states is given by [10]:

\[
s^h - s^f = \ln \left[ \frac{n^f_{\chi}}{n^h_{\chi}} \left( \frac{x^f}{x^h} \right)^{3/2} \right]
\]

where \(x = m_{\chi}/T\), \(T\) is the temperature of the gas. A region that fits with the conditions associated with the intermediate scale is the central region of halos (\(10^3\) pc within the halo core); evaluating the thermodynamical quantities at this region, using equation (2) and some assumptions more, it is possible to construct a theoretical estimate for \(s^h\) that depends on the nature of neutralinos (\(m_{\chi}\) and \(\langle \sigma v \rangle\)), initial conditions (given by \(x^f\)), cosmological parameters (\(\Omega_{\chi}\), the Hubble parameter, \(h\)) and structural parameters of the virialized halo (central values for temperature and density); for details of these and the following, see section IV of [10].

An alternative estimate for \(s^h\) can be made based on empirical quantities for observed structures in the present Universe using the microcanonical entropy definition in terms of phase space volume, but restricting this volume to the actual range of velocities accessible to the central particles, that is, up to a maximal escape velocity \(v_e(0)\) which is related to the central velocity dispersion of the halo (\(\sigma_h\)) by an intrinsic parameter \(\alpha\):

\[v_e^2(0) \sim \alpha \sigma_h^2(0)\].

The authors in [10] give an uncertainty range for the value of \(\alpha\) for actual galaxies: \(11.2 \leq \alpha \leq 24.8\). The range of values allowed for this parameter is of the highest importance to determine the allowed region of the parameter space in the mSUGRA model as will be clear in the results presented on next section.

Equating the theoretical an empirical estimates for the entropy per particle it is obtained a relation for the relic abundance of neutralinos using the EC criterion\(^2\):

\[
\ln(\Omega_{\chi} h^2) = 10.853 - x^f + \ln \left[ \frac{(x^f \alpha)^{3/2} m_{\chi}}{f^*_\gamma (x^f)} \right]
\]

\(^2\) This formula is a small modification to the one presented in [10]
RESULTS AND CONCLUSIONS

Using both AC and EC criteria, that have been briefly described above, we can compute the relic abundance of neutralinos and constrain the region in the mSUGRA parameter space where both criteria are fullfilled. In order to obtain a first result we will fix the value of two parameters in the mSUGRA model: $A_0 = 0$ and $\text{sgn}\mu = +$. In the left panel of figure (1), we present a region of the parameter space with these two values fixed and with $\tan\beta = 10$. The green region is where the $\tilde{\tau}$ is the LSP, the red and blue areas determine the WMAP allowed regions for the AC and EC criteria respectively. As we can see from figure (1) (left panel), the region where both criteria are fullfilled is very small, in fact only for the highest values of $\alpha$ there is an intersection between both criteria. The right panel of figure (1) shows the same regions as in the left panel but for $\tan\beta = 50$; contrary to the case of $\tan\beta = 10$ we see now a complete intersection of both AC and EC criteria.

We have followed the novel idea of [10] to introduce a new criterion to constrain the mSUGRA parameter space using the assumption of entropy consistency for the initial and final states of a neutralino gas. Using the program micrOMEGAs, we explored with precision which regions then satisfy this criterion and the usual AC criteria previously used several times. We found that for the regions so far explored, values with small $\tan\beta$ are not favored, leading to an insignificant allowed region satisfying both criteria. Values with $\tan\beta \gtrsim 50$ fulfill the requirement of both criteria and the WMAP constraints.
ther analysis, which is currently being done, is required to give more precise conclusions about this new method to constrain the parameter space of the mSUGRA model.

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