Gauge and Yukawa mediated supersymmetry breaking in the triplet seesaw scenario

Filipe R. Joaquim\textsuperscript{a,b,*} and Anna Rossi\textsuperscript{a,§}

\textsuperscript{a} Dipartimento di Fisica “G. Galilei”, Università di Padova I-35131 Padua, Italy
\textsuperscript{b} Istituto Nazionale di Fisica Nucleare (INFN), Sezione di Padova, I-35131 Padua, Italy

\textsuperscript{*}E-mail: joaquim@pd.infn.it
\textsuperscript{§}E-mail: arossi@pd.infn.it

ABSTRACT

We propose a novel supersymmetric unified scenario of the triplet seesaw mechanism where the exchange of the heavy triplets generates both neutrino masses and soft supersymmetry breaking terms. Our framework is very predictive since it relates neutrino mass parameters, lepton flavour violation in the slepton sector, sparticle and Higgs spectra and electroweak symmetry breakdown. The phenomenological viability and experimental signatures in lepton flavor violating processes are discussed.

Modern particle physics has been confronting the intriguing issue of neutrino mass generation and its phenomenological implications. From the theoretical point of view, the well-celebrated seesaw mechanism provides a natural explanation for the generation of neutrino masses and their suppression with respect to the other fermion masses of the Standard Model (SM). In its most popular versions, the seesaw mechanism is realized either by exchanging singlet fermions $N$ (type I) \cite{typeI}, or a $SU(2)_W$ scalar triplet $T$ with non-zero hypercharge (type II) \cite{typeII}, at a high scale $M_L$. An attractive feature of the supersymmetric version of the above scenarios is that lepton flavor violating (LFV) processes (otherwise unobservable) can be enhanced through one-loop exchange of lepton superpartners if their masses do not conserve flavor. Regarding this aspect, most of the literature has been focusing on the most conservative scenario of universal sfermion masses at high energy, as in minimal supergravity or gauge mediated supersymmetry.
(SUSY) breaking models. In such cases, flavor non-conservation in the sfermion masses arises from renormalization group (RG) effects due to flavor-violating Yukawa couplings \[ 3 \] \[ 4 \] \[ 5 \]. We recall that in the triplet seesaw the flavor structure of the slepton mass matrix \( m_L^2 \) after RG running can be univocally determined in terms of the low-energy neutrino parameters \[ 5 \]. This is in contrast with the type-I seesaw where the structure of \( m_L^2 \) cannot be unambiguously related to the neutrino parameters.

In this Letter we present a novel supersymmetric scenario of the triplet seesaw mechanism in which the soft SUSY breaking (SSB) parameters in the minimal supersymmetric extension of the SM (MSSM) are generated at the decoupling of the heavy triplets and the mass scale of such SSB terms is fixed only by the triplet SSB bilinear term \( B_T \). This scenario is highly predictive since it relates neutrino masses, LFV in the sfermion sector, sparticle and Higgs spectra and electroweak symmetry breaking (EWSB).

The supersymmetric version of the type-II seesaw requires introducing the triplets as super-multiplets \( T, \bar{T} \) in a vector-like \( SU(2)_W \times U(1)_Y \) representation, \( T \sim (3,1), \bar{T} \sim (3,-1) \). In order to preserve successful gauge coupling unification, we embed our framework in a \( SU(5) \) grand unified theory (GUT) \[ 3 \] where the triplet states fit into the 15 representation \( 15 = S + T + Z \) transforming as \( S \sim (6,1,-\frac{2}{3}), \ T \sim (1,3,1), \ Z \sim (3,2,\frac{1}{6}) \) under \( SU(3) \times SU(2)_W \times U(1)_Y \) (the \( 15 \) decomposition is obvious). The SUSY breaking mechanism is parametrized by a gauge singlet chiral supermultiplet \( X \), whose scalar \( S_X \) and auxiliary \( F_X \) components are assumed to acquire a vacuum expectation value through some unspecified dynamics in the secluded sector. It is suggestive for our discussion to assume that the \( SU(5) \) model conserves the combination \( B-L \) of baryon and lepton number. As a result, the relevant superpotential reads

\[
W_{SU(5)} = \frac{1}{\sqrt{2}}(Y_{15}\bar{\mathbb{5}} 15 \bar{5} + \lambda_{5H} \mathbb{T}_{\bar{15}} \bar{5}H) + Y_{\bar{5}H} \mathbb{5}H 10 \\
\quad + Y_{10} 10 \mathbb{5}H + M_{5H} \bar{5}H + \xi_{X} 15 \overline{\mathbb{T}_{15}},
\]

where the matter multiplets are understood as \( \bar{5} = (d^c, L) \), \( 10 = (u^c, e^c, Q) \) and the Higgs doublets fit with their coloured partners, \( t, \bar{t} \) like \( 5_H = (t, H_2), \bar{5}_H = (\bar{t}, H_1) \). The \( B-L \) quantum numbers are a combination of the hypercharges and the following charges:

\[ Q_{10} = \frac{1}{5}, \ Q_5 = -\frac{2}{5}, \ Q_{5H} = -\frac{2}{3}, \ Q_{\bar{5}H} = \frac{2}{3}, \ Q_{15} = \frac{6}{5}, \ Q_{\bar{15}} = \frac{4}{5} \] and \( Q_X = -2 \). The form of \( W_{SU(5)} \) implies that the \( 15, \overline{15} \) states play the role of messengers of both \( B-L \) and SUSY breaking to the visible (MSSM) sector thanks to the coupling with \( X \). Namely,
while ⟨S_X⟩ only breaks B−L, ⟨F_X⟩ breaks both SUSY and B−L. These effects are parametrized by the superpotential mass term M_{1515} \overline{15}, where M_{15} = \xi \langle S_X \rangle, and the bilinear SSB term −BM_{1515} \overline{15}, with BM_{15} = −\xi \langle F_X \rangle. Once SU(5) is broken to the SM group we find [5], below the GUT scale M_G,:

\[ W = W_0 + W_T + W_{S,Z} \]
\[ W_0 = Y_e e^c H_1 L + Y_d d^c H_1 Q + Y_u u^c Q H_2 + \mu H_2 H_1 \]
\[ W_T = \frac{1}{\sqrt{2}} (Y_T L T L + \lambda H_2 \bar{T} H_2) + M_T T \bar{T} \]
\[ W_{S,Z} = \frac{1}{\sqrt{2}} Y_S d^c S d^c + Y_Z d^c Z L + M_Z Z + M_S S S. \]

(2)

Here, W_0 denotes the MSSM superpotential, W_T contains the triplet Yukawa and mass terms, and W_{S,Z} includes the couplings and masses of the colored fragments S, Z. As in [5], we have relaxed the strict SU(5) symmetry relations for the Yukawa interactions and mass terms by allowing SU(5) breaking effects, induced, for example, by adjoint 24-insertions, such as \( Y_5 = Y_5^{(0)} + Y_5^{(1)} 24/\Lambda + \ldots \) with a cut-off scale \( \Lambda > M_G \). These insertions are necessary to correct the relation \( Y_e = Y_d^T \) and to solve the doublet-triplet splitting problem. For the sake of simplicity, we take \( M_T = M_S = M_Z \) and \( Y_S, Y_Z \ll Y_T \) at \( M_G \) (possibly due to 24-insertions), which does not alter the major point of our discussion. The SU(5) scenario with \( Y_S = Y_Z = Y_T \) implies correlations between LFV and quark flavour violation; this case will be considered in detail in [6]. In eq. (2), W_T is responsible for the realization of the seesaw mechanism. Actually, at the scale \( M_T \) the triplets act as tree-level messengers of lepton number and flavor violation via the symmetric Yukawa matrix \( Y_T \), generating the \( d = 5 \) effective operator \( \lambda Y_T (L H_2)^2 / M_T \). Subsequently, at the electroweak scale the Majorana neutrino mass matrix is obtained

\[ m^i_j = \frac{\lambda \langle H_2 \rangle^2}{M_T} Y_{ij}^T, \quad i, j = e, \mu, \tau. \]

(3)

In the basis where \( Y_e \) is diagonal, it is apparent that all LFV is encoded in \( Y_T \). Namely, the nine independent parameters contained in \( m_\nu \) are directly linked to the neutrino parameters according to \( m_\nu = U^* m_\nu^D U^\dagger \), where \( m_\nu^D = \text{diag}(m_1, m_2, m_3) \) are the mass eigenvalues, and \( U \) is the leptonic mixing matrix.

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1 Beneath the scale \( M_G \), baryon number is conserved since the colored partners \( t, \bar{t} \) are understood to be decoupled.
Regarding the SSB term one has, in the broken phase,

\[-B_T M_T (T\bar{T} + S\bar{S} + Z\bar{Z}) + \text{h.c.}, \tag{4}\]

where \(B_T \equiv B_{15}\). These terms lift the tree-level mass degeneracy in the MSSM supermultiplets. Indeed, at the scale \(M_T\), all the states \(T, \bar{T}, S, \bar{S}\) and \(Z, \bar{Z}\) are messengers of SUSY breaking to the MSSM sector via gauge interactions, as it happens in conventional gauge-mediation scenarios \[7\]. However, in our framework the states \(T, \bar{T}\) also transmit SUSY-breaking via Yukawa interactions. Finite contributions for the trilinear couplings of the superpartners with the Higgs doublets, \(A_e, A_u, A_d\), the gaugino masses \(M_a (a = 1, 2, 3)\) and the Higgs bilinear term \(-B_H\mu H_2 H_1\) emerge at the one-loop level:

\[
\begin{align*}
A_e &= \frac{3B_T}{16\pi^2} Y_e Y_T^\dagger Y_T, \\
A_u &= \frac{3B_T}{16\pi^2} Y_u |\lambda|^2, \\
A_d &= 0, \\
M_a &= \frac{7B_T}{16\pi^2} g_a, \\
B_H &= \frac{3B_T}{16\pi^2} |\lambda|^2, \\
\end{align*}
\tag{5}\]

\((g_a\) are the gauge coupling constants\)). As for the SSB squared scalar masses, the leading \(\mathcal{O}(F_X^2/M_T^2) = \mathcal{O}(B_T^2)\) contributions do not emerge at one-loop level \[8\], but instead at two-loop\(^2\):

\[
\begin{align*}
m_{L}^2 &= \frac{|B_T|^2}{(16\pi^2)^2} \left[ \frac{21}{10} g_1^4 + \frac{21}{2} g_2^4 - \left( \frac{27}{5} g_1^2 + 21 g_2^2 \right) Y_T^\dagger Y_T + 3 Y_T^\dagger Y_T Y_T^\ast Y_T + 18 (Y_T^\dagger Y_T)^2 \right. \\
&\quad \quad \quad \quad \quad + 3 \text{Tr}(Y_T^\dagger Y_T Y_T^\dagger Y_T) \\
m_{e}^2 &= \frac{|B_T|^2}{(16\pi^2)^2} \left[ \frac{42}{5} g_1^4 - 6 Y_e Y_T^\dagger Y_T^\ast Y_T \right] \\
m_{Q}^2 &= \frac{|B_T|^2}{(16\pi^2)^2} \left[ \frac{7}{30} g_1^4 + \frac{21}{2} g_2^4 + \frac{56}{3} g_3^4 - 3 |\lambda|^2 Y_u^\dagger Y_u \right] \\
m_{u}^2 &= \frac{|B_T|^2}{(16\pi^2)^2} \left[ \frac{56}{15} g_1^4 + \frac{56}{3} g_3^4 - 6 |\lambda|^2 Y_u^\dagger Y_u \right] \\
m_{d}^2 &= \frac{|B_T|^2}{(16\pi^2)^2} \left[ \frac{14}{15} g_1^4 + \frac{56}{3} g_3^4 \right] \\
m_{H_1}^2 &= \frac{|B_T|^2}{(16\pi^2)^2} \left[ \frac{21}{10} g_1^4 + \frac{21}{2} g_2^4 \right]
\end{align*}
\]

\(^2\)Such \(\mathcal{O}(F_X^2/M_T^2)\) two-loop contributions dominate over the \(\mathcal{O}(F_Y^4/M_T^6) = \mathcal{O}(B_T^4/M_T^2)\) one-loop ones for \(M_T > (4\pi Y_T/g^2)B_T\), which is indeed fulfilled in our analysis.
\begin{align}
    m_{H_2}^2 &= \frac{|B_T|^2}{(16\pi^2)^2} \left[ \frac{21}{10} g_1^4 + \frac{21}{2} g_2^2 - \left( \frac{27}{5} g_2^2 + 21 g_2^2 \right) |\lambda|^2 + 9 |\lambda|^2 \text{Tr}(Y_u Y_u^\dagger) + 21 |\lambda|^4 \right]. \tag{6}
\end{align}

The results (5) and (6) can be obtained either by diagrammatic computations or from generalization of the wave function renormalization method proposed in [9].

Notice that the generation of all the SSB gaugino masses requires the presence of the complete 15 representation. More specifically, \( M_1, M_2 \) and \( M_3 \) arise from the exchange of the \((T, S, Z), (T, Z)\) and \((S, Z)\) states, respectively. The expressions in eqs. (5) and (6) hold at the decoupling scale \( M_T \) and therefore are meant as boundary conditions for the SSB parameters which then undergo (MSSM) RG running to the low-energy scale \( \mu_{\text{SUSY}} \). In particular, we observe that the Yukawa couplings \( Y_T \) induce LFV to \( A_e \), to the scalar masses \( m_{\tilde{L}}^2 \) and to a much less extent in \( m_{\tilde{e}}^2 \). This feature makes the present scenario different from pure gauge-mediated models [7] where flavor violation comes out naturally suppressed (for other examples of Yukawa mediated SUSY breaking, see e.g [8, 10]). We suppose that possible gravity mediated contributions \( \sim F/M_{\text{Pl}} \) (where \( F^2 = \langle |F_X|^2 \rangle + \ldots \) is the sum of F-terms in the secluded sector) are negligible. This is the case if \( M_T \ll 10^{16} \text{ GeV} \) \( \xi \langle F_X \rangle / F \). Furthermore, it is necessary that \( \xi \langle F_X \rangle < M_T^2 \) (or \( B_T < M_T \)) to avoid tachyonic scalar messengers.

It is worth stressing that here the LFV entries \( (m_{\tilde{L}}^2)_{ij} (i \neq j) \) show up as finite radiative contributions induced by \( B_T \) at \( M_T \), and they are not essentially modified by the (MSSM) RG evolution to low-energy. This is different from a previous work [5] where a common SSB scalar mass \( m_0 \sim \mathcal{O}(100 \text{ GeV}) \) was assumed at \( M_G \) and the dominant LFV contributions to \( m_{\tilde{L}}^2 \) were generated by RG evolution from \( M_G \) down to the decoupling scale \( M_T \). In such a case, finite contributions like those in eqs. (5, 6) also emerge at \( M_T \), but they are subleading with respect to the RG corrections, since \( B_T \) is of the same order as \( m_0 \). Instead, in the present picture, there is a hierarchy between the SSB parameter \( B_T \) and the remaining ones [see eqs. (5, 6)], \( B_T^2 \gg (B_T g_2^2/16\pi^2)^2 \sim m_0^2 \). However, in both scenarios the flavor structure of \( m_{\tilde{L}}^2 \) is proportional to \( Y_T^\dagger Y_T \) and can be written by using eq. (3) in terms of the neutrino parameters (the terms \( \propto g^2 Y_T^\dagger Y_T \) are generically the leading ones):

\begin{align}
    (m_{\tilde{L}}^2)_{ij} \sim B_T^2 (Y_T^\dagger Y_T)_{ij} \sim B_T^2 \left( \frac{M_T}{\lambda \langle H_2 \rangle^2} \right)^2 \left[ U(m_\nu^D)^2 U^\dagger \right]_{ij}.
\end{align}

Consequently, the relative size of LFV in the different leptonic families can be univocally
predicted as:

\[
\frac{(m_L^2)_{\tau\mu}}{(m_L^2)_{\mu e}} \approx \rho \frac{s_{23} c_{23}}{s_{12} c_{12} c_{23}} \sim 40, \quad \frac{(m_L^2)_{\tau e}}{(m_L^2)_{\mu e}} \approx - \frac{s_{23}}{c_{23}} \sim -1,
\]

where \(\rho = (m_3/m_2)^2\), \(\theta_{12}\) and \(\theta_{23}\) are lepton mixing angles and \(\theta_{13} = 0\) is taken (the notation \(c_{ij} = \cos \theta_{ij}\ldots\) is used). A hierarchical neutrino mass spectrum is considered and the best-fit values for the parameters are used. By taking the present upper limit on \(\sin \theta_{13} = 0.2\), the above ratios become 3 and 0.8, respectively, while varying the other neutrino parameters within their experimental range affect these ratios by less than 10\% (see also [8]). The above relations imply that also the branching ratios (BR) of LFV processes such as the decays \(\ell_i \rightarrow \ell_j \gamma\) can be predicted

\[
\frac{\text{BR}(\tau \rightarrow \mu \gamma)}{\text{BR}(\mu \rightarrow e \gamma)} \sim 300, \quad \frac{\text{BR}(\tau \rightarrow e \gamma)}{\text{BR}(\mu \rightarrow e \gamma)} \sim 10^{-1}.
\]

Other LFV processes and related correlations [12] will be considered in [6]. (Connections between neutrino parameters and other observables can arise also in different scenarios, see e.g. [13]). Without loss of generality we take \(B_T\) to be real since its phase has not physical effect. However, a different approach was considered in [14] where a complex \(B_T\) could generate sizable electric dipole moments for quarks and leptons since there was a relative phase between the trilinear couplings \(A_{e,d,u}\) shown in eq. (5) and the gaugino masses. Moreover, the soft term \(B_T\) could play a significant role in generating the baryon asymmetry of the Universe in the context of resonant leptogenesis [15].

We shall now discuss the phenomenological viability of our scenario. Throughout our discussion we shall take \(M_T > 10^7\) GeV so that the gauge coupling constants remain perturbative up to \(M_G\). Our approach follows a bottom-up perspective where, for a given ratio \(M_T/\lambda\) and \(\tan \beta\), \(Y_T\) is determined at \(M_T\) according to the matching expressed by eq. (3) using the low-energy neutrino parameters. The Yukawa matrices \(Y_e, Y_u, Y_d\) are determined by the related charged fermion masses, modulo \(\tan \beta\). Although the \(\mu\)-parameter is not predicted by the underlying theory, it is nevertheless determined together with \(\tan \beta\) by correct EWSB conditions. Therefore, we end up with only three free parameters, \(B_T, M_T\) and \(\lambda\). In Fig. 1 we show the \((\lambda, M_T)\) parameter space allowed by the perturbativity and EWSB requirements, the experimental lower bound on the lightest Higgs mass\(^3\) \(m_h\) and the upper bound on \(BR(\mu \rightarrow e \gamma)\), for \(B_T = 20 (50)\) TeV in the

\(^3\)We include the low-energy radiative corrections to the Higgs masses by linking our code to \textsc{FeynHiggs} [16].
Figure 1: The $(\lambda, M_T)$ parameter space constrained by the perturbativity requirement (light-grey), correct EWSB from one-loop corrected Higgs potential, lower bound on the lightest Higgs boson mass $m_h$ and the upper bound on $BR(\mu \to e\gamma)$, for $B_T = 20(50)$ TeV in the upper (lower) panel. We also display the isocontours of $\tan\beta$ (solid) and $\mu$ (dashed). We have taken the top pole mass $m_t = 174$ GeV.
upper (lower) panel. First notice the light-grey regions excluded by the perturbativity requirement which are independent of $B_T$. For each value of $M_T$ there is a minimum value of $\lambda$, which scales as $\sim 2 \times 10^{-4}(M_T/10^{11} \text{ GeV})$, below which the couplings $Y_T$ reach the Landau pole below $M_G$. Similarly, there is a maximum value of $\lambda$ beyond which $\lambda$ itself blows up below $M_G$. The EWSB constraint excludes a region for $\lambda \sim 1 - 1.2$ and $M_T \gtrsim 10^{12} \text{ GeV}$ (independently of $B_T$) which is limited by the least achievable value of $\tan \beta$, $\tan \beta \sim 2.5$. As for the $\mu$-parameter (dashed lines), it slightly increases with increasing $M_T$ due to the large RG factor which affects $m^2_{H_d}(\mu_{\text{SUSY}})$ in the minimization condition, $\mu^2(\mu_{\text{SUSY}}) \approx -m^2_{H_d}(\mu_{\text{SUSY}})$, covering the range $\mu \sim 450 - 550 (1000 - 1200) \text{ GeV}$ for $B_T = 20 (50) \text{ TeV}$. We observe that $\lambda < 0.6 (0.7)$ for $B_T = 20 (50) \text{ TeV}$ is required by the constraint $m_h > 110 \text{ GeV}$. The related contour lies on the correspondent minimum value of $\tan \beta \sim 5 (3.5)$ for $B_T = 20 (50) \text{ TeV}$. When $B_T = 50 \text{ TeV}$, the sparticle spectrum is heavier and thus the radiative corrections $\sim \log(\mu_{\text{SUSY}}/m_r)$ to $m_h$ are larger and in the tree-level contribution $\sim M_Z |\cos 2\beta|$ smaller $\tan \beta$ can be tolerated.

The present bound on $BR(\mu \to e\gamma)$ provides a lower bound on $\lambda$ for each value of $M_T$. This stems from the fact that the LFV entries $(m^2_{\tilde{L}})_{ij}$ scale as $(M_T/\lambda)^2$ [eq. (6)]. Consequently, the allowed $\lambda$-range is wider for lower values of $M_T$ and, comparing the two panels, the whole parameter space is larger for $B_T = 50 \text{ TeV}$. In the allowed regions, the lightest MSSM sparticle is typically a charged slepton with mass around $100 - 200 (300 - 450) \text{ GeV}$ for $B_T = 20 (50) \text{ TeV}$, although for small $\tan \beta$ there could be a mass degeneracy with the lightest neutralino. However, either the lightest slepton or neutralino would decay into the gravitino which is most likely the lightest supersymmetric particle in our framework. Finally, we have checked that values of $B_T < 10 \text{ TeV}$ are phenomenologically unacceptable.

In Fig. 2 we display the branching ratios $BR(\ell_j \to \ell_i \gamma)$ as a function of $\lambda$ for $B_T = 20 \text{ TeV}$ and $M_T = 10^{13} (10^9) \text{ GeV}$ in the left (right) panel. The behaviour of these branching ratios is in remarkable agreement with the estimates of eq. (9). Hence, the relative size of LFV does not depend on the detail of the model, such as the values of $\lambda$, $B_T$, or $M_T$. This feature is not present for a very narrow range of $\lambda$ where $BR(\tau \to \mu \gamma)$ is strongly suppressed due to a conspiracy of the various contributions in $(m^2_{\tilde{L}})_{\tau \mu}$ which mutually cancel out [see eq. (6)].
The BRs of the lepton radiative decays are shown as a function of $\lambda$ for $B_T = 20$ TeV and for $M_T = 10^{13}(10^9)$ GeV in the left (right) panel. The horizontal lines indicate the present bound on each BR \cite{17}.

Before concluding, we would like to briefly mention that the tree-level exchange of the $T, \bar{T}$ states also generates the $L$-violating SSB operator $\lambda Y_T B_T (\bar{L} H_2)^2/M_T$ which induces a sneutrino/anti-sneutrino mass splitting $\Delta m_\tilde{\nu}^2 = B_T m_\nu$ at the EW scale. Since $B_T$ is much larger than the EW scale, we are led to think that this could render interesting effects for the phenomenology of sneutrino oscillations \cite{18}.

In conclusion, we have suggested a unified picture of the supersymmetric type-II seesaw where the triplets, besides being responsible for neutrino mass generation, communicate SUSY breaking to the observable sector through gauge and Yukawa interactions. We have performed a phenomenological analysis of the allowed parameter space emphasizing the role of LFV processes in testing our framework.

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References


