Pseudovector meson with strangeness and closed-charm

Ting-Wai Chiu\textsuperscript{1}, Tung-Han Hsieh\textsuperscript{2}

\textsuperscript{1}Department of Physics, National Taiwan University, Taipei, 10617, Taiwan
\textsuperscript{2}Physics Section, Commission of General Education, National United University, Miao-Li, 36003, Taiwan

(TWQCD Collaboration)

Abstract

We investigate the mass spectrum of $1^+$ exotic mesons with quark content $(c\bar{s}\bar{q})/(cq\bar{s})$, using molecular and diquark-antidiquark operators, in quenched lattice QCD with exact chiral symmetry. For the molecular operator $\{(\bar{q}\gamma_i c)(\bar{c}\gamma_5 s) - (\bar{c}\gamma_i s)(\bar{q}\gamma_5 c)\}$ and the diquark-antidiquark operator $\{(\bar{q}^T C\gamma_i c)(\bar{s} C\gamma_5 \bar{c}^T) - (\bar{q}^T C\gamma_5 c)(\bar{s} C\gamma_i \bar{c}^T)\}$, both detect a $1^+$ resonance with mass around $4010 \pm 50$ MeV in the limit $m_q \to m_{u,d}$.

PACS numbers: 11.15.Ha, 11.30.Rd, 12.38.Gc, 14.40.Lb, 14.40.Gx
Keywords: Lattice QCD, Exact Chiral Symmetry, Exotic mesons, Charmed Mesons, Diquarks
1 Introduction

Since the discovery of $D_s(2317)$ [1] by BABAR in April 2003, a series of new heavy mesons\footnote{For recent reviews of these new heavy mesons, see, for example, Ref. [2], and references therein.} with open-charm and closed-charm have been observed by Belle, CDF, CLEO, BABAR, and BES. Among these new heavy mesons, the most intriguing ones are the charmonium-like states, $X(3872)$ [3], $Y(3940)$ [4], $Y(4260)$ [5], $Z(3930)$ [6], and $X(3940)$ [7]. Evidently, one can hardly interpret all of them as orbital and/or radial excitations in the charmonium spectrum. Thus it is likely that some of them are exotic (non-$q\bar{q}$) mesons (e.g., molecule, diquark-antidiquark, and hybrid meson). Theoretically, the central question is whether the spectrum of QCD possesses these resonances, with the correct quantum numbers, masses, and decay widths.

Recently we have investigated the mass spectrum of closed-charm exotic mesons with $J^{PC}=1^{--}$ [8], and $1^{++}$ [9], in lattice QCD with exact chiral symmetry [10, 11, 12, 13, 14]. By constructing molecular and diquark-antidiquark operators with quark content $(c\bar{c}q\bar{q})$, we measured their time-correlation functions, and extracted the mass spectrum of the hadronic states overlapping with these exotic meson operators. Our results suggest that $Y(4260)$ and $X(3872)$ are in the spectrum of QCD, with $J^{PC}=1^{--}$ and $1^{++}$ respectively, and both with quark content $(c\bar{u}c\bar{u})$. Note that we have been working in the isospin limit (with $m_u=m_d$), thus our results [8, 9] also imply the existence of exotic mesons with quark content $(c\bar{d}c\bar{d})$, even though we cannot determine their mass differences from those with $(c\bar{u}c\bar{u})$. Moreover, we also observe heavier exotic mesons with quark contents $(c\bar{s}c\bar{q})$ and $(c\bar{c}c\bar{c})$, for $J^{PC}=1^{++}$ [9], and $1^{--}$ [8].

Now if the spectrum of QCD does possess exotic mesons with quark content $(c\bar{q}c\bar{q})$, then it is likely that there are also exotic mesons with other quark contents, e.g., $(c\bar{q}c\bar{s})$ and $(c\bar{c}c\bar{s})$. However, in general, whether any combination of two quarks and two antiquarks can emerge as a hadronic state relies on the nonperturbative dynamics between these four quarks.

In this paper, we investigate the masses of the lowest-lying states (with $J^P=1^{+}$) of molecular and diquark-antidiquark operators with quark content $(c\bar{s}c\bar{q})$ or $(c\bar{q}c\bar{s})$. For two lattice volumes $24^3 \times 48$ and $20^3 \times 40$, each of 100 gauge configurations generated with single-plaquette action at $\beta=6.1$, we compute point-to-point quark propagators for 30 quark masses in the range $0.03 \leq m_q a \leq 0.80$, and measure the time-correlation functions of these exotic meson operators. The inverse lattice spacing $a^{-1}$ is determined with the experimental input of $f_\pi$. The strange quark bare mass $m_s a = 0.08$, and the charm quark bare mass $m_c a = 0.80$ are fixed such that the masses of the corresponding vector mesons are in good agreement with $\phi(1020)$ and $J/\psi(3097)$ respectively [15, 16].
2 The Molecular Operator

In general, one can construct many different molecular operators with quark content \((cq\bar{c}\bar{s})\) or \((cs\bar{c}\bar{q})\) such that the lowest-lying state of each operator has \(J^P = 1^+\). However, not every one of them has good overlap with the lowest-lying hadronic state having the same quark content and quantum numbers. In the following we only present one of the good molecular operators \(^2\). Explicitly,

\[
M = \frac{1}{\sqrt{2}} \{(\bar{q}\gamma_i c)(\bar{c}\gamma_5 s) - (\bar{c}\gamma_i s)(\bar{q}\gamma_5 c)\} \tag{1}
\]

![Figure 1: The ratio of spectral weights of the lowest-lying state of the molecular operator \(M\), for \(20^3 \times 40\) and \(24^3 \times 48\) lattices at \(\beta = 6.1\). The upper-horizontal line \(R = (24/20)^3 = 1.728\), is the signature of 2-particle scattering state, while the lower-horizontal line \(R = 1.0\) is the signature of a resonance.](image)

We measure the time-correlation function,

\[
C(t) = \sum _\beta \langle M(\vec{x},t)M^\dagger(\vec{0},0)\rangle
\]

\(^2\)The operators we have investigated include \((\bar{q}\gamma_i c)(\bar{c}\gamma_5 s)\), \((\bar{c}\gamma_i s)(\bar{q}\gamma_5 c)\), and \([(\bar{q}\gamma_i c)(\bar{c}\gamma_5 s) \pm (\bar{c}\gamma_i s)(\bar{q}\gamma_5 c)\)], and they all yield compatible masses for the lowest-lying meson.
Figure 2: The mass of the lowest-lying state of $M$ versus the quark mass $m_q a$, on the $24^3 \times 48$ lattice at $\beta = 6.1$. The solid line is the linear fit.

where the $c \bar{c}$ annihilation diagrams are neglected such that $C(t)$ does not overlap with any conventional meson states. Also, $C(t)$ is averaged over $C_i$ (with $\gamma_i$) for $i = 1, 2, 3$, where in each case, the “forward-propagator” $C_i(t)$ and “backward-propagator” $C_i(T - t)$ are averaged to increase the statistics. The same strategy is applied to all time-correlation functions in this paper. Then the average of $C(t)$ over all gauge configurations is fitted to the usual formula

$$\frac{Z}{2ma} [e^{-mat} + e^{-ma(T-t)}]$$

to extract the mass $ma$ of the lowest-lying state and its spectral weight

$$W = \frac{Z}{2ma}.$$
In the following, we shall use the ratio of the spectral weights on two spatial volumes $20^3$ and $24^3$ with the same lattice spacing ($\beta = 6.1$) to discriminate whether any hadronic state under investigation is a resonance or not.

In Fig. 1, the ratio ($R = W_{20}/W_{24}$) of spectral weights of the lowest-lying state extracted from the time-correlation function of $M$ on the $20^3 \times 40$ and $24^3 \times 48$ lattices is plotted versus the quark mass $m_q a \in [0.03, 0.80]$. (Here $q$ ($\bar{q}$) is always taken to be different from $c$ ($\bar{c}$) and $s$ ($\bar{s}$), even in the limit $m_q \to m_c$ or $m_s$). Evidently, $R \simeq 1.0$ for the entire range of quark masses, which implies that there exist $J^P = 1^+$ resonances, with quark contents $(csc\bar{u})$ and $(cesd)$ respectively.

In Fig. 2, the mass of the lowest-lying state extracted from the molecular operator $M$ is plotted versus $m_q a$. In the limit $m_q \to m_{u,d} \simeq 0.00265 a^{-1}$ (corresponding to $m_\pi = 135$ MeV), it gives $m = 4007(34)$ MeV.

### 3 The Diquark-Antidiquark Operator

In general, one can construct many different diquark-antiquark operators with quark content $(cq\bar{c}s)$ or $(csc\bar{q})$ such that the lowest-lying state of each operator has $J^P = 1^+$. However, not every one of them has good overlap with the lowest-lying hadronic state having the same quark content and quantum numbers. In the following we only present one of the good diquark-antidiquark operators. Explicitly,

$$D_4(x) = \frac{1}{\sqrt{2}} \left\{ (q^T C \gamma_5 c)_xa (\bar{s} C \gamma_5 \bar{c}^T)_xa - (\bar{s} C \gamma_i^T \bar{c}^T)_xa (q^T C \gamma_5 c)_xa \right\}$$

(2)

where $C$ is the charge conjugation operator satisfying $C\gamma_\mu C^{-1} = -\gamma_\mu^T$ and $(C\gamma_5)^T = -C\gamma_5$. Here the “diquark” operator $(q^T \Gamma Q)_xa$ for any Dirac matrix $\Gamma$ is defined as

$$(q^T \Gamma Q)_xa \equiv \epsilon_{abc} q_{r\alpha} \Gamma_{\alpha\beta} Q_{\beta\bar{c}}$$

(3)

where $x$, $\{a,b,c\}$ and $\{\alpha, \beta\}$ denote the lattice site, color, and Dirac indices respectively, and $\epsilon_{abc}$ is the completely antisymmetric tensor. Thus the diquark (3) transforms like color anti-triplet. For $\Gamma = C\gamma_5$, it transforms like $J^P = 0^+$, while for $\Gamma = C\gamma_i$ ($i = 1, 2, 3$), it transforms like $1^+$.

In Fig. 3, the ratio ($R = W_{20}/W_{24}$) of spectral weights of the lowest-lying state extracted from the time-correlation function of $D_4$ on the $20^3 \times 40$ and $24^3 \times 48$ lattices is plotted versus the quark mass $m_q a \in [0.03, 0.8]$. Evidently, $R \simeq 1.0$ for the entire range of quark masses, which implies that there exist $J^P = 1^+$ resonances, with quark contents $(cd\bar{c}s)$ and $(cu\bar{c}s)$ respectively.

In Fig. 4, the mass of the lowest-lying state of the diquark-antidiquark operator $D_4$ is plotted versus $m_q a$. In the limit $m_q \to m_{u,d}$, it gives $m = 4015(25)$ MeV.

4
\[ R = \frac{W_{20}}{W_{24}} \]

Figure 3: The ratio of spectral weights of the lowest-lying state of diquark-antidiquark operator \( D_4 \), for \( 20^3 \times 40 \) and \( 24^3 \times 48 \) lattices at \( \beta = 6.1 \). The upper-horizontal line \( R = (24/20)^3 = 1.728 \), is the signature of 2-particle scattering state, while the lower-horizontal line \( R = 1.0 \) is the signature of a resonance.

4 Summary and Discussions

In this paper, we have investigated the mass spectra of the molecular operator \( M \) and the diquark-antidiquark operator \( D_4 \), in quenched lattice QCD with exact chiral symmetry. Our results for \( M \) and \( D_4 \) are summarized in Table 1, where in each case, the first error is statistical, and the second one is our crude estimate of combined systematic uncertainty.

Evidently, both the molecular operator \( M \) and the diquark-antidiquark operator \( D_4 \) detect a \( 1^+ \) resonance around \( 4010 \pm 50 \) MeV in the limit \( m_q \to m_{u,d} \). Since its mass is just slightly above \( Y(3940) \), high energy experiments should be able to see whether such a resonance, say, \( X_s \), exists in some decay channels, e.g., \( X_s \to K\pi J/\Psi \), in the near future.

Note that in our calculations, we have omitted all internal quark loops generated by vacuum fluctuations (i.e., the quenched approximation). However, we expect that the quenched approximation only results a few percent
systematic error in the mass spectrum, especially for charmed mesons [16]. If it turns out that high energy experiments do not find any charmonium-like resonances around 3960-4060 MeV, in particular, in the $K\pi J/\Psi$ channel, then it might imply that quenched QCD could be dramatically different from the unquenched QCD, even for the mass spectrum.

**Acknowledgement**

We thank Steve Olsen for his question regarding ($c\bar{u}\bar{c}\bar{s}$), which motivated our present study. This work was supported in part by the National Science Council, Republic of China, under the Grant No. NSC94-2112-M002-016 (T.W.C.), and Grant No. NSC94-2119-M239-001 (T.H.H.), and by the National Center for High Performance Computation at Hsinchu, and the Computer Center at National Taiwan University.
Table 1: Masses of the lowest-lying states \((J^P = 1^+)\) of the molecular operator \(M\) and the diquark-antidiquark operator \(D_4\). The last column \(R/S\) denotes resonance (R) or scattering (S) state.

<table>
<thead>
<tr>
<th>Operator</th>
<th>Mass (MeV)</th>
<th>R/S</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{\sqrt{2}}[(\bar{u}\gamma_5 c)(\bar{c}\gamma_5 s) - (\bar{c}\gamma_5 s)(\bar{u}\gamma_5 c)])</td>
<td>4007(34)(31)</td>
<td>R</td>
</tr>
<tr>
<td>(\frac{1}{\sqrt{2}}{(\bar{u}^T C\gamma_5 c)(\bar{s}C\gamma_5 c^T) - (\bar{u}^T C\gamma_5 c)(\bar{s}C\gamma_5 c^T)})</td>
<td>4015(25)(27)</td>
<td>R</td>
</tr>
</tbody>
</table>

References


