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A B S T R A C T

The problem of the compatibility of current algebra identities with the description of semi-weak scattering processes of currents on pions in terms of Regge poles or Veneziano models is investigated. Decomposing the scattering process into appropriate invariants and scalar amplitudes, it is shown that in the case of the commutator of axial currents it is possible to find a set of independent scalar amplitudes for which Regge poles in both channels or Veneziano model can be assumed, and which determines the remaining amplitudes and the electromagnetic form factor. As a result, fixed poles appear where they are required, and the asymptotic behaviour of the pion electromagnetic form factor is found to be of the form $t^{\alpha_s(s=m_\pi^2)-n}$, where $\alpha_s(s)$ is the trajectory exchanged in the channel of the incident current on the pion (i.e., the ρ trajectory), s is the corresponding Mandelstam variable, m_π is the pion mass and n is an integer. The dependence of the sum rule on the invariant masses associated to the currents is also investigated, and the consistency of the approach is checked by considering also the commutator of two-vector currents, which is found to give no new information on the form factor.

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INTRODUCTION

We reconsider the problem of exploiting the information contained in current algebra identities and, in particular, of their consistency with present ideas on the structure of scattering amplitudes, such as Regge theory, fixed poles, and Veneziano models. The current algebra identities consist in an equality between a form factor and a combination of invariant amplitudes describing the "scattering" of currents on hadrons. From the knowledge of the properties of the scattering amplitudes, we can learn in principle the properties of the form factors. The scattering amplitudes of currents on hadrons will be in the first approximation described by means of the same ingredients which appear in strong interactions, for instance by the same resonances and by similar analytic properties : in particular, we expect that the analytic structure in the complex angular momentum plane will be essentially the same apart from fixed poles. It is then natural to expect that the asymptotic behaviour of the form factors will be related to the singularities in the complex angular momentum plane of the scattering amplitudes. Moreover, if the asymptotic behaviour of the scattering amplitudes arises from a coherent sum of an infinite number of resonances, all those resonances contribute to the form factor and in a narrow width approximation it will be represented as a series of an infinite number of poles.

In general, as the currents carry spin, the "scattering" amplitude is decomposed in terms of several scalar amplitudes. These scalar amplitudes are not a priori independent, there are constraints among their imaginary parts which follow from current algebra itself.

The aim of this paper is to investigate the possibility of assuming a Regge description of these scalar amplitudes, allowing for occurrence of fixed poles only where they are required, and satisfying the constraints without introducing kinematical singularities. We show that it is indeed possible to find a set of scalar amplitudes, free from kinematical singularities or constraints, for which a pure Regge model can be assumed.

In the first Section, we consider the commutator of two axial currents between pion states. We find that a pure Regge behaviour for the invariant amplitudes which are not required to have fixed poles

determines the behaviour of the form factor. If these amplitudes behave like $t^{\alpha_p(s)}$, where t is the momentum transfer between the pions, s is the squared CMS energy of the "scattering" process, $\alpha_p(s)$ is the rho trajectory, the pion form factor has the behaviour $t^{\alpha_p(m_\pi^2)-n}$, where m_π is the pion mass, $n \geq 1$ a positive integer whose actual value depends on models.

More generally, we find that the form factor is given by a combination of scattering amplitudes, depending on s, t or u, t , with s and u frozen at the values $s = m_\pi^2$, $u = m_\pi^2$.

According to the present knowledge of Regge trajectories and the considerations of Lovelace¹⁾ and Ademollo, Veneziano and Weinberg²⁾, $\alpha_p(m_\pi^2) = \frac{1}{2}$ and we obtain for the form factor a half-integer power behaviour in the momentum transfer. This feature is allowed by the above-mentioned occurrence of an infinite number of poles contributing to the form factor (it is obvious that a finite number of poles will give anyhow an integer power behaviour).

We investigate also the dependence of current algebra identities on the invariant masses associated to the currents. We show a simple mechanism, in the narrow width approximation, for getting rid of the unwanted singularities on these masses. (This subject is completed in Section 2.) We show further, how a fixed pole contribution naturally appears in the invariant amplitude in which it is required, and how it disappears in the residue at the poles of the masses associated to the currents, corresponding to strong interaction processes.

In Section 3, the previously discussed features are verified in an explicit example, namely the Veneziano model. In the case of absence of satellites, the electromagnetic form factor of the pion is found to behave for large t like $t^{-\frac{3}{2}}$. We also reconsider in this model the Fubini-Furlan-Rossetti³⁾ sum rule and show how in this case the AVW²⁾ condition ensures the occurrence in the form factor of the correct singularities only.

In the last Section, we show the internal consistency of our approach to current algebra identities, by using the method of Section 1 for the commutator of two vector currents. In this case the current

algebra identity turns out to be automatically satisfied, once gauge invariance has been imposed on the related strong interaction processes. No new information on the structure of the electromagnetic form factor is obtained (and so, obviously, there are no contradictions with the result of Section 1).

1. We consider the commutator of two axial currents giving a vector current, sandwiched between two pion states. Following standard techniques, we introduce the "scattering amplitude" $M_{\mu\nu}$ of axial currents on pions, visualized on Fig 1 :

i.e.,

$$A_\mu(k) + \pi(p_1) \longrightarrow A_\nu(q) + \pi(p_2)$$

We introduce the following kinematics :

$$\begin{aligned}
 p_1 + k &= p_2 + q ; & P &\equiv p_1 + p_2 \\
 p_1^2 &= p_2^2 = m_\pi^2 \\
 s &= (p_1 + k)^2, & t &= (p_1 - p_2)^2, & u &= (p_1 - q)^2 \\
 \Sigma &= 2m_\pi^2 + q^2 + k^2 ; & s + t + u &= \Sigma.
 \end{aligned}
 \tag{1}$$

At the same time, we consider the amplitude X_ν of the related process in which we have a virtual pion at the place of the incoming axial current of momentum k . This "scattering" is visualized in Fig. 2

Finally, we define the matrix element of the vector current between two pions to be

$$V_\nu = F(t) P_\nu \quad . \quad (2)$$

The current algebra identity then reads :

$$k_\mu M_{\mu\nu} - X_\nu = V_\nu + (\text{Schwinger terms}) \quad (3)$$

In order to exploit the content of Eq. (3), we have to decompose $M_{\mu\nu}$ and X_ν into invariants. We are interested in deriving an equation involving the form factor, hence the index ν will be carried in the relevant invariants by the vector P_ν only (invariants containing k_ν, q_ν are related to Schwinger terms).

We will write X_ν as

$$X_\nu = X(s, t, u, q^2, k^2) P_\nu + X_1(s, t, u, q^2, k^2) q_\nu + X_2(s, t, u, q^2, k^2) k_\nu \quad (4)$$

and we will be interested in the scalar amplitude X only. In $M_{\mu\nu}$ the index μ can be carried by any combination of the vectors p_1, p_2, k, q , each combination corresponding to an equivalent way of decomposing into invariants. One could write, for instance :

$$M_{\mu\nu} = A(s, t, u, q^2, k^2) P_\mu P_\nu + B(s, t, u, q^2, k^2) q_\mu P_\nu + C(s, t, u, q^2, k^2) k_\mu P_\nu \\ + (\text{terms without } P_\nu) \quad (5)$$

so that, from Eq. (3), we obtain

$$(s - m_\pi^2 + \frac{1}{2}t - \frac{1}{2}q^2 - \frac{1}{2}k^2) A(s, t, u, q^2, k^2) - \frac{1}{2}(t - q^2 - k^2) B(s, t, u, q^2, k^2) + k^2 C(s, t, u, q^2, k^2) \\ - X(s, t, u, q^2, k^2) = F(t). \quad (6)$$

We recall that although each term of the left-hand side of Eq. (6), separately, depends on s, k^2, q^2 , the right-hand side is independent of these variables or, in other words, at given t , the identity is valid for any value of s (or u), k^2, q^2 . For simplicity, we take $k^2 = 0$, and we will omit k^2 from the arguments of the scalar amplitudes. (In Section 2 we shall sketch how to deal with the current algebra for $k^2 \neq 0$.) It is the aim of the present paper to investigate if the current algebra identity can be satisfied by assuming for the scalar amplitudes analytic properties in the complex angular momentum plane which are as similar as possible to those of strong interaction amplitudes.

It is not possible to assume just the same analytic properties as for strong interaction processes for all the scalar amplitudes of $M_{\mu\nu}$. For instance, it is well known that A must have a fixed pole ⁴⁾ in the angular momentum plane in the t channel. We want to see if there are other scalar amplitudes for which a pure Regge behaviour, i.e., moving singularities only, can be assumed. Fubini has shown that in the hypothesis of unsubtracted dispersion relations it is possible to derive from Eq. (6) the sum rule ⁵⁾

$$\frac{1}{2\pi i} \left[\int ds D_s A(s, t, \Sigma - s - t, q^2) - \int du D_u A(\Sigma - t - u, t, u, q^2) \right] = F(t) \quad (7)$$

where $D_s A$ ($D_u A$) is the s (u) discontinuity of A at fixed t . The discontinuities of A with respect to s, u are related to the discontinuities of the other scalar amplitudes, as it is clear from taking the discontinuity of the current algebra identity Eq. (3). With the decomposition of Eq. (5), we find for instance

$$D_s A(s, t, \Sigma - s - t, q^2) = \frac{\frac{1}{2}(t - q^2) D_s B(s, t, \Sigma - s - t, q^2) + D_s X(s, t, \Sigma - s - t, q^2)}{s - m_\pi^2 + \frac{1}{2}(t - q^2)} \quad (8)$$

Equation (8) shows a zero in the denominator [which would mean a kinematical singularity in $(t - q^2)$ at fixed s] which must be compensated by a corresponding zero in the numerator, implying an intimate correlation between $D_s B$ and $D_s X$. This suggests that the decomposition of Eq. (6) is not the most convenient one in order to do simple hypotheses on the analytic structure of the scalar amplitudes.

Let us now decompose $M_{\mu\nu}$ as

$$M_{\mu\nu} = 2 A p_{1\mu} p_{1\nu} + B_1 q_\mu q_\nu + C_1 k_\mu k_\nu + \dots \quad (9)$$

By comparison with Eq. (5), we have

$$\begin{aligned} A &= A, \text{ (unchanged)} \\ B_1 &= B - A, \\ C_1 &= C - A. \end{aligned} \quad (10)$$

In this case at $k^2 = 0$ Eqs. (6) and (8) become

$$(s - m_\pi^2) A(s, t, u, q^2) - \frac{1}{2}(t - q^2) B_1(s, t, u, q^2) - X(s, t, u, q^2) = F(t) \quad (11)$$

$$D_s A(s, t, \Sigma - s - t, q^2) = \frac{\frac{1}{2}(t - q^2) D_s B_1(s, t, \Sigma - s - t, q^2) + D_s X(s, t, \Sigma - s - t, q^2)}{s - m_\pi^2} \quad (12)$$

As all the discontinuities in s at fixed t are zero for s below a certain threshold $s_0 < m_\pi^2$, Eq. (12) does not show kinematical singularities and can be used for evaluating $D_s A$, once any reasonable model for $D_s B_1$ and $D_s X$ is given.

We want to generalize this argument for studying the discontinuities in the other variables. In general, let us assume that any scalar amplitude $f(s, t, u)$ can be written as

$$f(s, t, u) = f^{st}(s, t) + f^{ut}(u, t) + f^{su}(s, u)$$

where, by assumption, $f^{st}(s, t)$ has only the first Mandelstam spectral function (i.e., dynamical singularities for positive s, t only), etc.

Terms like $f^{su}(s, u)$, which possess only the third spectral function, can give at most a constant and contribute to the sum rule. This can be easily seen from Eq. (7). Write

$$A(s, t, u, q^2) = A^{st}(s, t, q^2) + A^{ut}(u, t, q^2) + A^{su}(s, u, q^2) \quad (13)$$

The discontinuity for $t < 0$ of the left-hand side of Eq. (7) is given by

$$\int ds D_u D_s A^{su}(s, \Sigma - s - t, q^2) - \int du D_s D_u A^{su}(\Sigma - t - u, u, q^2)$$

which obviously vanishes. This ensures that the form factor has no discontinuity for $t < 0$. We consider separately the contribution to the sum rule from scalar amplitudes of type $f^{st}(s, t)$, $f^{ut}(u, t)$, $f^{su}(s, u)$.

For the contribution of type $f^{st}(s, t)$ we will write $M_{\mu\nu}$ as in Eq. (9) and we will interpret Eq. (12) as an evaluation of $D_s A^{st}$ in terms of the other scalar amplitudes B_1^{st} , X^{st} , for which simple models can be assumed. So we have

$$D_s A^{st}(s, t, q^2) = \frac{\frac{1}{2}(t - q^2) D_s B_1^{st}(s, t, q^2) + D_s X^{st}(s, t, q^2)}{s - m_\pi^2} \quad (14)$$

For the contribution of type $f^{ut}(u,t)$ we write $M_{\mu\nu}$ as

$$M_{\mu\nu} = 2 A p_{2\mu} P_\nu + B_2 q_\mu P_\nu + C_2 k_\mu P_\nu + \dots \quad (15)$$

By comparison with Eq. (5) we have

$$A = A \text{ (unchanged)}$$

$$B_2 = B + A, \quad (16)$$

$$C_2 = C + A,$$

so that at $k^2=0$, Eq. (6) becomes

$$-(u-m_\pi^2) A(s,t,u,q^2) - \frac{1}{2}(t-q^2) B(s,t,u,q^2) - X(s,t,u,q^2) = F(t) \quad (17)$$

Correspondingly we have

$$D_u A^{ut}(u,t,q^2) = - \frac{\frac{1}{2}(t-q^2) D_u B^{ut}(u,t,q^2) + D_u X^{ut}(u,t,q^2)}{u - m_\pi^2} \quad (18)$$

and any reasonable assumption on B_2^{ut} , X^{ut} gives an acceptable A^{ut} .

Finally the contribution of type $f^{su}(s,u)$, which contains the third spectral function, has been seen to give at most a constant contribution to the sum rule Eq. (7). We then impose for simplicity

$$k_\mu M_{\mu\nu}^{su} = 0 \quad (19)$$

which is equivalent to state that this constant is zero.

Decomposing in this case $M_{\mu\nu}$ as

$$M_{\mu\nu} = 2 A_1 p_{1\mu} P_\nu + 2 A_2 p_{2\mu} P_\nu + C_3 k_\mu P_\nu + \dots \quad (20)$$

where, by comparison with Eq. (5), we have

$$\begin{aligned} A_1 &= \frac{1}{2} (A+B), \\ A_2 &= \frac{1}{2} (A-B), \\ C_3 &= B+C, \end{aligned} \quad (21)$$

our prescription reads

$$(s-m_\pi^2) A_1^{su}(s,u,q^2) - (u-m_\pi^2) A_2^{su}(s,u,q^2) - X^{su}(s,u,q^2) = 0. \quad (22)$$

We note that Eq. (22) does not contain explicitly q^2 , so that the same equation holds for any of the strong scattering processes which correspond to the singularities in q^2 .

As already noted, knowledge of A^{su} is not needed for the sum rule Eq. (7). For completeness, we can take X^{su} as given by Eq. (22) in terms of arbitrarily chosen A_1^{su}, A_2^{su} .

Summing up, we were able to specify the whole scattering process in terms of an independent set of amplitudes free of kinematical constraint, namely $B_1^{st}(s,t), X^{st}(s,t), B_2^{ut}(u,t), X^{ut}(u,t), A_1^{su}(s,u), A_2^{su}(s,u)$, for which we can safely assume, for instance, a Regge pole or Veneziano model. In order to be more explicit we summarize our result for the particular decomposition of Eq. (5), giving the actual expression of the amplitudes $A(s,t,u,q^2)$ B, X. Writing A as in Eq. (13), we can evaluate $D_s A^{st}, D_u A^{ut}$ and then A^{st}, A^{ut} , from Eqs. (14) and (18) and from Eq. (21) we have

$$A^{su}(s,u,q^2) = A_1^{su}(s,u,q^2) + A_2^{su}(s,u,q^2) \quad (23)$$

writing B as

$$B(s,t,u,q^2) = B^{st}(s,t,q^2) + B^{ut}(u,t,q^2) + B^{su}(s,u,q^2) \quad (24)$$

we have

$$\begin{aligned}
 B^{st} &= B_1^{st} + A^{st} \\
 B^{ut} &= B_2^{ut} - A^{ut} \\
 B^{su} &= A_1^{su} - A_2^{su}
 \end{aligned} \tag{25}$$

from Eqs. (10), (16) and (21), respectively. Finally, writing

$$X^s(s, t, u, q^2) = X^{st}(s, t, q^2) + X^{ut}(u, t, q^2) + X^{su}(s, u, q^2) \tag{26}$$

we have X^{st} , X^{ut} as independent amplitudes, and X^{su} given by Eq. (22) in terms of A_1^{su} , A_2^{su} .

By using Eqs. (12), (15), and having in mind the dispersive representations

$$B_1^{st}(s, t, q^2) = \frac{1}{2\pi i} \int \frac{D_s B_1(s', t, q^2)}{s' - s} ds' \quad \text{etc.} \tag{27}$$

the sum rule Eq. (7) reads

$$\frac{1}{2}(t - q^2) \left[B_1^{st}(s=m_\pi^2, t, q^2) + B_2^{ut}(u=m_\pi^2, t, q^2) \right] + X^{st}(s=m_\pi^2, t, q^2) + X^{ut}(u=m_\pi^2, t, q^2) = F(t) \tag{28}$$

According to the previous discussion, there are no particular restrictions for the analytic properties, for instance in the complex angular momentum plane, of the amplitudes entering Eq. (28). We can then assume a pure Regge behaviour $f(q^2)(-t)^{\alpha_s(s)}$, at large t and fixed s, q^2 for $B_1^{st}(s, t, q^2)$, $X^{st}(s, t, q^2)$ and a similar behaviour $f'(q^2)(-t)^{\alpha_u(u)}$ at large t and fixed u for B_2^{ut} , X^{ut} , where $\alpha_s(s)$ and $\alpha_u(u)$ are the Regge trajectories in the s and u channel. From Eq. (28) we reach the conclusion that the asymptotic behaviour of the vector form factor of the pion is determined by the trajectories which can be exchanged in the s and u channels of the axial current pion scattering, for s and u "frozen" at the squared pion mass.

We recall here the result of Ademollo, Veneziano and Weinberg²⁾, which seems to be approximately valid in nature, that the relevant trajectories in the s and u channels for the processes, visualized in Figs. 1 and 2, are equal to half-integers at the mass squared of the external particle. Due to that result, the form factor will have a half-integer power behaviour in the momentum transfer t .

If further the trajectories in the s channel form a family of infinite parallel trajectories, the actual trajectory of the sequence which will determine the asymptotic behaviour is a matter of models, depending on the coupling of the trajectories to the particular amplitudes which are relevant to the sum rule. For large t we will then have

$$F(t) \simeq \beta_1 (-t)^{\alpha_s(m_\pi^2) - n_1} + \beta_2 (-t)^{\alpha_u(m_\pi^2) - n_2} \quad (29)$$

(the q^2 dependence, which has anyhow to disappear, will be discussed in the following). We note the presence in such an equation of $(-t)^{\alpha_s(s)}$ only and not of

$$(-t)^{\alpha_s} \pm (t)^{\alpha_s} = t^{\alpha_s} (e^{-i\pi\alpha_s} \pm 1)$$

as usual in Regge theory. The missing term t^{α_s} , which gives the signature, comes in fact from the part of the amplitude containing the third spectral function, which we have seen not to contribute to the sum rule.

From now on, for simplicity, we will be concerned only with the $f^{st}(s, t, q^2)$ type terms of the amplitudes entering the sum rule; the $f^{ut}(u, t, q^2)$ terms can be treated in the same way.

We discuss now the q^2 dependence of Eq. (28). The simplest way of getting rid of it is the hypothesis of unsubtracted dispersion relations in q^2 . Writing the q^2 singularity, as a sum of poles at certain masses μ_n^2 , we have

$$B_1^{st}(s=m_\pi^2, t, q^2) = \sum_n \frac{\bar{B}_n^{st}(s=m_\pi^2, t)}{q^2 - \mu_n^2} \quad (30)$$

$$X^{st}(s=m_\pi^2, t, q^2) = \sum_n \frac{\bar{X}_n^{st}(s=m_\pi^2, t)}{q^2 - \mu_n^2}$$

and the cancellation of the q^2 dependence on Eq. (28) follows from the condition on the q^2 singularity :

$$\frac{1}{2}(t - \mu_n^2) \bar{B}_n^{st}(s=m_\pi^2, t) + \bar{X}_n^{st}(s=m_\pi^2, t) = 0 . \quad (31)$$

The sum rule reads in this case

$$-\frac{1}{2} \sum_n \bar{B}_n^{st}(s=m_\pi^2, t) = F(t) \quad (32)$$

(cf., the form of the Fubini sum rule).

If again $\alpha_s(s)$ is the Regge trajectory in the s channel, the scalar amplitude B_1^{st} , and hence each $\bar{B}_n(s, t)$ behaves in general as $t^{\alpha_s(m_\pi^2)-1}$.

While, in general, the asymptotic behaviour of an infinite series can be different from the behaviour of the single terms, in our case the same behaviour is nevertheless to be expected as we assumed a Regge behaviour for the whole amplitude $B_1^{st}(s, t, q^2)$.

We note that Eq. (31), for $t = \mu_n^2$, reduces to

$$\bar{X}_n^{st}(s=m_\pi^2, t=\mu_n^2) = 0 \quad (33)$$

which is the Adler self-consistency condition for the scattering of a zero momentum pion on the initial pion to give a meson of mass μ_n

and the final pion. We note that in the general case in which terms with all the three spectral functions are present the Adler self-consistency condition would read

$$\bar{X}_n^{st}(s=m_\pi^2, t=\mu_n^2) + \bar{X}^{ut}(u=m_\pi^2, t=\mu_n^2) + \bar{X}^{su}(s=m_\pi^2, u=m_\pi^2) = 0 \quad (34)$$

In our model, as can be seen by going to the pole in q^2 in Eq. (22), we have that

$$\bar{X}^{su}(s=m_\pi^2, u=m_\pi^2) = 0 \quad (35)$$

holds separately.

It is easy to see that, even if B_1^{st} and X^{st} have a pure Regge behaviour in both channels, A^{st} exhibits a fixed pole at $J=1$ in the angular momentum plane in the t channel. For this purpose it is sufficient to write

$$A^{st}(s, t, q^2) = \frac{1}{2\pi i} \int \frac{D_s A^{st}(s', t, q^2)}{s' - s} ds' \quad (36)$$

and then to substitute Eq. (12), so that

$$\begin{aligned} A^{st}(s, t, q^2) &= \frac{1}{2\pi i} \int \frac{ds'}{s' - s} \frac{\frac{1}{2}(t - q^2) D_s B_1^{st}(s', t, q^2) + D_s X^{st}(s', t, q^2)}{s' - m_\pi^2} \\ &= \frac{1}{s - m_\pi^2} \left[\frac{1}{2}(t - q^2) B_1^{st}(s, t, q^2) + X^{st}(s, t, q^2) \right] \\ &\quad - \frac{1}{s - m_\pi^2} \left[\frac{1}{2}(t - q^2) B_1^{st}(s = m_\pi^2, t, q^2) + X^{st}(s = m_\pi^2, t, q^2) \right] \end{aligned} \quad (37)$$

which, by use of Eq. (28), is equivalent to Eq. (11). If we assume Regge behaviour for B_1 and X it is seen that, as $s \rightarrow \infty$, the first term goes in general like $s^{\alpha(t)-n}$ while the second goes like $1/s$, i.e., corresponds to a fixed pole in the complex J plane of the t channel. The second term, on the other hand, does not depend on q^2 [Eq. (28)], so that the fixed pole disappears in the residue at the q^2 poles, corresponding to a strong and pure Regge process. It is also immediate to see from Eq. (37) that A does not possess the pole in the t channel ($D_s A \sim s^{\alpha(t)-2}$) as expected for a double spin flip amplitude.

2. A few words now on the k^2 dependence of Eq. (3). The k^2 singularities can be ascribed to pseudoscalar or axial vector particles. The pseudoscalar particles can contribute to the amplitude C only [see Eq. (5)]. The pion pole is cancelled by the counter-term X , as follows from the PCAC hypothesis. If other pseudoscalar particles of higher masses enter the game, they can be eliminated in the same way by means of a suitable generalization of the PCAC hypothesis.

The contribution of an axial vector particle, say the A_1 meson, to the full $M_{\mu\nu}$ amplitude will be of the form

$$M_{\mu\nu} \simeq \frac{\delta_{\mu\sigma} - \frac{k_\mu k_\sigma}{m_{A_1}^2}}{k^2 - m_{A_1}^2} \bar{M}_{\sigma\nu} \quad (38)$$

where $\bar{M}_{\sigma\nu}$ corresponds to the "scattering" of the A_1 on the initial pion to give an axial current and the final pion, and is regular at $k^2 = m_{A_1}^2$. We have

$$k_\mu M_{\mu\nu} = - \frac{1}{m_{A_1}^2} k_\mu \bar{M}_{\mu\nu}$$

so that Eq. (6) reads

$$- \frac{1}{m_{A_1}^2} \left[(s - m_\pi^2 + \frac{1}{2}t - \frac{1}{2}q^2 - \frac{1}{2}k^2) \bar{A}(s, t, u, q^2, k^2) - \frac{1}{2}(t - q^2 - k^2) \bar{B}(s, t, u, q^2, k^2) + k^2 \bar{C}(s, t, u, q^2, k^2) \right] - X(s, t, u, q^2, k^2) = F(t) \quad (39)$$

The way in which the residual k^2 dependence will be cancelled is a matter of models, i.e., it depends on the analytic continuation of Eq. (38) for $k^2 \neq m_{A_1}^2$. In the case of unsubtracted dispersion relations, $\bar{A}$, $\bar{B}$, etc., do not depend on k^2 (they are taken at $k^2 = m_{A_1}^2$) and then, once Eq. (31) has been imposed, we have simply the condition :

$$\frac{1}{2} \bar{A} + \frac{1}{2} \bar{B} - \bar{C} = 0$$

and we recover formally the situation for $k^2 = 0$ which has been discussed previously.

3. In order to show how the above considerations apply to a specific model, we consider Veneziano models ⁶⁾ for the reactions of Figs. 1 and 2. For definiteness, we consider the contribution from just a pole in q^2 , say the pole of the A_1 meson, and put $k^2 = 0$. We have already seen that if more poles are relevant, we have just to add their contributions.

Let us start from the amplitude X . It is spin flip in the s channel. As usual, we assume for both the s and t channels the trajectory and its daughters. Assuming the presence of the ρ pole in the t channel (which is necessary in order to have the ρ pole in the form factor) a simple Veneziano model will be

$$X^{st}(s, t, q^2) = g(q^2) \frac{\Gamma(n - \alpha_\rho(s)) \Gamma(1 - \alpha_\rho(t))}{\Gamma(n + 1 - \alpha_\rho(s) - \alpha_\rho(t))} \quad (40)$$

where $g(q^2) = \beta / (q^2 - m_{A_1}^2)$ and $\alpha_\rho(s)$ is the ρ trajectory.

The denominator of Eq. (40) is fixed once the numerator is given and the $s^{\alpha_\rho(t)-1}$ behaviour is required; a single Veneziano term will behave for large t like $t^{\alpha_\rho(s)-n}$, $n \geq 1$. The $t^{\alpha_\rho(s)}$ behaviour, together with the presence of the ρ pole in the t channel, can of

course be restored by means of satellites - in this example we disregard this possibility, and take for X^{st} the expression (40) for $n=1$. For the amplitude B_1^{st} we have the constraint (31) which we write, defining

$$B_1^{st}(s, t, q^2) = \frac{1}{q^2 - m_{A_1}^2} \bar{B}(\alpha_p(s), \alpha_p(t)) \quad (41)$$

and recalling the AVW result $\alpha_p(m_\pi^2) = \frac{1}{2}$, $\alpha_p(m_{A_1}^2) = \frac{3}{2}$,

$$\frac{\alpha_p(t) - \frac{3}{2}}{2\alpha'_p} \bar{B}(\alpha_p(s) = \frac{1}{2}, \alpha_p(t)) = -\beta \frac{\Gamma(\frac{1}{2}) \Gamma(1 - \alpha_p(t))}{\Gamma(\frac{3}{2} - \alpha_p(t))} \quad (42)$$

i.e.,

$$\bar{B}(\alpha_p(s), \alpha_p(t)) = 2\alpha'_p \beta \frac{\Gamma(\frac{1}{2}) \Gamma(1 - \alpha_p(t))}{\Gamma(\frac{5}{2} - \alpha_p(t))} \quad (43)$$

where α'_p is the slope of the ρ trajectory.

If we want to write B_1 as a single Veneziano term, we have

$$\bar{B}(\alpha_p(s), \alpha_p(t)) = 2\alpha'_p g(q^2) \frac{\Gamma(2 - \alpha_p(s)) \Gamma(1 - \alpha_p(t))}{\Gamma(3 - \alpha_p(s) - \alpha_p(t))} \frac{\Gamma(\frac{1}{2})}{\Gamma(\frac{3}{2})} \quad (44)$$

For $s \rightarrow \infty$ we have the expected behaviour $s^{\alpha_p(t)-1}$, while the ruling out of satellites gives, for large t , the behaviour $t^{\alpha_p(s)-2}$. In conclusion, from the sum rule (32), we get ⁷⁾

$$F(t) = -\frac{1}{2} \bar{B}(\frac{1}{2}, \alpha_p(t)) \simeq c \frac{\Gamma(1 - \alpha_p(t))}{\Gamma(\frac{5}{2} - \alpha_p(t))} \quad (45)$$

which behaves, at large t , as $t^{-\frac{3}{2}}$.

The amplitude A is then given by Eqs. (37), (40) and (44), and one can verify explicitly all the properties described at the end of the previous Section.

In this model the form factor $F(t)$ has an infinity of poles corresponding to the recurrences of the ρ trajectory, which is supposed as usual to be linear in t .

Our model shows indeed, besides the ρ pole at $\alpha_\rho(t)=1$, poles at $\alpha_\rho(t)=n$ which must be interpreted as the spin one daughters of the recurrences of the ρ . Like in the case of the related scattering amplitudes, the asymptotic behaviour in t is obtained by the contributions of an infinite number of resonances in the same channel. This can be compared to the idea of duality. A model of its kind, exhibiting an infinite number of poles and a half-integer power behaviour in the momentum transfer t , has been applied successfully to the description of the electromagnetic form factors of the nucleon ⁸⁾.

We recall, at this point, that taking $t=q^2$ in Eq. (28), we recover the Fubini, Furlan, Rossetti sum rule ³⁾

$$X(s=m_\pi^2, t, q^2=t) = F(t) \quad (46)$$

corresponding to the commutator of an axial charge with an axial current, i.e., the limit $k_\mu \rightarrow 0$ of Eq. (3). In the present model we have

$$F(t) = X(s=m_\pi^2, t, q^2=t) = \frac{\beta}{t-m_{A_1}^2} \frac{\Gamma(\frac{1}{2}) \Gamma(1-\alpha_\rho(t))}{\Gamma(\frac{3}{2}-\alpha_\rho(t))} \quad (47)$$

the unwanted pole at $t=m_{A_1}^2$ is cancelled by the zero of $1/\Gamma(\frac{3}{2}-\alpha_\rho(t))$, as $\alpha_\rho(m_{A_1}^2)=\frac{3}{2}$, which is the mechanism discussed by AVW. More generally, the unwanted pole is cancelled by virtue of the Adler self-consistency condition.

Finally, we want to give an example in terms of Veneziano models of amplitudes satisfying Eq. (22). To this end, we put

$$A_1^{su} = -A_2^{su} = h(q^2) \alpha'_\rho \frac{\Gamma(1-\alpha_\rho(s)) \Gamma(1-\alpha_\rho(u))}{\Gamma(2-\alpha_\rho(s)-\alpha_\rho(u))}$$

[as $s \rightarrow \infty$, $A_{1 \rightarrow s} \sim \alpha_p(u)^{-1}$, as it should be for a spin flip amplitude] and

$$X^{su} = h(q^2) \frac{\Gamma(1-\alpha_p(s)) \Gamma(1-\alpha_p(u))}{\Gamma(1-\alpha_p(s)-\alpha_p(u))}$$

Recalling once more $\alpha_p(m_\pi^2) = \frac{1}{2}$, we see that Eq. (22) is identically satisfied.

4. We consider now the commutator of two vector currents sandwiched between two pion states. We notice two main differences with respect to the case of two axial currents :

- i) the counter-term corresponding to the scattering of a virtual pion is absent;
- ii) the amplitude A has a pole at $s = m_\pi^2$, i.e., the external particle appears also as an intermediate state.

We take $k^2 = 0$ (for $k^2 \neq 0$ the discussion of Section 3 applies). We ignore for simplicity the u channel. Corresponding to Eq. (11) we have

$$(s - m_\pi^2) A(s, t, q^2) - \frac{1}{2} (t - q^2) B(s, t, q^2) = F(t) \quad (48)$$

We want to see if in this case the method of Section 1 can give new information.

To this end, we put $s = m_\pi^2$ in Eq. (48). In so doing, we are left with the residuum of A at $s = m_\pi^2$, which is just $F(q^2)$. We have then the identity

$$F(q^2) - \frac{1}{2} (t - q^2) B(s = m_\pi^2, t, q^2) = F(t) \quad (49)$$

(B has no pole at $s = m_\pi^2$ as it is a spin flip amplitude). It is clear that for $t = q^2$ [FFR ³⁾ sum rule], an identity is obtained.

Let us then study the q^2 structure of Eq. (49). Given a pole at $q^2 = m_n^2$ and for $q^2 \sim m_n^2$ write

$$F(q^2) \simeq \frac{c_n}{q^2 - m_n^2}$$

$$B(s=m_\pi^2, t, q^2) \simeq \frac{B_n(t)}{q^2 - m_n^2} \quad (50)$$

By taking the q^2 singularity of Eq. (49) we have

$$c_n - \frac{1}{2}(t - m_n^2) B_n(t) = 0$$

or

$$B_n(t) = \frac{2c_n}{t - m_n^2} \quad (51)$$

Equation (49) is not at all surprising : it states that if a term

is present, by gauge invariance the term

should also exist.

We note that, by summing over n , Eqs. (50) and (51) give

$$B(s=m_\pi^2, t=0, q^2=0) = 2 \sum_n \frac{c_n}{m_n^4} \equiv -2 \left. \frac{d}{dq^2} F(q^2) \right|_{q^2=0}$$

which is essentially the Cabibbo-Radicati sum rule ⁹⁾.

On the other hand, substituting back in Eq. (49) and summing on n , we have

$$\sum_n \frac{c_n}{t - m_n^2} = F(t)$$

which is essentially our previous definition of $F(t)$, Eq. (50). We are then led to the conclusion that our method, applied to the sum rule for the vector-vector commutator, imposes no restrictions on $F(t)$ - in particular it does not contradict the result of Section 1.

In our approach, the current algebra identity is automatically satisfied once one has imposed the gauge invariance constraint on the strong interaction process, namely the photoproduction of the particle of mass m_n . The amplitude $B(s, t, q^2)$ can well be, for instance, of Veneziano type, in which case the t poles [see Eq. (51)] would come from the sum of the poles in the s channel.

NOTE ADDED IN PROOF

After this work was completed, the authors became aware of a paper by M. Ademollo and E. Del Giudice, M.I.T. preprint (1969), who obtain for the electromagnetic form factor of the pion

$$F(t) \approx \frac{\Gamma(1 - \alpha_p(t))}{\Gamma(\gamma - \alpha_p(t))}$$

so that at large t , $F(t) \approx |t|^{-\gamma+1}$, γ being an undetermined parameter. This result is obviously consistent with Eqs. (29) and (45) of the present paper, if γ is taken to be $\alpha_p(m_\pi^2)$ - (an integer). We want to stress that, however, their approach and ours are completely different: Ademollo and Del Giudice in fact assume a new dynamical formula for the scattering of leptons and hadrons and derive from it a form factor which actually does not satisfy exactly the current algebra identities, while in the present paper we start from conventional Regge pole theory or Veneziano model for hadronic or semi-hadronic processes and then obtain the form factor by exploiting the content of current algebra identities.

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