Atom interferometer as a selective sensor of rotation or gravity

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In the presence of Earth gravity and gravity-gradient forces, centrifugal and Coriolis forces caused by the Earth rotation, the phase of the time-domain atom interferometers is calculated with accuracy up to the terms proportional to the fourth degree of the time separation between pulses. We considered double-loop atom interferometers and found appropriate condition to eliminate their sensitivity to acceleration to get atomic gyroscope, or to eliminate the sensitivity to rotation to increase accuracy of the atomic gravimeter. Consequent use of these interferometers allows one to measure all components of the acceleration and rotation frequency projection on the plane perpendicular to gravity acceleration. Atom interference on the Raman transition driving by non-counterpropagating optical fields is proposed to exclude stimulated echo processes which can affect the accuracy of the atomic gyroscopes. Using non-counterpropagating optical fields allows one to get new type of the Ramsey fringes arising in the unidirectional Raman pulses and therefore centered at the two-quantum line center. Density matrix in the Wigner representation is used to perform calculations. It is shown that in the time between pulses, in the non-inertial frame, for atoms with fully quantized spatial degrees of freedom, this density matrix obeys classical Liouville equations.

I. INTRODUCTION

Since atom interference [1] has been proposed as a sensor of inertial effects [2] and the use of Raman transition between atomic hyperfine sublevels [3] allowed tremendously increase the time separation between optical fields, unprecedented accuracy in the measurement of the Earth rotation [4], gravity gradients [5], and velocities [6] has also been achieved. The theoretical analysis [7, 8, 9, 10, 11] showed that the current level of interferometer phase measurements is sufficient to sense each source changing the atomic motion, i.e. rotation, acceleration, acceleration gradient and recoil effect [12], as well as the interplay between them, such as between rotation and acceleration [10], recoil effect and rotation [13], recoil effect and gravity gradient [10].

At the same time for navigation and geodetic applications one needs to measure separately a gravity acceleration g and rotation frequency Ω, i.e. one needs an interferometer whose phase is selectively sensitive only to one component of these vectors, while the sensitivity to others can be either excluded with sufficiently high accuracy or precisely taken into account. An example here is an atomic gyro [4], where the Ω projection on the gravity acceleration g has been measured, because influence of gravity has been excluded using spatially separated fields propagating in horizontal plane and using signals from two counterpropagating atomic beams.

In this article we consider the theory of atom interferometers that would allow one to measure separately an atom acceleration g and the rotation frequency component Ω which are perpendicular to g. We explore the fact that, when rotation, gravity gradient and recoil effect only slightly affect the atoms’ trajectory, parts of the interferometer phase associated with g and Ω evolve correspondingly as $T^2$ and $T^3$ while other contributions are precisely known or negligibly small. Using particular kinds of the atom interferometers one can exclude one of these dependences getting a sensor of acceleration or rotation only.

Usual time domain atom interferometer, consisting of three resonant pulses applied to the cold atom cloud at moments $t = 0$, $T_2$ and $T_3$ could not serve for these purposes because the only parameter, ratio of the time separation between pulses $T_3/T_2$, is already used to eliminate linear in time part of the phase. This Doppler phase vanishes at the echo point, $T_3 = 2T_2$. To eliminate another terms one needs at least a 4-pulses interferometer. It was found previously [14, 15] that double loop interferometer consisting of two $\pi/2$—pulses at the beginning and end and two $\pi$—pulses in between, i.e. the $(\pi/2, T_1 = 0) - (\pi, T_2) - (\pi, T_3) - (\pi/2, T_4)$ interferometer, has no sensitivity to the homogeneous acceleration if $T_2 = \frac{1}{4}T_4$ and $T_3 = \frac{3}{4}T_4$, see fig. 1a.

One can expect that the double-loop interferometer can serve as a sensor of rotation. It is correct only if pulses’ areas are precisely equal to assigned values. If one could not hold areas equal to $\pi$ or $\pi/2$, then stimulated echo processes also contribute (see Fig. 1b) to the interferometer phase and since stimulated echo is insensitive to the atomic motion between second and third pulses, their contribution is acceleration sensitive. Since typically contribution to the interferometers phase caused by the acceleration is 5-6 orders of magnitude larger than that caused by rotation [16], it is extremely important to find a technique for excluding stimulated echo. To achieve this goal one should violate the condition for stimulated echo. This opportunity exists if one uses Raman transitions, where the splitting of atomic momentum states occurs under the action of two optical fields having wave vectors $\mathbf{q}_1$ and $\mathbf{q}_2$, such as an effective wave vector associated with Raman transition is given by $\mathbf{k} = \mathbf{q}_1 - \mathbf{q}_2$. In principle one can use for this purpose
non-counterpropagating fields. Interferometers with \( q_1 \neq -q_2 \) have been previously created \[17\] and studied \[18\].

Since for Raman transition between hyperfine sublevels \( q_1 \approx q_2 \), \( |k| \approx k \sin \alpha \), where \( \alpha \) is the half-angle between vectors \( q_1 \) and \( q_2 \), and \( k \approx 2q_1 \) is the maximum value of the effective wave-vector corresponding to counterpropagating optical fields. If \( k_j \) is an effective wave vector associated with Raman pulse \( j \), than the phase matching condition for pulse sequences \( \pi/2 - \pi - \pi - \pi/2 \) (Fig. 1a) and \( \pi/2 - \pi/2 - \pi/2 - \pi/2 \) (Fig. 1b) are

\[
\begin{align*}
{k_4} - 2{k_3} + 2{k_2} - {k_1} &= 0, \\
{k_4} - {k_3} - {k_2} + {k_1} &= 0.
\end{align*}
\]

These conditions mean the same only for parallel pulses having effective wave vectors of the same absolute values \( k_j \). But even for pulses with slightly different \( k_j \) one can hold only one of this condition and violate another. In this article we propose and analyze an exclusion of the stimulated echo processes using for double loop atom interferometers Raman pulses with non-equal effective wave vectors.

While eliminating \( T^2 \) dependences one can use time domain interferometer as a gyroscope, to measure rotation frequency components perpendicular to acceleration, eliminating of \( T^3 \) dependences would guarantee that contributions of this order, which can be caused by \[10\] gravity gradient (for atoms with non-zero launch momentum), rotation (when effective wave vectors are not parallel to the acceleration), recoil effect and combinations of these factors, do not affect the interferometer phase at all. Absence of \( T^3 \) dependences should increase the accuracy of the acceleration measurement. We will show below that \( T^3 \) dependences disappear for the double-loop interferometer with non identical loops shown in Fig. 1c. Since in this case pulse separations \( T_2, T_3, T_4 \) are incommensurate, one can be sure that no other echo-like processes can affect the interferometer phase.
In this article we perform calculations of interferometers’ phases for 3- and 4-pulse cases, for arbitrarily directed gravity acceleration and rotation including Earth gravity gradient as a small correction. It would allow us to get phases’ terms evolving up to \( T^4 \) dependences. In our calculation we use equations for the density matrix in the Wigner representation, which are convenient to describe atomic clouds’ evolution between pulses [19], because we show that in this representation density matrix evolves exactly as the classical distribution function in the rotating frame in the presence of the homogeneous gravity and gravity gradient terms. This technique is mathematically equivalent to the pure quantum consideration of the atomic wave function evolution in the space between pulses elaborated in [2].

Using density matrix in the Wigner representation allows us to evaluate precisely all effects related to the atomic spatial motion quantization without calculating path integrals [20].

The article is arranged as follows. A simplified consideration of 4-pulses interferometer is performed in Sec. II. Evolution of the density matrix in the Wigner representation is considered in Sec. III. Atom trajectory in the Earth rotating frame, including gravity-gradient terms as perturbation, is calculated in Sec. IV. Density matrix jumps under the pulse action are obtained in Sec. V. In Sec. VI we calculate the phase of the \( \pi/2 - \pi - \pi/2 \) pulses interferometer.

Section VII is devoted to study 4-pulses interferometers as atomic gyroscope or gravimeter, while in the Sec. VIII we summarize our consideration and discuss further possible developments of the proposed multiple loop interferometers technique.

II. 4-PULSES ATOM INTERFEROMETERS

Consider an interaction of cold atoms with a sequence of 4 Raman pulses, \( \pi/2 - \pi - \pi/2 \), having the same effective wave-vector \( k \) and applied at moments \( T_1 = 0 \), \( T_2 \), \( T_3 \) and \( T_4 \). We will show bellow that for atoms with initial coordinate and momentum \( r \) and \( p \) the phase of interferometer associated with atomic motion is given by

\[
\phi_m = k \cdot \left[ R(r, p + \frac{\hbar k}{2}, T_4) - 2R(r, p + \frac{\hbar k}{2}, T_3) + 2R(r, p + \frac{\hbar k}{2}, T_2) - r \right],
\]

where \( R(r, p, t) \) is the position of an atom at the moment \( t \) under the classical motion in rotating frame. For small rotation frequency \( \Omega \) \( (\Omega t \ll 1) \) the atom trajectory is given by

\[
R(r, p, t) \approx r + \frac{p}{m}t + \frac{g}{2}t^2 - \Omega \times \left( r + \frac{p}{m}t + \frac{g}{3}t^3 \right) t,
\]

where \( m \) is the atomic mass, and for the phase \( \phi \) one arrives at the expression

\[
\phi_m = k \cdot \left( \frac{p + \frac{\hbar k}{2}}{m} - \Omega \times r \right) T_4 (1 - 2t_3 + 2t_2) + k \cdot \left( \frac{g}{2} - \Omega \times \frac{p}{m} \right) T_3^2 (1 - 2t_3^2 + 2t_2^2) + \frac{1}{3} (k \times g) \cdot \Omega T_4^3 (1 - 2t_3^3 + 2t_2^3),
\]

where

\[
t_j = T_j/T_4.
\]

If the purpose is to measure the rotation frequency, then one may put to 0 first and second terms in Eq. (4). System

\[
1 - 2t_3 + 2t_2 = 0, \quad 1 - 2t_3^2 + 2t_2^2 = 0
\]

has a solution \( t_2 = 1/4, t_3 = 3/4 \) corresponding to the double loop interferometer [14, 16] shown in Fig. 1a. In this case

\[
\phi_m = \frac{1}{16} (k \times g) \cdot \Omega T_4^3.
\]
If the gravity is measured, then the phase (7) can be used to measure perpendicular to gravity components of the rotation frequency. For example, for Cs interferometer having the effective wavelength 426 nm and total time separation between pulses $T_4 = 0.8 \, \text{s}$, at the latitude $41^\circ$ the maximum value of the phase (7) $\phi_m \approx 250$.

If the purpose is to eliminate cubic terms in order to increase the accuracy of the acceleration measurement, one should choose the solution of the system

$$
1 - 2t_3 + 2t_2 = 0,
$$

$$
1 - 2t_3^2 + 2t_2^2 = 0
$$

which is

$$
t_{2,3} = \sqrt{\frac{5}{4}} \pm \frac{1}{4},
$$

and for atoms having zero launch momentum, $p = 0$, the phase is given by

$$
\phi_m = -\frac{\sqrt{5} - 2}{4} \mathbf{k} \cdot \mathbf{g} T_4^2.
$$

For a given maximum separation between pulses phase (10), the absolute value of the phase (10) is approximately 4.2 times less than the signal in the usual 3-pulse interferometer. Therefore, it is a kind of trade of, when one should decide whether it is more important to get a larger signal or to eliminate $T^3-$corrections to the phase.

### III. DENSITY MATRIX EVOLUTION IN THE FREE SPACE

Starting with the Lagrangian for an atom in the frame rotating with constant rate,

$$
L = \frac{mv^2}{2} + m \mathbf{v} \cdot (\mathbf{\Omega} \times \mathbf{r}) + \frac{m}{2} (\mathbf{\Omega} \times \mathbf{r})^2 - \mathbf{U},
$$

where $\mathbf{U}$ is the potential, one gets generalized momentum and Hamiltonian

$$
\mathbf{p} = mv + m (\mathbf{\Omega} \times \mathbf{r});
$$

$$
H = \frac{\mathbf{p}^2}{2m} + \mathbf{p} \cdot (\mathbf{r} \times \mathbf{\Omega}) + \mathbf{U}.
$$

In the space free of the laser field, i.e. free space, the density matrix in the Wigner representation

$$
\rho (\mathbf{r}, \mathbf{p}, t) = \frac{1}{(2\pi \hbar)^3} \int d\mathbf{s} \rho \left( \mathbf{r} + \frac{1}{2} \mathbf{s}, \mathbf{r} - \frac{1}{2} \mathbf{s}, t \right) \exp (-i \mathbf{p} \cdot \mathbf{s} / \hbar),
$$

where

$$
\rho (\mathbf{r}, \mathbf{r}') = \psi (\mathbf{r}) \otimes \psi^\dagger (\mathbf{r}')
$$

is the density matrix in the coordinate representation, evolves as

$$
\partial_t \rho (\mathbf{r}, \mathbf{p}, t) = \frac{1}{i \hbar (2\pi \hbar)^3} \int d\mathbf{s} \exp (-i \mathbf{p} \cdot \mathbf{s} / \hbar) \left\{ \left[ -\frac{\hbar^2}{2m} \partial^2_{r_1} + i \hbar (\mathbf{\Omega} \times \mathbf{r}_1) + \mathbf{U} (\mathbf{r}_1) \right]
+ \frac{\hbar^2}{2m} \partial^2_{r_2} + i \hbar \partial_{r_1} (\mathbf{\Omega} \times \mathbf{r}_2) - \mathbf{U} (\mathbf{r}_2) \right\} \rho (\mathbf{r}_1, \mathbf{r}_2, t)\bigg|_{\mathbf{r}_1 = \mathbf{r} + \frac{1}{2} \mathbf{s}, \mathbf{r}_2 = \mathbf{r} - \frac{1}{2} \mathbf{s}.}
$$

Using the fact that

$$
\partial_{r_1} = \frac{1}{2} \partial_{\mathbf{r}} + \partial_{\mathbf{s}};
$$

$$
\partial_{r_2} = \frac{1}{2} \partial_{\mathbf{r}} - \partial_{\mathbf{s}}
$$

(16)
Hamiltonian equation for the classical momentum and coordinate is
\[ \{ \partial_t + \left( \frac{p}{m} + \mathbf{r} \times \Omega \right) \partial_r + \left[ (\mathbf{p} \times \Omega) - \partial_r U \right] \partial_p + Q \} \rho(\mathbf{r}, \mathbf{p}, t) = 0; \]  
\[ Q = -(i\hbar)^{-1} \left[ U \left( \mathbf{r} + \frac{1}{2} i\hbar \partial_p \right) - U \left( \mathbf{r} - \frac{1}{2} i\hbar \partial_p \right) \right] + \partial_r U. \]  
(17a)
(17b)

When the potential \( U \) is not higher than bilinear function of coordinates, quantum term disappears, \( Q = 0 \). Since the Hamiltonian equation for the classical momentum and coordinate is
\[ \dot{\mathbf{r}} = \frac{\mathbf{p}}{m} + \mathbf{r} \times \Omega; \]  
\[ \dot{\mathbf{p}} = -\partial_r U + (\mathbf{p} \times \Omega), \]  
(18a)
(18b)
one concludes that, if
\[ Q \sim \hbar^2 (\partial_r^2 U) \partial_r^3 \ll T^{-1}, \]  
(19)

where \( T \) is a typical time of the system’s evolution, then with an accuracy \( QT \) the quantum density matrix in the Wigner representation obeys the same equation as the classical density matrix
\[ (\partial_t + i\hbar \dot{\mathbf{r}} + \mathbf{p} \partial_r) \rho(\mathbf{r}, \mathbf{p}, t) = 0. \]  
(20)

If the density matrix is known at some previous moment \( t' < t \) then the solution of Eq. (20) is given by
\[ \rho(\mathbf{r}, \mathbf{p}, t) = \rho[R(\mathbf{r}, \mathbf{p}, t' - t), \mathbf{P}(\mathbf{r}, \mathbf{p}, t' - t), t'], \]  
(21)
where functions \( R(\mathbf{r}_0, \mathbf{p}_0, t), \mathbf{P}(\mathbf{r}_0, \mathbf{p}_0, t) \) are solutions of Eqs. (18) subjected to the initial condition \( (\mathbf{r}, \mathbf{p})_{t=0} = (\mathbf{r}_0, \mathbf{p}_0) \). Clearly these functions have to satisfy multiplication law
\[ R[R(\mathbf{r}, \mathbf{p}, t_1), \mathbf{P}(\mathbf{r}, \mathbf{p}, t_1), t_2] = R(\mathbf{r}, \mathbf{p}, t_1 + t_2), \]  
(22a)
\[ \mathbf{P}[R(\mathbf{r}, \mathbf{p}, t_1), \mathbf{P}(\mathbf{r}, \mathbf{p}, t_1), t_2] = \mathbf{P}(\mathbf{r}, \mathbf{p}, t_1 + t_2). \]  
(22b)

Solution (21) is the consequence of the phase-space invariance in time. This is correct only for canonical variables \( (\mathbf{r}, \mathbf{p}) \) and incorrect for variables \( (\mathbf{r}, \mathbf{v}) \). Solution (21) can be proved as follows
\[ \frac{\partial \rho(\mathbf{r}, \mathbf{p}, t)}{\partial t} = \frac{\partial \rho}{\partial R_i} \frac{dR_i(\mathbf{r}, \mathbf{p}, t' - t)}{dt} + \frac{\partial \rho}{\partial P_i} \frac{dP_i(\mathbf{r}, \mathbf{p}, t' - t)}{dt}. \]  
(23)

Since
\[ \frac{dR_i(\mathbf{r}, \mathbf{p}, t' - t)}{dt} = - \frac{dR_i(\mathbf{r}, \mathbf{p}, t' - t)}{dt'} = \frac{\partial H}{\partial P_i}, \]  
(24)
and
\[ \frac{dP_i(\mathbf{r}, \mathbf{p}, t' - t)}{dt} = - \frac{dP_i(\mathbf{r}, \mathbf{p}, t' - t)}{dt'} = \frac{\partial H}{\partial R_i}, \]  
(25)
one gets
\[ \frac{\partial \rho(\mathbf{r}, \mathbf{p}, t)}{\partial t} = - \{ H, \rho \}_R, \]  
(26)
where \( \{ H, \rho \} \) is a Poisson bracket, and using invariance of the Poisson brackets \( \{ H, \rho \}_R = \{ H, \rho \}_{RP} \) one arrives at Eq. (20).

**IV. ATOM CLASSICAL MOTION IN THE ROTATING FRAME**

In this section we calculate the atomic classical trajectory in the potential consisting of the linear part and gravity gradient part
\[ U(\mathbf{r}) = -ma \cdot \mathbf{r} + \delta U(\mathbf{r}), \]  
(27a)
\[ \delta U(\mathbf{r}) = \frac{m}{4} T_{zz} \left[ r^2 - 3 (\mathbf{n} \cdot \mathbf{r})^2 \right], \]  
(27b)
where \( \mathbf{a} = g - \Omega^2 \mathbf{R} \times (\mathbf{\nu} \times \mathbf{n}) \) is a sum of the gravity and centrifugal accelerations, \( \mathbf{\nu} = \Omega / \Omega \) is a unit vector along the rotation frequency, \( \mathbf{n} = \mathbf{R} / R \) is a unit vector along the system’s displacement \( \mathbf{R} \) from the Earth center, \( T_{zz} = -2g_z / R \), \( g_z \) is the vertical component of the gravity acceleration.

We will treat the gradient term \( 27b \) as a perturbation. In the zero-order the atom evolves as

\[
\dot{\mathbf{r}} = \mathbf{r} \times \Omega + \frac{\mathbf{p}}{m},
\]

subject to initial conditions \( \mathbf{r} (0) = \mathbf{r}_0, \mathbf{p} (0) = \mathbf{p}_0 \), while first-order corrections are given by

\[
\delta \mathbf{p} = -\frac{m}{2} T_{zz} \mathbf{p}_\gamma,
\]

\[
\delta \mathbf{r} = -\frac{1}{2} T_{zz} \mathbf{r}_\gamma,
\]

where \( \mathbf{r}_\gamma \) and \( \mathbf{p}_\gamma \) evolve as

\[
\dot{\mathbf{r}}_\gamma = \mathbf{r}_\gamma \times \Omega + \mathbf{p}_\gamma,
\]

\[
\dot{\mathbf{p}}_\gamma = \mathbf{p}_\gamma \times \Omega + \mathbf{r} - 3 \mathbf{n} (\mathbf{n} \cdot \mathbf{r}),
\]

subject to initial conditions \( \mathbf{r}_\gamma (0) = 0, \mathbf{p}_\gamma (0) = 0. \)

One sees that to get the atom trajectory it is necessary to solve equations of the type

\[
\dot{\mathbf{b}} = \Omega \mathbf{b} \times \mathbf{\nu} + \mathbf{f} (t).
\]

Directing temporarily \( z \) axis along \( \mathbf{\nu} \), one finds that variable \( \xi = b_x - ib_y \) is given by

\[
\xi = \exp (i\Omega t) \xi_0 + \int_0^t d\tau \exp [i\Omega (t - \tau)] [f_x (t) - if_y (\tau)],
\]

so that the solution of Eq. 31 can be found as

\[
\mathbf{b} = \cos \Omega t \left( b_{0x}, b_{0y}, 0 \right) + \sin \Omega t \left( b_{0y}, -b_{0x}, 0 \right)
\]

\[
+ \int_0^t d\tau \left\{ \cos \Omega (t - \tau) \left[ f_x (\tau), f_y (\tau), 0 \right] + \sin \Omega (t - \tau) \left[ f_y (\tau), -f_x (\tau), 0 \right] \right\} + \left[ 0, 0, b_{0z} + \int_0^t d\tau f_z (\tau) \right]
\]

Since for the chosen coordinate system \( (b_{0x}, b_{0y}, 0) = -\mathbf{\nu} \times \mathbf{b} \) and \( (b_{0y}, -b_{0x}, 0) = -\mathbf{\nu} \times \mathbf{b} \), then in the vector representation the solution of Eq. 31 is given by

\[
\mathbf{b} = \mathbf{b}_h + \mathbf{b}_p,
\]

\[
\mathbf{b}_h = -\mathbf{\nu} \times \left( \mathbf{\nu} \times \mathbf{b}_0 \cos \Omega t + \mathbf{b}_0 \sin \Omega t \right) + \mathbf{\nu} (\mathbf{\nu} \cdot \mathbf{b}_0),
\]

\[
\mathbf{b}_p = \int_0^t d\tau \left\{ -\mathbf{\nu} \times [\mathbf{\nu} \times \mathbf{f} (\tau) \cos \Omega (t - \tau) + \mathbf{f} (\tau) \sin \Omega (t - \tau)] + \mathbf{\nu} [\mathbf{\nu} \cdot \mathbf{f} (\tau)] \right\}.
\]

If in turn the driving term in Eq. 31 can be represented as

\[
\mathbf{f} (\tau) = \mathbf{\nu} \times \left[ \mathbf{\nu} \times \mathbf{s} q_1 (\tau) + \mathbf{s} q_2 (\tau) \right] + \mathbf{\nu} (\mathbf{\nu} \cdot \mathbf{s}) q_0 (\tau),
\]

then

\[
\mathbf{b}_p = \mathbf{\nu} \times \left\{ \mathbf{\nu} \times \int_0^t d\tau \left[ q_1 (\tau) \cos \Omega (t - \tau) - q_2 (\tau) \sin \Omega (t - \tau) \right] + \int_0^t d\tau \left[ q_1 (\tau) \sin \Omega (t - \tau) + q_2 (\tau) \cos \Omega (t - \tau) \right] \right\} + \mathbf{\nu} \int_0^t d\tau (\mathbf{\nu} \cdot \mathbf{s}) q_0 (\tau).
\]

Consecutive application of Eqs. 31, 32, 36 brought us to the following expression for the function \( \mathbf{R} (\mathbf{r}, \mathbf{p}, t) \)

\[
\mathbf{R} (\mathbf{r}, \mathbf{p}, t) = \mathbf{R}_0 (\mathbf{r}, \mathbf{p}, t) + \delta \mathbf{R} (\mathbf{r}, \mathbf{p}, t); \]

\[
(37a)
\]
\[ R_0(r, p, t) = \cos x \left( r + \frac{P}{m}t \right) - \sin x \nu \times \left( r + \frac{P}{m}t \right) + \left( 1 - \cos x \right) \nu \left[ \nu \cdot \left( r + \frac{P}{m}t \right) \right] \]
\[ + t^2 \left\{ \frac{a}{2} - \nu \times \left[ \nu \times a f_{0,1}(x) + a f_{0,2}(x) \right] \right\}, \quad (37b) \]
\[ f_{0,1}(x) = x^{-2} \left( \cos x + x \sin x - 1 \right) - \frac{1}{2} \]
\[ f_{0,2}(x) = x^{-2} \left( \sin x - x \cos x \right) ; \]
\[ \delta R(r, p, t) = -\frac{1}{2} \tau_{zz} r_{\gamma} ; \quad r_{\gamma} = r_1 + \ldots r_8 ; \quad (37c) \]
\[ r_1 = t^2 \left\{ \nu \left[ \nu \cdot \left( \frac{r}{2} + \frac{P}{6m}t \right) \right] \left( 1 - \cos x \right) + \left( \frac{r}{2} + \frac{P}{6m}t \right) \cos x - \nu \times \left( \frac{r}{2} + \frac{P}{6m}t \right) \sin x \right\} \]
\[ + t^4 \left\{ a f_{1,1}(x) + \nu \left( a \cdot \nu \right) \left[ \frac{1}{24} - f_{1,1}(x) \right] - \nu \times a f_{1,2}(x) \right\}, \quad (37d) \]
\[ f_{1,1}(x) = x^{-4} \left[ 1 - \cos x \left( 1 - 2^{-1}x^2 \right) - x \sin x \left( 1 - 6^{-1}x^2 \right) \right] , \]
\[ f_{1,2}(x) = x^{-4} \left[ x \cos x \left( 1 - 6^{-1}x^2 \right) - \sin x \left( 1 - 2^{-1}x^2 \right) \right] ; \]
\[ r_2 = 3t^2 \left( n \cdot \nu \right) \left\{ \nu \left( n \cdot v \right) \left( v \cdot r \right) f_{21}(x) - \frac{1}{2} \right\} + t \left( \nu \cdot \frac{P}{m} \right) \left[ f_{22}(x) - 6^{-1} \right] + t^2 \nu \cdot a \left( f_{23}(x) - \frac{1}{24} \right) \]
\[ - n \left[ \nu \left( r f_{21}(x) + \frac{P}{m} f_{22}(x) + a t^2 f_{23}(x) \right) \right] + \nu \times n \left[ \nu \cdot \left( r f_{02}(x) + \frac{P}{m} t f_{25}(x) + a t^2 f_{26}(x) \right) \right] \}. \quad (37e) \]
\[ f_{21}(x) = x^{-2} \left( \cos x + x \sin x - 1 \right) , \]
\[ f_{22}(x) = x^{-3} \left[ 2 \sin x - x \left( 1 + \cos x \right) \right] , \]
\[ f_{23}(x) = x^{-4} \left[ 3 \left( 1 - \cos x \right) - x \sin x - 2^{-1}x^2 \right] , \]
\[ f_{25}(x) = x^{-3} \left[ 2 \left( 1 - \cos x \right) - x \sin x \right] , \]
\[ f_{26}(x) = x^{-4} \left[ x \cos x - x \sin x \left( 1 + 2^{-1}x^2 \right) \right] ; \]
\[ r_3 = 3^{-3} t^2 \left[ \left( \nu \times n \right) \cdot \left( \nu \times r \right) \right] \left[ f_{30}(x) \nu \left( n \cdot \nu \right) + f_{31}(x) \nu + f_{32}(x) \nu \times n \right] \], \quad (37f) \]
\[ f_{30}(x) = x^{-2} \left[ 4 - \left( 4 + x^2 \right) \cos x - x \sin x \right] , \]
\[ f_{31}(x) = x^{-1} \left( \sin x + x \cos x \right) , \]
\[ f_{32}(x) = x^{-2} \left[ x \cos x - x \sin x \left( 1 + x^2 \right) \right] ; \]
\[ r_4 = -\frac{3}{4m} t^3 \left[ \left( \nu \times n \right) \cdot \left( \nu \times p \right) \right] \left[ f_{40}(x) \nu \left( n \cdot \nu \right) + f_{41}(x) \nu - \frac{1}{3} \sin x \nu \times n \right] , \quad (37g) \]
\[ f_{40}(x) = x^{-3} \left[ 7 \sin x - x \left( 3 + 3^{-1}x^2 \right) \cos x - 4x \right] , \]
\[ f_{41}(x) = x^{-3} \left[ \sin x - x \left( 1 + 3^{-1}x^2 \right) \cos x \right] ; \]
\[ r_5 = 3^{-3} t^2 \left[ \left( \nu \times n \right) \cdot r \right] \left[ f_{50}(x) \nu \left( n \cdot \nu \right) + f_{51}(x) \nu - f_{02}(x) \nu \times n \right] , \quad (37h) \]
\[ f_{50}(x) = x^{-2} \left[ x \left( 4 - \cos x \right) - x \sin \left( 3 + x^2 \right) \right] , \]
\[ f_{51}(x) = x^{-2} \left[ \sin x \left( 1 - x^2 \right) - x \cos x \right] ; \]
\[ r_6 = -\frac{3}{4m} t^3 \left[ \left( \nu \times n \right) \cdot p \right] \left[ f_{60}(x) \nu \left( n \cdot \nu \right) + 3^{-1} \sin x \nu + f_{62}(x) \nu \times n \right] , \quad (37i) \]
\[ f_{60}(x) = x^{-3} \left[ 6 \left( 1 - \cos x \right) - x \sin \left( 4 + 3^{-1}x^2 \right) \right] , \]
\[ f_{62}(x) = x^{-3} \left[ x \cos x \left( 1 + 3^{-1}x^2 \right) - \sin x \right] ; \]
\[
\mathbf{r}_7 = \frac{3}{4} t^4 \left[ (\mathbf{\nu} \times \mathbf{n}) \cdot \mathbf{a} \right] [f_{70} (x) \mathbf{\nu} (\mathbf{n} \cdot \mathbf{\nu}) + f_{71} (x) \mathbf{n} + f_{72} (x) \mathbf{\nu} \times \mathbf{n}] \\
(37j)
\]
\[
f_{70} (x) = x^{-4} \left\{ 2x \left[ 4 + (1 + 6^{-1} x^2) \cos x \right] - \sin x \left( 10 + x^2 \right) \right\},
\]
\[
f_{71} (x) = x^{-4} \left\{ 2x \cos x \left( 1 - 6^{-1} x^2 \right) - \sin x \left( 2 - x^2 \right) \right\},
\]
\[
f_{72} (x) = x^{-4} \left\{ x \cos x - \sin x \left( 1 - 3^{-1} x^2 \right) \right\};
\]
\[
\mathbf{r}_8 = \frac{3}{4} t^4 \left[ (\mathbf{\nu} \times \mathbf{n}) \cdot (\mathbf{\nu} \times \mathbf{a}) \right] \left[ f_{8,0} (x) \mathbf{\nu} (\mathbf{n} \cdot \mathbf{\nu}) + f_{8,1} (x) \mathbf{n} + f_{8,2} (x) \mathbf{\nu} \times \mathbf{n} \right] \\
(37k)
\]
\[
f_{8,0} (x) = x^{-4} \left\{ 8 \left[ 1 - (1 + 8^{-1} x^2) \cos x \right] - x \sin x \left( 1 + 3^{-1} x^2 \right) - 2x^2 \right\},
\]
\[
f_{8,1} (x) = x^{-4} \left\{ 4 \cos x \left( 4 - x^2 \right) - x \sin x \left( 3 - 3^{-1} x^2 \right) \right\},
\]
\[
f_{8,2} (x) = x^{-4} \left\{ \sin x \left( 2 - x^2 \right) - x \cos x \left( 2 - 3^{-1} x^2 \right) \right\};
\]
where \( x = \Omega t \).

V. DENSITY MATRIX EVOLUTION INSIDE RAMAN PULSE.

Consider atoms interacting with a field of two traveling waves
\[
\mathbf{E} = \frac{1}{2} \text{Re} \left\{ \mathbf{E}_1 e^{i(q_1 \cdot r - \omega_1 t + \psi_1)} + \mathbf{E}_2 e^{i(q_2 \cdot r - \omega_2 t + \psi_2)} \right\},
\]
(38)
where \( \mathbf{E}_j \), \( \omega_j \), \( q_j \) and \( \psi_j \) are amplitudes, frequencies, wave vectors and phases of waves respectively. We assume that the pulse of the field \( \mathbf{E} \) start and end times are \( t_0 \) and \( t_0 + \tau \), where \( \tau \) is the pulse duration, fields \( \mathbf{E}_1 \) and \( \mathbf{E}_2 \) are resonant to adjacent transitions \( |g \rangle \rightarrow |i \rangle \) and \( |e \rangle \rightarrow |i \rangle \) correspondingly, and \( g \) and \( e \) denote lower and upper hyperfine sublevels of the atomic ground state manifold, while \( i \) corresponds to the intermediate level in the Raman two-quantum transition. Regarding the pulse duration we assume that being larger than inverse fields’ detunings
\[
\tau \gg |\Delta|^{-1},
\]
(39)
where
\[
\Delta = \omega_1 - \omega_{ig} \approx \omega_2 - \omega_{ie},
\]
(40)
\( \omega_{ig} \) and \( \omega_{ie} \) are transition frequencies, it is also sufficiently small in respect to all other relevant time intervals
\[
\tau \ll \min \left\{ t_0, |\delta_{12}|^{-1}, (k\Delta p/m)^{-1}, (hk^2/m)^{-1}, |k \cdot g|^{-1/2}, |\Omega|^{-1} \right\},
\]
(41)
where
\[
k = q_1 - q_2
\]
(42)
is the effective wave vector,
\[
\delta_{12} = \omega_1 - \omega_2 - \omega_{eg}
\]
(43)
is the Raman detuning, \( \Delta p \) is the width of the atoms’ distribution over momenta.

Evolution of atomic levels amplitudes under assumption \( |\Delta| \) and in the absence of acceleration and rotation has been considered in review \( [21] \). From Eqs. (51, 52) in \( [21] \), using conditions \( [21] \) one finds that in the Shrödinger representation amplitudes of levels after the pulse action are given by
\[
c (e, p + \hbar k, t_0 + \tau) = \exp \left\{ -i \tau (\Omega_e^A + \Omega_g^A) / 2 \right\} \left\{ \cos \left( \frac{\theta}{2} \right) c (e, p + \hbar k, t_0) \right. \\
- i \exp \left\{ -i [\phi + \delta_{12} t_0 +] \right\} \sin \left( \frac{\theta}{2} \right) c (g, p, t_0) \right\},
\]
(44a)
\[
c (g, p, t_0 + \tau) = \exp \left\{ -i \tau (\Omega_e^A + \Omega_g^A) / 2 \right\} \left\{ -i \exp \left\{ i [\phi + \delta_{12} t_0] \right\} \sin \left( \frac{\theta}{2} \right) c (e, p + \hbar k, t_0) \\
+ \cos \left( \frac{\theta}{2} \right) c (g, p, t_0) \right\},
\]
(44b)
where $\Omega_{e}^{AC} = |\Omega_{e}|^2/4\Delta$ and $\Omega_{g}^{AC} = |\Omega_{g}|^2/4\Delta$ are ac-Stark shifts of levels $|e\rangle$ and $|g\rangle$, $\Omega_{e} = -\langle i|d\cdot E_{1}|e\rangle/\hbar$, $\Omega_{g} = -\langle i|d\cdot E_{1}|g\rangle/\hbar$, 
\begin{equation}
\phi = \psi_{2} - \psi_{1},
\end{equation}
\begin{equation}
\theta = \Omega_{e}ff
\end{equation}
is the pulse area, $\Omega_{e}ff = |\Omega_{e}g|/2\Delta$ is the effective Rabi frequency associated with the Raman transition $|e\rangle \rightarrow |g\rangle$. In the derivation of Eqs. (44) we assumed that Raman detuning is much smaller than the effective Rabi frequency, $|\delta_{12}| \ll \Omega_{e}ff$. For the optimal pulse areas $\theta \sim 1$, this condition follows from the inequality (41).

For the density matrix in the Wigner representation (13), which can be expressed through levels’ amplitudes as 
\begin{equation}
\rho_{ab}(r, p, t) = \int \frac{dq}{(2\pi)^{3}} c \left( \alpha, p + \frac{1}{2} h q, t \right) c^{*} \left( \beta, p - \frac{1}{2} h q, t \right) e^{i q \cdot r},
\end{equation}
one finds 
\begin{equation}
\rho_{ee}(r, p, t + \tau) = \cos^{2} \left( \frac{\theta}{2} \right) \rho_{ee}(r, p, t) + \sin^{2} \left( \frac{\theta}{2} \right) \rho_{gg}(r, p - \hbar k, t)
+ \text{Re} \left\{ i \sin \theta \exp \left[ -i \left( k \cdot r - \delta_{12} t - \phi \right) \right] \rho_{eg}(r, p - \frac{\hbar k}{2}, t) \right\};
\end{equation}
\begin{equation}
\rho_{gg}(r, p, t + \tau) = \sin^{2} \left( \frac{\theta}{2} \right) \rho_{ee}(r, p + \hbar k, t) + \cos^{2} \left( \frac{\theta}{2} \right) \rho_{gg}(r, p, t)
- \text{Re} \left\{ i \sin \theta \exp \left[ -i \left( k \cdot r - \delta_{12} t - \phi \right) \right] \rho_{eg}(r, p + \frac{\hbar k}{2}, t) \right\};
\end{equation}
\begin{equation}
\rho_{eg}(r, p, t + \tau) = \frac{1}{2} \sin \theta \exp \left[ i \left( k \cdot r - \delta_{12} t - \phi \right) \right] \left[ \rho_{ee}(r, p + \frac{\hbar k}{2}, t) - \rho_{gg}(r, p - \frac{\hbar k}{2}, t) \right]
+ \cos^{2} \left( \frac{\theta}{2} \right) \rho_{eg}(r, p, t) + \sin^{2} \left( \frac{\theta}{2} \right) \exp \left[ 2i \left( k \cdot r - \delta_{12} t - \phi \right) \right] \rho_{ge}(r, p, t).
\end{equation}

In the specific cases of $\pi$- and $\pi/2$-pulses Eqs. (48) reduces to 
\begin{equation}
\rho_{ee}(r, p, t + \tau) = \rho_{gg}(r, p - \hbar k, t);
\end{equation}
\begin{equation}
\rho_{gg}(r, p, t + \tau) = \rho_{ee}(r, p + \hbar k, t);
\end{equation}
\begin{equation}
\rho_{eg}(r, p, t + \tau) = \exp \left[ 2i \left( k \cdot r - \delta_{12} t - \phi \right) \right] \rho_{ge}(r, p, t),
\end{equation}
for $\theta = \pi$, and
\begin{equation}
\rho_{ee}(r, p, t + \tau) = \frac{1}{2} \left[ \rho_{ee}(r, p, t) + \rho_{gg}(r, p - \hbar k, t) \right]
+ \text{Re} \left\{ i \exp \left[ -i \left( k \cdot r - \delta_{12} t - \phi \right) \right] \rho_{eg}(r, p - \frac{\hbar k}{2}, t) \right\};
\end{equation}
\begin{equation}
\rho_{gg}(r, p, t + \tau) = \frac{1}{2} \left[ \rho_{ee}(r, p + \hbar k, t) + \rho_{gg}(r, p, t) \right]
- \text{Re} \left\{ i \exp \left[ -i \left( k \cdot r - \delta_{12} t - \phi \right) \right] \rho_{eg}(r, p + \frac{\hbar k}{2}, t) \right\};
\end{equation}
\begin{equation}
\rho_{eg}(r, p, t + \tau) = \frac{i}{2} \exp \left[ i \left( k \cdot r - \delta_{12} t - \phi \right) \right] \left[ \rho_{ee}(r, p + \frac{\hbar k}{2}, t) - \rho_{gg}(r, p - \frac{\hbar k}{2}, t) \right]
+ \frac{1}{2} \left[ \rho_{eg}(r, p, t) + \exp \left[ 2i \left( k \cdot r - \delta_{12} t - \phi \right) \right] \rho_{ge}(r, p, t) \right]
\end{equation}
for $\theta = \pi/2$.

\section{3 Pulses Atom Interferometer}

Consider first a $\pi/2 - \pi - \pi/2$ sequence of pulses applied at moments $T_{1} = 0$, $T_{2} = T_{3}/2$, $T_{3}$. If initially all atoms are in the lower state, where their distribution in the Wigner representation is $f(r,p)$, 
\begin{equation}
\rho(r,p,0) = \begin{pmatrix} 0 & 0 \\ 0 & f(r,p) \end{pmatrix},
\end{equation}
then applying consequently Eqs. 21, 49, 50 one finds that after the third pulse action, the upper state distribution is given by

\[
\rho_{ee}(\mathbf{r}, \mathbf{p}, T_3 + \tau) = \frac{1}{4} \left\{ f \left[ \mathbf{R} \left( \mathbf{r}, \mathbf{p}, T_2 - T_3 \right), \mathbf{P} \left( \mathbf{r}, \mathbf{p}, T_2 - T_3 \right) - \hbar \mathbf{k}, -T_2 \right], \right.
\]

\[
\left. \mathbf{P} \left( \mathbf{R} \left( \mathbf{r}, \mathbf{p}, T_2 - T_3 \right), \mathbf{P} \left( \mathbf{r}, \mathbf{p}, T_2 - T_3 \right) - \hbar \mathbf{k}, -T_2 \right) \right\} + f \left[ \mathbf{R} \left( \mathbf{r}, \mathbf{p} - \hbar \mathbf{k}, T_2 - T_3 \right), \mathbf{P} \left( \mathbf{r}, \mathbf{p} - \hbar \mathbf{k}, T_2 - T_3 \right) + \hbar \mathbf{k}, -T_2 \right], \right.
\]

\[
\left. \mathbf{P} \left( \mathbf{R} \left( \mathbf{r}, \mathbf{p} - \hbar \mathbf{k}, T_2 - T_3 \right), \mathbf{P} \left( \mathbf{r}, \mathbf{p} - \hbar \mathbf{k}, T_2 - T_3 \right) + \hbar \mathbf{k}, -T_2 \right) - \hbar \mathbf{k} \right\}
\]

\[
- \frac{1}{2} \cos \left[ \mathbf{k} \cdot \mathbf{r} - 2 \mathbf{k} \cdot \mathbf{R} \left( \mathbf{r}, \mathbf{p} - \frac{\hbar \mathbf{k}}{2}, T_2 - T_3 \right) + \mathbf{k} \cdot \mathbf{R} \left( \mathbf{r}, \mathbf{p} - \frac{\hbar \mathbf{k}}{2}, -T_3 \right) - \frac{\hbar \mathbf{k}}{2} \right]
\]

\[
- \delta_{12} (T_3 - 2 T_2) - \phi_3 + 2 \phi_2 - \phi_1 f \left[ \mathbf{R} \left( \mathbf{r}, \mathbf{p} - \frac{\hbar \mathbf{k}}{2}, -T_3 \right), \mathbf{P} \left( \mathbf{r}, \mathbf{p} - \frac{\hbar \mathbf{k}}{2}, -T_3 \right) - \frac{\hbar \mathbf{k}}{2} \right]. \tag{52}
\]

This expression can be used to calculate any response associated with atoms on the upper level. It can be used to study the spatial shift and deformation of atomic clouds. In this article we use it to get the total probability of atoms excitation

\[
w \equiv \int d\mathbf{r} d\mathbf{p} \rho_{ee}(\mathbf{r}, \mathbf{p}, T_3 + \tau). \tag{53}
\]

Choosing for each term in Eq. 52 arguments of the initial distribution \( f \) as integration variables and taking into account that phase space stays invariant under both the free atom motion and recoil kicks of the momentum, one arrives at the following probability of excitation

\[
w = \frac{1}{2} \left[ 1 - \int d\mathbf{r} d\mathbf{p} \cos \left( \phi_m - \phi_R - \phi_3 - 2 \phi_2 + \phi_1 \right) f \left( \mathbf{r}, \mathbf{p} \right) \right], \tag{54a}
\]

\[
\phi_m = \mathbf{k} \cdot \left[ \mathbf{R} \left( \mathbf{r}, \mathbf{p} + \frac{\hbar \mathbf{k}}{2}, T_3 \right) - 2 \mathbf{R} \left( \mathbf{r}, \mathbf{p} + \frac{\hbar \mathbf{k}}{2}, T_2 \right) + \mathbf{r} \right], \tag{54b}
\]

\[
\phi_R = \delta_{12} (T_3 - 2 T_2) \tag{54c}
\]

where \( \phi_j \) is a phase 13 associated with the pulse \( j \), and \( \phi_m \) is the phase caused by atomic motion and recoil effect, which reduces to the Doppler and recoil phase shifts in the absence of rotation and acceleration, and \( \phi_R \) is Ramsey phase 22.

Time independent and linear in \( T_j \) parts of the \( \phi_m \) cancel owing to the coincidence of wave vectors of pulses (phase-matching conditions) and proper relations between pulses’ separations. Other terms can be mapped out as those depending on \( \mathbf{r} \) and \( \mathbf{r} \)-independent. We assume that the atomic cloud size is sufficiently small to neglect \( \mathbf{r} \)-dependent terms. We accept the same assumption for nonlinear in \( T_j \) parts of the \( \phi_m \) regarding the momentum distribution, except that we allow atoms to be launched into some initial momentum \( \mathbf{p} \). Thereupon one can drop integration in Eq. 54a over phase space, and \( \phi_m \) becomes the phase of the interferometer.

All phase terms up to the order \( T_j^4 \) are presented in Table. They can be obtained by multiplying by \( \mathbf{k} \) corresponding vectors in Eq. 57, expanding functions \( f_{ij}(x) \) and replacing factor \( t^n \) \((n > 1)\) as

\[
t^n \to T_3^n \left( 1 - 2^{1-n} \right). \tag{55}
\]

We would like to mention that the 4th term in this Table is 1.5 times smaller than the corresponding part of the phase calculated in 8, which has to be corrected.

As an example, and to verify our calculations we present in columns 2 and 3 phase terms for the case considered in article 11, when atoms are launched from the coordinate system origin and gravity acceleration, launch velocity and wave vectors are all vertical,

\[
\mathbf{g} = g \hat{z}, \quad \mathbf{k} = k \hat{n}, \quad \mathbf{p} = mv_2 \hat{n}. \tag{56}
\]

The choice of the coordinate axes are also the same, \( z \) - vertical, \( y \) - south-north, \( x \) - west-east directions.

If Raman pulses are comprised from a non-counterpropagating fields then effective wave vectors still have to be parallel to one another, and for any ratio \( T_2/T_3 \) one can find appropriate magnitudes of absolute values of effective wave vectors and for \( T_2 \neq T_3/2 \) one gets non-zero Ramsey phase 54c.
TABLE I: 3 pulses interferometer phase shifts. Column 1: for an arbitrary acceleration $\mathbf{a}$, rotation frequency $\Omega$, and wave vector $\mathbf{k}$; column 2: for the vertical gravity acceleration ($\mathbf{g} = g_z \hat{\mathbf{n}}$), launch velocity ($\mathbf{p} = mv_z \hat{\mathbf{n}}$) and wave vector ($\mathbf{k} = k \hat{\mathbf{n}}$). Those dependencies which appear in the different cells of column 2 are underlined and summed up in the column 3.

\[
\begin{align*}
\frac{1}{4} \mathbf{k} \cdot \mathbf{a} T_3^2 & \quad \frac{1}{4} \mathbf{k} \Omega_y^2 R T_3^3 \\
\frac{1}{2m} (\Omega \times \mathbf{k}) \cdot \mathbf{p} T_3^2 & \quad \frac{1}{4} \mathbf{a} \cdot \mathbf{a} T_3^2 \\
\frac{3\Omega^2}{8m} \left[(\mathbf{k} \nu) (\mathbf{p} \nu) - (\mathbf{k} \mathbf{p}) \right] T_3^3 & \quad \frac{3}{8} g_z \Omega_y^2 T_3^3 \\
\frac{1}{4} \mathbf{k} \cdot (\mathbf{k} \times \mathbf{a}) T_3^3 & \quad 0 \\
\frac{3h}{16m} \left[(\mathbf{k} \cdot \nu)^2 - k^2\right] \Omega^2 T_3^3 & \quad \frac{3h k^2}{16m} \Omega_y^2 T_3^3 \\
\frac{7}{64} \left[(\mathbf{k} \cdot \mathbf{a}) - (\mathbf{k} \cdot \nu) (\mathbf{a} \cdot \nu)\right] \Omega^2 T_3^4 & \quad \frac{7}{64} k g_z \Omega_y^2 T_3^3 \\
\frac{1}{48m} (\nu \times \mathbf{k}) \cdot \mathbf{p} \nu^2 T_3^4 & \quad \frac{7}{64} k \nu^2 \Omega_y^2 T_3^4 \\
\frac{1}{16} \mathbf{k} \cdot \mathbf{p} T_3^2 T_{zz} & \quad \frac{1}{16} g_z T_3^3 T_{zz} \\
\frac{h k^2}{32m} T_3^3 T_{zz} & \quad \frac{h k^2}{16m} T_3^3 T_{zz} \\
\frac{7}{384} \mathbf{k} \cdot \mathbf{a} T_3^3 T_{zz} & \quad \frac{7}{384} k g_z T_3^3 T_{zz} \\
\frac{3}{16m} (\mathbf{n} \cdot \nu) (\mathbf{k} \cdot \mathbf{n}) (\nu \cdot \mathbf{p}) T_3^3 T_{zz} & \quad \frac{3}{16} \nu^2 g_z T_3^3 T_{zz} \\
\frac{3h}{32m} (\mathbf{n} \cdot \nu) (\mathbf{k} \cdot \mathbf{n}) (\nu \cdot \mathbf{k}) T_3^3 T_{zz} & \quad \frac{3h k^2}{32m} \nu^2 T_3^3 T_{zz} \\
\frac{7}{128} (\mathbf{n} \cdot \nu) (\mathbf{k} \cdot \mathbf{n}) (\nu \cdot \mathbf{n}) T_3^3 T_{zz} & \quad \frac{7}{128} \nu^2 g_z T_3^3 T_{zz} \\
\frac{7}{64m} (\mathbf{n} \cdot \nu) [\nu \cdot (\mathbf{n} \times \mathbf{k})] (\mathbf{k} \cdot \mathbf{n}) \Omega T_3^3 T_{zz} & \quad 0 \\
\frac{7}{128m} (\mathbf{n} \cdot \nu) [\nu \cdot (\mathbf{n} \times \mathbf{k})] (\mathbf{k} \cdot \mathbf{n}) \Omega T_3^3 T_{zz} & \quad 0 \\
\frac{3}{16m} [\nu \times \mathbf{n}] (\mathbf{k} \cdot \mathbf{n}) T_3^3 T_{zz} & \quad \frac{3}{16} \nu^2 g_z T_3^3 T_{zz} \\
\frac{3h}{32m} [\nu \times \mathbf{n}] (\mathbf{k} \cdot \mathbf{n}) T_3^3 T_{zz} & \quad \frac{3h k^2}{32m} \nu^2 T_3^3 T_{zz} \\
\frac{7}{64m} [\nu \times \mathbf{n}] (\mathbf{k} \cdot \mathbf{n}) \nu T_3^3 T_{zz} & \quad 0 \\
\frac{7}{128m} [\nu \times \mathbf{n}] (\mathbf{k} \cdot \mathbf{n}) \nu T_3^3 T_{zz} & \quad 0 \\
\frac{7}{128} [\nu \times \mathbf{n}] (\mathbf{k} \cdot \mathbf{n}) T_3^3 T_{zz} & \quad \frac{7}{128} \nu^2 g_z T_3^3 T_{zz} \\
\frac{7}{128} [\nu \times \mathbf{n}] (\mathbf{k} \cdot \mathbf{n}) \nu T_3^3 T_{zz} & \quad \frac{7}{128} \nu^2 \Omega^2 T_3^3 T_{zz}
\end{align*}
\]
Performing integration over the phase-space (using arguments of the term is responsible for the atom interference. Piecing together all these terms one arrives at the following expression are originated from the population transfer between fields [24]; they comprises the background of the signal. Only one consisting of 10 terms. Among them five terms associated with transferring coherences between different pairs of 

\[ w = \frac{1}{2} \left[ 1 - \cos \theta_1 \cos \theta_2 \cos \theta_3 - \sin \theta_1 \sin \theta_2 + \frac{\theta_2}{2} \sin \theta_1 \cos (\phi_m - \phi_R - \phi_3 - \phi_1 + 2\phi_2) \right], \]  

(57)

where \( \theta_j \) is an area of the pulse \( j \).

VII. 4-PULSES ATOMIC SENSORS

Consider now the interaction of atoms with the sequence of four pulses

\[ \left( \pi/2, T_1 = 0, k_1 \right) - \left( \pi, T_2, k_2 \right) - \left( \pi, T_3, k_3 \right) - \left( \pi/2, T_4, k_4 \right). \]  

(58)

Using Eqs. [24, 25, 50] one finds the upper state distribution

\[
\rho_{ee}(r, p, T_4) = \frac{1}{4} \left\{ \begin{array}{l}
R \left( R(r, p, T_3 - T_4), P(r, p, T_3 - T_4) - h k_3, T_2 - T_3 \right), \\
R \left( R(r, p, T_3 - T_4), P(r, p, T_3 - T_4) - h k_3, T_2 - T_3 \right) + h k_1, -T_2), \\
R \left( R(r, p, T_3 - T_4), P(r, p, T_3 - T_4) - h k_3, T_2 - T_3 \right) + h k_1, T_2 - T_3), \\
P \left( R(r, p - h k_4, T_3 - T_4), P(r, p - h k_4, T_3 - T_4) - h k_3, T_2 - T_3 \right) + h k_1, T_2 - T_3), \\
P \left( R(r, p - h k_4, T_3 - T_4), P(r, p - h k_4, T_3 - T_4) - h k_3, T_2 - T_3 \right) + h k_1, T_2 - T_3), \\
P \left( R(r, p - h k_4, T_3 - T_4), P(r, p - h k_4, T_3 - T_4) - h k_3, T_2 - T_3 \right) + h k_1, T_2 - T_3), \\
\end{array} \right. 
\]

(59)

Performing integration over the phase-space (using arguments of the \( f \) function as integration variables in each term) one arrives at the following excitation probability

\[
w = w_{bg} + \bar{w} \int d\mathbf{r} d\mathbf{p} \cos (\phi_m - \phi_R - \phi_4 + 2\phi_3 - 2\phi_2 + \phi_1) f(r, p), \]  

(60a)

\[
w_{bg} = \bar{w} = \frac{1}{2}, \]  

(60b)

\[
\phi_m = k_4 \cdot R \left( \left. r, p + \frac{h k_1}{2}, T_4 \right) - 2k_3 \cdot R \left( \left. r, p + \frac{h k_1}{2}, T_3 \right) \right) + 2k_2 \cdot R \left( \left. r, p + \frac{h k_1}{2}, T_2 \right) \right) - k_1 \cdot r, \]  

(60c)

\[
\phi_R = \delta_{12}(T_4 - 2T_3 + 2T_2), \]  

(60d)

where \( w_{bg} \) and \( \bar{w} \) are the background and the amplitude of the interference term correspondingly. Interferometer phase (60b) coincides with that given in Eq. (24) if all pulses have same effective wave vectors, \( k_i = k \).

For arbitrary areas of pulses one gets 34 terms, among them:

- 8 terms correspond to the transfer of level populations between fields, they comprise background of the excitation probability given by

\[
w_{bg} = \frac{1}{2} \left( 1 - \cos \theta_1 \cos \theta_2 \cos \theta_3 \cos \theta_4 \right). \]  

(61)

- 17 Ramsey terms, which are washed out after averaging over momenta.
• 6 terms correspond to the coherence transfer between two adjacent time intervals. They have structure similar to the case of the excitation by 3 pulses and are also washed out at least because the ratio of time intervals does not satisfy echo conditions.

• 2 terms \( w_{\pm} \) associated with the stimulated echo. They are given by

\[
\begin{align*}
w_{\pm} &= -0.25 \sin \theta_1 \sin \theta_3 \sin \theta_2 \sin \theta_1 \int dr dp \\
&\times \cos \left\{ k_4 \cdot R \left[ R \left( r, p + \frac{\hbar k_1}{2}, T_2 \right), P \left( r, p + \frac{\hbar k_1}{2}, T_2 \right) \pm \frac{\hbar k_2}{2}, T_3 - T_2 \right) \right. \\
&\left. - k_3 \cdot R \left[ R \left( r, p + \frac{\hbar k_1}{2}, T_2 \right), P \left( r, p + \frac{\hbar k_1}{2}, T_2 \right) + \frac{\hbar k_2}{2}, T_3 - T_2 \right] - \delta_{12} (T_4 - T_3) - \phi_4 + \phi_3 \right) \\
&\times \cos \left( k_2 \cdot R \left( r, p + \frac{\hbar k_1}{2}, T_2 \right) - k_1 \cdot r - \delta_{12} T_2 - \phi_2 + \phi_1 \right) f (r, p) ;
\end{align*}
\]

(62)

• a term associated with the double-loop interferometer. It coincides with the second term in Eq. (60a), except that the amplitude is equal to

\[
\bar{w} = \frac{1}{2} \sin \theta_1 \sin^2 \frac{\theta_3}{2} \sin^2 \frac{\theta_2}{2} \sin \theta_1.
\]

(63)

A. Atomic gyroscope

This is evident from Eq. (60c) that, for an arbitrary rotation and acceleration, one can eliminate phase shifts of the order \( T_j^n \) only if wave vectors and ratios of time intervals \( \theta_j \) satisfy the equality

\[
k_4 - 2k_3 t_1^n + 2k_2 t_2^n + k_1 t_3^n = 0.
\]

(64)

We always need to eliminate zero-order terms [phase matching condition (1a)] and first-order terms (to get interference). For the gyroscope one needs also to eliminate the second-order term. From the system of Eqs. (64) with \( n = 0, 1, \) and 2 one concludes that all effective wave vectors have to be collinear,

\[
k_j = s_j k,
\]

(65)

where \( k \) is comprised of counterpropagating traveling waves, \( s_j = \sin \alpha_j \), and \( \alpha_j \) is a half of the angle between optical fields wave vectors \( q_{1j} \) and \( q_{2j} \) presented in the pulse \( j \). Resolving the set of equations, for example, with respect to variables \( s_1, s_2 \) and \( s_3 \), one finds that

\[
\begin{align*}
s_1 &= s_4 \frac{(1 - t_2) (1 - t_3)}{t_2 t_3}, & (66a) \\
s_2 &= s_4 \frac{1 - t_2}{2 t_2 (t_3 - t_2)}, & (66b) \\
s_3 &= s_4 \frac{1 - t_2}{2 t_3 (t_3 - t_2)}. & (66c)
\end{align*}
\]

One sees that for given time delays between pulses one can always choose proper values of effective wave-vectors to construct a double-loop interferometer. Moreover, wave vectors can be scaled by a given factor. Since the interferometer phase (60c) is linear with respect to wave vectors and, therefore, proportional to this scale factor, one can choose it from conditions

\[
s_j \leq 1,
\]

i.e.

\[
s_4 = \min \left[ 1, \frac{t_2 t_3}{(1 - t_2)(1 - t_3)}, \frac{2 t_2 (t_3 - t_2)}{1 - t_3}, \frac{2 t_3 (t_3 - t_2)}{1 - t_2} \right].
\]

(68)
This expression guaranteed that all wave vectors satisfy the condition \((67)\) and at least one of them yields the maximum value \(k\) (or \(s_j = 1\)).

To get phase shifts of the atomic gyroscope, one can simply notice from comparison of Eqs. \((54b)\) and \((60c)\) that it is sufficient to replace \(T_3^n\) in the column 1 of the Table I by

\[
T_3^n \rightarrow T_4^n \frac{s_4 - 2s_3 t_2 + 2s_2 t_2}{1 - 2^{1-n}}. \tag{69}
\]

Since all \(T^2\)-terms are eliminated, for the launch momentum

\[
p \ll ma/\Omega T_4
\]

the main phase shift is presented in the fourth row of Table I. Saving only this term after the replacement \((69)\) one finds

\[
\phi_m = \Omega \cdot (k \times a) T_4^3 \psi, \tag{70a}
\]

\[
\psi = \frac{s_4}{3} (1 - t_2)(1 - t_3). \tag{70b}
\]

Phase dependence \((70)\) on the relative position of the second and the third pulses (on the parameters \(t_2\) and \(t_3\)) for the optimal value \((68)\) of the parameter \(s_4\) is plotted in Fig. 2.

One can see that the phase \((70)\) has only one maximum and that this maximum occurs for the double-loop interferometer shown in Fig. 1a, when

\[
s_j = 1 \text{ and } t_2 = 1/4, t_3 = 3/4 \tag{71}
\]
and $\phi_m$ is given by Eq. (7).

Nevertheless, one has to move out of the point (71) at least slightly to wash out contributions from the stimulated echo (92). Let us consider this case. If

$$s_j = 1 - \sigma_j$$

(72)

and $\sigma_j \ll 1$, then from Eqs. (66) one finds with accuracy up to second order terms in $\sigma_j$, that the second and the third pulses have to be located at

$$t_2 \approx \frac{1}{4} \left( 1 + \frac{9}{8} \sigma_1 - \sigma_2 - \frac{1}{8} \sigma_4 + \frac{3}{4} \sigma_1 \sigma_2 - \frac{3}{16} \sigma_1 \sigma_4 - \frac{1}{4} \sigma_2 \sigma_4 + \frac{9}{32} \sigma_1^2 - \sigma_2^2 + \frac{3}{32} \sigma_4^2 \right),$$

(73a)

$$t_3 \approx \frac{1}{4} \left( 3 - \frac{3}{8} \sigma_1 + \sigma_2 - \frac{5}{8} \sigma_4 + \frac{1}{4} \sigma_1 \sigma_2 - \frac{1}{16} \sigma_1 \sigma_4 - \frac{5}{4} \sigma_2 \sigma_4 - \frac{15}{32} \sigma_1^2 + \sigma_2^2 + \frac{13}{32} \sigma_4^2 \right).$$

(73b)

We found these values from Eqs. (66a) and (66b). Substituting these into Eqs. (66c) one gets the phase matching condition, in the first order over $\sigma_j$,

$$\sigma_4 - 2\sigma_3 + 2\sigma_2 - \sigma_1 = 0,$$

(74)

and the equality in the second order. We calculate, with the same accuracy, the main term (70) in the gyroscope phase,

$$\psi \approx \frac{1}{16} - \frac{1}{48} \sigma_4 - \frac{1}{24} \sigma_2 - \frac{45}{1024} \sigma_1^2 - \frac{13}{1024} \sigma_2^2 + \frac{3}{32} \sigma_1 \sigma_2 - \frac{3}{512} \sigma_1 \sigma_4 - \frac{1}{32} \sigma_2 \sigma_4,$$

(75)

while all terms in the zero-order over $\sigma_j$, calculated using Table I and the rule (69), are presented in the Table II. In last two columns of this Table and of the Table III below we calculated phase for Cs interferometer ($m = 2.21 \times 10^{-42}$ g, $k = 1.47 \times 10^9$ cm$^{-1}$) for $g_z = -980$ cm/s$^2$, $T_3 = 0.8$ s, $R = 6.72 \times 10^8$ cm, latitude 41°.

### B. Atomic accelerometer

One can use the 4-pulses interferometer to eliminate third order contributions to the interferometer phase in order to increase the accuracy of acceleration measurements. In this case wave vectors and relative positions of the second and the third pulses have to satisfy Eqs. (72) for $n = 0$, $1$ and $3$. Resolving these equations with respect to wave vectors of the first, second and third pulses and requiring that at least one of the pulse is comprised of counterpropagating waves, one finds

$$s_1 = s_4 \frac{(1 - t_3) (1 - t_2) (1 + t_2 + t_3)}{t_2 t_3 (t_2 + t_3)},$$

(76a)

$$s_2 = s_4 \frac{1 - t_2^2}{2 t_2 (t_2^2 - t_3^2)},$$

(76b)

$$s_3 = s_4 \frac{1 - t_3^2}{2 t_3 (t_2^2 - t_3^2)},$$

(76c)

$$s_4 = \min \left[ \frac{t_2 t_3 (t_2 + t_3)}{(1 - t_3) (1 - t_2) (1 + t_2 + t_3)}, \frac{2t_2 (t_3^2 - t_2^2)}{1 - t_3^2}, \frac{2t_3 (t_2^2 - t_3^2)}{1 - t_3^2}, 1 \right].$$

(76d)

To get the main contribution to the interferometer phase one should perform the replacement (69) for $n = 2$ in the very first cell of the Table II. It yields the following expression

$$\phi_m = -\frac{s_4 (1 - t_3) (1 - t_2)}{2 (t_2 + t_3)} k \cdot a T_4^2.$$

(77)

This dependence is shown in Fig. 8

One sees that only one maximum of the accelerometer’s phase corresponds to values of $t_2$ and $t_3$ given by Eq. (9). Substituting these values into Eqs. (66) one verifies that the maximum of accelerometer phase occurs when all pulses are comprised of counterpropagating fields, $s_j = 1$.

From Eq. (43) one can see that $T_4 - T_3 \neq T_2$ and therefore the contribution from stimulated echo processes given by Eq. (62) is washed out after averaging over momenta.

Different terms in the accelerometer phase can be obtained from Table I using the replacement (69). They are presented in Table III.
TABLE II: Atomic gyroscope phase shifts; column 1 - the general case; column 2 - for the wave vector \( \mathbf{k} \) and launch momentum \( \mathbf{p} \) in the west-east and vertical directions; columns 3 and 4 represent the absolute and the relative phase shifts when \( p = 0 \) for Cs-gyroscope

<table>
<thead>
<tr>
<th>Term</th>
<th>General Case</th>
<th>Relative Phase</th>
<th>Absolute Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3\Omega^2}{32m} ) ((\mathbf{k}\nu)\cdot(\mathbf{p}\nu)-(\mathbf{k}\mathbf{p}))</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{1}{16} \mathbf{\Omega} \cdot (\mathbf{k} \times \mathbf{a}) T_4^3 )</td>
<td>-16 ( k\nu g_s \Omega T_4^3 )</td>
<td>254</td>
<td>1</td>
</tr>
<tr>
<td>( \frac{3}{64m} ) ((\mathbf{k} \cdot \nu)^2 - \mathbf{k}^2 ) ( \Omega^2 T_4^3 )</td>
<td>-16 ( k\nu g_s^{3} \mathbf{R} T_4^3 )</td>
<td>0.92</td>
<td>3.6 \times 10^{-3}</td>
</tr>
<tr>
<td>( \frac{3h}{64m} ) ([\mathbf{k} \cdot \mathbf{a}] - (\mathbf{k} \cdot \nu) (\mathbf{a} \cdot \nu) ) ( \Omega^2 T_4^3 )</td>
<td>-3 ( h k^2 ) ( \Omega^2 T_4^3 )</td>
<td>-1.32 \times 10^{-5}</td>
<td>-5.2 \times 10^{-8}</td>
</tr>
<tr>
<td>( \frac{3}{16m} (\nu \times \mathbf{k}) \cdot \mathbf{p} \Omega^2 T_4^3 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{h k^2}{128m} T_4^3 T_{zz} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( -\frac{3}{64m} (\mathbf{n} \cdot \nu) (\mathbf{k} \cdot \mathbf{n}) (\nu \cdot \mathbf{p}) T_4^3 T_{zz} )</td>
<td>-3 ( \frac{3k p}{64m} \nu \nu_2 \Omega T_4^3 T_{zz} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( -\frac{3}{128m} (\mathbf{n} \cdot \nu) [\nu \cdot (\mathbf{n} \times \mathbf{k})] (\nu \cdot \mathbf{k}) T_4^3 T_{zz} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( -\frac{3}{128m} (\mathbf{n} \cdot \mathbf{p}) (\mathbf{k} \cdot \mathbf{n}) \Omega T_4^3 T_{zz} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( -\frac{3}{64m} (\mathbf{n} \cdot \mathbf{p}) (\nu \cdot \mathbf{k}) T_4^3 T_{zz} )</td>
<td>-3 ( \frac{k p}{64m} \nu \nu_2 \Omega T_4^3 T_{zz} )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
| VIII. DISCUSSION

In this article we showed that, in the presence of inertial forces and homogeneous acceleration gradient, the density matrix in the Wigner representation in the free space still obeys classical Liouville equation for the distribution function. It allows us to get interferometers’ phases without calculating path integrals.

For all interferometers under consideration we calculate phase shifts up to terms proportional to \( T_4^3 \) for an arbitrary constant in time acceleration and rotation.

Using double-loop interferometers \( \pi/2 - \pi - \pi - \pi/2 \) one can eliminate the phase dependence on the acceleration only and get an interferometer with the main phase term proportional to the cross product of acceleration and rotation. One can use this interferometer as a gyroscope to measure rotation frequency components perpendicular to the acceleration. Even for the rotation frequency as small as the Earth rotation rate, the phase of this gyroscope can achieve \( \phi_m \approx 100 \). Eliminating the acceleration term, which is typically 5-orders of magnitude larger, could be incomplete if pulses areas are not precisely equal \( \pi/2 \) or \( \pi \), because in this case stimulated echo processes, which
are extremely sensitive to the acceleration, contribute to the probability of the particles’ excitation. To exclude these processes we propose to drive Raman transitions between hyperfine atomic sublevels by a pulse of travelling non-counterpropagating optical waves. For this goal one can use even slightly non-counterpropagating fields having effective wave vectors given by Eqs. (65, 72), where the small parameter \( \sigma_j \) determines the deviation of the pulse \( j \) wave vector from the maximum value \( k \). If wave vectors satisfy the phase matching condition \( (1a) \) for double-loop interferometer but violate the phase-matching condition \( (1b) \) for stimulated echo, i.e. if

\[
\begin{align*}
\sigma_{dl} &= \sigma_4 - 2\sigma_3 + 2\sigma_2 - \sigma_1 = 0, \\
\sigma_{st} &= \sigma_4 - \sigma_3 - \sigma_2 + \sigma_1 \neq 0,
\end{align*}
\]  

where \( s_{\text{cloud}} \) is the initial atomic cloud size, in the case when the initial atom density is smooth in space, probabilities \( (62) \) become exponentially small with respect to the parameter \( \sigma_{st} s_{\text{cloud}} / \lambda \gg 1 \). In addition to arguments related to the difference in phase matching conditions, one can expect further decrease of probabilities \( (62) \) due to the fact that pulses are not properly positioned in time \( |1 - t_3| \neq |2| \), where \( t_j \) are given by Eqs. \( (73) \) to produce the stimulated echo. We obtained the required pulses’ positions and the main term in the gyroscope phase for arbitrary \( \sigma_j \), see Eqs. \( (73, 70a, 75) \). But it is sufficient for only two of effective wave-vectors to have \( \sigma_j \neq 0 \). If, for example,

\[
\sigma_3 = \sigma_4 = 0 \quad \text{and} \quad \sigma_2 = \frac{\sigma_1}{2} \neq 0,
\]  

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factors

and gyroscope phase is given by

\[ \sigma = \frac{1}{16} \left( 1 - \frac{1}{3} \sigma_1 - \frac{13}{64} \sigma_2^2 \right) (k \times a) \cdot \Omega T_3^3. \]  

A “side-effect” of using Raman pulses with non-equal effective wave vectors is that in this case one gets Ramsey fringes, i.e. excitation probability oscillating dependences on the Raman detuning \( \delta \), having period of the order of the inverse time delay between pulses \( T^{-1} \) [see Ramsey phases \( \phi_R \) for single- and double-loop interferometers]. Evidently, this new type of Ramsey fringes arises because even for Raman pulses with co-propagating effective wave-vectors, in the condition of the Doppler phase cancellation, time separations between pulses \( T_j \) enter with weight factors \( k_j \), while these factors are absent in expressions for Ramsey phase. In the contrast to previously proposed...
and observed Raman-Ramsey fringes on Doppler broadened two-quantum transitions based on the use of Raman pulses having counterpropagating effective wave-vectors, Ramsey fringes found here do not undergo recoil splitting, i.e. they are centered at the two-quantum line center.

Double-loop interferometer shown in Fig. 1 allows one to get the phase where all cubic in $T$ terms are eliminated and can serve as an accelerometer. We expect that owing to this cancellation, the accuracy of the acceleration measurement can be increased.

Thus in this article we propose the following scenario for the atomic gravity and rotation sensing. First one measures acceleration $a$ using 3 mutually perpendicular double-loop accelerometers. Afterwards, one uses the spatial-domain atomic gyroscope $\Omega$, orients it in the plane perpendicular to $a$ and measures the component of the rotation frequency $\Omega$ along $a$. Then one uses two mutually perpendicular time-domain double-loop gyroscopes to get components of $\Omega$ perpendicular to $a$.

Further obvious step here could be the consideration of multiple-loop interferometers. In general for an interferometer consisting of $\ell$ unidirectional pulses (first and last are $\pi/2$-pulses, others are $\pi$-pulses), having the same wave-vector, one has $\ell - 2$ parameters $t_j = T_j/T_\ell$ ($j = 2, \ldots, \ell - 1$), which can be chosen as a root of the system of $\ell - 2$ equations

$$1 - 2t_1^{u_1} + 2t_2^{u_2} + \ldots + (-1)^\ell t_\ell^{u_\ell} = 0$$

(83)

to eliminate phase terms evolving as $T_1^{u_1}, \ldots, T_\ell^{u_\ell}$. In the array $u = (u_1, \ldots, u_{\ell-2})$ one of parameters $u_j$ has to be equal 1, to eliminate the Doppler phase, while the choice of other parameters depends on phenomena one would like to sense using the interferometer. Remaining non-eliminated phase terms evolving as $T_\ell^{u_\ell}$ can be obtained from Table II by replacement

$$T_3^u \rightarrow \frac{1 - 2t_{\ell-1}^{u_{\ell-1}} + 2t_{\ell-2}^{u_{\ell-2}} + \ldots + (-1)^\ell t_\ell^{u_\ell}}{1 - 2^{1-\ell}}.$$  

(84)

Case of 4-pulses interferometers is considered above in detail. Using 5-pulses interferometers one can eliminate $T_5^4$ dependencies. For gyroscope $u = (1, 2, 4)$, we found numerically solution of system (83):

$$t_2 = 0.16615, \ t_3 = 0.54057, \ t_4 = 0.87442.$$  

(85)

This gyroscope phase is given by

$$\phi_m = -1.0143 \times 10^{-2} \Omega \cdot (k \times a) T_5^4 + O (T_5^5).$$  

(86)

For Cs gyroscope with same parameters as in the Table II $\phi_m \approx -41.2$.

For the 5-pulses accelerometer, $u = (1, 3, 4)$, one finds

$$t_2 = 0.20594, \ t_3 = 0.59437, \ t_4 = 0.88843,$$  

(87)

and the phase is given by

$$\phi_m = 2.1557 \times 10^{-2} k \cdot a T_5^4 + O (T_5^5).$$  

(88)

For the Cs accelerometer with the same parameters as in the Table III $\phi_m \approx -2.00 \times 10^{6}$.

Another example of 5-pulse interferometer is a “figure 8 1/2” interferometer proposed in [17] to eliminate $T_5^2$ and $T_5^3$ terms, such as the dominate contribution to the phase is caused by the influence of space-time curvature.

Consider also 6-pulses interferometer. One can use this pulses sequence to eliminate all phase terms arising as a result of the atom motion to the accuracy $T_6^6$ choosing $u = (1, 2, 3, 4)$. The numerical solution of the system (83) is given by

$$t_2 = 9.5492 \times 10^{-2}, \ t_3 = 0.34549, \ t_4 = 0.65451, \ t_5 = 0.90451.$$  

(89)

Of course, one cannot use this interferometer to sense atomic motion. But one can use it for the precise measurement of the other spectroscopic data. The accuracy of this measurement improves when the time of evolution $T_6$ increases. Earth gravity and rotation superimpose limitation on this time. This time can be increased in the microgravity environment. We actually propose to use the 6-pulses interferometer instead of transferring experiments into microgravity environment.

Possible examples here are measurement of levels polarizability [20] or observation of the Aharonov-Bohm effect [27]. We also believe that one can insert additional $\pi$-pulses in the scheme of the recoil frequency measurement [1] to make this measurement independent on the gravity, gravity gradient and the Earth rotation.
Recoil diagrams for 5- and 6-pulses interferometers are shown in Fig. 4.

Regarding the 6-pulses interferometer, one sees that it has to be protected from 2 kinds of the stimulated echo processes produced by the sequences of pulses \( \{1, 2, 5, 6\} \) and \( \{1, 3, 4, 6\} \). Similar note one can make regarding “figure 8 1/2” interferometer \[12\], where both stimulated echo produced by pulses \( \{1, 2, 4, 5\} \) and one-loop interferometer produced by pulses \( \{1, 3, 5\} \) have to be excluded. An effective technique of eliminating all these unwanted signals is using Raman pulses with non-equal effective wave vectors, as we demonstrated and analyzed above for the double-loop atomic gyroscope.

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