PULSATIONAL INSTABILITIES IN ACCRETING WHITE DWARFS

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ABSTRACT

The Cataclysmic Variable (CV) population harbors a diverse range of donor stars and accreting white dwarfs (WDs). A range of WD masses is expected, from low mass Helium core WDs, to massive WDs which have previously accreted at rates high enough for Hydrogen to burn steadily. Furthermore, a wide range of Helium enrichment is expected in the accreted material depending on the degree to which the donor star is evolved. We investigate the impact of this diversity on the range of effective temperatures ($T_{\text{eff}}$) for which g-modes are unstable. Motivated by earlier theoretical studies, we use a simple criterion for g-mode excitation: that the thermal time at the base of the convection zone becomes longer than (some multiple of) a fiducial shortest g-mode period. The critical $T_{\text{eff}}$ below which modes are unstable (“blue edge”) then depends on both surface gravity, $g$, and He abundance, $Y$. The Hydrogen/first Helium ionization instability strip is more sensitive to $g$ than $Y$. We find that (for solar composition envelopes), relative to a fiducial WD mass 0.6 $M_\odot$, the blue edge for a 0.4 $M_\odot$ He core WD shifts downward by $\approx 1000$ K, while that for a massive $\approx 1.2$ $M_\odot$ WD shifts upward by $\approx 2000$ K. The second Helium ionization instability strip exhibits strong dependences on both $g$ and $Y$. Surprisingly, increasing $Y$ by only 10% relative to solar creates an instability strip near 15,000 K. Hence CV’s below the period gap with evolved donor stars of $Y \gtrsim 0.4$ may have an “intermediate” instability strip well outside of the isolated DA and DB variables. This “intermediate” instability strip also occurs for low mass He WD with solar composition envelopes. The lack of pulsations in CV’s with $T_{\text{eff}}$ in the pure Hydrogen ZZ Ceti instability strip is also easily explained.

Subject headings: binaries: close—gravitational waves—novae, cataclysmic — variables— white dwarfs

1. INTRODUCTION

As they cool in isolation, WDs cross regions of pulsational instability and undergo non-radial oscillations, in particular g-modes (see Gaitschy & Saio 1996). Those with pure H atmospheres (DAV) cross the ZZ Ceti instability strip when $T_{\text{eff}} \approx 11,000 - 12,000$ K (see Mukadam et al 2004; Gianninas et al. 2005), whereas those with pure He envelopes (DBV) become unstable when $T_{\text{eff}} \approx 22,400 - 27,800$ K (Beaughean et al. 1999). The composition of these envelopes is rather pure, and the range of WD masses is that expected from isolated stellar evolution, favoring $M \approx 0.6 M_\odot$. Careful analysis of the adiabatic pulsations for these WDs yield accurate measurements of the WD masses ($M$), shell masses, and spin (see e.g. Bradley 2001; Kepler et al. 2003).

The recent discoveries of similar period pulsations from accreting WDs in CVs provides a window to their internal properties. The accretion of $> 0.1 M_\odot$ over 10^9 years impacts both the WD’s rotation rate and mass, parameters easily diagnosed through seismic studies. Starting with the initial discovery (van Zyl et al. 2000) of 3 oscillation periods in the WD primary of GW Lib, there are now 10 (Warner & Woudt 2003; Woudt & Warner 2004; Araujo-Betancor et al. 2003; Patterson et al. 2005; Szekody et al. 2005; Vanlandingham et al. 2004; Gänsicke et al. 2006) pulsating WDs in CVs. These are all found in CVs below the period gap, where a quiescent spectrum shows evidence for a WD with $T_{\text{eff}} < 25,000$ K (Sion 1991, 1999). Calculations of WD heating by prolonged accretion (Townsley & Bildsten 2004) explain the $T_{\text{eff}}$ at these orbital periods, and were employed for the first adiabatic analysis of GW Lib, yielding both the WD and accreted layer mass (Townsley et al. 2004).

We show here that the expected diversity of WD masses and accreted layer compositions (in particular, enriched He abundance from an evolved donor) in these systems will lead to a broad range of $T_{\text{eff}}$’s where WDs will pulsate. For example, low-mass He core WDs can pulsate at $T_{\text{eff}} \approx 10,000$ K, well below the ZZ Ceti instability strip. The enhanced Helium abundance expected from an evolved secondary can also push the instability strip to values as high as 20,000 K. These effects likely explain why many of the observed pulsators in CVs are “outside” of the conventional ZZ Ceti instability strip appropriate to pure H envelopes (Szkody et al. 2002).

In §2, we clarify the influence of WD mass and envelope composition on mode driving. Assuming that Brickhill’s convective driving mechanism operates, we explore the depth of the convection zone as a function of composition and surface gravity. A near-solar mix of elements allows the possibility of both H/HeI and HeII ionization zones, opening up additional frequency ranges for instability of hotter and/or rapidly rotating WDs. Section 3 contains a discussion of the expectations for the occurrence of He core WDs in CVs, and explains why many CVs can have an evolved donor that provides He rich material to the WD envelope. We close in §4 by summarizing our new understanding and discussing future work.

2. GRAVITY WAVE EXCITATION

Accreting WDs consist of a geometrically thin, mostly non-degenerate accreted envelope overlying a degenerate core. The envelope composition (of mean molecular weight $\mu$) is inherited from the donor star, while the core composition can be Helium for low mass (< 0.45 $M_\odot$) WDs or a mixture of Carbon and Oxygen for larger masses (0.6−1.1$M_\odot$). A luminosity from “compressional heating” of accretion is generated deep in the envelope and core (Townsley & Bildsten 2004), hence the luminosity is nearly constant at the shallow depths where mode driving (and damping) occurs.

The composition discontinuity at the base of the accreted envelope (where the temperature is $T_b$) separates the WD into two resonant cavities: the envelope and the core (Townsley et al. 2004). The highest frequency gravity wave (g-mode) lives mainly in the envelope (and has no radial nodes there) and has a frequency $\omega \sim (gH)^{1/2}k_z \sim 2\pi/(100 \text{ s})$ (for $l = 2$), where $H$ is the scale height at the base. $(gH)^{1/2} \sim (k_BT_\text{eff}/\mu_{mp})^{1/2}$ is roughly the sound speed at the base, and $k_z = [l(l + 1)^{1/2}/R$ is the horizontal wavenumber. Internal gravity waves trapped in the envelope have frequencies which are smaller by $\approx 1/n$, where $n$ is the number of radial nodes there.

Gravity waves are driven overstable by converting part of the escaping heat flux into mechanical motion. This occurs near ionization zones, either due to a rapid outward increase in opacity in radiative zones (“the $\kappa$ mechanism”, Dziembowski 1977), or more likely, due to a convection zone caused by such rapid increase in opacity (Brickhill’s “convective driving” mechanism, Brickhill 1983, Wu & Goldreich 1999). We focus on Brickhill’s mechanism, where the key quantity is $\omega_{\text{th}, \text{bcz}}$, the product of the mode frequency and the thermal time, $\tau_{\text{th}, \text{bcz}}$, at the base of the convection zone. When $\omega_{\text{th}, \text{bcz}} > 1/4$ (Wu & Goldreich 1999), the thermal adjustment time in the convection zone is longer than the mode period, and flux perturbations are absorbed by the convection zone, acting to put energy into the oscillation. In this paper, we adopt the instability criterion $P \leq 8\pi\tau_{\text{th}, \text{bcz}}$, where $P = 100 \text{ s}$ is the fiducial shortest period g-mode. We will not calculate the “red edge” $T_{\text{eff}}$ of the instability strip, below which the pulsations are unobservable. For isolated DAVs, the instability strip is quite narrow ($\approx 1000 \text{ K}$, Mukadam et al. 2004), and is much broader ($\approx 5000 \text{ K}$) for the DBVs (Beauchamp et al. 1999). Lastly, when two convection zones are present, we only consider driving by the inner convection zone as this closely resembles the theory derived for the DAV’s (Brickhill 1983, Wu & Goldreich 1999). However, driving may also be possible by the outer convection zone, if the radiative layer between the two convection zones does not contribute too much damping. We leave this issue for future studies.

The variation of $\tau_{\text{th}, \text{bcz}}$ with $g$ and $Y$ is found by constructing atmosphere models that use the OPAL opacity and equation of state (Iglesias & Rogers 1996) with solar metallicity ($Z = 0.02$). These are plane parallel, constant gravity and flux envelopes, using the ML2 mixing length prescription (Bergeron et al. 1992) in convective regions, in which the mixing length is set equal to the pressure scale height.

Figure 4 shows the longest driven mode period, $8\pi\tau_{\text{th}, \text{bcz}}$, as a function of $T_{\text{eff}}$ for $g = 10^8 \text{ cm s}^{-2}$ and a range of $Y$. At a given $T_{\text{eff}}$ there can be two distinct convection zones: an outer convection zone due to H and first He ionization (H/HeI) at short thermal times, and an inner convection zone due to second He ionization (HeII) at long thermal times. The six different lines show $Y = 0$ to $Y = 0.98$, from left to right. For solar abundance ($Y = 0.28$, leftmost dashed line), we find that $8\pi\tau_{\text{th}, \text{bcz}}$ exceeds 100 s for $T_{\text{eff}} < 12,000 \text{ K}$; the blue edge is not raised significantly by addition of a solar fraction of He relative to a pure H atmosphere. At solar He fraction, the inner HeII convection zone is only present at high temperatures ($\gtrsim 22,000 \text{ K}$) and only drives $\ell \gg 1$ modes with periods $P \lesssim 10 \text{ s}$, which are unobservable. However, increasing $Y$ above solar, even by a moderate amount, has a dramatic effect on the HeII convection zone. For $Y = 0.38$ (solid line), the inner HeII convection zone is present down to $T_{\text{eff}} \approx 14,000 \text{ K}$, and raises the blue edge to near 16,000 K. We call this the “intermediate” instability strip.

The $Y = 0.38$ line also indicates a feature of mode driving in CV WDs, in which a single object can have two instability strips. The upper strip in $T_{\text{eff}}$, arising from the HeII convection zone, has a red edge determined by the truncation of the HeII convection zone at lower $T_{\text{eff}}$, rather than Brickhill’s (1983) red edge due to vanishing flux perturbation at the surface. Instability can then reappear at $T_{\text{eff}} \lesssim 12,000 \text{ K}$. This only occurs over a limited range of $Y$. At $Y = 0.48$ (middle dashed line), the HeII convection zone is not truncated, and below the blue edge it extends across $8\pi\tau_{\text{th}} = 100 \text{ s}$. Rather than the H/HeI convection zone rising up to create another instability strip, the two convection zones merge when the radiative region between them disappears. The presence of H will, however, create an extremely wide instability strip by keeping the convection zone thin compared to the $Y = 0.98$ case (right dot-dashed line).

A similar experiment is shown in Figure 2, where $g$ varies from $g = 10^5 - 10^{7.5} \text{ cm s}^{-2}$ keeping solar composition ($X = 0.7, Y = 0.28, Z = 0.02$). A gravity of $g = 10^5 \text{ cm s}^{-2}$ corresponds to a $M \approx 0.4 M_\odot$ WD, while $g = 10^8 \text{ cm s}^{-2}$ gives $M \approx 0.6 M_\odot$, and $g = 10^9 \text{ cm s}^{-2}$ gives $M \approx 1.2 M_\odot$. As in Figure 1, the blue edge is the intersection of a curve with the horizontal line representing the fiducial shortest period g-mode. As discussed by Arras et al. 2003, the blue edge for the H/HeI ionization zone moves up (down) by $\approx 2000 \text{ K}$ for a factor of 10 increase (decrease) in gravity. Hence, a blue edge range of $\approx 4000 \text{ K}$ is expected from the lowest mass He-core WD to the massive $\gtrsim 1.1 M_\odot$ Carbon/Oxygen WD. In short, the range of WD masses expected in CVs should extend the H/HeI instability strip to be both well above and below the isolated WD ZZ Ceti instability strip. For the HeI ionization zone, note the appearance of the “intermediate” instability strip at $\approx 15,000 \text{ K}$ for low mass WDs. Since $\tau_{\text{th}, \text{bcz}}$ decreases as $g$ increases, the blue edge for HeI ionization decreases with higher $g$. This behavior is opposite that found in the nearly pure He case, where $\tau_{\text{th}, \text{bcz}}$ increases as $g$ increases (see figure 8).

To elucidate the sensitivity of the blue edge to the wide range of gravities and enrichments possible for CV WDs, Figure 8 shows contours of constant blue edge $T_{\text{eff}}$ in $\log g$ and $Y$ space. At small $Y$, driving occurs due to the H/HeI convection zone, while at large $Y$ driving is due to
the HeII convection zone. For low and moderate $g$, the transition occurs abruptly when the HeII convection zone extends above 100 s, causing the blue edge to jump up by several thousand K. The dashed line indicates the $Y$ above which the HeII convection zone is always present. As discussed above, there is a small region, indicated on the plot, where there are two separate instability strips. Dotted lines show the blue edge of the lower instability strip. We find that starting from solar abundance ($Y = 0.28$, dot-dashed line), $g$ must be increased significantly, half an order of magnitude, in order to increase the blue edge $T_{\text{eff}}$ by 1000 K. In contrast, a modest increase in $Y$ can effect a very sharp increase in the blue edge. There are thus two types of accreting pulsators, those with blue edge $T_{\text{eff}} \gtrsim 15,000$ K that are slightly enriched, and those closer to solar composition with blue edge $T_{\text{eff}} \lesssim 15,000$ K, more similar to normal ZZ Ceti stars. Observationally, these two instability strips may overlap in $T_{\text{eff}}$ depending on the location of the red edge.

3. HELIUM ENRICHMENT IN EVOLVED CATAclySMIC VARIABLES

Both theory and observation (see Townsley & Bildsten 2003) tell us that non-magnetic CVs need to be below the period gap ($P_{\text{orb}} \leq 2$ hr) to reach the $T_{\text{eff}} < 25,000$ K...
range where pulsations are possible. This is indeed where the known pulsating WDs in CVs occur. We showed in the previous section that, due to the dependence on \( Y \) and \( g \), the instability strips for accreting WDs will be richer than in isolated DA or DB WDs.

The first diversity to consider is the WD mass. The expectation (see Howell, Nelson & Rappaport 2001; hereafter HNR) is that WDs below the period gap will have a large range of masses, all the way down to low-mass He core WDs (\( M < 0.45 M_\odot \)). HNR’s population synthesis calculations showed that as many as 20% of the CVs with \( P_{\text{orb}} < 2 \) hr will have low mass He WDs. These will have surface gravities much lower than any of the isolated DAVs, for which the recent tabulations (Mukadam et al 2004; Gianninis et al. 2005), show no DAV with \( \log g \leq 7.7 - 7.8 \). The pulsator HS 2331+3905 has a low \( T_{\text{eff}} \approx 10,500 \) K (Araujo-Betancor et al. 2005), which from Figure 2 is quite reasonably inside the instability strip for solar abundance with \( \log g \lesssim 7.5 \). The inferred low mass of 0.4 \( M_\odot \) makes HS 2331+3905 a candidate He core WD.

The instability strip is also sensitive to the He abundance in the WD envelope. At accretion rates \( \gtrsim 10^{-14} M_\odot \) yr\(^{-1} \) for \( T_{\text{eff}} \gtrsim 11,000 \) K, there is no time for relative H/He separation (Paquette et al. 1986). Since even the small amount of quiescent disk accretion exceeds this rate, the envelope He abundance is set by the donor star. If the donors were unevolved, then the range

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Fig. 2.— Longest period driven mode, \( 8 \pi \tau_{\text{th, loc}} \), vs. \( T_{\text{eff}} \) for different gravity, but fixed (solar) composition \( Y = 0.28 \) and \( Z = 0.02 \). The gravity for each line is, from left to right at low \( T_{\text{eff}} \), \( g = 10^{7.0}, 10^{7.5}, 10^{8.0}, \) and \( 10^{8.5} \) cm s\(^{-2} \). Dotted lines only connect related curves and do not represent convection zone boundaries.
would be narrow, $Y = 0.25 - 0.3$, only depending on the initial metallicity of the donor. However, Pylyser & Savonije (1989) raised the distinct possibility of initial donor masses for CVs with $M \geq 1M_\odot$, which, depending on the age of the system at the onset of mass transfer, can have undergone nuclear burning in the core to raise the He abundance. Such donors are required to explain supersoft sources as stably burning WDs (van den Heuvel et al. 2002). By the time such donors are below the period gap and fully convective, such a core He enrichment will be evident at the surface and in the accreting matter. This gives reason to explore the range of $Y = 0.3 - 1.0$.

The prevalence of evolved donors amongst CVs with $P_{\text{orb}} < 8$ hours depends on many factors, including the efficiency of the common envelope phase, the outcome of the rapid thermal timescale mass transfer expected at the onset of Roche lobe filling, and the nature of magnetic braking in the pre-CV stage. (Schenker et al. 2002; Podsiadlowski, Han & Rappaport 2003; Andronov & Pinsonneault 2004; Kolb & Willems 2005). Podsiadlowski, Han & Rappaport (2003) performed a binary population synthesis calculation that allowed for evolved donors with a range of initial He abundances amongst initially massive ($M > M_\odot$) donors. They found that these could actually be the dominant population for $P_{\text{orb}} > 5$ hours, but that the large number of CVs injected at shorter orbital pe-
riods dilutes their fraction at shorter periods. Below the period gap, between one in ten and one in three would have evolved companions with $Y > 0.5$. The previously discussed observables for evolved donors are a later spectral type (at a fixed orbital period), and exposed material that is Nitrogen enhanced and Carbon poor due to the CN cycle. This latter hypothesis has been tested by Gänsicke et al. (2003) through measurements of UV line ratios in a diverse set of CVs, both above and below the period gap. They concluded that as many as 10-15\% of their sample could have evolved donors.

We are introducing here an additional probe of evolution, which is the impact of the He abundance on the instability strip. More evolved donors will show pulsators at hotter $T_{\text{eff}}$ than those which are unevolved. In the absence of more detailed population study predictions of the He abundance (typically calculated, but not reported in depth since, until now, no observational probe of He abundance was available), we can say very little. Due to mass transfer stability considerations, the donor stars for a He core WD should be initially low mass and therefore unevolved. Hence the correct low gravity models to constrain a He core WD should be initially low mass and therefore unevolved. In the opposite sense, we would expect highly evolved donors to have preferentially more massive WDs. Whether these correlations lead to a range of $T_{\text{eff}}$’s or other correlations remains to be seen.

4. CONCLUSIONS AND FUTURE WORK

We have shown that detecting pulsations in accreting WDs below the period gap has the promise to reveal the diversity expected in both the WD masses and the accreted He abundance. We do not expect one instability strip as known for isolated DA WDs, but rather a revealing diversity of $T_{\text{eff}}$’s. In particular, hot CV primaries like that in GW Lib with $T_{\text{eff}} \approx 14,000$ K can pulsate due to enhanced Helium abundance, or extreme surface gravity (large or small). Cool CV primaries like that in HS 2331+3905 with $T_{\text{eff}} = 10,500$ K, can pulsate due to low surface gravity. Both of these possibilities are naturally present in the diversity expected within the population of CVs. Indeed, some systems can have two separate instability regions in $T_{\text{eff}}$.

Little is known with certainty about the WD mass distribution, but evolutionary models provide a reasonable expectation for the level of Helium enrichment of the accreted material. CVs in which the companion is the remaining core of a slightly evolved star can have He significantly enriched above solar. Below the period gap, between one in ten and one in three CVs should have evolved companions with $Y > 0.5$ (Schenker et al. 2002).

Important work which remains is validation of our assumption that the numerical factor relating $\tau_{\text{th,bev}}$ and the driven mode period does not vary significantly with either $g$ or $Y$. A large variation (more than unity) is not expected, but quantifying this statement is necessary for using the presence or absence of modes to constrain properties of particular systems. Additional damping due to the opacity effects of metals in the radiation zone may affect this prefactor, shifting the blue edges evaluated here to lower $T_{\text{eff}}$. Inefficient convection in surface layers would decrease the rate of convective driving, again causing the blue edge to shift downward.

The WDs in CVs are expected to be rotating rapidly due to the angular momentum gain from long term disk accretion. This can qualitatively change the frequency spectrum of non-radial oscillations. Dziembowski (1977) showed that $g$-modes with large numbers of angular nodes, and hence large frequency, can be driven even in relatively hot WDs by the $\kappa$-mechanism acting in the He-II ionization zone. Such modes are unobservable in non-rotating stars due to averaging of the flux perturbation over the stellar disk. Rotationally modified $g$-modes, which are squeezed into an equatorial band, may be both unstable and observable, since they can have large horizontal wavenumber (and frequency) for a small number of angular nodes.

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