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COULOMB CORRECTIONS TO ADLER'S PCAC TEST IN NUCLEI

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A B S T R A C T

We estimate Coulomb corrections to the forward inelastic neutrino cross-section on nuclei which, according to Adler's theorem should be proportional to the pion cross-section. An eikonal wave function is used for the muon in the final state. The pion is treated in a simple optical model also in the eikonal approximation. Corrections $\lesssim 10\%$ are found for muon energy $E_2 \gtrsim 1$ GeV and energy transfer $\nu \gtrsim 0.5$ GeV in heavy nuclei.

1. INTRODUCTION

An interesting consequence of PCAC has been worked out by Adler ¹⁾. It relates the forward cross-section for inelastic neutrino reactions

$$\nu + I \rightarrow \mu^- + F \quad (1.1)$$

to the (hadronic) pion cross-section on the same target

$$\pi^+ + I \rightarrow F \quad (1.2)$$

The application of this theorem to nuclei as targets, as considered by Bell ²⁾, leads to the prediction that the forward neutrino cross-section on nuclei should be proportional to the total (hadronic) cross-section of π^+ on the same nucleus, and therefore should vary very roughly as $A^{2/3}$ where A is the mass number of the nucleus.

The inclusion of the strong Coulomb field could be expected to cause troubles, since the total cross-section of a charged particle in a Coulomb field is infinite. It turns out, however, that one never reaches the Coulomb singularity at momentum transfer $t=0$ since one is dealing with the scattering of virtual rather than real pions [compare Ref. 2)], so that, also in the forward direction, a longitudinal momentum transfer survives. It even turns out that the pure Coulomb contribution ^{*)} is negligible and that the leading corrections come from the Coulomb distortions of the muon, resp., pion wave in the final state of the nuclear contribution. These corrections are of typical order of magnitude V/E_2 , V/ν where V is the Coulomb potential, E_2 the muon energy and ν the energy transfer. Since V is around 20 MeV for heavy nuclei these effects are small in the high energy region.

^{*)} (The one which survives if the nuclear potential is set to zero.)

Our work is organized as follows: In Section 2 we relate the forward neutrino cross-section on a nucleus including the Coulomb field to pion amplitudes. The input here is a distorted muon wave of the eikonal type. In Section 3 these amplitudes are studied in a simple optical model using the eikonal approximation also for pion scattering. In Section 4 it is shown that even in the presence of the Coulomb field the weak amplitude can to a good accuracy be expressed entirely in terms of pion amplitudes. Section 5 is devoted to conclusions and numerical values of the corrections we found.

We use units $\hbar = c = 1$, metric conventions of Bjorken-Drell ³⁾, except for a change of sign in γ_5 .

2. FORMULATION OF THE PROBLEM; REDUCTION TO PION-SCATTERING AMPLITUDES

We shall study the cross-section for neutrino reactions on nuclei with the muon going in the forward direction:

$$\nu + \alpha \rightarrow \mu + \text{final state } \beta \quad (2.1)$$

where α denotes the ground state of the nucleus (see Fig. 1).

The kinematical variables are

$\vec{k}_1, E_1$	momentum and energy of neutrino
$\vec{k}_2, E_2$	momentum and energy of muon
$\nu = E_1 - E_2$	energy transfer
$\vec{q} = \vec{k}_1 - \vec{k}_2$	momentum transfer
$E_{\alpha, \beta}$	energy of states α, β .

Our considerations are based on the usual current x current weak Lagrangian. Then the S matrix element for the reaction (2.1) will be:

$$\langle \mu, \beta | S | \nu, \alpha \rangle = -i \frac{G}{\sqrt{2}} \frac{2\pi}{\Omega} \delta(E_\alpha + \nu - E_\beta)$$

$$\int d^3x \langle \beta | J_2(\vec{x}) | \alpha \rangle \ell^\lambda(\vec{x})$$

(2.2)

where G is the usual Fermi constant, Ω the normalization volume $\ell^\lambda(\vec{x})$ and $J^\lambda(\vec{x})$ are the lepton and hadron currents

$$\ell^\lambda(\vec{x}) = \bar{\psi}_\mu(\vec{x}) \gamma^\lambda (1 + \gamma_5) \psi_\nu(\vec{x}) \quad (2.3)$$

$\psi_\nu(\vec{x})$ is the incident neutrino plane wave, which, in our normalization, is:

$$\psi_\nu(\vec{x}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_1 \\ -\chi_1 \end{pmatrix} e^{i\vec{k}_1 \cdot \vec{x}} \quad (2.4)$$

χ_1 , the two-component spinor for the left-handed neutrino satisfies the following equations:

$$\vec{\sigma} \cdot \hat{k}_1 \chi_1 = -\chi_1$$

$$\hat{k}_1 = \vec{k}_1 / |\vec{k}_1| \quad (2.5)$$

$$\chi_1^\dagger \chi_1 = 1 \quad (2.6)$$

$\psi_\mu(\vec{x})$, the wave function of the muon, however, will be distorted by the Coulomb field of the nucleus. It satisfies the following Dirac equation

$$\left\{ \vec{\alpha} \cdot \vec{\nabla} + m_\mu \beta + V(\vec{x}) \right\} \psi_\mu(\vec{x}) = E_\mu \psi_\mu(\vec{x}) \quad (2.7)$$

with the boundary condition for an outgoing plane wave:

$$\psi_\mu(\vec{x}) \rightarrow \sqrt{\frac{1}{2}} \begin{pmatrix} \chi_2 \\ \frac{\vec{\sigma} \cdot \vec{k}_2}{E_\mu + m_\mu} \chi_2 \end{pmatrix} e^{i \vec{k}_2 \cdot \vec{x}}$$

$$\chi_2^\dagger \chi_2 = 1 \quad (2.8)$$

$V(\vec{x})$ is the Coulomb potential of the nucleus. If we assume the nucleus with Z protons to be a homogeneously charged sphere of radius R , then:

$$V(r) = \begin{cases} Z\alpha \frac{r^2 - 3R^2}{2R^3} & r < R \\ -Z\alpha / r & r > R \end{cases} \quad (2.9)$$

From (2.7) we can deduce the following relation for $\ell_\lambda(\vec{x})$:

$$-i(\gamma + V(\vec{x}))\ell_0 + \vec{\nabla} \cdot \vec{\ell}(\vec{x}) = i m_\mu \ell(\vec{x}) \quad (2.10)$$

$$\ell(\vec{x}) = \bar{\psi}_\mu(\vec{x}) (1 + \gamma_5) \psi_\nu(\vec{x}) \quad (2.11)$$

Turning to the hadron current $J_\lambda(\vec{x})$, we know that CVC plus PCAC impose, in the absence of an electromagnetic field, the following relation

$$\partial^\lambda g_\lambda = m_\pi^2 f_\pi \phi \quad (2.12)$$

where m_π , f_π , ϕ are the pion mass, decay constant and field operator. The question how Eq. (2.12) has to be modified in presence of an electromagnetic field was answered in Ref. 4). We simply have to make a minimal substitution in (2.12):

$$(\partial_0 + iV(\vec{x}))g_0(\vec{x}) + \vec{\nabla} \cdot \vec{g}(\vec{x}) = m_\pi^2 f_\pi \phi(\vec{x}) \quad (2.13)$$

It is convenient to separate the pion pole contribution explicitly:

$$g_\lambda = g'_\lambda - f_\pi (\partial_\lambda + iV(\vec{x})\delta_{\lambda 0}) \phi \quad (2.14)$$

$$\begin{aligned} & (\partial^\lambda + iV(\vec{x})\delta^{\lambda 0}) g'_\lambda \\ &= f_\pi \{ -(\nu + V(\vec{x}))^2 - \Delta + m_\pi^2 \} \phi(\vec{x}) \\ &= f_\pi j_\pi^{(h)}(\vec{x}) \end{aligned} \quad (2.15)$$

where $j_\pi^{(h)}(\vec{x})$ is the hadronic part of the pion current.

Substituting Eq. (2.14) in Eq. (2.2) and using Eq. (2.10) we can write the S matrix element as follows:

$$\begin{aligned}
 \langle \mu \beta | S | \nu \alpha \rangle = & -i \frac{G}{\sqrt{2}} \frac{2\pi}{\Omega} \delta(E_\alpha + \nu - E_\beta) \\
 & \int d^3x \left\{ \langle \beta | \gamma_\lambda(\vec{x}) | \alpha \rangle l^\lambda(\vec{x}) \right. \\
 & \left. + i m_\mu f_\pi \langle \beta | \phi(\vec{x}) | \alpha \rangle l(\vec{x}) \right\}
 \end{aligned}
 \tag{2.16}$$

The reason for this decomposition is that we will, as usual neglect the muon mass in the first term and keep it only in the pion pole term [compare Refs. 1), 5)].

The next step is to derive expressions for $l_\lambda(\vec{x})$ and $l(\vec{x})$ using an eikonal wave function for $\psi_\mu(\vec{x})$ ^{*)}.

In this quasi-classical approximation we can speak of the path of the muon as it leaves the nucleus (see Fig. 2). The phase of the wave is given by the eikonal $S(\vec{x})$:

$$S(\vec{x}) = k_2 z + \frac{E_2}{k_2} \int_z^\infty dz' V(x, y, z')
 \tag{2.17}$$

where we have specialized to the case that the final muon momentum is parallel to the incident neutrino momentum which we choose as the z axis of our co-ordinate system. The muon momentum at any point is then

$$\vec{k}_2(\vec{x}) = \vec{\nabla} S(\vec{x})
 \tag{2.18}$$

*) Compare Ref. 6) for a treatment of a Dirac particle in the eikonal approximation. We shall not, however, use the expansion around the origin given there, since we need the wave functions at the edge of the nucleus.

its kinetic energy

$$\tilde{E}_2 = E_2 - V(\vec{x}) \quad (2.19)$$

Moreover, the muon wave becomes defocused as it travels out of the nucleus. This is taken in account by a normalization factor $N(\vec{x})$, which, on the down-stream edge of the nucleus is given by

$$N(\vec{x}) = 1 - \frac{1}{2} \frac{V(\vec{x})}{E_2} \quad (2.20)$$

whereas the average value on the up-stream edge is

$$N(\vec{x}) = 1 - 2 \frac{V(\vec{x})}{E_2} \quad (2.20a)$$

The spin of the muon finally remains parallel to the local momentum to order m_μ/E_2 as can be seen from the Bargman-Michel-Telegdi equation ⁷⁾. Putting everything together we find to leading order in m_μ/E_2

$$l_\lambda(\vec{x}) = \frac{2N(\vec{x})}{\tilde{\nu}} \tilde{\ell}_\lambda e^{i[\vec{k}_1 \vec{x} - S(\vec{x})]} + \begin{pmatrix} 0 \\ \vec{\ell}_\perp(\vec{x}) \end{pmatrix} \quad (2.21)$$

$$\tilde{\ell}^\lambda = \begin{pmatrix} \tilde{\nu} \\ \vec{\ell}_\perp(\vec{x}) \end{pmatrix} = \begin{pmatrix} \nu + V(\vec{x}) \\ \vec{k}_1 - \vec{k}_2(\vec{x}) \end{pmatrix} \quad (2.22)$$

$$\vec{\ell}_\perp(\vec{x}) \vec{\ell}_\perp(\vec{x}) = 0 \quad |\vec{\ell}_\perp(\vec{x})| \propto \frac{V(\vec{x})}{E_2} \quad (2.23)$$

$$l_{\text{nonflip}}(\vec{x}) = \frac{m_\mu}{\tilde{E}_2} N(\vec{x}) e^{i[\vec{k}_1 \vec{x} - S(\vec{x})]} \quad (2.24)$$

$$l^{\text{flip}}(\vec{x}) = 2 \sin \frac{\vartheta(\vec{x})}{2} e^{i[\vec{k}_1 \vec{x} - S(\vec{x})]} \quad (2.25)$$

$$\vartheta(\vec{x}) = \angle \vec{k}_1, \vec{k}_2(\vec{x}) \quad (2.26)$$

$$|l^{\text{flip}}(\vec{x})| \propto \frac{V(\vec{x})}{E_2} \quad (2.27)$$

A spin flip amplitude occurs because the muon may be created at an angle different from zero even if its final direction coincides with the neutrino incident direction.

In the cross-section the spin flip amplitude will by (2.27) be proportional to $(V/E_2)^2$ which we can neglect, since we are only calculating to order V/E_2 .

Substituting (2.21) and (2.24) in (2.16) we find:

$$\begin{aligned} \langle \mu, \beta | S | \nu, \alpha \rangle &= -i \frac{G}{\sqrt{2}} \frac{2\pi}{\Omega} \delta(E_\alpha + \nu - E_\beta) \\ &\int d^3x \left\{ - \langle \beta | \vec{y}'(\vec{x}) | \alpha \rangle \vec{\ell}_1(\vec{x}) \right. \\ &+ \langle \beta | y'_2(\vec{x}) | \alpha \rangle \frac{2N(\vec{x})}{\tilde{v}} \tilde{g}^{\lambda} e^{i[\vec{k}_1 \vec{x} - S(\vec{x})]} \\ &\left. + im_\mu f_\pi \langle \beta | \phi(\vec{x}) | \alpha \rangle \ell(\vec{x}) \right\} \end{aligned} \quad (2.28)$$

The second term of Eq. (2.28) can be written as

$$\int d^3\vec{x} \frac{2N(\vec{x})}{\tilde{\nu}} \left\{ \langle \beta | \vec{y}'_0(\vec{x}) | \alpha \rangle \tilde{\nu} - \langle \beta | \vec{y}'(\vec{x}) | \alpha \rangle \frac{1}{i} \vec{\nabla} \right\} e^{i[\vec{k}_1 \cdot \vec{x} - S(\vec{x})]} \quad (2.29)$$

A partial integration, neglecting the derivative of the amplitude factor $N(\vec{x})/\tilde{\nu}$ and using the PCAC equation (2.15) leads to

$$\int d^3\vec{x} (-if_\pi) (-\tilde{\nu}^2 - \Delta + m_\pi^2) \langle \beta | \phi(\vec{x}) | \alpha \rangle \frac{2N}{\tilde{\nu}} e^{i[\vec{k}_1 \cdot \vec{x} - S(\vec{x})]} \quad (2.30)$$

Substituting in (2.28) gives:

$$\begin{aligned} \langle \mu\beta | S | \nu\alpha \rangle &= -i \frac{G}{\sqrt{2}} \frac{2\pi}{\Omega} \delta(E_\alpha + \nu - E_\beta) \\ &\int d^3\vec{x} \left\{ -\frac{2if_\pi}{\nu} \langle \beta | \phi(\vec{x}) | \alpha \rangle \xi(\vec{x}) \right. \\ &\quad \left. - \langle \beta | \vec{y}'(\vec{x}) | \alpha \rangle \vec{\ell}_1(\vec{x}) \right\} \end{aligned} \quad (2.31)$$

$$\xi(\vec{x}) = \left\{ (-\tilde{\nu}^2 - \Delta + m_\pi^2) - \frac{m_\mu^2 \tilde{\nu}}{2E_2} \right\} N \frac{\nu}{\tilde{\nu}} e^{i[\vec{k}_1 \cdot \vec{x} - S(\vec{x})]} \quad (2.32)$$

From (2.31) we find for the differential cross-section with the muon going forward and summed over all final hadronic states (to order ν/E_2):

$$\left. \frac{\partial^2 \sigma}{\partial \nu \partial \Omega} \right|_{\Omega=0} = \frac{E_2^2}{(2\pi)^3} \frac{G^2}{2} \left\{ \left(\frac{2f_\pi}{\nu} \right)^2 \Gamma + \Gamma' \right\} \quad (2.33)$$

$$\Gamma = \sum_{\beta} 2\pi \delta(E_{\alpha} + \nu - E_{\beta}) \int d^3\vec{x} d^3\vec{y} \xi^*(\vec{x}) \langle \alpha | \phi^{\dagger}(\vec{x}) | \beta \rangle \langle \beta | \phi(\vec{y}) | \alpha \rangle \xi(\vec{y}) \quad (2.34)$$

$$\Gamma' = \frac{4f_{\pi}}{\nu} \text{Im} \sum_{\beta} 2\pi \delta(E_{\alpha} + \nu - E_{\beta}) \int d^3\vec{x} d^3\vec{y} \xi^*(\vec{x}) \langle \alpha | \phi^{\dagger}(\vec{x}) | \beta \rangle \langle \beta | \vec{y}'(\vec{y}) | \alpha \rangle \vec{\ell}_{\perp}(\vec{y}) \quad (2.35)$$

3. THE PION CONTRIBUTION

In this Chapter we shall study the quantity Γ defined by Eq. (2.34). In the terminology of Refs. 2), 5) it is related to the total cross-section of an off-shell pion. Following Ref. 2), we introduce the Green's function for the pion field:

$$G(\vec{x}, \vec{y}; \nu) = \int dt e^{-i\nu t} i \langle \alpha | T(\phi^{\dagger}(\vec{x}), \phi(\vec{y}, t)) | \alpha \rangle \quad (3.1)$$

G satisfies a Lippman-Schwinger equation

$$G = G_0 - G_0 U G \quad (3.2)$$

where G_0 is the pion propagator in vacuo, which we approximate by its pole term

$$G_0 = (m_{\pi}^2 - \Delta - \nu^2 - i\varepsilon)^{-1} \quad (3.3)$$

U is the optical potential, containing a nuclear part and a Coulomb part. Making the usual ansatz for the nuclear part, we find

$$U(\vec{x}) = U_N(\vec{x}) - 2\nu V(\vec{x}) \quad (3.4)$$

$$U_N(\vec{x}) = -i v \sigma_\pi \rho(\vec{x}) \quad (3.5)$$

where σ_π is the total pion nucleon cross-section and $\rho(\vec{x})$ the nuclear density function. We have furthermore assumed that the pion nucleon scattering amplitude is purely imaginary in the forward direction.

Now, Γ Eq. (2.34) can be related to G :

$$\Gamma = 2 \operatorname{Im} \langle \xi | G | \xi \rangle \quad (3.6)$$

It is convenient to separate Γ , in an elastic part, containing as final states the coherently produced pions (no nuclear excitation) and the remainder, which is described by absorption in the optical model. By repeated use of Eq. (3.2) we find

$$\Gamma = \Gamma_a + \Gamma_{el} \quad (3.7)$$

$$\Gamma_a = -2 \langle \xi | G^\dagger \operatorname{Im} U G | \xi \rangle \quad (3.8)$$

$$\Gamma_{el} = \int \frac{d^3 p}{(2\pi)^2} \delta(p^2 - v^2 + m_\pi^2) / \langle \chi_r | \xi \rangle^2 \quad (3.9)$$

where $\chi_r(\vec{x})$ is the wave function for an outgoing pion:

$$\{-\Delta - v^2 + m_\pi^2 + U^\dagger\} \chi_r(\vec{x}) = 0 \quad (3.10)$$

$$\chi_r(\vec{x}) \rightarrow e^{i\vec{p}\vec{x}} \quad \text{for } \vec{p}\vec{x} \rightarrow \infty \quad (3.11)$$

i) The absorptive contribution

Remembering the definition of ξ , Eq. (2.32), and Eq. (3.2), for G , we can write

$$G|\xi\rangle = N \frac{\nu}{\tilde{\nu}} e^{i[\vec{k}_1 \vec{x} - S(\vec{x})]} - \varphi(\vec{x}) \quad (3.12)$$

$$\begin{aligned} \{-\Delta - \nu^2 + m_\pi^2 + \mathcal{U}\} \varphi(\vec{x}) = \\ \left\{ \mathcal{U}_N + \frac{m_\pi^2 \tilde{\nu}}{2\tilde{E}_2} \right\} N \frac{\nu}{\tilde{\nu}} e^{i[\vec{k}_1 \vec{x} - S(\vec{x})]} \end{aligned} \quad (3.13)$$

$\varphi(\vec{x})$ is the coherently produced pion wave which interferes with the direct reaction, represented by the first term in Eq. (3.12) (see Fig. 3).

We can solve Eq. (3.13) in the eikonal approximation. This leads to the following result:

$$\begin{aligned} G|\xi\rangle = e^{\int_{-\infty}^z \frac{\mathcal{U}_N}{2i\tilde{\nu}} dz''} \left(1 - \frac{m_\pi^2 \tilde{\nu}}{2(m_\pi^2 \tilde{\nu} + m_\pi^2 \tilde{E}_2)} \right) \\ \cdot N \frac{\nu}{\tilde{\nu}} e^{i[\vec{k}_1 \vec{x} - S(\vec{x})]} \end{aligned} \quad (3.14)$$

where the quantities $\tilde{\nu}$, $\tilde{E}_2$, $N(\vec{x})$ have to be taken at the upstream end of the nucleus. Inserting this in (3.8) leads finally to

$$\begin{aligned} \Gamma_a = 2\nu \int d^2b \left(1 - e^{-\int_{-\infty}^{\infty} \tilde{\sigma}_\pi \rho(\vec{b}, z) dz} \right) \\ \cdot \left(1 - \frac{m_\pi^2 \tilde{\nu}}{2(m_\pi^2 \tilde{\nu} + m_\pi^2 \tilde{E}_2)} \right)^2 N^2 \cdot \frac{\nu}{\tilde{\nu}} \end{aligned} \quad (3.15)$$

which we can also write as

$$\Gamma_a = 2\nu \sigma_{\pi}^{(a)}(A) \left(1 - \frac{m_{\pi}^2 \tilde{\nu}}{2(m_{\pi}^2 \tilde{\nu} + m_{\pi}^2 \tilde{E}_2)}\right)^2 N^2 \frac{\nu}{\tilde{\nu}} \quad (3.16)$$

where $\sigma_{\pi}^{(a)}(A)$ is the absorption cross-section for pions on the nucleus A in the absence of the Coulomb field.

ii) The elastic contribution

To determine the matrix element $\langle \chi_p | \xi \rangle$ appearing in Eq. (3.9) we insert ξ from Eq. (2.32). This gives:

$$\begin{aligned} \langle \chi_p | \xi \rangle &= - \langle \chi_p | U_N | N \frac{\nu}{\tilde{\nu}} e^{i[\vec{k}_1 \cdot \vec{x} - S(\vec{x})]} \rangle \\ &\quad - \langle \chi_p | \frac{m_{\pi}^2 \tilde{\nu}}{2 \tilde{E}_2} | N \frac{\nu}{\tilde{\nu}} e^{i[\vec{k}_1 \cdot \vec{x} - S(\vec{x})]} \rangle \end{aligned} \quad (3.17)$$

In the second term we will substitute the following expression for χ_p derived from Eq. (3.10)

$$\chi_p = \chi_p^c - \frac{1}{-\Delta - \tilde{\nu}^2 + m_{\pi}^2} U_N^{\dagger} \chi_p \quad (3.18)$$

where χ_p^c is the wave function for a pion in the pure Coulomb field.

$$\begin{aligned} \langle \chi_p | \xi \rangle &= \\ &= - \langle \chi_p | U_N \left(1 - \frac{1}{-\Delta - \tilde{\nu}^2 + m_{\pi}^2} \frac{m_{\pi}^2 \tilde{\nu}}{2 \tilde{E}_2}\right) | N \frac{\nu}{\tilde{\nu}} e^{i[\vec{k}_1 \cdot \vec{x} - S(\vec{x})]} \rangle \\ &\quad - \langle \chi_p^c | \frac{m_{\pi}^2 \tilde{\nu}}{2 \tilde{E}_2} | N \frac{\nu}{\tilde{\nu}} e^{i[\vec{k}_1 \cdot \vec{x} - S(\vec{x})]} \rangle \end{aligned} \quad (3.19)$$

Evaluating the integrals of Eq. (3.19) in the eikonal approximation leads finally to

$$\begin{aligned}
 \langle \chi_r | \mathcal{F} \rangle = & 2i\nu \int d^2b e^{-i\vec{p}_\perp \cdot \vec{b}} \left(1 - e^{-\int_{-\infty}^{\infty} \frac{\sigma_\pi \rho(b,z)}{2} dz} \right) \\
 & \cdot \left(1 - \frac{m_\mu^2 \tilde{\nu}}{2(m_\mu^2 \tilde{\nu} + m_\pi^2 \tilde{E}_2)} \right) N N_\pi \frac{\nu}{\tilde{\nu}} \\
 & - \frac{m_\mu^2 \nu^2}{2(m_\mu^2 \nu + m_\pi^2 E_2)} \left(\frac{m_\pi^2}{\nu^2} - \frac{m_\mu^2}{E_2^2} \right) \int d\vec{x} e^{i(\vec{k}_1 - \vec{k}_2 - \vec{p}) \cdot \vec{x}} V(\vec{x})
 \end{aligned}
 \tag{3.20}$$

where

$$N_\pi = 1 + \frac{1}{2} \frac{V(\vec{x})}{\nu}
 \tag{3.21}$$

is a normalization factor for the π wave analogous to the one for the μ wave defined in Eq. (2.20).

By numerical comparison of the first and second term in Eq. (3.20) we found that the second term is of the order of per cents of the first. Since the first is purely imaginary, the second purely real, the contribution of the latter in the absolute square is negligible. Taking this into account, we find for Γ_{el} :

$$\begin{aligned}
 \Gamma_{el} = & 2\nu \int d^2b \left(1 - e^{-\int_{-\infty}^{\infty} \frac{\sigma_\pi \rho(b,z)}{2} dz} \right)^2 \\
 & \cdot \left(1 - \frac{m_\mu^2 \tilde{\nu}}{2(m_\mu^2 \tilde{\nu} + m_\pi^2 \tilde{E}_2)} \right)^2 N^2 N_\pi^2 \left(\frac{\nu}{\tilde{\nu}} \right)^2 \\
 = & 2\nu \sigma_\pi^{(el)}(A) \left(1 - \frac{m_\mu^2 \tilde{\nu}}{2(m_\mu^2 \tilde{\nu} + m_\pi^2 \tilde{E}_2)} \right)^2 N^2 N_\pi^2 \left(\frac{\nu}{\tilde{\nu}} \right)^2
 \end{aligned}
 \tag{3.22}$$

where here the value at the down-stream end of the nucleus has to be taken for $\tilde{\nu}$, $\tilde{E}_2$, N , N_π . $\sigma_\pi^{el}(A)$ is the elastic cross-section for pions on the nucleus A without Coulomb effect.

Inserting the values of N , Eq. (2.20) and N_π , Eq. (3.21) in Γ_{el} and N , Eq. (2.20a) in Γ_a we find:

$$\Gamma_a = 2\nu \sigma_\pi^{(a)}(A) \left(1 - \frac{m_\pi^2 \tilde{\nu}}{2(m_\pi^2 \tilde{\nu} + m_\pi^2 \tilde{E}_2)}\right)^2 \cdot \left(1 - 4 \frac{\nu}{\tilde{E}_2} - \frac{\nu}{\nu}\right) \quad (3.23)$$

$$\Gamma_{el} = 2\nu \sigma_\pi^{el}(A) \left(1 - \frac{m_\pi^2 \tilde{\nu}}{2(m_\pi^2 \tilde{\nu} + m_\pi^2 \tilde{E}_2)}\right)^2 \cdot \left(1 - \frac{\nu}{\tilde{E}_2} - \frac{\nu}{\nu}\right) \quad (3.24)$$

If the nucleus is an opaque sphere to the pion, then $\sigma_\pi^{(a)}(A) = \sigma_\pi^{el}(A)$. In reality this is not exactly true, however, we certainly can assume this in the correction term of order $\nu/\tilde{E}_2$. Doing so we arrive at

$$\Gamma = 2\nu \sigma_\pi^{tot}(A) \left(1 - \frac{m_\pi^2 \tilde{\nu}}{2(m_\pi^2 \tilde{\nu} + m_\pi^2 \tilde{E}_2)}\right)^2 \cdot \left(1 - \frac{5}{2} \frac{\nu}{\tilde{E}_2} - \frac{\nu}{\nu}\right) \quad (3.25)$$

where $\sigma_\pi^{tot}(A)$ is the total nuclear cross-section of pions on the nucleus A , without inclusion of the Coulomb scattering.

4. THE INTERFERENCE TERM

We still have to estimate the interference term Γ' , Eq. (2.35). Since at our moderate energy transfers ν coherent propagation of vector mesons in the nucleus is expected to be unimportant, the sum over final states $|\beta\rangle$ in Eq. (2.35) will be a sum over the states produced by the current $\vec{J}'\vec{l}_\perp$ on the single nucleons. By the optical theorem we can relate this sum to the production amplitude for a pion of momentum $\vec{q}(\vec{x})$ by the current $\vec{J}'\vec{l}_\perp(\vec{x})$ where $\vec{x}$ is the position of the nucleon. This amplitude, however, vanishes by virtue of Eq. (2.23). Therefore, we find that $\Gamma' = 0$.

5. CONCLUSION

We have performed an estimate of the Coulomb corrections to the Adler PCAC test in nuclei for energy transfers ν large enough so that effects from going off the pion mass shell can be neglected but below or only slightly above threshold for vector meson production. A typical range for ν would be [compare Ref. 5) for the lower limit]

$$0.4 < \nu < 1.2 \text{ GeV}$$

(5.1)

In a qualitative way we can describe our calculation as follows: the muon inside the nucleus is created with a different wave number than it has at infinity. This implies that the lepton current essentially becomes proportional to the local four-momentum transfer $\tilde{q}_\lambda(\vec{x})$. But this is exactly what is needed to convert the hadron current to the pion field using the PCAC equation modified for the presence of the Coulomb field. Therefore, the problem could again be described by the scattering of a virtual pion on the nucleus, where the driving term, however, was not a plane wave but a Coulomb distorted one. The pure Coulomb contribution to the scattering of this virtual pion was found non-singular, since it involved even for forward production a momentum transfer $\neq 0$. So, one never reaches the Coulomb singularity. Moreover, there is a cancellation between the contributions from diagrams of type (a) and (b) of Fig. 4, which makes this contribution negligible.

The remaining corrections can be summarized in a multiplicative factor:

$$\left(\frac{\partial^2 \sigma}{\partial \nu \partial R} \right)_{R=0} = \left(\frac{\partial^2 \sigma}{\partial \nu \partial R} \right)_{R=0}^{(0)} (1 + \delta) \quad (5.2)$$

$$1 + \delta = \left(1 - \frac{m_\mu^2 \tilde{\nu}}{2(m_\mu^2 \tilde{\nu} + m_\pi^2 \tilde{E}_2)} \right)^2 \cdot \left(1 - \frac{m_\mu^2 \nu}{2(m_\mu^2 \nu + m_\pi^2 E_2)} \right)^{-2} \left(1 - \frac{5}{2} \frac{V}{E_2} - \frac{V}{\nu} \right) \quad (5.3)$$

$\tilde{\nu} = \nu + V$, $\tilde{E}_2 = E_2 - V$ and $V = -Z\alpha/R$ is the Coulomb potential at the edge of the nucleus.

In Fig. 5 we give the result of a calculation of δ [Eq. (5.3)] for the case of lead. It is seen that for $E_2 \geq 1.0$ GeV and $\nu \geq 0.5$ GeV, $\delta \leq 10\%$. The implications of this result for the recent experiment ⁸⁾ done at CERN to detect the $A^{2/3}$ behaviour of the forward neutrino cross-section will be discussed elsewhere.

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FIGURE CAPTIONS

- Figure 1 :
Neutrino reaction on nucleus.
- Figure 2 :
Propagation of the muon out of the nucleus.
- Figure 3 :
Direct (left) and pion induced reaction (right).
- Figure 4 :
Pion production by neutrinos in the Coulomb field
to lowest order.
- Figure 5 :
Coulomb correction factor δ [Eq. (5.3)] for lead
as function of energy transfer ν , for muon energies
 $E_2 = 0.5, 1.0, 1.5$ GeV.

FIG.1

FIG.2

FIG.3

FIG.4

FIG.5