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TOTAL CROSS SECTIONS FOR PIONS ON PROTONS IN
THE MOMENTUM RANGE 10 TO 20 GEV/C

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SUMMARY

The total cross sections for positive and negative pions on protons have been measured in the momentum range 10 - 20 GeV/c, using the same methods as applied in the momentum range below 10 GeV/c. The monotonic decrease of both cross sections with increasing momentum, previously observed between 4.5 and 10 GeV/c, is found to persist over the whole momentum range covered, but the rate of decrease is considerably smaller above 10 GeV/c than in the region below 10 GeV/c. The difference between $\sigma(\pi^- + p)$ and $\sigma(\pi^+ - p)$ is slowly decreasing as the momentum is increased and is of the order of 1 mb at 20 GeV/c. The results are consistent with the Pomeranchuk theorem and with the sum rule for pion-nucleon scattering.

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In a previous paper we reported on cross section measure-
ments for positive and negative pions on protons in the momentum
range 4.5 to 10 Gev/c. Contrary to the current assumption that the
cross sections for positive and negative pions are constant and
equal above 2 Gev/c, the experiment revealed a monotonic decrease
of the cross sections with momentum and a difference between \( \sigma(\pi^+ + p) \)
and \( \sigma(\pi^+ - p) \) of about 2 mb. Lindenbaum et al.\(^2\) reported on measure-
ments of pion-proton cross sections in the momentum range 7 - 20 Gev/c,
which indicated that the monotonic decrease of the cross sections
persists up to 20 Gev/c and that the difference between them shows
no sizeable decrease with increasing momentum. This finite dif-
ference of the cross sections at relatively high momentum is interest-
ing in view of two important theorems.

a) According to a theorem first formulated by Pomeranchuk\(^3\), the
cross sections \( \sigma_+ (E) \) and \( \sigma_- (E) \) for a particle and its anti-
particle on any object should become equal, as \( E \to \infty \). This
theorem has recently been derived under relatively weak as-
sumptions\(^4\): neither of the two cross sections must increase
faster than \( (\ln E)^{3/2} \) as \( E \to \infty \), and the difference \( \Delta \sigma = \sigma_- - \sigma_+ \)
must not change sign an infinite number of times.

b) The sum rule for pion-nucleon scattering

\[ \frac{1}{6\mu} (1+\mu/M)(a_1-a_2) = c_1^2 \mu^2 + \frac{1}{8\pi^2} \int_\mu^\infty (\sigma_-(\omega) = c_1^2(\omega)) \frac{d\omega}{\omega} \]  

(1)

which connects the s-wave scattering lengths \( a_1 \) and \( a_2 \) with the re-
normalized pion-nucleon coupling constant \( c_1^2 \) and an integral over \( \Delta \sigma \).
This sum rule which follows from the dispersion relations for forward pion-nucleon scattering has recently been evaluated by Spearman under the assumption, that $\Delta \sigma = 0$ for $p > 2$ Gev/c. The result was $f^2 = 0.082 \pm 0.008$, which is in good agreement with independent determinations of the coupling constant. Since recent experiments indicate a finite $\Delta \sigma$ above 2 Gev/c there will be a correction to $f^2$ given by

$$\delta(f^2) = \frac{2}{f^2} \int^{\infty}_{2 \text{Gev/c}} \Delta \sigma \frac{d\omega}{p}$$  \hspace{1cm} (2)$$

which will give a finite modification to the coupling constant if the Pomeranchuk theorem is satisfied and which is infinite otherwise.

As a continuation of our previous work and in order to check the behaviour of pion-proton cross sections in view of the above theorems, the measurements were extended into the momentum range 10 - 20 Gev/c. The method used was essentially the same as described earlier, with the only modification that, in addition to a differential gas Čerenkov counter, a 10 m long threshold Čerenkov counter was used to identify the incident pions. The rejection against other strongly interacting particles was better than 1% and the electron composition of the beam was estimated from previous measurements, to be less than 1%. The $\mu$ meson component was eliminated from the measurements by a combined anticoincidence absorber technique, as described previously, with an Fe absorber of 1.6 m length. The uncertainty introduced by beam contamination was therefore negligible at all momenta. For each cross section the transmission was measured in at least 3 geometries, the counters subtending average solid angles at the target between 0.2 and 1 msr at 10 Gev/c and between 0.1 and 0.5 msr at 20 Gev/c.

The results are given in Table I.
The errors given include, in addition to the statistical uncertainties, the errors due to rate effects, and to the extrapolation procedure. The absolute values of the cross sections may have an additional common error of 0.5 mb, due to uncertainties in the dummy calibration, the effective target length, and other systematic sources. The rate of decrease of both cross sections is considerably smaller above 10 Gev/c than at lower momenta. In this region the cross section for positive pions is constant within the limit of error, but seems to decrease slowly. The same is true for the difference \( \Delta \sigma \), which is given in column 4 of Table I.

One may try to fit the total cross section data to some assumed functional law in order to extrapolate them to higher energies. Theory does not give much guidance on what law should be used, except that the Pomeranchuk theorem requires the difference between the positive and negative cross sections to decrease faster than \( 1/\ln E \). Lindenbaum et al.\(^2\) found that a law of the type \((a+b/p)\) fitted both their data and our previous data up to 10 Gev/c. The constant \( a \), which they find is different for positive and negative pions, and this functional behaviour would therefore violate Pomeranchuk's theorem and the sum rule (1).

Our data between 4.5 and 20 Gev/c can be fitted very well with a functional behaviour

\[
\sigma(p^{\pm}p) = \sigma_{\infty} + b^{\pm}_p - a
\]

which satisfies the Pomeranchuk theorem and makes the sum rule convergent. The cross section at infinite energy \( \sigma_{\infty} \) and the coefficient \( b^{\pm}_p \) for positive and negative pions are given in Table I for different values of the exponent \( a \). From the value of \( \chi^2 \), whose expected value is 13 when the 3 parameter expression (3) is fitted to our 16 measurements, it is seen that a wide range of exponents between 0.3 and 1 give excellent fits to our measurements. The fact
that the minimum values $\chi^2$ are considerably below the expected value, and that the fit is thus rather better than one would expect, is an indication that the errors are not entirely statistical but also include some systematic errors.

The goodness of the fit between the data and the expression (3) is also demonstrated by Fig. 1, where the momentum scale is chosen such that a linear relation is obtained. This figure also includes our previous measurements from 4.5 to 10 Gev/c. The data of Lindenbaum et al\(^2\) between 7 and 20 Gev/c are shown for comparison. For negative pions they agree within the errors with our points and with the functional behaviour. For positive pions their measurement in the range between 11 and 14 Gev/c is lower than ours, but outside this range there is good agreement. The data of Longo\(^6\) below 4 Gev/c fit very well to the energy dependence (3) extrapolated with the same constants as ours down to 2.7 Gev/c. Below this momentum the cross section levels off to a constant value of about 30 mb.

The good fit to a momentum dependence of the type (3) over this wide momentum range may be taken as a strong support of Pomeranchuk's prediction that positive and negative total cross sections approach the same value at high energies, and that the approach to this limit is sufficiently rapid to be consistent with the sum rule. The latter fact is apparent from the last column in Table I, which gives the correction to the coupling constant contributed by the energy range above 2 Gev/c. These corrections were calculated by taking $\Delta \sigma(p) = \Delta \sigma(4.5 \text{ Gev/c})$ for $2 \text{ Gev/c} < p < 4.5 \text{ Gev/c}$ and using (3) for $p > 4.5 \text{ Gev/c}$. Since the error in the sum rule from the uncertainty in the low-energy quantities which enter the sum rule is about 0.008, the high-energy behaviour has very little influence, if the exponent $\alpha$ is larger than 0.2.
It should be emphasized that while our measurements within the experimental accuracy support the assumption of equal cross sections at infinite energy, they do not, of course, prove conclusively that this assumption is indeed correct. One can certainly imagine other functional laws which fit the data up to 20 Gev/c and still behave quite differently at infinite energy.

A conclusive proof, if ever possible, will require very accurate measurements at much higher energies than are presently available.

We are grateful for the whole-hearted co-operation of the PS machine division in the operation of the synchrotron and of our hydrogen target. We also want to thank Messrs. G. Gendre and L. Velati for their assistance in preparing and running the experiment.
### TABLE I
Total Cross Sections for $(\pi^\pm + p)$

<table>
<thead>
<tr>
<th>Momentum (Gev/c)</th>
<th>$\sigma(\pi^- , p)$ (mb)</th>
<th>$\sigma(\pi^+ , p)$ (mb)</th>
<th>$\Delta\sigma$ (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>26.5 ± 0.35</td>
<td>25.0 ± 0.5</td>
<td>1.5 ± 0.45</td>
</tr>
<tr>
<td>12</td>
<td>26.0 ± 0.25</td>
<td>24.8 ± 0.3</td>
<td>1.2 ± 0.4</td>
</tr>
<tr>
<td>14</td>
<td>26.0 ± 0.20</td>
<td>24.7 ± 0.3</td>
<td>1.3 ± 0.4</td>
</tr>
<tr>
<td>17</td>
<td>25.7 ± 0.20</td>
<td>24.6 ± 0.2</td>
<td>1.1 ± 0.3</td>
</tr>
<tr>
<td>20</td>
<td>25.6 ± 0.50</td>
<td>24.1 ± 0.5</td>
<td>1.5 ± 0.7</td>
</tr>
</tbody>
</table>

### TABLE II
Best Fits to the Data for $\sigma_T(\pi^\pm , p)$ 4.5 - 20 Gev/c

$\sigma_T^{\pm} = \sigma_0 + b_\pm \cdot p^{-\alpha}$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\sigma_0$</th>
<th>$b_+^{\text{mb(Gev/c)+} \alpha}$</th>
<th>$b_-^{\text{mb(Gev/c)} \alpha}$</th>
<th>$\chi^2$</th>
<th>$-\delta\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>12.92</td>
<td>19.87</td>
<td>22.37</td>
<td>18</td>
<td>0.0079</td>
</tr>
<tr>
<td>0.3</td>
<td>17.39</td>
<td>16.03</td>
<td>19.18</td>
<td>15</td>
<td>0.0062</td>
</tr>
<tr>
<td>0.5</td>
<td>20.94</td>
<td>13.97</td>
<td>19.04</td>
<td>9</td>
<td>0.0049</td>
</tr>
<tr>
<td>0.7</td>
<td>22.48</td>
<td>14.37</td>
<td>22.10</td>
<td>5</td>
<td>0.0042</td>
</tr>
<tr>
<td>1.0</td>
<td>23.63</td>
<td>17.19</td>
<td>31.28</td>
<td>7</td>
<td>0.0036</td>
</tr>
</tbody>
</table>

Expected $\chi^2 = 16 - 3 = 13$. 
References


* * *
Figure Caption

Fig. 1 - Total $(\pi^- + p)$ and $(\pi^+ + p)$ cross sections versus $p^{-0.7}$. The solid line represents the least squares fit with $\alpha = 0.7$ (see Table II) to the data of Ref. 1 and of the present experiment.

The results of other authors are shown for comparison. The half-height of the symbols indicate the standard deviations.

* * *

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