Lepton number violation via intermediate black hole processes

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Black holes at the TeV scale are investigated in the extra large dimension scenario. We interpret the lightest black hole excitation as a singlet scalar field, and show how interaction terms can be appended to the standard model at the dimension five non-renormalizable level. Lepton family number violation is natural in this model. Muon magnetic moment, and neutrino masses are investigated. We also present a quantization scheme in n dimensions.

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I. INTRODUCTION

The existence of black holes at the \( M_P \sim \text{TeV} \) range has been conjectured within recently proposed scenarios of extra large dimensions. In light of planned searches at the LHC, an effective field theory for black hole processes has been designed. This approach is superior to other avenues for black hole analysis in the respect that it allows one to calculate interference effects of black hole processes with relative ease.

In this paper we show how black hole excitations can be incorporated into the standard model at the dimension five non-renormalizable level. We calculate the corrections that the muon magnetic moment receives from a black hole at the TeV scale and show that black hole corrections do not spoil the smallness of the neutrino masses.

It is assumed that there are n additional compact spatial dimensions possibly at the size of a millimeter and that the fundamental Planck scale is comparable to the electroweak scale. Black hole solutions on such higher dimensional backgrounds are rare. However, in the case where \( R_s \ll R \) where \( R_s \) is the Schwarzschild radius and \( R \) the characteristic size of the extra dimensions, we can approximate the dimensions as asymptotically flat and then use the results of Myers and Perry. To wit
\[ R_s \sim \frac{1}{M_{ew}} \left( \frac{M_{bh}}{M_{ew}} \right)^{1/(n+1)}, \quad \text{and} \quad R \sim 10^{30/n} \text{ TeV}^{-1}, \]
so we would need a black hole of mass \( \gtrsim 10^{30} \text{ TeV} \) before this assumption breaks down. Typically, we will be dealing with black holes of \( M_{bh} \sim 1 \text{ TeV} \).

The black hole in this paper is treated as a particle quantized in mass. Originally this quantization was constructed in 4 dimensions. In Appendix 1 we present the analogous result for a charged spin zero black hole in n+3 spatial dimensions.

II. SUMMARY OF THE MODEL

In essence the philosophy of the model is to interpret each quantized black hole excitation as an independent quantum field. The lightest such excitation has zero charge and zero angular momentum and thus corresponds to a neutral scalar field. The original ansatz was constructed for the interaction between two charged fermions and a doubly charged black hole. It consisted of a Yukawa Lagrangian of the type:
\[ L_{\text{int}} = i k_{\text{eff}} M_{bh} \phi_{bh} \overline{\psi}_f \hat{C} \psi_f + \text{h.c.}, \]
where \( \hat{C} = i \gamma_2 \mathcal{K} \) is the charge conjugation operator, \( \mathcal{K} \) being complex conjugation and \( \psi_f \) is the fermion field. Similarly, one can write an effective Lagrangian for a neutral scalar black hole by removing the \( \hat{C} \) operator:
\[ L_{\text{int}} = i k_{\text{eff}} M_{bh} \phi_{bh} \overline{\psi}_f \psi_f + \text{h.c.}. \]

Photon-black hole interactions have also previously been considered:
\[ L_{\text{int}} = \frac{k_{\text{eff}}}{M_p} \phi_{bh} F_{\mu \nu} F^{\mu \nu}. \]
In this paper we present a rationale for the choice of these black hole interactions. That is, they all arise from non-renormalizable dimension five operators that respect the standard model symmetries. Thus one imagines that this effective theory is the remnant of some quantum gravity theory broken at the $M_p$ scale.

For simplicity we focus on the phenomenology of a single neutral scalar black hole, $\phi_{bh}$. This is a singlet under the standard model gauge group. Therefore at the dimension five level we have:

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{g'}{\Lambda} \mathcal{L}_{SM} \phi_{bh} + (D\phi_{bh})^2 - V(\phi_{bh}) + h.c.,$$

(4)

where $g' L_{SM} \phi_{bh}$ is symbolic for all standard model operators times a $\phi_{bh}$ operator with possibly different coupling constants $g'$ on each term and D is the covariant derivative. We expect that the theory will be cut off at the fundamental planck scale $\Lambda = M_p$, where a new theory that accommodates gravity will take over.

It is the second term of (4) that contains the black hole interactions used in previous work. For example one sees that the interaction in equation (2) can be obtained from $\frac{g'}{M_p} \bar{\Psi} H e_R \phi_{bh}$ term after Higgs breaking $<\phi_H> = v$,

$$\frac{g'}{M_p} \bar{\Psi} H e_R \phi_{bh} + h.c. \rightarrow k_{eff} M_{bh} \bar{\Psi} e \Psi_{e} \phi_{bh},$$

if we make the identification $k_{eff} = \frac{g'}{M_p M_{bh}}$.

The authors [8] have determined $k_{eff}$ by comparing the black hole effective production cross section with the geometrical cross section $\sigma_{geom} = \pi R^2_{bh}$ on resonance, i.e multiplied by a dimensionless generalized function peaked at the com energy $\sqrt{s} = M_{bh}$:

$$\sigma_{geom} = \pi R^2_{bh} M_{bh} \delta(\sqrt{s} - M_{bh}).$$

(5)

The relevant effective production process is shown in FIG. 1:

![FIG. 1: Black hole production.](image)

This has the cross section,

$$\sigma_{eff} = \frac{1}{4} |k_{eff}|^2 M_{bh} \delta(\sqrt{s} - M_{bh}).$$

(6)

After equating $\sigma_{eff}$ and $\sigma_{geom}$ the relationship $k_{eff} = 2R_s$ is made. The radius of the $n+3$ dimensional Schwarzschild black hole is solved in [10], thus,

$$k_{eff} = \frac{2}{\sqrt{\pi M_p}} \left[ \frac{M_{bh} 8\Gamma(\frac{n+3}{2})}{M_p^{n+2}} \right]^{1/(n+1)}.$$

(7)

This gives the same $k_{eff} \propto \frac{1}{M_p}$ dependence that we expect if the theory is cut off at the $\Lambda = M_p$ scale. The two approaches are consistent if the black hole mass is at the electroweak scale. From here on we assume that $k_{eff} = g'/M_p$. Where $g'$ is a coupling constant dependent on the particular interaction under investigation.

### III. LEPTON NUMBER VIOLATION

It has been conjectured that black hole processes will violate certain approximate global symmetries like lepton number [12, 13, 14]. Presently, we wish to extend the original work [8, 9] to include lepton family number violation. This is implemented in equation (4) by allowing the usual 3 generations of fermion fields in $\mathcal{L}_{SM}$. Consider the terms:

$$\mathcal{L} = \ldots + \frac{1}{M_p} \bar{\Psi} H e_i e_{Rj} \phi_{bh} + \bar{\Psi} H \lambda_{ij} e_{Rj},$$

(8)

where a summation over generations is understood ($i, j \in 1, 2, 3$). Now after Higgs breaking and rotating the weak eigenstates into mass eigenstates, i.e $m = U \lambda U^\dagger$, this Lagrangian becomes:

$$\mathcal{L} \rightarrow \bar{\Psi} L_i k_{ij} e_{Rj} \phi_{bh} + m_i \bar{\Psi} L_i e_{Rj}.$$

(9)

Importantly $k = \frac{g'}{M_p} U U^\dagger$ is not in general diagonal. Thus lepton family violating interactions like $k_{\mu e} \bar{\Psi} e_{Rj} \phi_{bh}$ arise naturally in this picture as off diagonal terms in $\bar{\Psi} L_i k_{ij} e_{Rj} \phi_{bh}$.

We now consider the decay mode $\mu^{-} \rightarrow e^{-} e^{+} e^{-}$, which proceeds through the processes shown in FIG. 2. It has been said [9] that the experimental bound on this process would require a Planck scale at the 100 TeV range. In our approach we have an extra parameter in $g'$, recall $k = \frac{g'}{M_p}$, if we want a TeV scaled Planck mass then we must also tolerate $g' \sim 10^{-3} - 10^{-4}$ sized couplings.
The decay rate is:
\[ \Gamma(\mu^- \rightarrow e^- e^+ e^-) = m_\mu^5 k_{ee}^2 k_{\mu e}, \]
which puts a bound on the product of the two couplings,
\[ k_{ee} k_{\mu e} < 1.5 \times 10^{-7} \text{ TeV}^{-2}. \]

Since we take \( M_\mu \sim 1 \text{ TeV} \) this means
\[ g_{ee \mu e} < 1.5 \times 10^{-7}, \]
which is consistent with a TeV scaled Planck mass. It is also interesting that with tolerable tuning we could have one \( g \) rather large \( \sim 10^{-1} \) and the other small \( \sim 10^{-6} \). Our model would therefore be able to accommodate black holes that favor certain processes. This does not contradict thermal arguments as our understanding of Hawking radiation breaks down at the Planck scale.

**FIG. 2:** Feynman diagram for the muon decay via the neutral scalar black hole interaction.

**IV. MUON MAGNETIC MOMENT CORRECTION**

In this section we calculate the correction to the muon magnetic moment from the diagram shown in FIG. 3. The matrix element for this process is:

\[ \mathcal{M} = e k_\phi^2 M_{bh}^2 \bar{U}(p') \Lambda' U(p) A_{\mu} (p' - p), \]

where,
\[ \Lambda' = \frac{-i}{(2\pi)^4} \int \frac{dk^4}{k^2 - M_{bh}^2 + i\epsilon} \frac{p' - k + m_1}{(p' - k)^2 - m_1^2 + i\epsilon} \gamma^\mu \frac{p - k + m_1}{(p - k)^2 - m_1^2 + i\epsilon}, \]

and we have allowed for lepton number violating vertices, ie the internal fermion lines in FIG. 3 could be non-muonic charged leptons of mass \( m_1 \). This gives a corresponding correction to the muon magnetic moment of
\[ a_{\phi h} = \frac{k_\phi^2 m_\mu^2}{8\pi^2} \int_0^1 dz \frac{z^2 (\beta - z)}{(1 - z) + \alpha z^2}, \]
where $\beta = 1 + \frac{m_{\nu l}}{m_{\nu l}}$ and $\alpha = \left(\frac{m_{\nu l}}{m_{\nu l}}\right)^2$.

If one assumes a $g_{\mu\mu} \lesssim 1$ the correction is of order $a_{bh} \sim 10^{-10}$, which is close to the current level of deviation between standard model and experiment $|a_{\text{exp}} - a_{\text{theory}}| < 42.6 \times 10^{-10}$ [10]. Choosing $g_{\mu\mu} \lesssim 1$ is not inconsistent with the muon decay result [11], however it does require some tuning. Assuming that all couplings are of the same order brings us to corrections of the size $10^{-15}$. In any case it is not possible for perturbative $g$ to produce corrections that would contradict experimental results. Specifically, an overestimate can be obtained by taking $g = 1$ and multiplying the correction for a tauon in the loop by 3 since this contributes the most of the 3 leptons. This leads to $a_{bh} < 1.5 \times 10^{-10}$.

**V. NEUTRINO MASSES**

It is interesting that black hole process at the TeV scale can induce small neutrino masses. Consider the process shown below:

![Figure 4: Neutrino mass term](image)

This loop has a momentum integral:

$$\int_0^{M_p} d^4k \frac{1}{k^2 + M_{bh}^2}$$

which induces a mass $m \sim g^2 M_p$. Taking $g$ of the order of $\sim 10^{-6}$ would give masses of $\sim 1$ eV size. If one is willing to believe that black hole processes at the TeV range are the dominate source of neutrino mass then one could place constraints on the $g'_{ij}$ couplings using the neutrino mass differences and the mixing angles.

**VI. CONCLUSION**

In this work we have shown that the phenomenology of a single scalar black hole excitation at the TeV scale can be introduced into the standard model without spoiling current experimental results either for the muon magnetic moment or the neutrino masses. All calculations made in this paper are done within the effective non-renormalizable theory. One imagines that these operators are remnant radiative effects of some high energy theory like string theory. The true worth of our approach lies in its ability to attain sensible results that are comparable with experiment. Our approach has the flexibility to accommodate alternative black hole phenomenology that would be dependent on the results of forthcoming searches planned for the LHC.

**Appendix 1**

In this appendix we present the quantization scheme of a charged spin zero black hole in $n+3$ spatial dimensions. The full Kerr solution in higher dimensions is not analytically tractable, nevertheless scalar excitations are the most important in the present work and these we can solve for. In units with $\hbar = c = 1$ we have the Einstein-Hilbert-Maxwell action:

$$S = \frac{1}{16\pi G_{n+2}} \int d^{n+4}x \sqrt{-g} \left(R - F_{\mu\nu} F^{\mu\nu}\right).$$

The solution [10] is,

$$f = g^{-1} = \left(1 - \frac{C}{r^{n+1}} + \frac{D^2}{r^{2(n+1)}}\right)^{1/2},$$

where $C = \frac{16\pi G_{n+2} m_{bh}}{S_{n+3}(n+2)}$ and $D^2 = \frac{2Q^2G_{n+2}}{(n+2)(n+1)}$.

From equation (16) there is an event horizon if:

$$r_{n+1} = \frac{C}{2} \pm \sqrt{\frac{1}{4} C^2 - D^2}.$$  

We now want to perform the canonical quantization on the area of the outer horizon ($A = A(r_+)$) in the same spirit as Bekenstein [11]. Recall that the irreducible mass of a black hole, $M_{ir}$, is related to its area via:

$$M_{ir}^2 = \frac{A}{16\pi G_{n+2}}.$$

Quantizing the irreducible mass $M_{ir} = n_b g_{\mu}p$ and the charge $Q = q e$ and rearranging for $M_{bh}$ we find:

$$M_{bh} = M_p c_1 (n_b g_{\mu}p)^{n+2/4} \left[1 + \frac{1}{4} (n_b g_{\mu}p)^{n+2/4}\right].$$

Where,

$$c_1 \equiv (n+2) \left(\frac{S_{n+3}}{16\pi}\right)^{1/2},$$

$$c_2 \equiv 2 \sqrt{2(n+2)(n+1)} \left(\frac{S_{n+3}}{16\pi}\right)^{n+2/4},$$

$S_m = \frac{2\pi^{n+2}}{n+2}$ is the surface area of a unit $m$-sphere (ie in the case of an $(n+3)$-sphere $A = S_{n+3}(n+2)$ and $n_b (\in N)$ and $q (\in Z)$ are the quantization numbers for mass and charge respectively. $n_b$ is not to be confused with $n$ the number of extra dimensions; we have adopted the
notation used in [8]. The black hole mass gap, \( g_p \), is controversial. Some authors use \( g_p = 0.614/\pi \), which is calculated in a loop quantum gravity framework [17]. In the \( n=0 \) case \( c_1 \) and \( c_2 \) are equal to one and we recover the 3 dimensional quantization scheme:

\[
\frac{M_{bh}}{M_p} = (n_b g_p)^{1/2} \left[ 1 + \frac{1}{4} \frac{q^2 \alpha_{em}}{n_b g} \right].
\]

In quantizing, see equation (18), we have introduced an infinite tower of black holes labelled by the numbers \( n_b, q \) (and if we took the realistic case with angular momentum \( J \) also) each with a definite mass, \( M_{bh}^{(n_b, q, J)} \), and therefore a different propagator. Thus, to calculate the \( \mathcal{M} \) matrix for any given process we would need to sum over all the black hole modes that can contribute. In the current work the tower is naturally cut off at the \( M_p \) scale. The exact number of modes that can participate will therefore depend on the value of \( g_p \) and the number of extra dimensions. As \( \phi_{bh} \) is an effective 4 dimensional field for a higher dimensional black hole Kaluza-Klein modes will also be present of mass \( M_{bh}^n = \sqrt{M_{bh}^2 + n^2/R} \). For simplicity we choose a black hole mass such that the higher modes can be pushed above the cut off.

In this paper we have focused on the observable effects that a single scalar excitation will produce. If in new experiments black hole effects are discovered one can then use equation (18) to determine both \( g_p \) and \( n \) and the related black hole phenomenology.

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