Experimental Study of a 1.5-MW, 110-GHz Gyrotron Oscillator

by

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Abstract

This thesis reports the design, construction and testing of a 1.5 MW, 110 GHz gyrotron oscillator. This high power microwave tube has been proposed as the next evolutionary step for gyrotrons used to provide electron cyclotron heating required in fusion devices. A short pulse gyrotron based on the industrial tube design was built at MIT for experimental studies. The experiments are the first demonstration of such high powers at 110 GHz.

Using a 96 kV, 40 A electron beam, over 1.4 MW was axially extracted in the design \(\text{TE}_{22,6}\) mode in 3 \(\mu\)s pulses, corresponding to a microwave efficiency of 37 \%. The beam alpha, the ratio of transverse to axial velocity in the electron beam, was measured with a probe. At the high efficiency operating point the beam alpha was measured as 1.33. This value of alpha is less than the design value of 1.4, possibly accounting for the slightly reduced experimental efficiency. The output power and efficiency, as a function of magnetic field, beam voltage, and beam current, are in good agreement with nonlinear theory and simulations with the MAGY code.

In another phase of the experiment, a second tube was built and tested. This tube used the same gun and cavity but also incorporated an internal mode converter to transform the generated waveguide mode into a free-space propagating beam. The gun was tested to full power and current in the experiment. Preliminary results were obtained. A mode map was generated to locate the region of operating parameters for the design mode, as well as for neighboring modes. Scans of the output microwave beam were also taken using a power-detecting diode. Future work will focus on generating high power, as well as operating the collector at a depressed voltage for even higher efficiency.

A study is also presented of the 96 kV, 40 A magnetron injection gun. A critical parameter for the successful application of this electron gun is the uniformity of electron emission. The current-voltage response, at a series of temperatures, is measured for two separate cathodes. Analysis indicates that the work function of the first emitter is 1.76 eV with a (total) spread of 0.04 eV. The second emitter has a spread of 0.03 eV, centered around 1.88 eV. Measurement of the azimuthal emission uniformity with
a rotating probe indicates that the work function variation around the azimuth, the
global spread, is 0.03 eV for the first cathode, 0.02 eV for the second. The spread due
to local (microscopic scale) work function variations is then calculated to be around
0.03 eV for both cathodes. Based on the beam azimuthal measurements, tempera-
ture variation is ruled out as the cause of emission nonuniformity. In another part
of the current probe experiment, current-voltage curves were measured at azimuthal
locations in 30° increments for several cathode temperatures. From this extensive set
of data the work function distribution parameters were identified over small sections
of the cathode for the entire cathode surface.

In addition, a formulation is presented of the irradiance moments applied to the
determination of phase profiles of microwave beams from known amplitudes. While
traditional approaches to this problem employ an iterative error-reduction algorithm,
the irradiance moment technique calculates a two-dimensional polynomial phasefront
based on the moments of intensity measurements. This novel formulation has the
important advantage of identifying measurement error, thus allowing for its possible
removal. The validity of the irradiance moment approach is shown by examining a
simple case of an ideal Gaussian beam with and without measurement errors. The
effectiveness of this approach is further demonstrated by applying intensity measure-
ments from cold-test gyrotron data to produce a phasefront solution calculated via
the irradiance moment technique. The accuracy of these results is shown to be compa-
rable with that obtained from the iteration method. This algorithm was then applied
to the design of the phase correcting mirrors used in the internal mode converter
experiment.

The results of this investigation are promising for the development of an industrial
version of this gyrotron capable of long pulse or continuous-wave operation.

Thesis Supervisor: Richard J. Temkin
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Dedicated to the memory of James E. Anderson (10/7/32 – 5/29/95)
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Chapter 1

Introduction

1.1 High Power Microwave Tubes

Vacuum Electron Devices (VEDs) were first demonstrated in 1883, when Thomas Edison measured an electric current from a heated filament cathode to a positively-biased plate, or anode, in an evacuated bulb [1]. However, it was not until 1920 when the magnetron was invented [2] that significant microwave radiation was being generated from VEDs. The advent of World War II further served as a catalyst for the development of microwave tubes. The need for high power microwave signals used for radar detection systems brought about the 3-GHz, 10-kW magnetron in 1939 [3], and the klystron, invented by the Varian brothers in 1937 [4]. Ever since then, the microwave tube industry has been rapidly advancing due to an ever increasing demand for higher powers, greater efficiency, and higher frequencies.

New and varied applications for these devices have allowed the industry to evolve and thrive. The U.S. high power tube industry was initially driven and supported by the military, which was interested in reliable reusable klystrons and magnetrons for radar systems. But commercial applications followed soon after, making microwave tubes ubiquitous. Due to the popularity of the microwave oven, almost every household in the U.S. contains a magnetron. Satellite communication systems, broadcasting, and air traffic control radar all rely heavily on the travelling wave tube (TWT) amplifier [5]. New ground- and space-based radar systems for atmospheric
imaging are being developed at higher frequencies for greater resolution. For example, at the Naval Research Lab (NRL), a 35-GHz gyroklystron has been developed for radar applications [6], as well as a 94-GHz gyroklystron, which was recently designed and successfully tested [7]. The high energy physics community is interested in these devices as well. The klystron has been the workhorse in many particle accelerators and colliders, notably the Stanford Linear Accelerator (SLAC) and, later, the Stanford Linear Collider (SLC), which used amplified signals produced by 65-MW S-band klystrons to accelerate particles to high energies [8]. Many new high energy experiments are being proposed, including the Next Linear Collider (NLC), which will require very high energy gradients. NLC is planning to use an improved 75-MW X-band klystron design [9], and possibly even gyroklystrons [10], which operate at higher frequencies. Many nuclear fusion experiments require gyrotrons to provide high power microwaves over long pulses for heating plasmas to very high temperatures, as will be discussed. The proposed International Thermonuclear Experimental Reactor (ITER) will require many of these gyrotrons producing powers which have never been reached before. Gyrotrons are also being seriously considered for imaging in medical applications, such as Electron Paramagnetic Resonance (EPR) used in Dynamic Nuclear Polarization (DNP) spectroscopy. At MIT, a 250 GHz gyrotron has been demonstrated for use in DNP [11], and a 460 GHz second-harmonic gyrotron is being developed to provide greater resolution [12]. In addition, high power tubes are being used for processing materials to make them stronger and more reliable, which may lead to whole new products and industries [13].

The common perception is that microwave tubes are based on old or archaic technologies, and will eventually be entirely supplanted by solid-state devices. As long as there is a need for efficient high power sources, however, this will not happen. Solid-state devices are simply unable to reach the power levels achieved by modern vacuum electron devices (Fig. 1-1 [14].) And although the basic principles were established before the solid-state devices were developed, new theories are being advanced and design tools are being created as the industry continues to grow. Certainly some of the types of tubes are older and better understood than others. However, even
magnetrons and klystrons, which were among the first microwave tubes to be developed, are still being investigated and experimented on, with a goal of improving their performance and enhancing efficiency. There is an ongoing effort, in research laboratories and universities, to improve existing tubes, as well as to develop novel electron devices. Understanding the basic physical mechanisms behind them is the key to doing this.

### 1.2 Classification of Sources

Vacuum tubes are generally divided into two classes: fast wave and slow wave. Most of the tubes discussed above (TWTs, magnetrons, and backward wave oscillators) may be classified as slow wave devices. In these tubes, the phase velocity \( v_{ph} = \omega / k_z \) of the wave being excited is slowed down to match the velocity of the electrons in the beam. In this case \( v_{ph} \approx v_z < c \). This is typically accomplished by using periodic wave guiding structures, such as gratings, helices, and folded waveguides. In a slow wave device, the electrons must pass near the guiding structure, where the field is
Figure 1-2: Effectiveness of high power sources in wavelength regimes. At high frequencies, photonic devices are efficient sources. Magnetrons, klystrons, TWTs, and other electron devices can produce large amounts of power at lower frequencies. The gyrotron may fill the source gap between these two types of devices.

concentrated. These tubes have proved very efficient with large bandwidths in the frequency range of 500 MHz to 20 GHz.

In fast wave devices, the electrons oscillate in either a strong periodic or homogeneous magnetic field, and the phase velocity of the electromagnetic wave is approximately the speed of light, or $v_{ph} \approx c$. The excitation occurs without slowing down the wave. Free electron lasers (FELs) and gyro-devices such as gyrotron oscillators, gyroklystrons, gyro-TWTs, and cyclotron auto-resonance masers (CARMs) are fast wave tubes. The beam-wave interactions in these devices must be described in relativistic terms.

1.3 Electron Cyclotron Masers

As has been mentioned, slow wave tubes are very efficient devices for producing microwaves at low frequencies. However, since the interaction structures scale as the wavelength for these devices, higher frequencies necessitate smaller structures. Significant engineering issues arise when designing slow wave devices to produce power at high frequencies. For example, it becomes increasingly difficult to pass a high
energy electron beam near structures as their dimensions shrink without damaging the structure itself. In addition, the power density creates a problem since breakdown may occur. Finally, manufacturing such a complex geometry becomes challenging due to the detailed features of the structure. Micro-machining manufacturing processes such as LIGA (a German acronym for lithography, electroforming, and injection molding) and wire EDM (electrical discharge machining) are being investigated [15], but, for the moment, many are time-intensive and costly. Some recent designs based on microfabrication technology include the 560 GHz folded-waveguide travelling wave tube [16], and the 95 GHz klystrino [17], which was recently built and tested.

Conversely, at high frequencies, photonic devices such as lasers have been proven to be effective. Generally, it becomes difficult to use lasers to provide high powers at lower frequencies due to limiting quantum effects, although a quantum-cascade laser has recently been operated at as low as 3.8 THz [18]. Nonetheless, a significant gap exists between these two frequency ranges (Fig. 1-2.) New types of devices are currently being investigated to fill in this source gap.

One of the more promising devices is the gyrotron (Fig. 1-3.) A gyrotron is classified as an electron cyclotron maser (ECM), a fast wave relativistic device which generates microwaves based on the cyclotron resonance maser (CRM) instability (described in Chapter 2.) Gyrotrons generate power at microwave, millimeter, and sub-millimeter wavelengths by extracting radiated energy from the rotational motion of the electrons in the beam. The electrons rotate at a cyclotron frequency dependent on the externally applied magnetic field, typically produced by a superconducting magnet. The rotating electrons enter an interaction region where the electrons in the beam bunch in their gyro-phase, or Larmor orbits. The bunched beam then transfers power to an electromagnetic mode supported in the structure and which resonates near the cyclotron frequency, or near one of its harmonics. Because of this type of interaction between the beam and wave, the frequency of the electromagnetic radiation produced by the bunched electrons is not dependent on the size of the structure. Since the fields do not necessarily have to be located near the walls of the resonator, very high powers may be generated at high frequencies. The resonator may be overmoded,
such that most of the field resonating at the design frequency is near the center of the structure. In this case, power lost due to ohmic heating at the walls is minimal.

Although considered a relatively new device, the gyrotron has been in existence for almost half a century [19],[20]. The electron cyclotron maser was first proposed in 1958 by Twiss [21], and independently by Gaponov [22] and Schneider [23] in 1959. Experimental results in 1959 by Pantell later verified the fast wave CRM interaction [24], although the first electron cyclotron maser was demonstrated in 1966 by Wachtel and Hirshfield [25]. A Bitter magnet was then used for ECM experiments in 1968 at MIT by Robinson [26]. In the early 1970’s, the first practical gyrotron oscillator was invented and developed in the U.S.S.R., which was used for cyclotron resonance heating [27].

Technological breakthroughs and new materials have since allowed the gyrotron to become a powerful radiation source at high frequency. For example, the magnetron injection gun (MIG), already used in klystrons, made it possible to produce a high quality electron beam at high voltages for CRM devices. Another major breakthrough was the helical-cut radiating launcher, developed by Vlasov in 1975 [28],
which provided an efficient method for coupling the overmoded guided waves to a Gaussian propagating beam. The launcher was later enhanced in 1992 by Denisov with the inclusion of rippled walls [29]. The recent use of CVD (chemical vapor deposition) diamond RF windows has significantly increased the output power capability of gyrotrons [30].

Novel features and further enhancements are being proposed for the next generation of gyrotrons. Single- and multi-stage depressed collectors are now being incorporated to improve efficiency and reduce the beam power loading. Coaxial resonators are being considered to reduce voltage depression and limit mode competition. A 140 GHz coaxial gyrotron was experimentally demonstrated in 1999 at MIT [31], and researchers at FZK are planning to use a coaxial cavity for a 2 MW, 170 GHz gyrotron [32]. Other novel methods for mode suppression in gyrotron oscillators and amplifiers are being investigated at MIT. A photonic bandgap (PBG) structure consisting of an array of metallic rods has been successfully demonstrated in a 140 GHz gyrotron experiment [33]. A 140 GHz gyro-TWT amplifier using an open-edged confocal cavity to suppress competing modes has also been recently tested [34].

1.4 Electron Cyclotron Heating

Gyrotron oscillators have numerous applications. Fusion devices, in particular, have a need for high power microwave tubes as sources of plasma heating. Multi-megawatt plasma heating is currently being carried out in numerous plasma experiments, such as the DIII-D tokamak at General Atomics [35] (Fig. 1-4), the Large Helical Device (LHD) at the NIFS in Japan [36], the JT-60 tokamak at JAERI in Japan [37], the TCV tokamak in Lausanne, Switzerland [38], and the ASDEX tokamak in Garching, Germany [39]. A ten megawatt heating system is planned for the Wendelstein 7X stellarator and a 27 MW system for the proposed ITER machine. All of these devices currently use, or plan to use, gyrotrons for electron cyclotron resonance heating (ECRH) and electron cyclotron current drive (ECCD.)

For the Wendelstein 7X stellarator, 140 GHz gyrotrons have been built which
Figure 1-4: Drawing of the DIII-D Tokamak at General Atomics. (Courtesy of General Atomics.)

Figure 1-5: Schematic of the DIII-D Tokamak system for ECH heating. Three of the six gyrotrons pictured are 1 MW, 110 GHz CPI tubes; the other three are 750 kW, 110 GHz tubes from GYCOM.
currently provide over 500 kW for pulses of several minutes and 1 MW for about 10 s [40],[41]. The DIII-D tokamak at General Atomics is using three 1 MW, 110 GHz gyrotrons operating at up to 10 s pulse lengths [42] (Fig. 1-5.) In addition, a gyrotron generating high power at 170 GHz is currently under development for providing the heating system for the ITER reactor, which will ultimately require a large array of gyrotrons [43].

For these heating systems requiring about 10 MW or more, it would be advantageous to increase the power-per-tube ratio to the 1.5 to 2 MW range. It is with this goal in mind that the U.S. gyrotron program began working on the development of a 1.5 MW, 110 GHz gyrotron in 1999 [44]. This design is based on extending the successful 1 MW design with a new cavity, a new electron gun for higher voltage (96 kV), and with a single-stage or two-stage [45] depressed collector for higher net efficiency. To validate the new design, MIT has built an experimental, short-pulse version of the tube, which is described here. Progress is also being made worldwide on higher power gyrotrons. Coaxial gyrotrons, for example, have been built in Europe with 2.2 MW of output at 165 GHz, currently operating in short pulses [46]. A long pulse / CW version of this tube is being planned. In Japan, an experimental 110 GHz gyrotron tube has been operated up to 1.2 MW for 4 seconds [47], and is being tested up to 1.5 MW. In Russia, output power levels above 1 MW have been obtained in a coaxial gyrotron at a frequency of 140 GHz [48]. A 170 GHz gyrotron has demonstrated up to 1.5 MW in short pulse operation in the TE_{28,8} mode at MIT [49].

1.5 Thesis Outline

This thesis is organized into seven chapters. The first two chapters provide background information and introductory material. Chapter 2 describes the theory behind gyrotron operation, and also discusses some of the different tools which may be used to analyze gyrotrons. Chapter 3 presents the investigations of the electron gun used in the gyrotron experiments in terms of emission uniformity. There is a discussion of the gun design, and the basic theory describing thermionic cathode emission, fol-
ollowed by experimental results obtained by both the first SpectraMat cathode, and its
replacement, also from SpectraMat. The activation process for both cathodes is also
briefly discussed. The 1.5 MW, 110 GHz gyrotron experiment results for the axial
configuration are presented in Chapter 4. These results are also compared with the-
tory and simulation. The irradiance moment phase retrieval technique is outlined in
Chapter 5. This method is useful for internal and external gyrotron mode converters.
The method is demonstrated for theoretical cases using manufactured intensity data,
as well as for a case based on actual cold-test gyrotron intensity measurements. The
results are compared with the previously existing iterative phase retrieval method. A
discussion of the effectiveness and accuracy of each method is also included. Chapter
6 details the design for the 1.5 MW, 110 GHz gyrotron experiment in the inter-
nal mode converter configuration. The phase-correcting mirrors used in the internal
mode converter, which were designed using a combination of the irradiance moment
technique and the iterative phase retrieval method, are presented. This experiment
also includes a collector which may be held at a depressed potential to enhance effi-
ciency. The preliminary results of the experiment are analyzed and discussed. The
final chapter draws conclusions based on data collected from all the experiments and
provides a discussion for future experiments.
Chapter 2

Theory of Gyrotrons

Gyro-devices are based on the cyclotron resonance maser instability, a relativistic phenomenon which provides an energy transfer mechanism between the rotating electrons in the beam and the excitation field. The formalism may be presented in terms of classical physics, or using a quantum approach [50]. In this chapter, the beam-wave interaction is explained in the classical/relativistic sense, which is physically more descriptive and intuitive than using a quantum model.

2.1 General Principles

To better understand the conversion of electron orbital kinetic energy into RF output, a summary of the beam phase bunching mechanisms, common to all ECM devices is presented [51],[52]. Although by no means complete, this brief discussion should be sufficient for the purposes of this thesis.

Microwave amplification results from a two stage process. In the first stage the RF fields bunch the electrons together in phase so that they are no longer uniformly distributed around the orbit. The second stage consists of positioning the phase bunches in a phase with respect to the RF field so that the electrons as a group lose energy to the field. While these two stages progress simultaneously in actual devices, they will be discussed individually.

Phase bunching can most easily be understood from the reference frame of the
Figure 2-1: Evolution of annular beam of electrons (radius $r_b$) in time-varying TE$_{01}$ electric field. The electrons eventually rotate in phase with the electric field.
electron, where the axial velocity vanishes. A basic example of phase bunching can be demonstrated by studying the behavior of a large annular beam of radius \( r_b \) consisting of electrons in a magnetic field \( B_o \) with small Larmor radius \( r_L = v_{\perp}/\omega_c \), where \( v_{\perp} \) is the rotational velocity and \( \omega_c \equiv eB_o/m\gamma \) is the rotational frequency for an electron with charge \( e \), mass \( m \), and relativistic mass factor \( \gamma \). This annular beam is then placed in the presence of a TE_{0n} circular mode, where the \( E \) field components are purely azimuthal and have a large magnitude near the beam radius \( r_b \) such that maximum coupling occurs (Fig. 2-1.) Initially, at \( t = 0 \), and \( z = -\sqrt{3}/2L \), where \( L \) is the length of the cavity, the relative phases of the electrons in their cyclotron orbits are random (Fig. 2-1(a)) such that there is no net energy exchange between the electrons and the electromagnetic wave.

Representing the electron distribution in phase space, then, electrons of Larmor radius \( r_L \) rotate around a common guiding center. The initial random distribution is illustrated by 50 equally distributed test electrons in Fig. 2-2(a). When the TE fields are present, phase bunching is initiated (Fig. 2-2(b) and Fig. 2-2(c).) In this frame of reference, the azimuthal \( E \) field of Fig. 2-1 is localized and in one direction. Specifically, a TE field with frequency \( \omega(=\omega_c) \) will decelerate electrons moving with the field, causing them to lose energy and spiral inward. However, this loss of energy also decreases the relativistic mass factor \( \gamma \):

\[
\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1 + eV/mc^2
\]  

(2.1)

where \( V \) is voltage of the beam, \( v \) is the particle's velocity, and \( c \) is the speed of light.

Since \( \omega_c = eB_0/m\gamma \), the rotational frequency will increase. Electrons moving against the field undergo the opposite effect, accelerating due to the presence of the field. These electrons gain energy, which causes them to spiral outward and rotate more slowly. After a number of field periods, the electrons bunch in phase within the Larmor orbit, which can best be seen in Fig. 2-1(d) and Fig. 2-1(e), where most of the electrons are grouped together. (Note for the simulations shown in Fig. 2-1 and Fig. 2-2, the parameters are: \( V = 96 \text{ kV}, r_b = 1.05 \text{ cm}, r_L = 706 \mu \text{m}, B_o = 4.45 \)
Figure 2-2: Phase bunching illustrated by 50 test electrons in a Larmor orbit, radius $r_L$. 
$T, f_c = \omega_c/2\pi = 104.7 \text{ GHz}$, $L = 2.39 \text{ cm}$, $|E| = 0.256 \text{ V/cm}$, and $f = \omega/2\pi = 110 \text{ GHz}$.)

At the same time that the electron bunching progresses, a slight detuning between the RF and cyclotron frequencies causes an increasing phase shift, which brings the bunched electrons into a phase where energy extraction can occur. If there were no frequency detuning ($\omega = \omega_c$), the same number of electrons would be accelerated and decelerated, and no net energy would be exchanged between the electrons and the field. However, energy is transferred if the wave frequency is somewhat larger than the initial value of the cyclotron frequency:

$$\omega - \omega_\infty > 0 \quad (2.2)$$

where $\omega_\infty = eB_0/m\gamma_0$ is the initial cyclotron frequency. In this case, more electrons are traveling with the direction of the wave. Therefore, more electrons are decelerated over a field period than are accelerated and the net energy is transferred to the wave. The wave amplitude increases and wave energy grows exponentially in time for each field period.

The mechanisms involved in this process of transferring energy become evident through examination of the dispersion relation for fast wave ECMs:

$$\omega = n\omega_c + k_z v_z, \quad n = 1, 2, 3, \ldots \quad (2.3)$$

where $\omega$ is the electromagnetic frequency, $k_z$ and $v_z$ are the axial wavenumber and electron drift velocity, respectively, $\omega_c$ is the cyclotron frequency, and $n$ is an integer, allowing for interactions at harmonics of the cyclotron frequency.

For the gyrotron oscillator, the Doppler upshifting (product term $k_z v_z$) of the cyclotron frequency in Equation 2.3 is typically ignored since these devices operate near cutoff ($k_z \approx 0$) to reduce the effect of beam velocity spread $\Delta v_z$. This simplification is not valid, however, for some high voltage beam devices which operate away from the cutoff condition (such as the CARM.) All terms in the dispersion relation must be included for these devices.
To understand the behavior of this relation as the beam loses energy to microwave fields, Equation 2.3 is differentiated, with \( n = 1 \):

\[
\frac{d\omega}{dt} = k_z \frac{dv_z}{dt} + n \frac{d\omega_c}{dt} = k_z \frac{dv_z}{dt} - n\omega_c \left( \frac{1}{\gamma} \frac{d\gamma}{dt} \right).
\]  

Equation 2.4 highlights the significance of \( \gamma \) (or more precisely, \( d\gamma/dt \)) in ECM devices. Furthermore, substitutions may be made from the Lorentz force equation for a particle with momentum \( p \)

\[
\frac{dp}{dt} = -e (E + v \times B),
\]  

the energy equation

\[
m\gamma c^2 \frac{d\gamma}{dt} = -ev \cdot E,
\]

and Faraday's law for the transverse \( E \) and \( B \) fields in a TE waveguide mode

\[
B_\perp = \frac{k_z}{\omega} \hat{z} \times E_\perp.
\]

These substitutions yield the following result:

\[
\frac{d\omega}{dt} = \frac{e\omega}{\gamma mc^2} v_\perp \cdot E_\perp \left[ 1 - \left( \frac{k_z c}{\omega} \right)^2 \right].
\]  

Equation 2.8 contains two terms, which are designated (1) and (2). The first term (1) comes from the rate of change of \( \gamma \) (the azimuthal energy change due to acceleration or deceleration of the electron, \( d\gamma/dt \)). It is this term which produces the phase bunching associated with the CRM mechanism. Term (2) comes from the Lorentz force term \(-e(v_\perp \times B_\perp)\), which moves electrons in the axial direction. This is the force that produces the phase bunching commonly referred to as the Weibel mechanism. The CRM and Weibel processes occur simultaneously in ECM devices, yet one will tend to dominate over the other, depending on the region of operation. When the phase velocity, \( v_p(\equiv \omega/k_z) \), is greater than the speed of light, or \( v_p/c > 1 \) (the fast wave
region where gyro-devices operate), the CRM term dominates the Weibel instability term, and when $v_p/c < 1$, the Weibel term dominates.

### 2.2 Nonlinear Single-Mode Theory

Although the theory outlined above describes the phase bunching mechanism, a nonlinear theory based on generalized pendulum equations is typically used for an accurate prediction of gyrotron efficiency and power [53],[54].

The perpendicular efficiency $\eta_\perp$ of a CRM device is the amount of transverse electron energy given up to the resonator field. For a single mode it may be computed entirely from the three normalized parameters $F$, $\mu$, and $\Delta$:

\[
F \equiv \frac{E_0}{B_0} \beta_{\perp 0}^{-n-4} \left( \frac{n^{n-1}}{n!2^{n-1}} \right) J_{m\pm n}(k_\perp r_b), \tag{2.9}
\]

\[
\mu \equiv \pi \left( \beta_{\perp 0}^2 / \beta_{\| 0} \right) (L/\lambda), \tag{2.10}
\]

\[
\Delta \equiv 2\delta_0 / \beta_{\perp 0}^2 \tag{2.11}
\]

where $\lambda$ is the wavelength, $L$ is the effective cavity length, $n$ is the harmonic number, $\beta_{\| 0} = v_{\|} / c$ is the normalized initial axial velocity, and $\beta_{\perp 0} = v_{\perp} / c$ is the normalized initial perpendicular velocity. $r_b$ is the electron beam radius, and $k_\perp = \nu_{mp} / a$, where $\nu_{mp}$ is the $p$th nonvanishing zero of $J'_m(x)$ and $a$ is the cavity radius. Field parameter $F$, which contains the electric and magnetic field amplitudes, $E_0$ and $B_0$, describes the strength of the field in the cavity. The $\mu$ parameter represents the normalized length of the cavity. The magnetic field detuning between the field’s frequency and the cyclotron frequency of the electrons is described by $\Delta$, since $\delta_0 \equiv 1 - n\omega_{co}/\omega$, where $\omega_{co} = eB_0/mc\gamma_0$. Fig. 2-1 and Fig. 2-2 were generated by using a judicious choice of these normalized gyrotron parameters: $F = 0.14$, $\mu = 17.5$, and $\Delta = 0.5$.

If we further define normalized energy variable $u$ and normalized axial position $\zeta$ as

\[
u \equiv \frac{2}{\beta_{\perp 0}^2} \left( 1 - \frac{\gamma}{\gamma_0} \right) \tag{2.12}
\]
Figure 2-3: Initial location of 6 electrons evenly distributed within one Larmor orbit. Also shown is the change in phases of the 6 electrons as the beam traverses the cavity. The gyro-phases of the 6 electrons begin to coincide.

\[ \zeta \equiv \pi \frac{\beta_{||}^2 z}{\beta_0 \lambda^1} \]  \hspace{1cm} (2.13)

it can be shown [55] that for an electron moving in TE\(_{mp}\) fields near cutoff, the coupled equations of motion simplify to

\[ \frac{du}{d\zeta} = 2F f(\zeta)(1 - u)^{n/2} \sin \theta \]  \hspace{1cm} (2.14)

\[ \frac{d\theta}{d\zeta} = \Delta - u - nF f(\zeta)(1 - u)^{n/2-1} \cos \theta \]  \hspace{1cm} (2.15)

where the axial field profile is described by \( f(\zeta) \). This system of differential equations may be solved numerically using a Runge-Kutta method. Equation 2.14 relates the change in beam energy as the electrons traverse a cavity with a Gaussian field profile. The phase bunching seen in Fig. 2-2 evolves through the change of the phase variable \( \theta = \omega t - n\phi \) in Equation 2.15, where \( \phi \) is the electron phase in its Larmor orbit. This phase bunching may be further illustrated by examining the evolving phases of six electrons in a single Larmor orbit which are initially evenly distributed (Fig. 2-3(a)). By tracking the phase trajectories as the particles interact with the field (Fig. 2-3(b)),

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Figure 2-4: Growth of perpendicular efficiency as the bunching electrons traverse
the cavity. The perpendicular efficiency reaches a maximum when the electrons are
closest together in their gyro-phases.

it is evident that bunching occurs, since the phases begin to overlap.

As the beam bunches, the interaction efficiency grows. The total efficiency is given
by
\[ \eta \equiv \frac{\gamma_0 - \gamma}{\gamma_0 - 1} = \left[ \beta_{10}^2/2 \left( 1 - \gamma_0^{-1} \right) \right] \eta_{\perp} \quad (2.16) \]
where the optimized transverse efficiency (shown in Fig. 2-4 using ideal parameters)
is the electron energies at the end of the resonator, \( \zeta = \zeta_{out} \), averaged over their initial
phase \( \theta = \theta_0 \)
\[ \eta_{\perp} = \langle u(\zeta_{out}) \rangle_{\theta_0} \cdot \quad (2.17) \]
Contour plots of optimized efficiency for a range of values of normalized current
(which may be determined from field parameter \( F \)) and normalized cavity length \( \mu \)
have been generated up to fifth harmonic in [55].

The dissipated power from the electrons, which is the power transferred to the
electromagnetic mode to be extracted from the cavity, is expressed in terms of the
efficiency:
\[ P = \eta I_A V = \frac{mc^2 \gamma_0 \beta_{10}^2}{e} I_b \eta_{\perp} \quad (2.18) \]
where \( I_b \) is the beam current.
From a linear theory of gyrotron oscillators [56], the starting current for oscillations of a particular mode may be determined from the normalized parameters as well. If the axial field profile is approximated by a Gaussian function, \( f(\zeta) = e^{-(2\zeta/\mu)^2} \), the start oscillation current is given by the following expression:

\[
I_{\text{start}}(\Delta, \mu) = \frac{4}{\pi \mu (\mu \Delta - n)} \frac{e^{2x^2}}{I_0}
\]

where \( x = \mu \Delta/4 \), and the normalized current \( I_0 \) is

\[
I_0 = \left( \frac{\pi}{2} \right)^{5/2} \left( \frac{\epsilon_0 m_e c^3}{e} \right) \frac{\gamma L}{Q \lambda^2} \beta_{\perp}^{2(3-n)} \left( \frac{2^n n!}{n^n} \right)^2 \\
\times \frac{\left( \nu_{mp}^2 - m^2 \right) J_m^2(\nu_{mp})}{J_{m \pm n}^2(k \tau_b)}.
\]

\( Q \) in Equation 2.20 is the total resonator quality factor, and \( \epsilon_0 \) is the permittivity of free space.

### 2.3 Multi-Mode Theory

The analysis in the previous section is sufficient for a preliminary gyrotron design. However, the formalism does not take into account mode competition, a crucial design issue for overmoded gyrotron resonators. Due to mode suppression, where the operating conditions favor the growth of one mode which in turn suppresses the growth of other competing modes, operation is often single-moded even for cavities with dense mode spectra. However, experiments have shown that the mode density does affect the efficiency and the accessibility of certain modes.

Many gyrotron resonator designs are increasingly overmoded due to higher power requirements. For these devices, multi-mode theory must be used to accurately predict power and efficiency. The approach, detailed in [57], involves solving the coupled, nonlinear, time-dependent equations for the transverse electric field mode amplitudes.
\( (a_n) \) and phases \( (\psi_n) \) of the form

\[
\frac{da_n}{dt} + \frac{\omega_0 a_n}{2Q_n} = -\frac{\omega_0}{2\epsilon_0} \text{Im} P_n(t) \tag{2.21}
\]

\[
\frac{d\psi_n}{dt} + \omega_0 = \omega_{n0} - \frac{\omega_0}{2\epsilon_0 a_n} \text{Re} P_n(t) \tag{2.22}
\]

where \( P_n(t) \) is the complex slow time-scale component of the electron beam polarization for the mode \( n \), \( \omega_{n0} \) is the mode cold-cavity resonating frequency, \( \omega_0 \) is a nearby reference frequency, \( Q_n \) is the mode quality factor, and \( \epsilon_0 \) is the permittivity of free space.

Equation 2.21 and Equation 2.22 are analogous to the amplitude and phase evolution equations for single-mode theory for a particular mode. In practice, a set of modes are chosen for the time-dependent interaction simulation. Each mode is assigned a small initial amplitude and arbitrary phase. The equations are then integrated for each mode at a single time step in the iterative process. Overall power and efficiency may then be calculated using expressions derived for energy stored in each mode.

The issues of mode suppression and stability have been further investigated using a similar multi-mode approach in [58]. In this analysis, the equations of motion are expanded using a superposition of modes. The regions of operation for which a large-amplitude mode may suppress competing modes may then be identified. These regions are largely dependent on cavity length, represented by normalized length parameter \( \mu \) (Equation 2.10.) In general, an equilibrium mode with \( \mu = 10 \) has a large stable operating range. In contrast, when \( \mu \approx 17 \), the maximum efficiency equilibrium is likely to be unstable due to the growth of competing modes.

### 2.4 Computer Codes

Many tools may be used to analyze and predict gyrotron behavior. Historically, accurately calculating the efficiency for a gyrotron design has been challenging due to the complexity of the interaction equations, particularly for multi-mode simulations.
Designing a gyrotron using solely computational methods has proven impractical. However, as processor speeds have rapidly increased, gyrotron beam-wave interaction codes have become more sophisticated. The efficiency of a gyrotron cavity may now be realistically maximized by optimizing design parameters.

A code has been developed to evaluate the single-mode self-consistent nonlinear equations in Section 2.2 [59]. For this code, a stationary wave, a wave structure which is fixed in the presence of the electron beam, is used to accelerate the calculations. The code is very useful in generating a first-order “rough cut” gyrotron design.

For multi-mode interactions (Section 2.3), a code developed by the University of Maryland, called MAGY, may be used [60]. MAGY has proven very accurate in predicting efficiency for highly overmoded gyrotron cavities. The code solves for efficiency using either single-mode, doublet-mode, or triplet-mode calculations. In addition, the user may include the initial cold-cavity field profiles and specify beam parameters. To reduce computation time, non-ideal beam effects such as beam velocity spread and nonuniform emission are typically not included in the simulations. Unfortunately, these beam characteristics can and usually will have a major effect on the efficiency, as will be demonstrated.
Chapter 3

Emission Uniformity Studies

One of the main issues of concern which arises during gyrotron operation is the emission uniformity of the electron gun. Theoretical studies [61],[62] and experimental research [63],[31] of gyrotrons have shown that the emission uniformity of the annular beam produced from a thermionic cathode can have a significant effect on the amount of energy available for exciting a particular electromagnetic mode. Unwanted mode competition can be generated due to nonuniform emission of electrons. This mode competition significantly decreases the microwave efficiency of the device. Nonuniform emission also leads to nonuniform heating or hot spots in the collector which may cause excessive outgassing or even melting. Therefore, to reduce mode competition and improve gyrotron performance, a good understanding of the uniformity of emission from the cathode surface is necessary.

One previous investigation of nonuniform cathode emission in a gyrotron was motivated by the observation of a large-scale azimuthal emission asymmetry in the electron beam. The cathode had a partial failure of its heater resulting in emission of only a half-circle of beam. This resulted in a large reduction in gyrotron efficiency [64]. That gun was rebuilt and an improved efficiency was obtained. Later studies of gyrotron emission uniformity at MIT indicated that even in the presence of good thermal uniformity, cathode emission was still nonuniform [65]. Studies of the power distribution in the collector of megawatt power level gyrotrons also indicate an azimuthal asymmetry. Recent research by Glyavin et al. [63] and Advani et al.
have identified work function variation as an issue in gyrotron cathode emission uniformity.

In this chapter, we focus on the design of the electron gun which will be used for the gyrotron experiments. In addition, following a brief discussion of the theory behind electron emission from a thermionic cathode, measurements are presented of the electron beam emission uniformity produced from the gun which was fabricated based on this design. The results of these measurements, some of which were presented in [66] and [67], are then compared with those of similar cathodes.

3.1 Electron Gun Design

The 1.5 MW, 110 GHz gyrotron design began with a parametric study in order to identify the key gyrotron features. We considered a variety of constraints including cavity ohmic losses, mode competition, and power supply limitations on the beam voltage and current. The design is based on a tapered cylindrical cavity operating in the $\text{TE}_{22,6,1}$ mode. This mode, which is the same mode used in the 1 MW CPI tube, will provide acceptable ohmic losses even at the 1.5 MW level using a redesigned cavity. We anticipate no mode competition problems if the cavity length is properly chosen. Past experiments at MIT [49], IAP (N. Novgorod, Russia) [68], and FZK (Karlsruhe, Germany) [69] have shown that 1 MW [68] and 1.5 MW [49], [69] power levels can be generated in extremely high order modes with good efficiency and minimal problems from competing modes.

An important design parameter is the beam velocity ratio, $\alpha$, which is defined as the ratio of the transverse velocity $v_{\perp}$, to the parallel velocity $v_{\parallel}$, or $v_{\perp}/v_{\parallel}$. Typically a high ratio is desirable for efficient operation, but this can lead to trapped electrons between the collector and gun regions that can degrade performance. Since most of the parallel electron energy can be recovered, it is actually preferable to operate at lower $\alpha$ to avoid trapped electrons. The expected total efficiency is shown in Fig. 3-1 for the design velocity ratio, $\alpha = 1.4$. The voltage difference $V_{\text{DEP}}$ between the cavity and the collector represents the beam energy that is recovered during
Figure 3-1: Total efficiency for a 1.5 MW, 110 GHz gyrotron with a velocity ratio of 1.4. $V_{DEP}$ represents the beam energy that is recovered during depressed collector operation.

Figure 3-2: Geometry, beam trajectory, equipotentials and magnetic field for the 110 GHz diode gun design.
<table>
<thead>
<tr>
<th>Frequency</th>
<th>110 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Power</td>
<td>1.5 MW</td>
</tr>
<tr>
<td>Beam Voltage</td>
<td>96 kV</td>
</tr>
<tr>
<td>Beam Current</td>
<td>40 A</td>
</tr>
<tr>
<td>Velocity Ratio</td>
<td>1.4</td>
</tr>
<tr>
<td>Velocity Spread</td>
<td>2.5 %</td>
</tr>
<tr>
<td>Cavity B Field</td>
<td>4.3 T</td>
</tr>
<tr>
<td>Compression</td>
<td>22.13</td>
</tr>
</tbody>
</table>

Table 3.1: 1.5 MW, 110 GHz gyrotron MIG electron gun design

depressed collector operation. Although higher efficiencies are possible at higher $\alpha$, this also requires operating closer to the limiting current. The design parameters for the gun are listed in Table 3.1. Using the 1 MW CPI gun cathode geometry as a starting point, the gun design was completed using EGUN [70]. The electron gun geometry and beam trajectory are shown in Fig. 3-2. A typical simulation of the velocity ratio and perpendicular velocity spread is shown in Fig. 3-3 where an $\alpha = 1.4$ produces a velocity spread of 2.5 % ($\delta v_\perp/v_\perp = 0.025$) due to beam optics. Additional sources of velocity spread include 1.2 % from surface roughness, 0.9 % from machining or misalignment errors, and 0.6 % from thermal spread and nonuniform emission [49],[71]. EGUN simulations were performed to verify the effect on the velocity spread due to mechanical misalignment, machining errors, voltage ripple, magnetic field inhomogeneities, and current nonuniformity. As a result, the final beam spread is predicted to be about 5 %.

### 3.2 Electron Gun Activation

#### 3.2.1 Old Cathode

The 96 kV, 40 A electron gun described in the previous section originally used a thermionic M-type cathode built by SpectraMat. The first cathode was impregnated with a 5:3:2 mixture of BaO:CaO:Al₂O₃. In addition, to assure uniform emission, the surface finish of the emitter was specified as 16 microinches RMS.

Although this cathode activated properly and functioned well when separated from
the experiment, it failed to fully activated when connected to the tube. Only partial activation was accomplished for the cathode in the current probe experiment (shown in Fig. 3-9.) The cathode was completely dormant in the gyrotron axial experiment (Fig. 4-10.) In both of these configurations the tube and gun were thoroughly baked out at close to 200 °C. The final pressure in the vacuum was $\sim 10^{-8}$ torr.

Various techniques were tried to activate the cathode, including extensive cleaning, the removal of the gate valve near the gun, and the use of additional ion pumps. It was discovered that the gun activated successfully with the gate valve closed, and a 2 l/s ion pump evacuating the gun area. Whenever the gate valve was opened, however, the activation current rapidly decreased, falling to zero within 10 minutes. This phenomenon occurred even though the total pressure improved. The results indicated that the surface of the cathode was becoming poisoned by some gas present in the tube.

A residual gas analyzer (RGA) was then attached to the system. RGA measurements were taken when the gate valve was closed during activation, and after the gate valve was opened. The scan (Fig. 3-4) detected changes in the partial pressures of H$_2$O, CO, and CO$_2$, gasses which are all known causes of cathode poisoning [72]. The measurements indicated that there was a net flux of these gases from the tube.
3.2.2 New Cathode

The RGA results showed the cathode was very susceptible to poisoning. Therefore it became evident that a new cathode would be necessary for the vacuum conditions in the experiments.

A new cathode was built at SpectraMat based on the original cathode design, but with several differences. First the surface finish requirement was loosened from 16 to between 32 and 64 microinches RMS, which was less restrictive than for the previous cathode, and more typical for gyrotron cathodes. It was possible that the surface of the original cathode had emitting pores which were partially blocked as a result of the smooth surface finish. In addition, the impregnate ratio was changed to 4:1:1. Studies have indicated this ratio is much less susceptible to poisoning than 5:3:2 [72]. One final minor change was the final cathode surface coating. An osmium M coating was applied by SpectraMat instead of the typical ternary alloy (TA) coating applied by
Figure 3-5: Installation of the 96 kV, 40 A electron gun (pictured with Ivan Mastovsky.)

<table>
<thead>
<tr>
<th></th>
<th>Old SpectraMat Cathode</th>
<th>New SpectraMat Cathode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emitter Surface Finish</td>
<td>16 microm. RMS</td>
<td>32 - 64 microm. RMS</td>
</tr>
<tr>
<td>Impregnate Ratio</td>
<td>5:3:2</td>
<td>4:1:1</td>
</tr>
<tr>
<td>Coating</td>
<td>TA (CPI)</td>
<td>M (SpectraMat)</td>
</tr>
</tbody>
</table>

Table 3.2: Comparison of old and new SpectraMat cathodes.

CPI for the original cathode. This change was made since previous gyrotron cathodes at MIT used the M coating. The differences between the two cathodes is provided in Table 3.2.

The new cathode replaced the original cathode in the same gun (Fig. 3-5.) After being tested with the gate valve closed, the cathode was exposed to the rest of the tube, which had been baked out as before. The cathode was activated and operated successfully at 96 kV and 40 A for all subsequent experiments.
3.3 Emission Uniformity Theory

As previously stated, the electron beam quality is largely dependent on the quality of the cathode. For a gyrotron cathode the quality is determined by both its temperature uniformity and its work function uniformity. Cathode emission is generally described by two equations: the Child-Langmuir law, in the low-voltage space-charge limited regime, and the Richardson-Dushman equation, in the high-voltage temperature limited regime. Numerous studies have attempted to model the transition between these two regimes of operation [73],[74]. In this section, we discuss one approach by first describing the well-known idealized current laws [75]-[77], and then by demonstrating how these equations are transformed in the more realistic case, when there is a spread in the work function. Moreover, we make the differentiation between the local and global amounts of work function spread observed in thermionic cathodes.

The ideal annular cathode has no variations in work function or temperature. For this case, there is an abrupt transition between space-charge limited electron emission at lower voltages, and temperature limited emission. The current density averaged over the cathode surface in the space-charge limited region is governed by an equation derived from the Child-Langmuir law for a diode [78]:

\[ J_{SCL} = \kappa V^{3/2} \quad V \leq V_T \]  

(3.1)

where the constant \( \kappa \) is defined as the perveance over the area of the cathode \((A/cm^2/V^{3/2})\) and therefore depends on the cathode geometry. Past a threshold voltage, \( V_T \), the emitted averaged current density follows the Richardson-Dushman equation in the temperature limited region [78]:

\[ J_{TL} = A_o T^2 \exp \left[ \frac{-e}{kT} \left( \phi - \sqrt{\frac{eE}{4\pi\varepsilon_0}} \right) \right] \quad V > V_T \]  

(3.2)

where \( \phi \) is the work function and \( E \) is the electric field which contributes to the Schottky effect, \( T \) is the temperature of the cathode, \( k \) is Boltzmann’s constant, and the constant \( A_o = 120 \text{ A/cm}^2 \text{ deg}^2 \). The threshold voltage, where \( J_{SCL} = J_{TL} \), is
determined by the work function and temperature of the cathode. If the total current density $J$ is plotted for increasing voltage, the current linearly increases as $V^{3/2}$ until the threshold voltage ($V_T$ in Fig. 3-6(b) for the single work function curve) is reached. Past this voltage, the current is saturated and becomes nearly constant. There is a small slope due to the Schottky effect.

In reality, however, a cathode has a spread in the work function and temperature. Therefore, electron emission varies from one particular location of the cathode to another. In this case, there is a smooth transition between space-charge limited operation and temperature limited operation (Fig. 3-6(f).) The spread in the work function may be determined by examining current behavior in this transition region. Conversely, the total current at a given voltage is determined from a known work function distribution by summing up the portion of the cathode which is still emitting in the space-charge limited region and the cathode region which is temperature limited. Mathematically this may be stated as [79]:

$$J_V = \int_{\phi_{min}}^{\phi_V} J_{SCL}(\phi)d\phi + \int_{\phi_V}^{\phi_{max}} J_{TLD}(\phi)d\phi$$

where $D(\phi)$ is the normalized work function distribution, and $\phi_V$ is defined as:

$$\phi_V = \sqrt{\frac{eE}{4\pi\varepsilon_o}} - \frac{kT}{e} \ln \left( \frac{k V^{3/2}}{A_o T^2} \right).$$

As an example, for a cathode with a Gaussian work function distribution centered at $\phi_o$ with standard deviation $\sigma$, the averaged current density is given by the following equation (Appendix A):

$$J_V = \frac{\kappa V^{3/2}}{2} \left[ 1 + \text{erf} \left( \frac{\phi_T - \phi_o}{\sigma \sqrt{2}} \right) \right] + \frac{A_o T^2}{2} \left[ 1 - \text{erf} \left( \frac{\phi_T - \phi_o + e\sigma^2}{kT} \right) \right]$$

$$\times \exp \left[ -\frac{e}{kT} \left( \phi_o - \sqrt{\frac{eE}{4\pi\varepsilon_o}} - \frac{e\sigma^2}{2kT} \right) \right]$$

where $\phi_T$ is $\phi(V_T)$, or the transitional work function, as illustrated in Fig. 3-6(b).
Figure 3-6: Work function distribution and their I-V curves. There is an abrupt transition for the uniform work function's I-V curve at $V_T^{3/2}$. A slight slope occurs in the temperature limited region with the inclusion of the Schottky effect.
Figure 3-7: The annular cathode (top) has a much larger area than sections which contribute to the local spread (bottom.) An example of surface spatial variations over a small section of the cathode (~500 \( \mu m \) on a side), which give rise to a local spread in the work function, is shown in the bottom figure. The current variations are on a similar spatial scale.

In this research we analyze the I-V curves by assuming that the work function distribution may be characterized as a Gaussian with central value \( \phi_o \) and standard deviation \( \sigma \), which we refer to as the work function spread. Equation 3.5 will be used to fit the measured data to determine these two free parameters [79]. We also assume the cathode has a spread only in work function, not in temperature. As will be shown later, emission nonuniformity due to a temperature distribution is small compared to nonuniformities caused by a spread in the cathode's work function.

In addition, a distinction may be made between the global and local work function
spreads. It is known that small areas of the cathode (as small as 500 µm on a side) show an effect due to work function variation [80],[81]. We denote this variation as a "local" spread in work function, meaning that it occurs over a microscopic area of the emitter (Fig. 3-7.) In addition to the local spread, different regions of the emitter, separated on a large scale length of order centimeters, may have a different local mean value of the work function. We denote this variation to be the "global" spread in work function. We assume that the local and the global spreads are uncorrelated. In that case, the two effects may be summed:

$$\sigma_{total}^2 = \sigma_{global}^2 + \sigma_{local}^2.$$  

(3.6)

One method of determining the local spread is by taking current measurements over very small locations of the cathode [80],[82]. Another method, such as the one employed at MIT, determines the global spread first, from which the local spread may be calculated using the total spread and Equation 3.6.

### 3.4 Emission Uniformity Measurements

#### 3.4.1 Old Cathode

According to previous emission studies [79], a Gaussian distribution is a reasonable approximation for the work function of most thermionic cathodes. Therefore, a Gaussian distribution is assumed for the cathode analyzed at MIT. In the first step, the electron beam current is measured at several different cathode temperatures over a wide range of voltages (Fig. 3-8.) Next, the data are fit using Equation 3.5, which yields a form for the total current in terms of the Gaussian distribution parameters, $\phi_o$, and $\sigma$. For the I-V data shown in Fig. 3-8, the work function center was found to be $\phi_o = 1.76$ eV, with a deviation of $\sigma = \sigma_{total} = 0.04 \pm 0.01$ eV. Since the current measurements were taken for the entire beam, this deviation represents the total spread in the work function, $\sigma_{total}$. Note that these values provide a reasonably good fit at all temperatures.
Figure 3-8: The current-voltage data is taken for various cathode temperatures. The current measured at each voltage point is accurate to within 1%. The best Gaussian work function fit to the measured I-V data taken at various temperatures is when the central work function value is $\phi_0 = 1.76$ eV, and the work function spread is $\sigma_{Total} = 0.04$ eV.

To determine the global spread, detailed measurements of the cathode emission around the azimuth must be taken. Therefore the emitted electron beam is examined at many different azimuthal locations using a rotating current probe as part of the beam collector (the far right end of the experiment shown in Fig. 3-9.) The setup of this diagnostic tool is shown in Fig. 3-10. Most of the annular beam enters a metal cylinder and impacts on the inner wall of the structure as the beam expands in a region of decreasing magnetic field. Some of the beam, however, passes through a narrow (10°) slot, from which the current is sampled. The probe is rotated to measure the current at many different angles around the gyrotron axis. A normalized scan taken of the beam emitted from the cathode used at MIT is plotted in Fig. 3-11(a).

The global work function spread is determined from this normalized scan by first noting that each data point represents a ratio of the current density at that location to a maximum current value, or $J/J_{max}$. Since the temperature-limited current in the Richardson-Dushman equation (Equation 3.2) has an $\exp(-e\phi/kT)$ dependence,
Figure 3-9: Schematic of current probe experiment.

Figure 3-10: The diagnostic used for measuring the beam current at a particular azimuthal angle consists of a conducting tube, which the electron beam enters, and a narrow beam slot, through which a small angular section of the beam exits. This portion of the beam current is measured by a current plate, placed above the beam slot. The structure is rotated around the azimuth to sample the beam current at various angles.
the maximum current density, $J_{\text{max}}$, occurs at the location where the work function is smallest, $\phi_{\text{min}}$. The ratio of the two currents is related to the work function:

$$\frac{J}{J_{\text{max}}} = \exp \left[ -e \frac{\phi - \phi_{\text{min}}}{kT} \right]. \quad (3.7)$$

This may be rewritten such that it is possible to determine the work function difference from the measured ratio:

$$\phi - \phi_{\text{min}} = -\frac{kT}{e} \ln \left( \frac{J}{J_{\text{max}}} \right). \quad (3.8)$$

Using this equation, the normalized current values, such as those shown in Fig. 3-11(a), may be converted to work function differences. The distribution of these work function differences are shown in Fig. 3-11(b). If we assume a Gaussian distribution, the standard deviation of $D(\phi - \phi_{\text{min}})$ is the same as $D(\phi)$. This deviation represents the global work function spread, $\sigma_{\text{global}}$. For the distribution shown in Fig. 3-11(b), the global work function spread is $\sigma_{\text{global}} = 0.03 \pm 0.01$ eV. This value of $\sigma_{\text{global}}$ is not surprising since the standard deviation of $\phi$ in Equation 3.7 must be comparatively smaller than $kT$ (which is $0.11$ eV at $T = 1000^\circ$ C) to produce the variations in the
current seen in Fig. 3-11(a).

To this point we have assumed that current emission variations are due to work function spread and not temperature spread. We can justify this assumption by noting what the value of the temperature spread would have to be in order to cause the same type of azimuthal current variations seen by the current probe. For any two data points, if we assume the work function was identical at each location but the temperature varies, we find

$$\frac{J_1}{J_2} = \exp \left[ \frac{-e\phi}{k} \left( \frac{1}{T_1} - \frac{1}{T_2} \right) \right].$$  \hspace{1cm} (3.9)

For the current data shown in Fig. 3-11(a), the maximum current density variation is $J_1/J_2 = 5.2$. Assuming a nominal work function value of 1.76 eV and a temperature of around 1000° C, then the temperature difference to cause this much variation in the current would have to be at least 84° C. Since direct temperature measurements of the cathode indicate a temperature uniformity of better than ±5° C, the dominant cause of the current variations is not temperature spread, but work function spread.

Since the total work function spread is determined from the current density data shown in Fig. 3-8, and the global spread is determined from the current probe measurements plotted in Fig. 3-11, the local spread may be calculated by applying Equation 3.6. Using the numbers for the cathode at MIT, the local work function spread is $\sigma_{local} = 0.03\pm0.01$ eV. The observed local work function spread may be the result of several effects. One effect is that local emission varies with the amount of barium surface coverage at the microscopic level, which in turn may be influenced by the pore sizes, which are less than ten microns. Another possibility is that the emitter material consists of small crystals whose surfaces have different crystal orientations. Since the work function depends on the crystal orientation, a spread in work function would arise [78]. A third possibility is that surface roughness gives rise to a variation in the effective electric field strength at the emitter surface. Estimates of the variation in electric field have been previously carried out in analyzing the origin of velocity spread in electron guns [83]. The variation can be large because of the presence of
points on the emitter surface; a typical value of the local electric field variation is $E_{\text{max}} = 4E_{\text{min}}$. Using this result in Equation 3.1 yields an estimated variation in emission current density, through the Schottky effect, of $J_1/J_2 = 2.2$. This current variation is not due to a variation in work function; however, it is the same size as the variation caused by a work function RMS spread of about 0.03 eV. This spread is smaller than the measured value but is not negligible. There may also be other effects that cause the observed variation in local work function spread.

### 3.4.2 New Cathode

As has been mentioned, the original SpectraMat cathode was removed due to its inability to activate properly. The cathode was replaced by a nearly identical cathode. Extensive measurements were taken of the new cathode using the same experimental setup (Fig. 3-9.) The data points shown in Fig. 3-12 were recorded by guiding a beam to the current probe and measuring the total current over a range of voltages. The cathode temperature was also varied. The data set was fit using Equation 3.5 to determine the central work function value $\phi_0$ and spread $\sigma$ for the cathode at each temperature. In this case, the total work function spreads $\sigma_{\text{Total}}$ are represented, since the entire collector current was measured and used in the calculations. The central work function value $\phi_0$ for this M-type tungsten cathode was around 1.88 eV for each temperature. For the data shown in Fig. 3-12, the fit using Equation 3.5 gave a work function spread of 0.046 eV at $T = 965^\circ$ C, 0.042 eV at $T = 975^\circ$ C, and 0.033 eV at $T = 1000^\circ$ C. This decrease in work function spread with increasing temperature agrees with previous observations [79].

Next, to examine local and global work function variations, the probe was mechanically rotated. The azimuthal change in probe current was measured. In the first part of this experiment, the temperature was fixed at $T = 1000^\circ$ C while the voltage was varied for each scan. Fig. 3-13 shows the results of these measurements. Note each scan was performed multiple times to verify the repeatability of the experiment. The voltage was not increased past 70 kV, mainly due to the noise present in the measurement system at higher voltages. As the results show, the voltage, above
50 kV, did not affect the azimuthal uniformity of the beam. This was because the cathode at those voltages was fully temperature limited. When the voltage decreased below 50 kV, emission became more uniform. In this case, more sections of the cathode had become space-charge limited. Neither the cathode’s work function nor its work function spread have an effect on current emission in the space-charge limited regime (Equation 3.1.) When the voltage became very small, ~2.5 kV, the current azimuthal distribution was almost completely uniform. A small sinusoidal variation was observed, which may be explained by an offset error of 0.3 mm in the transverse positioning of the cathode with respect to the anode.

The high-voltage data in Fig. 3-13 may be used to estimate the cathode’s global work function spread. As described earlier, each normalized current probe value \( J/J_{\text{max}} \) may be converted into a work function difference \( \phi - \phi_{\text{min}} \) by applying the Richardson-Dushman equation. This equation is used since the data was taken at high voltage, where the cathode is fully temperature limited. The minimum work function value \( \phi_{\text{min}} \) occurs at the maximum current \( J_{\text{max}} \). A distribution in these
Figure 3-13: Probe current uniformity for various applied voltages at $T = 1000 \, ^\circ C$. Below 50 kV the emission is more uniform since the cathode becomes progressively space-charge limited. The global work function spread may also be determined from the data. For this scan, $\sigma_{\text{global}} = 0.2 \, \text{eV}$.

work function differences is then generated and a Gaussian spread $\sigma_{\text{global}}$ is estimated from the distribution. Here the work function spread is global since the distribution represents variations over the entire cathode surface. For this particular cathode, the global work function spread was around 0.02 eV at $T = 1000 \, ^\circ C$. Recalling that the total spread was 0.033 eV, Equation 3.6 was applied to determine the local spread, 0.026 eV.

An extensive set of data was then taken to generate I-V curves of the current probe at each azimuthal angle in 30° increments. A sampling of the data is shown in Fig. 3-14(a) at $T = 1000 \, ^\circ C$. Similar measurements were taken at $T = 975 \, ^\circ C$ and $T = 965 \, ^\circ C$. The azimuthal variations shown in Fig. 3-14(b) were measured by the probe when the cathode was fully temperature limited for each of the three temperatures. The work function distribution parameters $\phi_o$ and $\sigma$ at each location were then determined by using Equation 3.5 to fit each curve, using the same procedure as for the total work function distribution parameters. Since the slot width of the probe was 10°, the current could not be measured over microscopic dimensions. Therefore the spread
Figure 3-14: Sampling of the probe current at various azimuthal angles. I-V measurements were taken every 30 degrees of rotation. For the data shown in (a), \( T = 1000 \) °C. In (b), the azimuthal variations as measured by the probe are represented for all three temperatures. The cathode was fully temperature-limited for all scans in (b).

Figure 3-15: Work function variations (in eV) at each 30° of rotation for various cathode temperatures.
Table 3.3: Summary of work function spreads from beam measurements of both MIT gyrotron cathodes and various other cathodes.

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Cathode} & \phi_o \text{ (eV)} & \sigma_{\text{Total}} \text{ (eV)} & \sigma_{\text{Global}} \text{ (eV)} & \sigma_{\text{Local}} \text{ (eV)} \\
\hline
\text{MIT 110 GHz (Cathode I)} & 1.76 & 0.04\pm0.01 & 0.03\pm0.01 & 0.03\pm0.01 \\
\text{MIT 110 GHz (Cathode II)} & 1.88 & 0.03\pm0.01 & 0.02\pm0.01 & 0.03\pm0.01 \\
\text{Glyavin, et. al. (Cathode I) [63]} & 2.29 & 0.023 & 0.013 & 0.020 \\
\text{Glyavin, et. al. (Cathode II) [63]} & 2.36 & 0.054 & 0.044 & 0.031 \\
\text{Tonnerre, et. al. [79]} & 1.8 & 0.05 & \\
\text{Fortgang [82]} & 1.8 & & 0.06 & \\
\text{Gibson, et. al. [80]} & 1.6 & & 0.03 & \\
\hline
\end{array}
\]

in this case was not precisely \(\sigma_{\text{local}}\). However, the analysis allowed a closer look at the variation in work function and spread of small sections around the cathode. A plot was generated to illustrate how the work function \(\phi_o\) changed over the cathode surface (Fig. 3-15.) These work function variations mirrored the azimuthal current probe data shown in Fig. 3-14(a). The work function values were largely independent of temperature.

As has been documented in [84] and [85], the work function is a function of barium surface coverage, or \(\phi(\Theta)\), where \(\Theta\) is the percentage of barium coverage, typically expressed in terms of barium-oxide (BaO) density and the moments of the individual BaO dipoles [86]. By extension, the work function distribution parameters must also have the same dependence. Furthermore, as these measurements have indicated and previous experiments have demonstrated [79], the spread \(\sigma\) is also dependent on temperature. It would be useful in the future to investigate the relationship between work function spread and barium coverage as well as temperature for these cathodes.

Results for the MIT cathodes are summarized in Table 3.3. Research on similar cathodes has produced comparable values for work function spreads. For example, the local work function spread of a scandate cathode measured in [80] was 0.03 eV. In [79], the total work function spread of an M-type tungsten cathode was 0.05 eV. Finally, in [82], the total work function spread of a large thermionic cathode was 0.08 eV, while the local spread was around 0.06 eV.
3.5 Summary and Discussion

We have presented the design of a 96 kV, 40 A electron gun which is designed to produce 1.5 MW of microwave power at 110 GHz in gyrotron experiments at MIT. The design is based on previous gyrotron tube designs.

In addition to the gun design, the theory behind electron emission from a thermionic cathode has been presented. The current density produced from an ideal emitter is described by two basic equations (Equation 3.1 and Equation 3.2) and is dependent on the voltage, work function, and temperature of the cathode. When the work function is distributed these equations are combined into one equation (Equation 3.3.) The emission quality of the cathode is determined from the spread of the distribution. Both local and global effects contribute to the total work function spread.

The work function distributions have been estimated from measurements of both cathodes used in the gun constructed from the design. Specifically, the total work function spreads were calculated from fitting the current density data with Equation 3.5, which assumes a Gaussian distribution for the work function. Next, a current probe was used to determine the global spreads and, by applying Equation 3.6, the local spreads were determined. Also, changes in the work function distribution parameters over the cathode surface were observed (Fig. 3-15.)

It is possible to import this distribution data into a 3-D electron particle tracking code (such as OMNITRAK [87]), or even a gyrotron simulation code (such as MAGY [60].) Therefore, the azimuthal beam current variations may be accounted for in modeling the electron beam quality and gyrotron efficiency. This will allow for more accurate prediction of existing gyrotron behavior, and the opportunity to design higher efficiency gyrotrons in the future. Future research will attempt to determine the origin of the observed global work function spread, which is undesirable for gyrotron operation.
Chapter 4

Gyrotron Axial Experiment

The main goal of the experiment described in this chapter is to examine such issues as mode competition, output power, mode purity, mode start-up scenarios, changes in the beam velocity ratio, and the operating regions of neighboring modes for the 1.5 MW, 110 GHz gyrotron. In 2002, the initial successful short-pulsed operation of the experiment using an older triode gun was reported. That experiment, although limited due to the use of a gun not designed specifically for the experiment, provided some encouraging preliminary results, with over 1 MW of output power obtained [88]. A new diode gun has since replaced the triode gun. This new gun, which had previously been separately tested for emission uniformity (as described in the previous chapter and in [66]), has been rebuilt with a new cathode. Much of the results of the gyrotron experiment using this new cathode are presented in [89].

This chapter will first describe the setup of the experiment, show the results obtained using the old and new guns, and compare them with theory and simulation, and then discuss these results, along with plans for future experiments.

4.1 Design

The crucial design issues of a multi-megawatt gyrotron involve the essential elements of the gyrotron: the gun and the cavity. As described in Chapter 3, the gun for this experiment has been designed to produce a 40 A electron beam at 96 kV with the
Table 4.1: Design parameters for the 1.5 MW, 110 GHz axial gyrotron experiment.

The design parameters for the gyrotron, listed in Table 4.1, are chosen such that a 96 kV, 40 A electron beam generates 1.5 MW of power in the TE\(_{22,6}\) cylindrical waveguide mode (Fig. 4-2), corresponding to an efficiency of 39 %. The TE\(_{22,6}\) mode was chosen for several reasons. First, the points of highest intensity (the first Bessel function maxima) of the mode pattern are away from the walls of the structure, towards the center of the cavity. This is an important design characteristic since the ohmic wall losses and heating are minimized with such a mode. The loading for this
Figure 4-1: The cavity profile is shown by the solid line. The dotted line represents the field structure. For the TE$_{22,6}$ mode, the resonant cold-cavity frequency is 110.08 GHz, and the $Q$ is 935.

Figure 4-2: Cross-section of intensity ($|E|^2$) pattern at 110 GHz for the TE$_{22,6,1}$ cylindrical resonator mode.
cavity design is predicted to be < 1.5 kW/cm$^2$. Second, theory and simulation have shown that this mode provides high coupling with the beam, such that it is a highly efficient mode for energy transfer. Third, the distribution of modes in the frequency spectrum suggests that the $\text{TE}_{22,6}$ mode is comparatively isolated from other modes, implying a stable operating region with minimal mode competition. Finally, we have previous experience with other tubes which use the $\text{TE}_{22,6}$ mode, including the robust 1 MW, 110 GHz gyrotron, upon which this experiment is based. These tubes have been highly successful in the past [90].

The alpha, $\alpha = v_\perp/v_\parallel$, is a beam parameter measuring the ratio of the perpendicular velocity of electrons to the parallel velocity. The beam alpha is a very useful parameter since it indicates how much rotational (perpendicular) energy is available for mode excitation. If the alpha is large, there will be considerable energy transfer in the interaction region since most of the electron energy is rotational. However, it is very difficult to fabricate an electron gun capable of producing high alpha with low velocity spread. Furthermore, if the design value of the alpha is too large, then imperfections in beam uniformity, alignment, and/or velocity spread can result in some electrons being reflected back toward the gun. The designed alpha for this experiment is 1.4 at the cavity with a perpendicular velocity spread ($\delta v_\perp/v_\perp$) of 2.5 % from optical sources. (A discussion of other sources of velocity spread is in Section 3.1 and [66].)

4.1.2 Mode Competition

In the design and operation of a gyrotron, mode competition must be considered to assure the proper mode is excited. Oscillation in a competing mode can result in a decrease of interaction efficiency, degraded performance of the mode converter system, shift in oscillation frequency, and increase in window reflections.

To examine mode competition for this experiment, we must identify which modes have similar characteristics as the $\text{TE}_{22,6}$ mode. These characteristics include coupling conditions and starting currents. The expression for the coupling coefficient $C_{mp}$
between the electron beam and the electric field of a mode is given by

\[ C_{mp} = \frac{J_{m+n}^2(k \cdot r_b)}{\pi (\nu_{mp}^2 - m^2) J_m^2(\nu_{mp})}. \]  

(4.1)

Fig. 4-3 shows the coupling coefficients for the design mode and several nearby modes. Note Equation 4.1 is a term contained in the expression for starting current (Equation 2.19.) Most mode competition can be avoided by proper positioning of the beam radius. The TE_{21,6} and TE_{23,6} modes, however, have almost equal coupling coefficients at the same radial position. In this instance, the start-up scenario, or order of mode excitation, becomes important. These start-up conditions are discussed in the following section.

If the quality factors of competing modes are not significantly different from that of the design (TE_{22,6}) mode, the coupling coefficient gives a good estimate of the relative starting current. However, to further explore the regions of operation for competing modes, start oscillation current curves are generated using Equation 2.19 from the single-mode theory in Section 2.2 (Fig. 4-4.) The multi-mode gyrotron
Figure 4-4: Start oscillation curves for various modes, including the design (TE_{22,6}) mode. The curves were calculated based on single-mode theory (Equation 2.19), assuming $V_b = 96$ kV, $\alpha = 1.435$, and $r_b = 1.05$ cm.

Figure 4-5: Start oscillation curves calculated using the MAGY code. (Courtesy of Steve Cauffman, CPI.)
Figure 4-6: Start current contours for the TE_{22,6} mode. The left plot assumes a Gaussian field profile; the right uses the cold-cavity field. The voltage rise is shown for the case where $\alpha = 1.435$ at $V_b = 96$ kV.

simulation code MAGY obtained similar results (Fig. 4-5.) In each case, the starting currents of several modes are plotted as a function of the cavity magnetic field. For all calculations the beam voltage and current were 96 kV and 40 A, the beam alpha was 1.4, and the beam radius was 1.024 cm for maximum coupling with the TE_{22,6} mode. Both figures show that the mode spectrum near the TE_{22,6} mode is very dense. Since many of the nearby modes have similar starting requirements, mode competition will be an issue during gyrotron operation.

4.1.3 Start-up Conditions

As has been discussed, many modes may be excited under similar operating conditions for a gyrotron cavity with a dense mode spectrum. For these highly overmoded cavities, many modes will begin to oscillate with small amplitudes until one mode dominates and suppresses the others. The temporal evolution of the beam parameters during the start-up cycle becomes very important in determining the dominating mode.

To examine what occurs during various start-up scenarios, it is instructive to identify regions of excitation for competing modes in the $\alpha - E$ parameter space, where $\alpha \equiv v_\perp/v_\parallel$ is the pitch angle of the electrons and $E$ is the energy of the electron beam.
Figure 4-7: $I_{\text{start}} = 20$ A contours for various modes as the $B$ field varies. A Gaussian field profile was assumed with $r_b = 1.05$ cm. The voltage rise curve is also shown.

[91],[92]. The start-up path depends on the method in which the cathode voltage is raised from zero to the nominal value, or 96 kV for this case. The choice of the start-up scenario determines the evolution of the beam energy, current, and velocity pitch angle during the initial rise time of the electron beam pulse. Depending on the path taken, one of the competing modes will be excited first and will grow to large amplitude. This mode can suppress all other competing modes. The final beam parameters may be reached using other start-up paths, resulting in other dominating modes.

In Fig. 4-6, the start-up scenario is shown for a diode gun where $\alpha = 1.4$ at $E = 96$ keV. The start oscillation current contours for Fig. 4-6(a) are calculated for the $TE_{22,6}$ mode using Equation 2.19 and assuming the field profile is Gaussian with
Figure 4-8: $I_{\text{start}} = 20$ A contours for various modes as beam radius varies. $B = 4.52$ T for all cases.

$Q = 935$. For a more accurate start-up plot, the cold-cavity field profile was used as $f(\zeta)$ for the calculations in Fig. 4-6(b). A comparison shows that the Gaussian approximation is just as valid since the two results only slightly differ. Therefore the Gaussian field profile was used in subsequent calculations to reduce computational times.

To identify operating conditions where mode competition becomes an issue, start current contours of other nearby modes were superimposed onto the $\alpha - E$ plot for the design mode. The $I_{\text{start}} = 20$ A contour was calculated and plotted for each mode. Since operating conditions are rarely ideal, plots were generated for a range of parameters. The contours in Fig. 4-7 all demonstrate a simple shift as the cavity magnetic field varies. Note in practice, changes to the magnetic field take place over a long time scale because of the magnet’s large inductance. The relative positions of
Figure 4-9: Mode amplitudes for start-up conditions based on MAGY calculations at cross-section $z = 8.5$ cm when $V_b = 64$ kV. (Courtesy of Steve Cauffman, CPI.)

the contours change, however, when the beam radius is altered, as in Fig. 4-8. The start-up conditions are shown to be sensitive to changes in the beam radius, and of course to finite beam thickness as well.

Mode suppression may also be simulated by MAGY, where small-signal amplitudes are arbitrarily assigned to modes of interest. The evolution of the mode amplitudes are then plotted on a small time scale. The modes either grow or are damped based on the start-up conditions. A MAGY start-up simulation is shown in Fig. 4-9 for the $TE_{22,6}$ and competing modes. In the start-up scenario chosen for this particular simulation, the $TE_{22,6}$ mode dominates and other modes are suppressed.

4.2 Experimental Setup and Diagnostics

The schematic for the short-pulsed gyrotron experiment in its axial configuration is shown in Fig. 4-10. The electron gun in this case is a diode gun. A gate valve is located between the gun and the rest of the tube to allow rapid changes to be made to the experiment while maintaining vacuum conditions for the gun. The superconducting magnet is able to produce 4.3 T at its center, where the cavity is located to take advantage of the flat field profile. In addition, a magnetic coil is placed at the cathode
Figure 4-10: A schematic shows the 1.5 MW, 110 GHz gyrotron experiment in its axial configuration.

Figure 4-11: Schematic and photo of the cavity section of the experiment.
location. This gun coil provides an additional small field at the cathode which has two effects. Both the alpha and beam radius are altered by changing the compression ratio, so the gun coil may be used to fine-tune these two beam parameters. The microwave resonator (Fig. 4-11), as mentioned previously, is an open-ended cylindrical cavity, with a linear downtaper from the beam tunnel and a rounded uptaper to the output waveguide. This cavity has been designed based on cold-cavity codes, which solve for the electromagnetic fields in the absence of the electron beam, such that the TE$_{22,6}$ mode resonates at 110 GHz with a quality factor, $Q$, of 935. Fig. 4-1 shows the wall profile and cold-cavity field profile for the TE$_{22,6}$ mode. Although it is not shown in the schematic, an alpha probe [93] is located immediately before the cavity. The collector, in this configuration where the power is extracted axially, is a hollow copper pipe which also acts as a cylindrical overmoded waveguide which guides the generated microwaves to a window at the end of the tube. The fused-quartz window is of a thickness designed to allow full transmission at the mode bounce angle at 110 GHz. Most of the power is radiated in a directed cone determined by the bounce angle of the wave in the waveguide.

The position of the tube must be carefully adjusted in order to make sure the cavity is properly aligned with respect to the beam and magnetic field. The gyrotron has freedom of movement in the horizontal and vertical directions at both the collector end and the gun end of the tube. The tube position may be axially adjusted as well.

Numerous diagnostics were used to take measurements of the axial configuration gyrotron experiment. To measure the power, a calibrated calorimeter placed near the output window is used. This calorimeter indicates the averaged power heating its surface over the duty cycle, typically 1 s for this experiment. The pulse shape and duration is measured by a horn antenna fed to a power-detecting diode located at a distance and angle from the window. Including the relative error in the pulse width, the relative error for the power measurement is ± 5 %. A horn-and-waveguide network is also used to deliver output radiation to a heterodyne receiver system. This system, composed of a local oscillator, harmonic mixer, and several filters, makes it possible to measure the frequency content of the output. By tuning the local oscillator, a broad
range of frequencies may be scanned with very high accuracy (± 10 MHz.) Finally, the alpha probe, a small annular metal ring which measures an induced electric potential from the radial electric field of the beam, is used to estimate the beam velocity ratio near the cavity, where the alpha is unchanging. This type of probe has been used successfully in previous experiments [49]. The accuracy of the calibrated alpha probe is ± 10 %.

4.3 Experimental Results

4.3.1 82 kV, 50 A Gun

Initial results of the axial experiment were obtained in 2002 using a triode electron gun from an earlier gyrotron tube at MIT [88]. This gun was originally used with the 170 GHz experiment which demonstrated 1.5 MW of power in the $\text{TE}_{28,8}$ mode [49]. For the 110 GHz gyrotron experiment, the gun operated at 82 kV and 50 A. The main goal of this brief experiment was to test the cavity design. The tube, after
only a cursory alignment, demonstrated over 1 MW of power at a frequency near 110 GHz, corresponding to an efficiency of 25 % (Fig. 4-12.) The results, although preliminary, were very promising for future experiments.

4.3.2 96 kV, 40 A Gun

Subsequent to those initial investigations, the gun was replaced by the 96 kV, 40 A diode gun (Fig. 4-13.) This gun, unlike the earlier triode gun, was designed specifically for the present 110 GHz experiment, producing an electron beam with parameters with match those listed in Table 4.1.

Frequency Measurements

The tube was fixed at a location which provided optimal power by aligning the tube in the magnet. After this alignment had been accomplished, the goal of the first part of the experiment was to locate frequencies of oscillation and identify the modes by varying the operating parameters. A list of some of the frequencies of the radiation measured by the receiver system and their corresponding modes is provided in Table 4.2. The frequencies were also compared with the resonating frequencies in the cavity.
<table>
<thead>
<tr>
<th>Mode</th>
<th>$f_{th}$ (GHz)</th>
<th>$f_{meas}$ (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE$_{17,7}$</td>
<td>103.55</td>
<td>103.64</td>
</tr>
<tr>
<td>TE$_{18,7}$</td>
<td>106.60</td>
<td>106.70</td>
</tr>
<tr>
<td>TE$_{21,6}$</td>
<td>107.14</td>
<td>107.15</td>
</tr>
<tr>
<td>TE$_{19,7}$</td>
<td>109.63</td>
<td>109.70</td>
</tr>
<tr>
<td>TE$_{22,6}$</td>
<td>110.08</td>
<td>110.08</td>
</tr>
<tr>
<td>TE$_{20,7}$</td>
<td>112.64</td>
<td>112.65</td>
</tr>
<tr>
<td>TE$_{23,6}$</td>
<td>113.01</td>
<td>113.01</td>
</tr>
<tr>
<td>TE$_{21,7}$</td>
<td>115.64</td>
<td>115.65</td>
</tr>
<tr>
<td>TE$_{22,7}$</td>
<td>118.63</td>
<td>118.64</td>
</tr>
</tbody>
</table>

Table 4.2: A list of measured frequencies compared with cold-cavity theory.

The majority of measured frequencies were very close to the predicted values, as shown by the table.

**Mode Map**

The next part of the experiment was to chart the regions of parameter space in which the various possible excitation modes are observed. This “mode map” locates the operational limits of each mode and identifies the nearest competing modes at these limits. For the typical mode map two parameters are varied, namely the main magnetic field and the cathode magnetic field. The field at the cavity affects the cyclotron frequency of the electrons in the beam, on which the detuning parameter for the excited mode is based, and also affects the beam radius and the beam alpha. Changing the cavity magnetic field will thus affect the interaction efficiency, and can also cause a shift from one mode and frequency of operation to another. The magnetic field at the cathode, which is fine-tuned by adjusting the gun coil current, affects the beam alpha, as well as the beam radius. These effects can limit the effective operating region of the device.

When the electric and magnetic fields vary slowly over the distance the electrons...
travel during one cyclotron period, $z_L$, stated mathematically as

$$z_L^2 \left| \frac{\partial^2 B}{\partial z^2} \right| \ll B, \quad z_L^2 \left| \frac{\partial^2 E}{\partial z^2} \right| \ll E,$$

$$z_L \left| \frac{\partial B}{\partial z} \right| \ll B, \quad z_L^2 \left| \frac{\partial E}{\partial z} \right| \ll E,$$  \hspace{1cm} (4.2)

then the beam evolution may be treated according to adiabatic theory. Under these adiabatic conditions, the beam alpha increases as the cathode field decreases, since

$$\alpha = \left( \frac{\gamma^2 - 1}{\gamma^2 \beta_\perp^2 - 1} \right)^{-1/2}$$  \hspace{1cm} (4.3)

where $\gamma$ is the relativistic factor dependent on the accelerating voltage of the beam, and the perpendicular velocity component, $\beta_\perp$, is given by

$$\beta_\perp = \left( \frac{E_g}{c B_g} \right) \sqrt{\frac{B_o}{B_g}}.$$  \hspace{1cm} (4.4)

Here $E_g$ and $B_g$ are the electric and magnetic fields at the gun cathode, and $B_o$ is the cavity magnetic field. Eventually the alpha may become so high that the electrons begin to reflect back into the gun. In addition, adjusting the magnetic compression ratio affects the beam radius, causing the mode to change (Fig. 4-3), and the beam radius may become so large that interception may occur at the structure walls.

The mode map shown in Fig. 4-14 identifies the location of the design ($\text{TE}_{22,6}$) mode as well as several nearby modes. During these measurements, the beam voltage and current were maintained at 96 kV and 40 A. The $\text{TE}_{21,6}$, $\text{TE}_{22,6}$ and $\text{TE}_{23,6}$ modes all appear in a band of modes following a linear upward slope as the cavity magnetic field increases. Likewise the $\text{TE}_{19,7}$ and $\text{TE}_{20,7}$ modes lie along a line, although underneath the $\text{TE}_{m,6}$ modes ($m = 21 - 23$.) In addition, the points of highest efficiency were located for several modes. Highest modal efficiency generally occurred at the lower limits of the cavity magnetic field where the mode was still excited. For the regions on the mode map where the cathode magnetic field was low, the alpha was high and the beam reflected so there was no stable operation. Above
Figure 4-14: The mode map is shown for the design mode and nearby competing modes. Also included are the points of highest measured mode efficiency.
Figure 4-15: Shown are the traces for the applied voltage, the current at the collector, a typical voltage pulse from the alpha probe, and a typical voltage pulse from the power-detecting diode. Note the width of the RF pulse on the diode, 2.6 $\mu$s.

the band of labeled modes, where the cathode magnetic field was high, the body current increased, indicating that beam interception was occurring. For other regions on the mode map where no modes are labeled there was unstable mode operation, or oscillation between two or more modes.

**Power Measurements**

The power and efficiency were extensively measured for the TE$_{m,6}$ modes. For these scans a calorimeter was used to estimate the average power as the main magnetic field was swept, and the efficiency calculated based on beam voltage and current of 96 kV and 40 A. A typical voltage pulse shape produced by the power supply at 96 kV is shown in Fig. 4-15(a). The resulting current at the collector (Fig. 4-15(b)) was 40 A. The radiation pulse length used to calculate the actual power was typically around 2.6 $\mu$s. An example of a flat-top pulse shape measured by the power diode is
Figure 4-16: A power scan of three $\text{TE}_{m,6}$ modes is plotted for various magnetic field values. A comparison with MAGY simulations is given for the $\text{TE}_{22,6}$ mode (dotted line).

shown in Fig. 4-15(d). For each data point in Fig. 4-16, the cathode magnetic field was adjusted to maximize the amount of power in the mode. As plotted in Fig. 4-16, the $\text{TE}_{21,6}$ and $\text{TE}_{23,6}$ modes each produced a maximum of 1.1 MW, corresponding to an efficiency of around 30%. The highest power in the design mode was a little more than 1.4 MW, with 37% efficiency. This data point corresponds with the point of highest efficiency marked in Fig. 4-14. The multi-mode gyrotron simulation tool MAGY was used to generate simulated results for the $\text{TE}_{22,6}$ mode over several magnetic field values. These results are overlaid on the figure for comparison. The experimental results agreed well with the simulation, indicating that the cavity was aligned properly within the magnet, and that the gun was operating at close to the design parameters. It should be noted, however, that these particular simulations did not include the effects of emission nonuniformity, although this effect has been studied at MIT using MAGY [94].
Figure 4-17: The alpha values are shown by the solid line as measured by the alpha probe for a range of magnetic field values. The alpha values computed based on EGUN results are indicated by the dotted line.

**Beam Alpha**

The beam alpha was determined based on voltage measurements from the calibrated alpha probe, the beam voltage, and the beam current. The axial energy may be estimated using a capacitive probe which encircles the beam near the cavity [93]. The induced potential on the probe from the beam is

\[
V_{probe} = \frac{e \lambda_e}{2\pi \varepsilon_0} \ln \left( \frac{r_b}{r_{probe}} \right) = \frac{e \lambda_e}{C_{probe}}
\]  

(4.5)

where \( \lambda_e \) is the electron density per unit length, \( r_b \) is the radius of the electron beam, \( r_{probe} \) the inner radius of the metallic probe, and \( C_{probe} \) is defined as the probe capacitance. Knowing the beam current \( I_b = e \lambda_e v_\parallel \), the axial beam velocity \( v_\parallel \) may be calculated, since

\[
v_\parallel = \frac{I_b}{V_{probe} C_{probe}}.
\]  

(4.6)
The relativistic mass factor $\gamma$ may be calculated from the beam voltage:

$$\gamma = 1 + \frac{e(V_b - V_{dep})}{m_e c^2}. \quad (4.7)$$

$V_{dep}$ in Equation 4.7 is the voltage depression given by [95]

$$V_{dep} \approx \frac{1}{2\pi \varepsilon_0} \frac{I_b}{v_{||}} \ln \left( \frac{r_w}{r_b} \right) \quad (4.8)$$

where $r_w$ is the radius of the resonator wall. The perpendicular velocity is

$$v_\perp = c \sqrt{1 - \frac{1}{\gamma^2} - \frac{v_{||}^2}{c^2}}. \quad (4.9)$$

The beam pitch factor may now be determined, since $\alpha \equiv v_{||}/v_\perp$. Methods for reliable calibration of the probe are described in [71]. The measurement error in $\alpha$ from the probe is around 20 % [93].

A typical trace measured by the alpha probe for this experiment is shown in Fig. 4-15(c). The alpha probe data taken for the magnetic field sweep of the TE$_{22,6}$ mode are plotted in Fig. 4-17. Also shown are the alpha values calculated from adiabatic theory using the same parameters based on the EGUN simulation results at $\alpha = 1.43$. The measured alpha for each operating point was very close to the calculated value. The point of highest efficiency, at $B = 4.35$ T, corresponded to a calculated alpha of 1.45, while the measured alpha was around 1.33. At $\alpha = 1.33$, the theoretical efficiency is predicted to be 36.4 %. MAGY results at the lower alpha are not available, but a study is currently being done at MIT using MAGY with inputs that match the beam parameters measured in the experiment [94].

**Start-up Conditions**

Once the optimal magnetic field values were identified for excitation of the desired mode, the effects of varying other operating parameters, such as the beam voltage and current, could be explored. These parameters were varied to determine the range of values over which the desired mode would be excited and to determine the
Figure 4-18: A scan was taken of the power as a function of current for the TE$_{22,6}$ mode. For this particular plot, $V_b = 96$ kV and $B = 4.4$ T. The MAGY results are shown by the dotted line.

Figure 4-19: A similar plot was generated to determine the variation of power with voltage. The MAGY simulations are shown by the dotted line. Here $I_b = 40$ A and $B = 4.4$ T.
operating point with optimal efficiency. In one experiment, once the TE_{22,6} mode was located at 96 kV and 40 A, and the magnetic field was fixed at 4.4 T to stabilize the mode, with a beam alpha of 1.2, the current was slowly decreased. The decrease in power was recorded and plotted in Fig. 4-18. A comparison with MAGY for this magnetic field value is also provided. The slight discrepancy in power levels between the experimental results and MAGY are due to the fact that the beam alpha for the experiment at this operating point differs from the alpha used in the simulations. In Fig. 4-19, the results of a similar experiment are shown, except this time the current was fixed, while the voltage was decreased until the mode power was negligible. While MAGY predicted the TE_{22,6} mode would appear around 68 kV, the beginning voltage measured in the experiment was 75 kV, although this may also be explained by the difference in the alpha measured in the experiment and the alpha used in the simulations.

Related to this last experiment was one which measured the mode content and
alpha as the voltage was varied. In this study, the TE$_{22,6}$ mode was first optimized at 96 kV, and 40 A, with a magnetic field of 4.4 T. For this magnetic field, the beam alpha at 96 kV, 40 A operation was measured by the alpha probe as 1.2. The voltage was then varied while the mode frequency and beam alpha were monitored. The results are shown in Fig. 4-20, which also contains start-up current contours for a beam current of 40 A, as determined by linear start-up theory for an idealized Gaussian field profile in the cavity [91]. Along with the TE$_{22,6}$ mode, both the TE$_{21,6}$ mode and the TE$_{23,6}$ mode were measured as the voltage was varied. The limits of mode operation were found to match well with theory.

**Window Reflections**

Gyrotron output windows have become a key issue for determining how much power the device may generate. Windows may become damaged or even crack if the output power becomes too high. The duration of the pulse may also be limited, since imperfections in the window cause the surface to heat up. Recently, CVD diamond windows have been used due to their unparalleled thermal properties. However, as gyrotron
power requirements increase, window manufacturing remains an issue. These issues will not be explored with this short-pulse experiment. Only properties common to all gyrotron windows are examined.

The influence of reflections on gyrotron operation has been studied in several papers. Hayashi et al. pointed out that wave reflection at the gyrotron window can have an effect on mode competition in the gyrotron cavity [96]. The first general theoretical approach to studying effects of window reflection on gyrotron operation was presented by Antonsen et al. [97]. Dammertz et al. observed that if a significant part of the wave is reflected directly toward the cavity the generated power was reduced considerably, even for relatively small reflections [98]. The specific problem of window reflections on mode competition is addressed in Dumbrajs et al. [99],[100].

The window used for this gyrotron axial experiment is made of UV-grade fused silica. It may be modeled as a dielectric plate with thickness \( d \) and permittivity \( \varepsilon_r \), which is assumed to be real. Since the coupling between transverse guide modes is absent, the window behaves like a simple Fabry-Perot interferometer. The reflection and transmission coefficients depend on the plate thickness and mode bounce angle [101]:

\[
|R|^2 = \frac{(1 - \gamma_r)^2 \sin^2 \chi}{4\gamma_r \cos^2 \chi + (1 + \gamma_r)^2 \sin^2 \chi} \tag{4.10}
\]

where \( \gamma_r = (\varepsilon_r - \sin^2 \theta_B) / \cos^2 \theta_B \), \( \chi = (\omega/c) d \sqrt{\varepsilon_r - \sin^2 \theta_B} \), and \( \theta_B = \tan^{-1}(k_{\perp}/k_{||}) \) is the incidence angle, which in this case is the Brillouin, or bounce angle of the TE\(_{22,6}\) wave in the waveguide. Zero reflection may be obtained for a window when the plate thickness \( d \) is equal to half-integer number of longitudinal wavelengths in the dielectric.

The relative permittivity for fused silica is \( \varepsilon_r = 3.8267 \). The radius of the waveguide is 2.223 cm, and the Brillouin angle is 62.6°. The window was originally designed for the 170 GHz experiment, which used the TE\(_{28,8}\) mode [71]. For that experiment, the window thickness \( d \) was chosen as 0.391 cm. Fortuitously, the same window is also purely transmissive for the TE\(_{22,6}\) mode at 110 GHz (Fig. 4-21.)

Since the frequency bandwidth of the window is not infinite, reflections unavoid-
Figure 4-22: Schematic showing distance between cavity and window.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$f_1$(meas) (GHz)</th>
<th>$f_2$(meas) (GHz)</th>
<th>$f_2$(th) (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE$_{21,6}$</td>
<td>107.15</td>
<td>107.20</td>
<td>107.19</td>
</tr>
<tr>
<td>TE$_{22,6}$</td>
<td>110.08</td>
<td>110.13</td>
<td>110.12</td>
</tr>
<tr>
<td>TE$_{23,6}$</td>
<td>113.01</td>
<td>112.96</td>
<td>112.97</td>
</tr>
</tbody>
</table>

Table 4.3: Measured mode frequency shifts due to window reflections compared with theoretical shifts.

Relatively occur. These reflections alter the effective $Q$ of the cavity, causing frequency shifts. Frequency shifts due to window reflections can be calculated by

$$k_{z2} - k_{z1} = \pm \frac{\pi}{2L_w}$$

(4.11)

where $k_{z1}$ is the axial propagation constant for the mode at the design or predicted frequency, and $k_{z2}$ is the axial propagation constant at the shifted frequency. $L_w$ is the distance between the end of the resonator and the window (Fig. 4-22.)

Frequency shifts were observed during the operation of the axial experiment. These shifts were recorded, and compared with the theoretical shift the mode would experience due to window reflections. The results, listed in Table 4.3, show that the shifts between two measured frequencies ($f_1$(meas) and $f_2$(meas)) agree well with the predictions for the TE$_{21,6}$, TE$_{22,6}$, and TE$_{23,6}$ modes.
Radiation Pattern

Far field pattern scans were performed to examine the mode content of the radiation leaving the gyrotron. The receiving unit, originally described in [102], was formed by a WR6 21 dB pyramidal horn, a calibrated attenuator, and a WR6 diode detector. This unit was incrementally rotated along the polar angle (θ) at a constant radius, with the pivot point directly beneath the end of the launching waveguide. At each angle the variable attenuator was adjusted to keep the diode signal at a constant angle. The horn was vertically centered at the window and mounted in the orientation for detecting $E_\phi$, which is the dominant component of the far field radiation pattern for the $\text{TE}_{22,6}$ and neighboring modes.

The theoretical far field radiation pattern for a mode from an open cylindrical waveguide is [103]

$$P(\theta) = P_0 \left[ \frac{(k_z + k \cos \theta) J'_m(k r_w \sin(\theta))}{1 - (k \sin \theta/k_\perp)^2} \right]$$

(4.12)

where $P_0$ is a normalization constant, $k$ is the free space propagation constant, $k_z$ the axial propagation constant, $k_\perp = v_{mn}/r_w$, and $r_w$ is the waveguide radius at the window, 2.223 cm. Unfortunately, the radiation angles were large for this experiment, with the main lobe corresponding to the wide waveguide bounce angle, $\theta_B = 62.6^\circ$. Due to physical limitations, the measurements could only be taken up to around 70° on each side of the window. Not all mode content was captured by the scan.

The far field scan is shown in Fig. 4-23. For this scan, $V_b = 96$ kV and $I_b = 40$ A, and the operating parameters were set such that the $\text{TE}_{22,6}$ mode, as measured by the frequency system, was excited at 110.08 GHz. The operating point was stable and constant from pulse to pulse. For this case, the mode content matches well with the theoretically predicted far field pattern for the $\text{TE}_{22,6}$ mode. The results indicate that the mode being generated was a pure $\text{TE}_{22,6}$ mode, and little mode conversion has occurred.
Figure 4-23: The radiation pattern was measured over a wide range of angles. Also shown are the calculated maximum intensity angles for $\text{TE}_{22,n}$ modes. The solid lines represent the limits of the measurement system.

### 4.4 Summary and Discussion

The design issues addressed in this experiment include the identification of various modes near the design mode, the operational limits of the design mode, its start-up conditions, the location of competing modes, and the beam alpha. In all cases, the measured results have been compared with theoretically predicted values or values obtained with numerical simulations. The results agree well with theory and simulations. Discrepancies with MAGY may be explained by the fact that the alpha values used for the simulations were not the same as those measured in the experiment. In addition, the actual beam is not ideal. There is some velocity spread, radial spread, and beam nonuniformity which were not included in the MAGY simulations. These beam parameters were later added to MAGY input and the results are published in [94].

It is important to note that very little evidence was found for multi-moding. The gyrotron was stable in the sense that the design mode, at its optimal operating point,
was a pure mode, without the presence of any other frequencies. Therefore any competing modes were successfully suppressed by the $\text{TE}_{22,6}$ mode.

The highest power achieved in the design mode was slightly more than 1.4 MW at a beam voltage and current of 96 kV and 40 A, which corresponded to a total efficiency of 37%. Thus the 1.5 MW power level at 39% efficiency predicted by theory was not attained. There are several reasons which explain why the gyrotron did not operate at full power and efficiency. First, the beam alpha measured by the alpha probe at the highest efficiency point was only 1.33, possibly due to an axial misalignment of the tube. Therefore, not as much transverse energy was available from the electron beam. If the alpha reached the design value for the gun, 1.43, while still operating in the $\text{TE}_{22,6}$ mode, the efficiency of the gyrotron would have in theory increased to 39%. It may have been possible to increase the alpha by lowering the magnetic field at the cathode. However, as seen in the mode map in Fig. 4-14, doing so would have caused the mode to shift to the $\text{TE}_{19,7}$ mode. A second reason is due to other beam effects such as velocity spread, radial spread, and nonazimuthally-symmetric effects (nonuniform emission and current distribution, as well as misalignment.)

Additional research has been conducted using a new tube incorporating an internal mode converter. This mode converter includes a dimpled wall radiating launcher and four internal mirrors to convert the $\text{TE}_{22,6}$ mode to a free-space Gaussian beam which propagates through a window transverse to the gyrotron axis. Eventually this new tube will use voltage depression with a single stage collector to boost the net efficiency to over 50%. This experiment will be presented in Chapter 6.
Chapter 5

Phase Retrieval Based on Irradiance Moments

Microwaves generated from a gyrotron in high order waveguide modes must be transformed into a low order symmetrical mode for transmission and application. Either a Vlasov launcher [28] or its modification, a rippled-wall Denisov launcher [29], radiates the microwave energy extracted from bunched electrons. The microwave beam is then shaped and directed using a series of internal mode converter reflectors comprised of phase correctors and simple focusing mirrors. The configuration shown in Fig. 5-1, which represents the layout of a 1 MW, 110 GHz gyrotron [90], uses a system of two doubly-curved reflectors (M1 and M2) and two phase-correcting reflectors (M3 and M4) [104]. After the resultant beam propagates through the output window, the beam typically undergoes additional phase correction and/or focusing to further convert it to a Gaussian beam. Gaussian beams are advantageous since they may be focused and redirected using simple optical components. Additionally, the beam profile of a Gaussian beam matches well with the fundamental HE_{11} mode profile in a corrugated waveguide.

A common approach to design the phase correctors is to use either analytical or numerical techniques to approximate the radiation pattern emitted from the launcher. Attempts have been made, for example, to model the radiated fields generated from the rippled-wall launcher [105]. However, experimental results have shown that the
Figure 5-1: The gyrotron internal mode converter schematic.
measured field profile does not agree with predicted beam behavior at the gyrotron window [90]. The discrepancies between theory and experiment may occur from problems with launcher alignment or from problems with the launcher theory itself.

To fully account for the field radiated from the internal mode converter, cold-test intensity measurements are taken initially, without the presence of an electron beam in vacuum, using a high order mode generator. The low power quasi-optical microwave beam radiated from the launcher can be used to design the correcting mirrors. These mirror designs, however, require a knowledge of the free-space propagation behavior of the beam, characterized by both the amplitude and phase. While the amplitude profile can be directly measured at low power using a spatial scanner with a receiving horn and detector, the phasefront (at frequencies above 100 GHz) cannot be so easily determined. Therefore, numerical methods are ordinarily employed to retrieve the phase based on a series of measured intensity data taken at several planes located past the launch point. Fig. 5-2 shows a schematic of a cold-test gyrotron experimental setup which may be used to design appropriate external phase correcting mirrors. A series of measurement planes, shown as dotted lines, lie immediately before and after the window location where the beam is the narrowest (the beam waist) [106]. In the actual cold-test, the window is omitted from the set-up. Examples of discretely-
sampled measured data on a series of planes are presented in a later section of this chapter. Alternatively, in hot-test, in which high power microwaves are generated from an electron beam, data can be taken on a series of planes beyond the window plane with an infrared camera viewing a microwave absorbing screen.

Several methods have been developed to retrieve the microwave output phase based on intensity measurements. The more traditional approach, based on the Gerchberg-Saxton formulation [107], uses an iterative algorithm. This method attempts an initial approximation of the phase at the first measurement plane and then propagates this paraxial beam forward to the next measurement plane using a Fourier transform inter-plane wave propagation routine [108]. Using the assumed phase of the resultant beam with the measured intensity at this plane, the beam is then propagated back to the initial plane. The initial phase approximation is then modified to compensate for the error between the measured and reconstructed amplitudes [109],[110]. Through numerous repetitions the phase solution generally converges until the amplitude error is minimized [110],[111]. This error-reduction approach, known as the iteration method (or “phase-retrieval method”), although numerically intensive, has proven to be successful in designing accurate internal phase correcting mirrors [106],[112] and external mirrors [112]- [114].

An added advantage of using intensity measurements is the valuable information gained from computing the normalized weighted moments, or expectation values. The moments are useful to improve the reliability of the data and for phase retrieval, especially when the data set is taken using an infrared camera in hot-tests, since phase retrieval methods are typically sensitive to misalignment. Since measurements at different planes require moving the camera and target, alignment errors are not uncommon. Moreover, the work space is typically limited, which forces frequent re-positioning of the system. In [115], the moments of infrared images of gyrotron radiation were calculated to determine the accuracy of the spatial alignment of the images. The iterative phase retrieval procedure and internal mirror synthesis [106] were employed using only the well-aligned images.

A phase retrieval algorithm based solely on these moments may prove to yield even
more accurate results while being computationally more efficient [116]. This “irradiance moment” approach assumes an initial two-dimensional polynomial phasefront, which is predominately parabolic but with additional higher-order phase aberrations. The coefficients of the polynomial are calculated from the weighted moments based on intensity distributions over several measurement planes. Although employed in optics for applications such as characterizing the beam quality of high-power multimode lasers [117], this numerical method, to our knowledge, has never before been applied to the phase retrieval of a microwave beam. The microwave problem is recognized as more difficult than optical applications because of diffraction.

In the following chapter, as in [118], the theory behind the irradiance moment technique will be briefly presented, along with the generalized approach for retrieving the phase. The success of this scheme is then demonstrated numerically using both a simple Gaussian beam and previously generated gyrotron cold-test data. The final section will discuss future developments for the irradiance moment technique.

5.1 Irradiance Moment Theory

The basic idea behind any phase retrieval method is to determine the initial phase from intensity measurements near the beam waist. While commonly used Gerchberg-Saxton iteration methods rely on repeatedly improving an initial phase approximation to reduce amplitude error [109], the irradiance moment technique attempts to solve the initial phase polynomial coefficients from the weighted moments. Both approaches make use of Fresnel diffraction theory to propagate the beam, assuming the beam is linearly polarized and propagates paraxially. However, the iteration method advances the amplitude and phase of the beam, whereas the irradiance moment technique propagates the moments of the beam. Using the moment approach, we present the relationship between the phase at a fixed plane and irradiance moments propagating orthogonally from this plane. Although this relationship is generally nonlinear, we may form a set of solvable relations from the linear terms of the moments. After calculating the moments and predicting how they propagate, we then apply these
linear relations to determine the phase.

To be consistent with the geometry shown in Fig. 5-1 and Fig. 5-2, we will assume the paraxial beam propagates along the y direction. The z-axis is reserved as the axis of the gyrotron. The behavior of the wave at a particular axial location, y, which includes amplitude \( A_y \) and phase \( \Phi_y \), can be described by its wavefunction, or complex amplitude, \( \psi \), where \( \psi(x, y, z) = A_y(x, z)e^{i\Phi_y(x, z)} \).

The moments of the wavefunction are based on the normalized weighted intensity integrated over the finite measurement plane at an axial location, \( y \). They are defined as:

\[
M_{pq}(y) = \langle x^p z^q \rangle_y = \int\int x^p z^q A_y^2(x, z) dx dz
\]  

(5.1)

where the integration is performed over all \( x \) and \( z \).

It is important to note that a numerical approximation of the irradiance moment technique (and, indeed, of any phase retrieval method relying on intensity measurements) must be made from the fact that the measurement planes are finite. Theoretically, the moments are calculated by integrating over an infinite plane. For the integrals in the irradiance moment technique to be completely accurate, the amplitude at the plane edges must be zero. In practice, beam information outside of the plane limits is lost. This truncation introduces an error into the calculation of the moments, particularly higher order moments. For example, if a Gaussian beam is truncated to 20 dB below the peak value, then the error in the fourth-order moments is over 10 %. If the measurements are truncated to 28 dB, this error decreases to 2 %. Of course, if the measurement plane is chosen very large, such that the truncation is at 35 dB, then the fourth-order moment error is a very small number, around 0.5 %.

The Kirchhoff-Huygens diffraction integral in its Fresnel approximation describes a paraxial beam at any distance in terms of the initial wavefunction [119]:

\[
\psi(x, y, z) = \frac{i}{\lambda y} \int\int d\alpha' dz' \left[ \psi(x', 0, z') \exp \left( -i\frac{k}{2y} \left( (x' - x)^2 + (z' - z)^2 \right) \right) \right]
\]

(5.2)

where \( \lambda \) is the wavelength and \( k = 2/\lambda \) is the wavenumber. By applying the definition
of moments (Equation 5.1) to the diffraction integral, we can show how the moments change as the beam propagates, as derived in Appendix B and in [120],[121]:

\[
M_{pq}(y) = \left( -\frac{iy}{k} \right)^{p+q} \int \int dx dz \left[ \psi(x, 0, z) \exp \left( -i \frac{k(x^2 + z^2)}{2y} \right) \right] \times \frac{\partial^p}{\partial x^p} \frac{\partial^q}{\partial z^q} \left[ \psi^*(x, 0, z) \exp \left( \frac{i k(x^2 + z^2)}{2y} \right) \right]
\]  

(5.3)

where \( \psi(x, 0, z) = A_0(x, z)e^{i\Phi_0(x, z)} \) is the analytic complex amplitude at the initial \((y = 0)\) plane and \( \psi^*(x, 0, z) \) is its complex conjugate. This formula yields an expression for the propagation behavior of the moments in terms of the initial wave amplitude and phase. Furthermore, it relates the moments at any given axial location to the initial phase. Explicit expressions are listed in Appendix C for the first- and second-order moments.

From Equation 5.3, the moment \( M_{pq}(y) \) is in general a one-dimensional polynomial along the direction of propagation, \( y \), with order \( p+q \) and coefficients \( C_{pq}^{(m)} \) as follows:

\[
M_{pq}(y) = M_{pq}(0) + \sum_{m=1}^{p+q} C_{pq}^{(m)} y^m = \sum_{m=0}^{p+q} C_{pq}^{(m)} y^m.
\]  

(5.4)

The moment polynomial coefficients, \( C_{pq}^{(m)} \), are determined by a combination of the initial amplitude and phase and their derivatives integrated over the measurement plane. In practice, the data set is analyzed by finding a series of \( M_{pq} \) values at several planes along \( y \). The calculated \( M_{pq} \) values are then applied to a one-dimensional polynomial fit in \( y \) to obtain the moment polynomial coefficients \( C_{pq}^{(m)} \) up to \( m = p+q \), the polynomial order. This technique is illustrated in a later section.

From Equation 5.3 and Equation 5.4, it can be shown (Appendix B) that the linear coefficient of any moment order \( M_{pq} \), which measures the slope of the moment at the retrieval plane, may be expressed as a product of the initial intensity and weighted initial phase derivative integrated over the \( xz \)-plane:

\[
C_{pq}^{(1)} = -\frac{1}{k} \int \int dx dz \left[ \frac{\partial \Phi_0}{\partial x} \frac{\partial}{\partial x} (x^p z^q) + \frac{\partial \Phi_0}{\partial z} \frac{\partial}{\partial z} (x^p z^q) \right] A_0^2. 
\]  

(5.5)
Figure 5-3: Propagation behavior of the moments.
A set of linear equations may be created from the linear moment coefficients by assuming an appropriate form for the phasefront. For a spatially directed microwave beam, the phasefront can be expanded as a 2-D polynomial at the reference transverse \((xz-)\) plane with coefficients \(\phi_{ij}\):

\[
\Phi_0(x, z) = \phi_{10}x + \phi_{01}z + \phi_{20}x^2 + \phi_{11}xz + \phi_{02}z^2 + \phi_{30}x^3 + \ldots
\]  

Equation 5.6

The series in Equation 5.6 converges for beams decaying exponentially in the transverse direction. One way to show this is by applying the uniform stationary phase method \([122]\).

The linear moment coefficients \(C_{pq}^{(1)}\) are linear functions of the phase expansion coefficients and the initial amplitude moments, represented here as the intercept coefficients \(C_{pq}^{(0)}\), where \(C_{pq}^{(0)} = M_{pq}(0)\). This linear dependence is shown in the example provided in Appendix C. Linearized equations are formed because, from Equation 5.5 and the selection of a suitable polynomial phasefront expansion, the slope of each moment propagating in \(y\) is linear with respect to the phase expansion coefficients in the transverse \((xz-)\) plane.

The phasefront expansion of \(\Phi_0\) must be truncated to form a set of solvable linear equations from the expressions relating the linear moment coefficients to the phase coefficients. This approximation is valid due to the fact that the gyrotron beam is directed and paraxial. For well-behaved paraxial beams, the phasefront described by Equation 5.6 is primarily parabolic and nearly symmetrical. Therefore, the \(\phi_{02}\) and \(\phi_{20}\) coefficients are the dominant terms in the polynomial phasefront expansion. Higher order aberrations are included to provide an accurate phasefront solution. Generally, the solution is more accurate if many phase expansion terms are included. As mentioned earlier, however, the series in Equation 5.6 will converge for directed beams. Only the first few higher order phase expansion terms are necessary to provide an accurate solution.

The set of equations is closed and solvable by requiring that the maximum moment order of the calculated linear \(C_{pq}\) coefficients, or \(C_{pq}^{(1)}\), be equal to the order of the
truncated 2-D transverse plane phase expansion in \( \Phi_0 \), which is defined as \( N \). In other words, \((p + q)_{\text{max}} = (i + j)_{\text{max}} = N\). The number of equations which are formed is simply the summation of the number of unknown phase expansion coefficients. Since each \((i + j)\)th expansion in \( \Phi_0 \) has \( i + j + 1 \) coefficients:

\[
\text{Number of Equations} = \sum_{n=1}^{N} (n + 1) = \frac{N(N + 3)}{2}.
\]

The phase is calculated by first truncating the phasefront at the initial plane, \( \Phi_0 \), to order \( N \), which specifies the number of phase expansion coefficients \( \phi_{ij} \). The moments \( M_{pq}(y) \) are directly calculated from intensity measurements at several planes along \( y \) for every \( p \) and \( q \) up to \( p + q = N \). Each of these moments is then fitted over \( y \) to a one-dimensional polynomial of order \( p + q \) to determine the moment coefficients \( C_{pq}^{(m)} \) up to \( m = p + q \). Only the linear coefficients \( C_{pq}^{(1)} \) and the coefficients of zero rank \( C_{pq}^{(0)} \), which represent the slope and moment intercept, respectively, are required to form a series of linear equations which ultimately determine the phase expansion coefficients.

Because each of the moments is fitted over \( y \) to polynomials of order \( p + q \), the number of planes (\( y \) data) required by the irradiance moment technique must be at least \( p + q + 1 \). Since \((p + q)_{\text{max}} = (i + j)_{\text{max}} = N\), then the minimum number of planes is \( N + 1 \) for an \( N \)th order 2-D polynomial phasefront solution. More data planes may be used to insure a better moment fit and thus a more accurate solution. If fewer planes are used, however, the solution is indeterminate and an erroneous solution may occur. Since there will not be enough data in this instance to fit to the moment polynomials, the solution will not be unique.

After the phasefront has been determined at the initial plane, external reflectors may be designed to correct for the phase and amplitude of the gyrotron output, and to effectively couple power to the fundamental \( \text{HE}_{11} \) mode of a corrugated waveguide for guided transmission. In the procedure of reflector synthesis, we can take advantage of the irradiance moment method approach’s phasefront solution, which is in analytical form. Therefore, the numerical procedure of phase unwrapping, required for reflector
synthesis using the iteration method [106], is not needed using irradiance moments. This improves the mirror shaping because phase unwrapping is ambiguous.

5.2 Outline of the Irradiance Moment Approach

The procedure for finding a unique phasefront solution using the irradiance moment technique is fairly straightforward. The general outline for finding an Nth order unique phase solution is as follows:

1. Normalize all the measured intensity planes such that $M_{00}(y) = 1$ at each $y$ location.

2. Calculate the moment $M_{pq}(y)$ numerically at each plane, including the initial plane ($y = 0$) by applying a discrete integration routine to the weighted amplitude data.

3. Generate a one-dimensional polynomial fit in $y$ of order $p + q$ for the $M_{pq}$ moment.

4. Repeat steps 2 and 3 for each moment $M_{pq}$ up to $p + q = N$. 

Figure 5-4: General algorithm for irradiance moment approach.
5. Propagate the moments analytically by applying Fresnel diffraction theory. Specifically, the linear moment coefficients $C_{pq}^{(1)}$ must be expressed in terms of the initial phase (Equation 5.5.)

6. Expand and truncate the initial phasefront $\Phi_0$ as an $N$th order 2-D polynomial in the $xz-$plane.

7. Find a linear analytical expression for each of the linear moment coefficients $C_{pq}^{(1)}$ from step 5 in terms of the phase expansion coefficients $\phi_{ij}$ from step 6 and the moments at the initial plane $M_{pq}(0)$ represented by $C_{pq}^{(0)}$.

8. Solve the set of linear equations for the phase expansion coefficients $\phi_{ij}$ using the moment coefficients from the polynomial moment fits in $y$.

### 5.3 Irradiance Moment Technique Results: Gaussian Beam

#### 5.3.1 Ideal Gaussian Beam

The advantages and limitations of the irradiance moment approach are demonstrated by testing the algorithm with both an ideal Gaussian beam and a series of existing measured gyrotron intensity data. The results obtained from the iteration method are also presented for comparison.

For the ideal Gaussian case, a beam is chosen with $\lambda = 0.273$ cm and a waist $w_0$ of 2.0 cm at $y = 0$. Five planes containing discretized ideal Gaussian amplitude data are generated for the phase retrieval calculations: at $y = 20, 30, 40, 50$ and 60 cm. After the phase is retrieved at the initial plane $y = 20$ cm, the amplitude and computed phase are propagated to an observation plane $y = -30$ cm. A propagation routine based on the 2-D discretized fast Fourier transform calculates the beam wavefunction at any axial location given the initial amplitude and phasefront solution. The reconstructed complex amplitude at the observation plane may then be compared with the theoretical Gaussian amplitude and phase.
Since five amplitude planes are used in the irradiance moment technique phase retrieval algorithm, it is possible to assume a fourth-order phasefront 2-D polynomial at the initial \( y = 20 \) cm plane. However, this expansion is not necessary since the phasefront of an ideal Gaussian beam is parabolic and will not contain higher order aberrations. A second order phasefront polynomial \( (N = 2) \) is more appropriate for the calculations. A set of five linear equations (Equation 5.7) based on the five first- and second-order moments are required to solve the unknown phase coefficients \( \phi_{10}, \phi_{01}, \phi_{02}, \phi_{11}, \) and \( \phi_{02} \). These equations are derived explicitly in Appendix C. Note that this algorithm in principle requires only three data planes, since \( N = 2 \). Five planes are included to provide more data with which to accurately fit the moments and to be consistent with the other example in this section.

To understand how the irradiance moment approach works in this simple Gaussian case, we must examine the behavior of the moments. The first-order moments \( M_{10} \) and \( M_{01} \) are measures of the expectation values, or \( \langle x \rangle \) and \( \langle z \rangle \), respectively. They indicate where the center of the beam is located. For the ideal Gaussian case, they are at zero for all planes since the beam does not drift. The values of the linear polynomial coefficients are zero everywhere, i.e., the slopes are zero \( (C^{(1)}_{10} = C^{(1)}_{01} = 0) \) and the intercepts are zero \( (C^{(0)}_{10} = C^{(0)}_{01} = 0) \).

The second-order moments \( M_{20}, M_{11}, \) and \( M_{02} \) are not as simple to understand. Physically, these moments represent the size of the beam in \( x \) and \( z \). For a symmetric beam, \( M_{11} = 0 \) everywhere since there is no coupling between \( x \) and \( z \). The coefficients for this moment polynomial \( (C^{(0)}_{11}, C^{(1)}_{11}, \text{and } C^{(2)}_{11}) \) are zero. The other second-order moments \( M_{20} \) and \( M_{02} \) follow a quadratic describing the beam growth over distance. The calculated \( M_{20} \) moments at each of the five planes are shown in Fig. 5-5. Note the predicted \( M_{20} \) moment value at \( y = 0 \) cm is 1.0 cm, which accurately reflects the beam waist \( (w_0 = 2.0 \) cm). As seen in the example in Fig. 5-5, the polynomial coefficients of each moment at the phase retrieval plane \( (y = 20 \) cm) are easily determined. Recall only \( C^{(0)}_{pq} \) and \( C^{(1)}_{pq} \) are needed to calculate the phasefront.

The irradiance moment 2-D phasefront solution which uses the fitted moment
coefficients in the set of five linear equations is $\phi_{10} = \phi_{01} = 0, \phi_{20} = \phi_{02} = 0.0914$ cm$^{-2}$, and $\phi_{11} = 0$. The calculated phasefront at $y = 20$ cm has the form:

$$\Phi_0(x, z) = (0.0914 \text{ cm}^{-2})x^2 + (0.0914 \text{ cm}^{-2})z^2.$$  \hspace{1cm} (5.8)

From the radius of curvature formula based on the theory of Gaussian optics [119], the phasefront should have the following solution:

$$\phi_{20} = \phi_{02} = \frac{2ky}{4y^2 + (kw_0^2)^2}.$$  \hspace{1cm} (5.9)

Evaluating this expression at $y = 20$ cm using the given beam parameters yields $\phi_{20} = \phi_{02} = 0.0914$ cm$^{-2}$. Therefore, by testing the moment method with artificial Gaussian beam data, we have verified the irradiance moment technique produces an accurate solution for this simple example.

The intensity at the observation plane ($y = -30$ cm) was reconstructed using the
Figure 5-6: The mid-plane ideal Gaussian intensity profile at the observation plane \((y = -30 \text{ cm})\) is compared with the reconstructed amplitudes using the iteration method and irradiance moment technique near the edge of the beam.

phasefront computed by the irradiance moment approach as described above. In addition, we obtained a phasefront solution by applying the iteration method algorithm to the artificial Gaussian data. The reconstructed intensity based on this converged solution was also calculated at the observation plane. The results of both methods are compared in Fig. 5-6. Both algorithms accurately reconstruct the amplitude, although the irradiance moment approach produces a more accurate beam below -40 dB. This is not surprising since the iteration method attempts to reduce the error below an acceptable tolerance level, whereas the moment method provides a more exact one-step numerical solution.

5.3.2 Gaussian Beam With Uncorrected Offset Measurement Error

While the ideal Gaussian case confirms the validity of the irradiance moment approach, it is a trivial exercise. More insight may be gained from manufacturing a
simple case where measurement error has occurred. Errors in the form of shifted or offset data are common when using laboratory devices such as an infrared camera to measure intensity. It is important, therefore, to examine how such an error affects the phase retrieval process.

To examine offset error, the same five planes containing ideal Gaussian amplitude data are used as in the previous example. Error is introduced at the \( y = 40 \) cm plane, where the beam is shifted in the \( x \)-direction by \(+0.7\) cm, a distance greater than two wavelengths. Such a shift is not physical since the beam must travel in a straight line, yet the error could occur in an experiment if an infrared camera and viewing screen were misaligned. As before, a second order polynomial phasefront form is solved at the initial \( y = 20 \) cm plane.

Because the beam is shifted, \( M_{10}(y) \) is zero for all \( y \) except at \( y = 40 \) cm, where \( M_{10}(40 \text{ cm}) = 0.7 \) cm. The best linear fit of the first-order \( M_{10} \), or \( <x> \), moment is also shifted to reflect the increased average value over all five \( M_{10} \) moments. Therefore, while the \( C_{10}^{(1)} \) coefficient, or slope, does not change, the \( C_{10}^{(0)} \) value is raised slightly to the averaged value of the \( M_{10} \) moments, i.e., 0.14 cm. Since there is no shift in the \( z \)-direction, \( M_{01}(y) \) is still zero everywhere.

Since the introduction of an offset error does not change the size of the beam, the second-order moments are exactly the same as before. The \( M_{20} \) moments, for example, follow a smooth quadratic with or without an offset in the data (Fig. 5-5.) The \( C_{20}^{(0)} \) and \( C_{20}^{(1)} \) coefficients that are determined from this fit do not change.

The phasefront solution computed from the set of equations using the artificial
data with offset becomes

$$\Phi_0(x, z) = -(0.026 \text{ cm}^{-1})x + (0.0929 \text{ cm}^{-2})x^2 + (0.0914 \text{ cm}^{-2})z^2.$$  \hspace{1cm} (5.10)

Note that in Equation 5.10 there is a term linear in $x$ which arises from the offset error and is missing in the ideal case of Equation 5.8. The phase solution now has a slight tilt and asymmetry.

The theoretical Gaussian intensity at the observation plane is again compared with the reconstructed amplitude obtained from the irradiance moment technique (Fig. 5-9.) A phasefront solution and reconstructed intensity were also obtained by applying the iteration method algorithm to the same data. The iteration method intensity profile is shown for comparison.

Although the beam which is reconstructed using the irradiance moment algorithm has little width distortion, it does shift in the positive $x$—direction, albeit by a small amount. To understand the reason this shifting occurs in the irradiance moment phasefront solution, each moment must be examined. As mentioned earlier, the second-order moments do not change with the addition of an offset error. The first-
order moment polynomial fits, on the other hand, are altered when an offset error occurs at one or more planes. In this case, the $M_{10}(40 \text{ cm})$ moment increases to 0.7 cm due to the $x$–direction shift in the beam centroid at that plane.

The reconstructed profile from the iteration method in Fig. 5-9 is shifted more noticeably in the $+x$–direction and the width more distorted than the irradiance moment reconstructed profile. The shift arises from a tilt introduced in the phasefront solution at the initial plane as part of the iteration algorithm's attempt to compensate for the offset plane. The width distortion arises from the ellipticity of the beam seen by viewing the superposition of non-concentric circles. In addition, more noise is present due to the fact that the algorithm has difficulty converging to a unique solution for this case. In fact, for offset measurement errors greater than three wavelengths, the iteration algorithm does not converge at all.
5.3.3 Gaussian Beam With Corrected Offset Measurement Error

Although the results of the previous case including offset measurement error are not dramatically different from the ideal case, it is possible to correct for these errors by examining the moments. This error-correcting feature of the irradiance moment technique is its main advantage over other phase-retrieval methods. For example, the $M_{10}$ moment at $y = 40$ cm is easily identified as measurement error since it is not physically possible for the beam centroid to travel in a path other than a straight line. The intensity at this plane may be shifted in the $-x$-direction such that the $M_{10}(40 \text{ cm})$ moment is aligned with the other four moments. The phase retrieved by the irradiance moment approach in this corrected case reverts back to the original solution obtained using the ideal Gaussian beam (Equation 5.8.)

It is also possible to improve the irradiance moment phase solution by selective data omission. If the $y = 40$ cm plane is not included in the data set, only four planes are used to retrieve the phase. These four planes, however, will still be able to produce the ideal Gaussian second order polynomial phasefront solution since the data set will now lack measurement error.

5.4 Irradiance Moment Technique Results: Cold-Test Gyrotron Data

The irradiance moment approach is further explored by testing the algorithm with measured gyrotron intensity data. Namely, the input is taken from the cold test results of a 1 MW, 110 GHz gyrotron built by CPI. The discretely sampled data form a set of eight intensity measurements at various locations from the window: $y = -10, -5, 0, 5, 10, 20, 40, 60 \text{ cm}$ (Fig. 5-10.) The paraxial beam radiated by the internal mode converter (Fig. 5-1), is centered at $(x, z) = (0 \text{ cm, } 37.4 \text{ cm})$, which is on axis with the window center.

In principle, a seventh-order polynomial phase expansion may be applied to the
Figure 5-10: Contour intensities of the eight measurement planes are shown from the CPI gyrotron in cold test. The window center is at $z = 37.4$ cm, $x = 0$ cm. Contours of constant intensity are at 3 dB intervals from peak. All axial distances, $y$, are referenced from the window plane.
algorithm since eight planes are available. However, the phasefront is only expanded to fourth order \((N = 4)\) to simplify the system to a set of 14 equations (Equation 5.7.) In addition, the accuracy of a seventh-order polynomial moment fit, which would be required for a seventh-order 2-D phasefront solution, would be limited using only eight data planes.

To find the solution to a unique fourth-order 2-D phase expansion with the irradiance moment approach requires at least \(N + 1\), or five, planes. To obtain accurate polynomial fits of the various moments, the first seven planes are used in the calculations. The phase is retrieved at the initial \((y = -10\ \text{cm})\) plane by fitting each moment to an appropriate polynomial in \(y\) using the seven planes. It is convenient here to reserve the final \(y = 60\ \text{cm}\) plane as the observation or check plane.

After the initial phase is constructed, the approximated wavefunction may then be propagated from the \(y = -10\ \text{cm}\) plane using the measured initial intensity and the 2-D phasefront expansion computed from the irradiance moment technique. To test the accuracy of the solution computed by the fourth-order irradiance moment scheme, the beam is advanced to the observation plane \(y = 60\ \text{cm}\) (Fig. 5-12(a).) A normalized error, \(E\), is then calculated between the method’s computed amplitude \(A_y^{(c)}(x, z)\) and the measured amplitude \(A_y^{(m)}(x, z)\) at this plane

\[
E = 1 - \frac{\left| \iint dx dz A_y^{(c)}(x, z) A_y^{(m)}(x, z) \right|^2}{\iint dx dz \left| A_y^{(c)}(x, z) \right|^2 \iint dx dz \left| A_y^{(m)}(x, z) \right|^2}.
\]

\[(5.11)\]
Figure 5-12: Comparison of phase retrieval methods and measured intensity at \( y = 60 \) cm. The iteration method results are on the left side and the irradiance moment method results are on the right.
Figure 5-13: The y = 60 cm profile at mid-plane is shown along x for the measured and reconstructed intensities.

Applying the error equation to the beam which was reconstructed from the irradiance moment technique, we find $E = 0.019$ at $y = 60$ cm. Using the same data in the iteration method algorithm, the error $E$ is 0.015. Both methods yield a very small error at the observation plane, with the iteration method error somewhat smaller than the error from the irradiance moment approach. While this result is not immediately obvious in a comparison of the reconstructed intensity contour plots at $y = 60$ cm (Fig. 5-12(b)), the intensity profiles along the $x$ transverse direction shown in Fig. 5-13 demonstrate that the irradiance moment phasefront solution predicts a slightly narrower beam at the observation plane than measured.

The reason the reconstructed beam has a narrower waist than expected may be explained by the second-order moments, which indicate the beam size. The quadratic fit of $M_{20}$, for example, if extended to the observation plane, predicts a lower moment value than the moment value of the measured intensity pattern. The reason for this discrepancy is small measurement errors.

It is evident from the first- and second-order moments in $x$, $M_{10}$ shown in Fig.
Figure 5-14: Beam centroid $< x >$ and beam size $< x^2 >$ are plotted versus $y$. Deviations from a straight line in (a) and a quadratic in (b) represent measurement errors.

5.5 Summary and Discussion

The theory and general method for retrieving the phase of gyrotron beams based on weighted amplitude moments has been presented. In addition, the success of this moment approach has been demonstrated by testing the algorithm with an ideal Gaussian beam, a Gaussian beam with one offset plane, and with measured data.
Irradiance Moment Approach

- Approximates an initial guess for the phase front.
- Numerically propagates beam. Through iteration, phase solution converges.
- Requires computationally-intensive iterative calculations.
- Produces a phase solution in numerical form.
- Ambiguous process of phase unwrapping is required.
- Does not compensate for measurement errors. For large errors, does not converge.

- Calculates the irradiance moments of the intensity data.
- Forms a set of linear equations from Fresnel integral.
- Solves small set of linear equations.
- Produces an analytical form for the phase solution.
- Unwrapping is not required.
- May be used to locate measurement errors and compensate for them.

<table>
<thead>
<tr>
<th>Iteration Method</th>
<th>Irradiance Moment Approach</th>
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<tbody>
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</tr>
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Table 5.1: Comparison of phase retrieval methods.

Furthermore, the irradiance moment approach has been benchmarked against the previously developed iteration method. The amplitude errors produced from the irradiance moment approach were comparable with those from the iteration method in each case, even without error correction.

The main advantage of the phase-retrieving irradiance moment technique is its ability to locate and compensate for significant measurement errors. In addition, the irradiance moment approach produces an analytical solution and eliminates the need for computationally-intensive iterative calculations to produce accurate results. The differences between the iteration method and the irradiance moment approach are summarized in Table 5.1.

As discussed in this chapter, the irradiance moment technique is a powerful tool for determining the accuracy of the images used in a phase retrieval analysis. In the preceding section, the example of a gyrotron cold test beam was found to have relatively small errors. In that case, the retrieved phase is quite accurate. There is very little improvement in accuracy, if any, to be gained by attempting to reject data planes that are slightly misaligned. In previous work on external phase correcting mirrors for a gyrotron in use at the large helical device (LHD) stellarator, National Institute for Fusion Science, Toki, Japan, we found that a large improvement was
possible if planes that were out of alignment were rejected. That work is described in [115].

While initial results indicate the effectiveness of the irradiance moment approach, modifications may be made to enhance the algorithm and improve its performance. A nonlinear approach to solving the phase coefficients may increase the accuracy of the solution. This nonlinear irradiance moment technique would require the moment coefficients of higher rank discarded in the linear version. Additionally, a formalism is being developed which directly synthesizes the phase correcting mirrors using irradiance moments and intensity data [123]. Finally, an "iterative irradiance moment method" may be explored which incorporates ideas from both phase retrieval algorithms. These topics could be the subject of future research.

The present examples show that the irradiance moment technique, previously limited to phase-retrieval problems in the optical regime, can be successfully applied to retrieving the phase of microwave beams. It is a promising new approach to an old problem. This method may become a novel and powerful numerical method to predicting gyrotron beam behavior and shaping phase correcting mirrors. However, further research is needed to estimate the accuracy of the technique, including research on random noise, more complex measurement errors and beams with more complex amplitude forms.
Chapter 6

Internal Mode Converter

Experiment

In the axial gyrotron experiment (described in Chapter 4), the microwave power generated in the cavity was coupled to an output waveguide. The waveguide, like the cavity, was overmoded at 110 GHz, and therefore able to support the TE$_{22,6}$ circular rotating mode. The output system of the tube was designed such that the mode propagated to the end of the waveguide, where a fused silica window was located. The mode was then radiated into free space, as described by the formula for an open-ended waveguide antenna (Equation 4.12.)

For most gyrotron applications, however, this beam output system is not practical or useful. Electron cyclotron resonance heating, for example, requires that the guided mode be converted into a free-space propagating Gaussian beam, in a configuration referred to as radial output coupling. For radial power extraction, a quasi-optical mode converter is used. This mode converter consists of a helical-cut waveguide launcher antenna and a mirror system for focusing and phase correcting of the radiated beam. Since the mode converter is contained inside the gyrotron tube, it is commonly referred to as an internal mode converter.

The mode conversion efficiency is especially important when designing converters for high-power long-pulse gyrotrons, which must be able to contain and suppress heating due to uncoupled and reflected microwave power in the tube. Megawatt-level
gyrotron mode converters have historically been difficult to design and test. Recently, however, significant progress has been made with theory and numerical techniques, in conjunction with experiments, such that the design process has become simplified and more reliable. It is not uncommon for mode converters in modern gyrotrons to achieve conversion efficiencies of over 95%. The internal mode converter for the experiment discussed in this chapter was in fact built based on an existing design for a 1 MW gyrotron tube.

An experimental gyrotron with an internal mode converter is significantly closer physically to an industrial tube compared to the experiment using axial power extraction. The goal of the MIT 1.5 MW gyrotron internal mode converter experiment, however, is not to mimic the performance of an industrial tube, but to examine some of the physics issues involved with such a design. The components of the quasi-optical mirror system for the 1.5 MW design are for the most part new and untested. Although the beam / mode interactions in the cavity are not expected to change significantly, the problem of microwave reflections at the window propagating back into the interaction region will be avoided. Finally, the characteristics of the beam collector will differ if radial extraction is used. Indeed, one of the main advantages of using the internal mode converter is that the collector may be electrically isolated from the other components. This feature allows a voltage depression to be applied to the collector to increase the electrical efficiency of the gyrotron. Depressed collectors are commonly used in many other microwave tubes; only recently has the idea been proposed and tested in gyrotrons. The design issues and physics involved with a depressed collector configuration in this experiment will be discussed later.

This chapter is similar in structure to Chapter 4, which discussed the axial gyrotron experiment. The chapter will first describe the setup of the internal mode converter experiment, present and discuss the preliminary results, and then propose plans for future experiments.
6.1  Design

6.1.1  Design Parameters

The 1.5 MW 110 GHz gyrotron internal mode converter experiment employs an entire new tube from the axial gyrotron experiment and the electron beam uniformity experiment. New components such as the launcher, mirror system, window, and collector (each of which will be discussed in the subsequent subsections) have been designed, built, and installed. However, the same cavity and 96 kV, 40 A electron gun are used as before. Additionally the superconducting magnet, gun coil electromagnet, and hence magnetic field profile are the same. Since these basic gyrotron elements are retained, the electron beam characteristics, namely alpha, trajectory, and beam quality, are expected to be identical in the new tube. Moreover, since the cavity is the same, the physics of the beam-wave interaction will not change. The start-up conditions, mode coupling, and mode competition are not expected to vary significantly from the axial experiment. Therefore the design parameters for the internal mode converter experiment are the same as previous design parameters for the axial experiment (Table 4.1.) As in the earlier experiment, the theoretically predicted power is 1.5 MW, and the efficiency (without depressed collection) is 39 %.

Ultimately, the experimental results should be similar to the results obtained for the axial experiment. Assuming the internal mode converter is properly designed and has a high mode conversion efficiency, the experiment should produce the 1.4 MW, and 37 % efficiency (without depressed collection) seen in the axial experiment.

Even higher efficiency may be possible with voltage depression on the electrically isolated collector, as will be discussed in Section 6.1.4. With the proper voltage depression on the collector, the efficiency could increase to over 50 %.

6.1.2  Internal Mode Converter

One of the crucial elements in megawatt level gyrotron tubes being developed for electron cyclotron resonance heating is the internal mode converter. The converter
typically consists of a launcher and three or more focusing and shaping mirrors. The 1 MW, 110 GHz gyrotron tube designed by Communications and Power Industries (CPI) uses a Vlasov-type dimpled wall launcher to radiate the generated TE$_{22,6}$ circular waveguide mode to free space [90]. The resulting beam is then guided and focused using four mirrors placed along the gyrotron axis. The purpose of these mirrors is not only for guidance, but also to shape the beam such that it is Gaussian. Moreover, the mode converter is designed to produce a beam with a minimum waist size at the window of the gyrotron.

Mirrors 1 and 2 for this internal mode converter were designed at MIT using the Stratton-Chu vector diffraction theory [105]. These mirrors, although slightly ellipsoidal, are mostly parabolic, to narrow the launched beam in the azimuthal direction. To determine these mirror shapes, the fields at the launcher are first theoretically derived. These fields are then used to predict the beam shape launched into free space. This beam is then numerically propagated through the mirror system to the third mirror location. Since the theoretical beam intensity pattern is known at the third mirror location, and the output is stipulated as an ideal Gaussian beam with a waist at the window location, the surfaces of mirrors 3 and 4 may be designed.

In theory, mirrors 3 and 4 may be flat reflectors which simply guide the Gaussian beam from mirror 2 to the window. It is fairly straightforward to design these mirrors based on the predicted field pattern at mirror 3. However, due to approximations and limitations in the theory, and perhaps also to inaccuracies in the launcher fabrication process, the actual field does not match the field predicted by theory. This was shown when the field after mirror 2 was experimentally determined during a cold-test of the 1 MW CPI tube’s internal mode converter. This measured beam was then numerically propagated to the mirror 3 location. Since the theory did not precisely match the measured data, it was concluded that a simple, flat-surface design of mirrors 3 and 4 would not accurately produce a Gaussian beam at the window.

A process was developed at MIT to design phase-correcting surfaces for mirrors 3 and 4, given the input amplitude and desired output amplitude [106]. This mirror synthesis technique used a phase-retrieval scheme, which is a numerical method used
to estimate the phase at any plane along the direction of propagation based on two or more amplitude data planes. Either the iteration method or the irradiance moment approach may be used. The technique also approximated the mirror surfaces as thin lenses in an equivalent optical system (Fig. 4-21.) The process for the synthesis of these mirror surfaces, given the measured amplitude incident on mirror 3 and the desired amplitude after mirror 4, is listed below.

1. Run the iteration phase-retrieval code or the irradiance moments code to estimate the phase at the mirror 3 location from cold-test intensity measurements.

2. Run the code to estimate the phase at the mirror 4 location.

3. Find the necessary phase differences at these locations which will produce the desired phase after mirror 3, and before mirror 4.

4. Generate the mirror surfaces based on these differences.

5. Check the design by using a numerical propagation code.

By using this process, the phase-correcting mirrors designed for the internal mode converter used in CPI's 1 MW gyrotron tubes were very successful in producing a Gaussian beam at the window, with the correct waist. For these tubes, the viewing diameter of the diamond window was 5.08 cm, or 2 inches. If we stipulate that the window aperture should allow the -21 dB contour, the beam waist should then be around 1.5 cm.
As mentioned, the 1.5 MW tube is based on the successful 1 MW gyrotron. Since there is a significant increase in power, the beam power must be redistributed such that the beam may pass through the diamond window without destroying it. Therefore, the new CVD diamond window for the 1.5 MW industrial gyrotron will have a viewing diameter of 8.89 cm, or 3.5 inches. Fortunately, the cost of a larger diameter window is not much higher than the original window, so this modification is acceptable. In addition, the window in the new design is somewhat closer to the gyrotron axis, from \( y = 29 \) cm to \( y = 25.4 \) cm.

Both of these mirror design alterations imply that the internal mode converter must also be redesigned. Efforts are being made to completely change the mirror system based on new numerical codes which accurately predict the launcher fields [124]. This technique is still in development, however, and has not been fully tested yet. Our approach at MIT is to keep the launcher and first two mirrors the same, and simply redesign mirrors 3 and 4, therefore reducing the number of modifications to the existing 1 MW design. It is possible to change the phase-correcting surfaces of these mirrors using the same techniques as before, and the same input amplitude data as before, to produce a larger beam at the new window location.

Our initial attempt to design the new phase-correctors was to simply change the radii of curvature of the mirrors, where the curvature was determined from the original mirrors and Gaussian optics. The surface perturbations were kept exactly the same. This effort, however, proved unsuccessful. It can be shown that this technique is most effective if the positions of the mirrors are changeable and the mirrors are scalable. Since we are basing our design on the previous mode converter, the mirrors are already fixed, so the design based on this method was not satisfactory.

After the first attempt failed, we returned to the original design process that proved so successful for the mode converter used in the 1 MW tube: phase retrieval and mirror synthesis. New cold-test data was obtained by Michael Shapiro and Sam Chu at CPI in 2003. The iteration phase retrieval method was employed in conjunction with the irradiance moments (as described in Chapter 5.) This approach allowed for the correction of errors in the cold-test measurements. After several modifications
Figure 6-2: Phase-correcting surfaces for mirror 3 and mirror 4 of the internal mode converter.

Figure 6-3: The predicted beam intensity pattern in 3 dB increments at the window. The window is outlined in bold. An ideal Gaussian beam pattern is also shown as a comparison (dashed lines.)
were made to the phase retrieval process and different variations were tested, an optimized design was achieved. The final mirror 3 and 4 phase-correcting surfaces are shown in Fig. 6-2. Fig. 6-3 shows the predicted resulting beam intensity pattern at the window using similar mirror surfaces. Since the new window of the 1.5 MW industrial tube design has a diameter of 8.89 cm, the desired beam waist at this location should also increase. If the mirrors are designed such that the -21 dB contour passes through the window, then the new beam waist is around 2.67 cm. For the beam shown in Fig. 6-3, the waist in the x-direction is 2.63 cm, and 2.62 cm in the z-direction. Using Equation 5.11, the Gaussian beam content of the beam, based on its numerically predicted amplitude and phase is 98.94 %.

One final consideration in the feasibility of this design is the amount of power lost due to the finite sizes of the mirrors. Mirror 3, in particular, is limited in size since
Figure 6-5: The exiting beam after mirror 4 at the mirror 3 location. Only the -27 dB contour is clipped by the edge of mirror 3. Furthermore, less than 0.08 % of the power falls on the mirror.
Mirror 3  \((x, y, z)\) in cm

-3.0, 10.94, 23.96
5.5, 10.80, 23.96
-3.0, 11.12, 32.46
5.5, 10.58, 32.46

Mirror 4  \((x, y, z)\) in cm

-6.0, -11.02, 31.27
6.0, -10.87, 31.27
-6.0, -8.23, 43.43
6.0, -8.54, 43.43

Table 6.1: The coordinates of each corner of mirrors 3 and 4. The coordinate system used in the table is shown in Fig. 6-6.

The dimensions for the new mirror 3 and mirror 4 designs are provided in Table 6.1, where the coordinates of each corner of the mirrors are given. The \(z = 0\) reference plane is at the beginning of the launcher.

The complete internal mode converter design is shown in Fig. 6-6. By using the same methods as before at MIT, the new phase-correcting mirrors 3 and 4 are numerically shown to produce an acceptable Gaussian beam with a flat phasefront at the window location. Further, the predicted losses due to limited mirror sizes are small.
Figure 6-6: Mirror layout of the internal mode converter. Mirrors 1 and 2 are toroidal and Mirrors 3 and 4 have phase-correcting surfaces.
Figure 6-7: Window reflectivity $|R|^2$ of fused silica for a wave at normal incidence. The thickness $d = 0.7$ cm.

6.1.3 Window

As mentioned during the discussion of the internal mode converter design, the window diameter of the 1.5 MW industrial gyrotron tube will be 8.89 cm (3.5 inches.) This window will be made of CVD diamond in order to withstand the high amount of microwave energy transmitted through the aperture. The experimental internal mode converter gyrotron, however, is operated in short pulses, so fused silica may be used for the window material. In Section 4.3.2, which contained a discussion of the fused silica window in the axial gyrotron experiment, the relative permittivity of this material was reported to be $\varepsilon_r = 3.8267$. The new window diameter was chosen as 12.192 cm (4.8 inches); the diameter of the aperture was 10.795 cm (4.25 inches.) This diameter was chosen for the design since it was the closest standard size (as manufactured by Dynasil Corporation) to the industrial window diameter. Indeed it is slightly larger, which is ultimately advantageous for the experimental tube design. With a larger window, the design becomes more flexible and accommodating since realistically the exiting beam may be larger than predicted by the numerical codes used for the internal mode converter design, or the beam may be off-center.
The window thickness $d$ was calculated as previously, where the window was modeled as a dielectric plate, exhibiting behavior similar to a Fabry-Perot interferometer. For normal incidence, when the bounce angle $\theta_B = 0^\circ$, the original equation for the reflection coefficient (Equation 4.10) is simplified to:

$$|R|^2 = \frac{(1 - \varepsilon_r)^2 \sin^2 \chi_o}{4 \varepsilon_r \cos^2 \chi_o + (1 + \varepsilon_r)^2 \sin^2 \chi_o}$$

(6.1)

where $\chi_o = (\omega/c)d\sqrt{\varepsilon_r}$. The thickness $d$ for this window is set at 0.699 cm (0.275 inches), since the reflectivity at 110 GHz is close to 0 (Fig. 6-7.) This value is not the minimum thickness which may be used; the thickness was chosen also for a robust window which could withstand the high pressures created by vacuum conditions.

Window reflections in the axial experiment were discussed in terms of their impact on mode interaction efficiency and mode competition. These influences for gyrotrons in the axial configuration are also discussed in detail in [100]. However, the interaction region of the internal mode converter experiment is not expected to encounter such adverse effects due to window reflections. Efficiency should not decrease, and frequency shifting, described in Equation 4.11, and exhibited in Table 4.3, should not be observed.

### 6.1.4 Collector

One of the advantages of the internal mode converter configuration is that the collector may be electrically isolated from the rest of the tube. As a consequence, a voltage may be applied to the collector for enhancing the electrical efficiency. When voltage depression is used, the collector is typically referred to as a depressed collector.

For depressed collectors, the remaining kinetic energy of the spent electron beam is partly converted into electrical energy. In this case, the beam loading on the collector is also greatly reduced since the electrons have much less energy when impacting on the collector surface. Dissipating the heat generated at the collector becomes much easier.

This technique of using voltage depression at the collector to slow down electrons is
not new. Other types of high power tubes such as TWTs and klystrons currently use depressed collectors. However, only recently have depressed collectors been explored for gyrotrons [68],[125]-[127] and gyroklystrons [128]. One of the issues involved with using depressed collectors for energy recovery in gyrotrons is that only axial electron energy may be recovered. The electrons in a gyrotron beam generally have a large amount of rotational energy instead of axial energy. However, after exiting from the interaction region, most of transverse velocity of the electrons is transformed into axial velocity in the decreasing magnetic field. One limitation of depressed collectors in general is that energy spread in the beam makes it difficult to recover energy. Only electrons with a particular energy are recovered for a collector at a single potential. Indeed, the main limitation of the efficiency of depressed collectors is beam energy spread. Multi-stage depressed collectors are currently being developed to compensate for this energy spread and recover energy from more electrons in gyrotron beams [129]. One final design issue is that electrons are created from the beam impacting on the collector surface. These secondary electrons may be reflected back into the interaction region, severely limiting the microwave efficiency [45]. The potential depression on the collector must be small enough to avoid reflection of these emitted particles.

The efficiency of the depressed collector $\eta_{coll}$ is given by [130]:

$$\eta_{coll} = \frac{P_{coll}}{P_b - P_{RF}}$$

(6.2)

where $P_{coll}$ is the power recovered by the depressed collector. In a single-stage depressed collector $P_{coll} = V_{coll} \cdot I_{coll}$ with $V_{coll}$ the applied collector voltage and $I_{coll}$ the collector current. $P_b$ is the incident beam power, and $P_{RF}$ is the generated RF power. $P_b - P_{RF}$ is the remaining beam power after the interaction cavity. The total or wall-plug efficiency $\eta_t$ is defined as [131]:

$$\eta_t = \frac{\eta_{el}}{1 - \eta_{coll}(1 - \eta_{el})}$$

(6.3)

where $\eta_{el} = P_{RF}/P_b$ is the electronic efficiency as given by Equation 2.16.

Typically, a power supply is used to provide the voltage depression of the collector
in an industrial CW gyrotron tube. For short pulse experiments, however, a power resistor may be used to create an instantaneous voltage at the collector (Fig. 6-8.) In this configuration the beam current is $I_b = I_{\text{coll}} + I_{\text{body}}$. For a well-aligned tube the body current is generally very low.

A simple cylinder geometry was used for the design of the collector in the internal mode converter experiment. The diameter was set at 12.7 cm (5 inches), which allowed the collector to fit inside an outer assembly having a 17.8 cm (7 inch) diameter. One of the design issues was the avoidance of breakdown due to high electric fields generated during voltage depression. Therefore to reduce the electric fields, the edge of the cylinder was designed with a rounded lip.

This collector geometry was modeled in a simulation to determine the electric fields. A 2-D finite difference method was used to find the potential distribution by solving Poisson's Equation. Due to the geometry of the tube, which consisted of two intersecting cylinders, two separate planes were examined (Fig. 6-9.) For these simulations, a 25 kV voltage was applied to the collector. In this configuration, the simulations show that the maximum electric field would be 15.5 kV/cm in one
plane and 19.6 kV/cm in the other. These values are within the limits for avoiding breakdown in a vacuum.

Finally, safety becomes an issue when considering the depressed collector, particularly in a laboratory environment. High voltages would be present at the collector end of the tube, not just the gun end. Shielding must be used to protect the researcher from exposure to this high voltage. The collector was therefore encased and properly shielded using a fiber glass housing.

### 6.2 Experimental Setup and Diagnostics

A schematic for the internal mode converter experiment is shown in Fig. 6-10. The gun, beam tunnel, and cavity remain the same as in the axial gyrotron experiment, with the distance between the cathode and cavity (48.2 cm) remaining the same (Fig. 6-11(a).) In addition, the same test stand and power supply are used. The superconducting magnet is therefore also the same, along with the electromagnetic coil located at the cathode for changing the beam alpha and adjusting the beam radius. The polarity of the magnet was chosen such that the co-rotating (not counter-
rotating) waveguide modes would be excited. Due to the helical cut of the launcher, only co-rotating modes are efficiently radiated into a directed beam.

A gate valve separates the gun from the tube to isolate the gun and allow for changes to be made to the experiment if necessary. The gate valve has been modified such that a 2 l/s ion pump provides additional pumping at the gun end of the tube. In addition the pump may be used to activate the gun with the gate valve closed. This feature becomes important if vacuum conditions in the tube cause poisoning of the cathode.

The components of the internal mode converter immediately follow the resonator. The helical cut launcher is the same as the cold-test launcher that was formerly used at CPI for testing mirror configurations and taking phase retrieval data. This launcher is directly coupled to the cavity, since its beginning radius matches the radius of the final uptapered portion of the cavity section. Mirrors 1 and 2, as mentioned earlier, have the same design as in the original 1 MW internal mode converter. They were fabricated using a multi-axis computer numerical control (CNC) milling machine to create the smooth parabolic surfaces in copper. The more complex phase-correcting
surfaces of mirrors 3 and 4 were created in blocks of copper by a more precise CNC machine at CPI, which was able to take as input the appropriate data files containing the surface details (Fig. 6-2.)

The internal mode converter assembly is shown in Fig. 6-11(b). A stainless steel holder for mirrors 1 and 2 attaches to the launcher sheath. An additional stainless steel piece is used to hold and position the phase-correctors, mirrors 3 and 4, in accordance with the positioning coordinates given in Table 6.1. This mirror holder is then connected to the rest of the assembly.
The electron beam collector (Fig. 6-12(a)) is made of rolled copper with a thickness of 30 mils. It is designed with a 12.7 cm diameter and a rounded lip at the edge to reduce the electric field, as discussed in the depressed collector design (Fig. 6-9.) The collector is axially positioned after the fourth mirror, such that the electron beam clears the mirror and impacts on the inner surface of the collector. The collector is attached to the end of the outer assembly which has a 17.8 cm inner diameter. This outer assembly consists of a cylindrical ceramic ring brazed between cylindrical sections of stainless steel. The ceramic ring acts as an electrical isolator by separating the collector from the rest of the tube. During depressed collector operation, an appropriate power resistor is connected between the collector lead and ground. Additionally, a fiber glass shield may be placed around the collector assembly. This shield is used for safety purposes to protect from exposure to high voltage.

The fused silica window is positioned such that it is centered at \( z = 37.4 \) cm, referenced from the beginning of the launcher. It is located at \( y = 25.4 \) cm from the gyrotron axis. The diameter of the window is 12.192 cm. Since the window is sealed in the tube using an assembly containing viton O-ring seals, the actual window aperture is 10.795 cm.

In addition to the 2 l/s ion pump at the gun end, a 220 l/s ion pump and a 60 l/s ion pump are attached at a port behind mirror 4, opposite the window port. A getter pump was initially attached at a small port at the top of the tube. Since the effectiveness of the getter pump proved inconclusive, it was eventually removed. It was replaced by a residual gas analyzer (RGA) to provide a means of examining the vacuum conditions inside the tube. The final pressure reached in the tube after it had been cleaned and baked out to close to 200 °C was around \( 2 \times 10^{-8} \) torr.

The final assembly of the tube is shown in Fig. 6-12(b). The tube was eventually placed on a stand which allowed for vertical, horizontal, and axial alignment within the magnet. For the best microwave efficiency, such that the cavity was positioned at the center of the magnet where the magnetic field profile was flat, the cathode was placed at 23.1 cm from the beginning edge of the magnet, with the gun coil centered at the cathode.
Table 6.2: A list of measured frequencies for the internal mode converter experiment \( f_{\text{mc}} \) compared with cold-cavity theory \( f_{\text{thy}} \) and the frequencies measured in the axial experiment \( f_{\text{axial}} \).

<table>
<thead>
<tr>
<th>Mode</th>
<th>( f_{\text{thy}} ) (GHz)</th>
<th>( f_{\text{mc}} ) (GHz)</th>
<th>( f_{\text{axial}} ) (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE(_{18,7})</td>
<td>106.60</td>
<td>106.70</td>
<td>106.70</td>
</tr>
<tr>
<td>TE(_{21,6})</td>
<td>107.14</td>
<td>107.20</td>
<td>107.15</td>
</tr>
<tr>
<td>TE(_{19,7})</td>
<td>109.63</td>
<td>109.70</td>
<td>109.70</td>
</tr>
<tr>
<td>TE(_{22,6})</td>
<td>110.08</td>
<td>110.12</td>
<td>110.08</td>
</tr>
<tr>
<td>TE(_{20,7})</td>
<td>112.64</td>
<td>112.66</td>
<td>112.65</td>
</tr>
<tr>
<td>TE(_{23,6})</td>
<td>113.01</td>
<td>113.03</td>
<td>113.01</td>
</tr>
</tbody>
</table>

The diagnostics of the experiment include the power-detecting diode used to determine the shape and width of the output microwave pulse, the horn-and-waveguide network used to deliver output radiation to a heterodyne receiver system, and the calorimeter used to measure averaged power. These measurement systems are the same as in the axial gyrotron experiment and are described in Section 4.2. The beam alpha probe used in the axial experiment is also present in the internal mode converter experiment. However, an output port has not yet been made available to measure the probe results. Additionally, a residual gas analyzer (RGA) has been installed to examine the vacuum conditions.

### 6.3 Experimental Results

#### 6.3.1 Frequency Measurements

During the preliminary experiments of the internal mode converter gyrotron, several frequencies were observed with the receiver system. After the tube was properly aligned vertically and horizontally, these frequencies were recorded and their corresponding modes were identified (Table 6.2.) The frequencies were close to the cold cavity frequencies as predicted by electromagnetic theory.

Not surprisingly, since the interaction region and gun dynamics are identical to the axial gyrotron experiment, the frequencies were very close to those measured in the previous experiment (Table 4.2.) The slight frequency discrepancies for the $\text{TE}_{21,6}$
Figure 6-13: Typical scope traces for the applied voltage, the collector current, and a sample voltage pulse from the power-detecting diode. The width of the RF pulse on the diode is about 2.5 $\mu$s.

and TE$_{22,6}$ modes may be easily explained by the lack of window reflections which caused a frequency shift in several modes for the axial gyrotron experiment (Table 4.3.)

### 6.3.2 Mode Map

Following tube alignment and during the frequency measurements, the gun performance and electron beam propagation were verified. The design current, 40 A, was reached and maintained at an applied voltage of 96 kV, with less than 10 mA of body current. An example of the traces can be seen in Fig. 6-13(a) and Fig. 6-13(b). Fig. 6-13(c) shows one type of pulse shape measured by the power-detecting diode for a stable TE$_{22,6}$ mode. The pulse width for this particular pulse was around 2.5 $\mu$s. The
pulse shapes and widths were required for determining the amount of power in the mode.

Next, the operating parameters of the experiment were varied to find the limits of the predominant modes. A mode map (Fig. 6-14) was charted by maintaining a 40 A electron beam at 96 kV, while changing the magnetic field of the superconducting magnet and the gun coil magnetic field value, and observing which modes were excited. As discussed in Chapter 4, the main magnetic field in essence sets the cyclotron frequency, and hence the amount of frequency detuning of a mode. Adjusting the gun coil current, which perturbs the magnetic field at the cathode position, varies the beam alpha and beam radius. If the magnetic field becomes too high, and, as a result, the compression ratio too low, the beam radius increases such that beam interception occurs at the beam tunnel. This effect is easily observed by an increase in the body current. Conversely, if the magnetic field at the cathode is low enough, the beam alpha increases to a point where a large amount of electrons in the beam are reflected. Reflection is observed when the collector current pulse starts to increase.

Figure 6-14: Mode map for the internal mode converter experiment.
and the collector trace exhibits a rippling effect.

The mode map seen in Fig. 6-14 for the internal mode converter experiment has similar characteristics to the mode map for the axial gyrotron experiment (Fig. 4-14.) The limits of the $\text{TE}_{21,6}$, $\text{TE}_{22,6}$, and $\text{TE}_{23,6}$ modes are about the same. However, the $\text{TE}_{19,7}$ mode is absent, and the region of operation for the $\text{TE}_{20,7}$ mode is very small. Both of these modes were in fact observed, but the power in each was relatively small, and the modes themselves were unstable. The reason for this phenomenon is most likely due to the fact that these modes are counter-rotating and not co-rotating. The helical launcher is cut such that only co-rotating modes are efficiently radiated into free space.

### 6.3.3 Power Measurements

After the operating regions of the modes were identified, the power in each mode was estimated. The energy over 1 s was measured by the calorimeter placed in front of the window. At the same time, the pulse width of the radiation was recorded. From these measurements the power in each individual pulse was calculated. The power in the $\text{TE}_{21,6}$, $\text{TE}_{22,6}$, and $\text{TE}_{23,6}$ modes were tracked as the magnetic field was changed.
The results of these power measurements can be seen in Fig. 6-15. The power in each mode exhibited the same behavior as in the axial gyrotron experiment. In each case the power increased as the magnetic field (and mode detuning) was slowly decreased. After the power came to a maximum level the mode disappeared as the gyrotron entered the operating region of another mode. The amount of power in these modes, however, was significantly smaller than exhibited in the previous experiment. The maximum power achieved was 530 kW in the TE_{21,6} mode with an efficiency of 14 %. In the design mode the power was 460 kW, corresponding to an efficiency of 12 %. 300 kW maximum power was measured in the TE_{23,6} mode. Early measurements were taken before a correction in the position of mirror 3 was made along the x direction. Those early results, not shown in Fig. 6-15, indicated the power in the TE_{22,6} mode to be around 680 kW in that configuration.

### 6.3.4 Output Beam Scans

In the next part of the experiment, the power-detecting diode was used to take scans of the microwave beam transmitted through the window. The diode horn was initially positioned horizontally and vertically at the window center, at a location 15 cm after...
Figure 6-17: Comparison of predicted beam intensity patterns on the surface of mirror 2. Figure (a) shows the theoretical beam pattern for the correct alignment of mirrors. The predicted pattern for the incorrect alignment is shown in (b). For this mirror system, the beam is too low. Most of power misses the mirror.

The parameters of the gyrotron were then set such that the TE_{22,6} mode was excited in a stable region of operation. While the diode position was varied in both the horizontal and vertical directions, the attenuator of the diode was adjusted such that the level of the diode pulse shape remained constant. In this manner the shape of the radiated beam at the output window was determined.

Fig. 6-16 shows the results of one typical scan in both the horizontal and vertical directions at the mid-beam location. Although the horizontal profile of the beam was fairly close to a Gaussian beam, the beam in the vertical direction exhibited several significant sidelobes. The scans were performed several times with similar results, which verified the repeatability of the experiment. These results were an indication that one or more of the components of the internal mode converter may have been misaligned. A misalignment is one possible explanation for the low efficiency measured by the calorimeter at the window (Fig. 6-15.)

As a consequence of these results, the design of the internal mode converter was reviewed. It was found that, like mirror 3, which had been corrected earlier, the positions of mirrors 1 and 2 were shifted around the x axis. Instead of being centered with a +x offset, the mirrors were offset in the −x direction. Mirror 4 was not
misaligned since it was centered at \( x = 0 \). This error was most likely the result of translating the coordinate system from the numerical code to the design coordinate system. The offset in the center of the mirrors would have a large effect on the behavior of the beam passing through the mirror system. Most of the beam, in fact, would miss mirror 2 in the \(-x\) direction (Fig. 6-17.) The remaining part of the beam would not fall on mirror 3, or reach the window.

The components of the internal mode converter may not be shifted while in vacuum. To correct the mirror misalignment, it will be necessary to open the tube to physically change the positions of mirror 1 and 2. The correction also requires that the mirror holder must be rebuilt.

### 6.4 Summary and Discussion

The design of the 1.5 MW internal mode converter gyrotron experiment has been presented in this chapter. Since the gun and cavity were the same as in the axial experiment, the design parameters of the basic experiment were the same as well. However several new components and features were added in this experiment. The design of these components were discussed individually: the quasi-optical internal mode converter, including the launcher and four mirrors, the window, and the collector, which may be operated with voltage depression to increase efficiency.

The new tube and components were built and assembled based on these designs. During initial testing of the experiment, the gun was operated successfully at full voltage and current. Several frequencies were observed, and the modes corresponding to the frequencies were identified. The results matched closely with theory, as well as the previous experiment. A mode map was generated which identified the regions of operation for the predominant modes, as well as regions of possible mode competition. The mode map was similar to the mode map with axial extraction, although the presence of the counter-rotating modes was greatly reduced. Power measurements were taken of the \( \text{TE}_{m,6} \) modes using a calorimeter. The preliminary results showed efficiencies significantly lower than theoretical predictions and results measured from
the previous experiment. Finally, the output beam was scanned using a power detecting diode. The scans indicated a beam which was not purely Gaussian. Some sidelobes and beam reflections were evident.

The reason for the low efficiency was most likely due to the fact that the first and second mirrors of the internal mode converter components were found to be not in the proper position. Such a misalignment would drastically affect the output beam and ultimately the power transmitted through the window. As evidenced by the diode scans, the output beam of the launched TE_{22,6} mode is of poor quality and not purely Gaussian. The mirrors will have to be shifted to their correct positions. It also may be possible to externally cold-test the mode converter using a TE_{22,6} mode generator.

Future experiments will be conducted after correcting the alignment of the internal mode converter mirrors. Following this realignment, the goal of obtaining 1.5 MW at 110 GHz should be achievable. In addition, further output beam measurements will be taken, as well as measuring the beam alpha from the alpha probe, and varying the electron beam current and voltage to examine the effects of these operating parameters on mode efficiency. Finally, after the tube has been optimized for highest efficiency, voltage depression will be applied to the collector to investigate the physics issues involved with the gyrotron depressed collector configuration.
Chapter 7

Conclusion

The experimental investigation of a 1.5 MW, 110 GHz gyrotron design was conducted. The design consisted of a 96 kV, 40 A electron gun which was fabricated for the experiments. A significant effort was made to describe the quality of beam emitted from the surface of the gun’s thermionic cathode. This is a critical issue for gyrotron efficiency and mode competition. A theory was developed to describe emission nonuniformity in terms of the work function spread of the cathode. This theory linked the behavior of space-charge limited emission and temperature limited emission by treating the cathode’s work function as a distributed quantity. A formula was derived for the work function with a Gaussian distribution. The work function spread was discovered to contain both local and global effects.

Experiments were performed on the gun to measure the emission nonuniformity. Two cathodes were used during these experiments. For each cathode, current-voltage data were taken to determine the amount of total work function spread by fitting an appropriate curve to the data points. For the old cathode, the total work function spread was 0.04 eV distributed around a central value of 1.76 eV; the work function spread for the new cathode was 0.03 eV centered around 1.88 eV.

A rotatable current probe was installed to directly scan the azimuthal nonuniformities of the emitted beam. From these measurements it was concluded that the nonuniformities were caused by work function spread and not a temperature spread on the cathode surface. Moreover, it was possible to estimate the amount of global
spread, and hence calculate local spread. The global spread of the original cathode was 0.03 eV, from which a local spread of 0.03 eV was calculated. For the second cathode, the global spread was 0.02 eV, with a local spread of 0.03 eV. These results are comparable to other studies performed on other types of cathodes. However, this was the first time that gyrotron cathodes had been fully characterized in terms of contributions from the local and global work function spreads.

Additional data was taken for the second cathode using the current probe. Current-voltage curves were generated for the emitted beam at discrete azimuthal positions. It was observed that there were changes in these calculated work function spreads over the surface of the cathode. This may be explained by localized effects such as variations in the amount of barium surface coverage.

The 96 kV, 40 A gun was installed in the 1.5 MW 110 GHz axial gyrotron experiment. The experiment, running in short (3 $\mu$s) pulses, demonstrated over 1.4 MW of power in the design (TE$_{22,6}$) mode with an efficiency of 37 %. This is a new and higher power regime for gyrotrons used for electron cyclotron resonance heating. A maximum of 1.1 MW of power was also measured in both the TE$_{21,6}$ and TE$_{23,6}$ neighboring modes, corresponding to an efficiency of 30 %. The alpha probe indicated that the beam alpha at the point of highest efficiency was 1.3, which was lower than the design value of 1.4. This could explain why the design goal of 1.5 MW was not reached. Beam nonuniformities are also a possible cause for the reduced efficiency. A major effort is being made to understand the effects of beam nonuniformities on gyrotron efficiency using codes such as MAGY [94] and OMNITRAK.

A mode map was generated to examine the regions of operation for the design mode and neighboring modes. It was found that mode competition could be occurring from the TE$_{19,7}$ mode. Further measurements were made by varying the beam voltage and current of the gun. These results were found to match well with theory and predictions from the MAGY code. The radiation pattern at the window also matched with the predicted pattern for radiation from an open-ended waveguide antenna. Finally, frequency shifting of several modes was observed during operation of the experiment. It was concluded that the shifting was due to reflections at the window.
Ultimately the experimental results form a solid basis for the 1.5 MW industrial tube to proceed.

A novel phase retrieval method was developed to design phase-correctors based on the irradiance moments of a propagating microwave beam. This phase retrieval method used the Fresnel diffraction theory to look at the propagating behavior of the wave's moments instead of its amplitude and phase. The formalism made it possible to solve for an analytical phasefront solution, instead of a numerical solution typical in an iterative phase retrieval approach. This feature would allow for phase correctors to be built with smooth and simple features based on a 2-D polynomial. Furthermore, the irradiance moments technique allows for correction and elimination of measurement errors in the data set.

The irradiance moment phase retrieval approach was used for an ideal Gaussian beam, and then a Gaussian beam with an offset error. By using the irradiance moments, the offset error was easily identified and corrected. In a final test case, actual gyrotron cold-test data was used. The phase retrieval solution from the irradiance moment technique was as accurate as the solution produced by the commonly used iteration method.

The irradiance moment approach has since been used for other aligning components in other experiments, including LHD [133], as well as the design of mirrors in the internal mode converter experiment at MIT.

For the final experiment, a new tube was built which incorporated an internal mode converter. Although the gun and cavity were the same as in the axial gyrotron experiment, several new components were designed and built. These components included the launcher, ellipsoidal mirrors, and phase-correcting mirrors for the internal mode converter, which were designed to produce a free-space Gaussian beam from the TE_{22,6} mode generated in the cavity. The mirrors were built using a combination of the previously developed iteration phase retrieval method and the new irradiance moments approach. The mode converter efficiency was predicted to be close to 99 %.

A fused silica window was also designed and built. The window diameter and thickness were chosen to permit the microwave beam to be fully transmitted through
the window. Finally, a new collector was fabricated. This collector was electrically isolated from the rest of the tube to allow for voltage depression. Calculations showed that the electric fields generated during operation of the depressed collector should be low enough to avoid breakdown.

The internal mode converter experiment produced over 500 kW in the TE_{21,6} mode, 680 kW in the TE_{22,6} mode, and 300 kW in the TE_{23,6} mode. The power was lower than the design value and lower than the power measured for the axial experiment. Scans of the output beam were performed using a power-detecting located near the window. The scans showed that the output beam contained non-Gaussian features, mostly in the vertical direction. These results led to the discovery that mirrors 1 and 2 of the internal mode converter components were not positioned properly. This misalignment will be corrected for future experiments. The goal of demonstrating 1.5 MW of power should be attained following this correction.

A mode map was also generated to identify regions of operation for the design mode and neighboring modes. This map matched well with the mode map for the axial experiment. However, frequency shifting of the modes was not observed in the internal mode converter since the window did not produce any reflections back to the interaction region.

To conclude, this thesis explores the design issues of a 1.5 MW, 110 GHz gyrotron through experimental investigations. The overall results of the experiments are promising for an industrial CW version of the tube. Furthermore, these experiments will help us to better understand the complex issues involved when building future multi-megawatt gyrotrons. It is also hoped that these results will ultimately contribute to the ongoing evolution of the gyrotron.
Appendix A

Derivation of the Current Equation for a Gaussian Work Function Distribution

The mathematical derivation of Equation 3.5 begins with the general equation for the current emission at a particular voltage $V$ for a cathode with a normalized work function distribution $D(\phi)$ [79]:

$$J_V = \int_{\phi_{\text{min}}}^{\phi_V} J_{\text{SCL}} D(\phi) d\phi + \int_{\phi_V}^{\phi_{\text{max}}} J_{\text{TL}} D(\phi) d\phi$$

where $\phi_V$ is defined as:

$$\phi_V = \sqrt{\frac{eE}{4\pi \varepsilon_0}} - \frac{kT}{e} \ln \left( \frac{\kappa V^{3/2}}{A_o T^2} \right).$$

From the Child-Langmuir law (Equation 3.1) describing current emission in the space-charge limited regime, and the Richardson-Dushman equation (Equation 3.2) for the temperature limited regime of operation, the generalized current equation may be expanded to

$$J_V = \kappa V^{3/2} \int_{\phi_{\text{min}}}^{\phi_V} D(\phi) d\phi + A_o T^2 \exp \left[ \frac{e}{kT} \sqrt{\frac{eE}{4\pi \varepsilon_0}} \right] \int_{\phi_V}^{\phi_{\text{max}}} D(\phi) \exp \left( \frac{-e\phi}{kT} \right) d\phi.$$
For a cathode with a Gaussian work function centered at $\phi_o$ with standard deviation $\sigma$, the normalized distribution function is

$$D(\phi) = \sqrt{\frac{1}{2\pi \sigma}} \exp \left[ -\frac{(\phi - \phi_o)^2}{2\sigma^2} \right].$$

So therefore

$$\int_{\phi_{\text{min}}}^{\phi_{\text{max}}} D(\phi) d\phi = 1,$$

where $\phi_{\text{min}}$ is approximated as $-\infty$, and $\phi_{\text{max}}$ is $+\infty$. In actuality, the work functions of a cathode are always positive and finite. However, these are reasonable approximations since the distribution function tapers off rapidly from the central work function value.

The current equation with the Gaussian work function distribution becomes

$$J_V = kV^{3/2} \int_{-\infty}^{\phi_v} \sqrt{\frac{1}{2\pi \sigma}} \exp \left[ -\frac{(\phi - \phi_o)^2}{2\sigma^2} \right] d\phi + A_o T^2 \exp \left[ \frac{eE}{kT} \right] \int_{\phi_v}^{\infty} \sqrt{\frac{1}{2\pi \sigma}} \exp \left[ -\frac{(\phi - \phi_o)^2}{2\sigma^2} \right] \exp \left( \frac{-e\phi}{kT} \right) d\phi.$$

From [132]:

$$\frac{1}{2\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp \left[ -\frac{(t - m)^2}{2\sigma^2} \right] dt = \frac{1}{2} \text{erf} \left( \frac{x - m}{\sqrt{2}\sigma} \right) + \frac{1}{2},$$

which transforms the first integral to

$$kV^{3/2} \int_{-\infty}^{\phi_v} \sqrt{\frac{1}{2\pi \sigma}} \exp \left[ -\frac{(\phi - \phi_o)^2}{2\sigma^2} \right] d\phi = kV^{3/2} \left[ \frac{1}{2} \text{erf} \left( \frac{\phi_v - \phi_o}{\sqrt{2}\sigma} \right) + \frac{1}{2} \right].$$

The second integral may also be rearranged:

$$\int_{\phi_v}^{\infty} \sqrt{\frac{1}{2\pi \sigma}} \exp \left[ -\frac{(\phi - \phi_o)^2}{2\sigma^2} \right] \exp \left( \frac{-e\phi}{kT} \right) d\phi = \frac{1}{\sigma\sqrt{2\pi}} \int_{\phi_v}^{\infty} \exp \left[ \frac{-1}{2\sigma^2} \phi^2 + \left( \frac{\phi_o}{\sigma^2} - \frac{e}{kT} \right) \phi - \frac{\phi_o^2}{2\sigma^2} \right] d\phi.$$
Upon completing the square, the integral becomes

\[
\int = \exp \left( \frac{-e\phi_o}{kT} + \frac{e^2 \sigma^2}{2k^2 T^2} \right) \frac{1}{\sqrt{2\pi}} \int_{\phi_v}^{\infty} \exp \left\{ -\frac{1}{2} \left[ \frac{1}{\sigma} \phi - \left( \frac{\phi_o - e \sigma}{kT} \right) \right]^2 \right\} d\phi.
\]

Let

\[ s = \frac{1}{\sigma} \phi - \left( \frac{\phi_o - e \sigma}{kT} \right) \]

and

\[ ds = \frac{1}{\sigma} d\phi. \]

The integral then becomes

\[
\int = \exp \left( \frac{-e\phi_o}{kT} + \frac{e^2 \sigma^2}{2k^2 T^2} \right) \frac{1}{\sqrt{2\pi}} \int_{\phi_v - (\phi_o - e \sigma/kT)}^{\infty} \exp \left( -\frac{s^2}{2} \right) ds.
\]

Again, from [132]:

\[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp \left[ -\frac{t^2}{2} \right] dt = \frac{1}{2} - \frac{1}{2} \erf \left( \frac{x}{\sqrt{2}} \right).
\]

Using this formula, then the integral is

\[
\int = \exp \left( \frac{-e\phi_o}{kT} + \frac{e^2 \sigma^2}{2k^2 T^2} \right) \left[ \frac{1}{2} - \frac{1}{2} \erf \left( \frac{\phi_v - \phi_o + e \sigma/kT}{\sqrt{2}\sigma} \right) + \erf \left( \frac{\phi_o - e \sigma/kT}{\sqrt{2}\sigma} \right) \right].
\]

Making these substitutions, the total current equation is then

\[
J_V = \frac{kV^{3/2}}{2} \left[ 1 + \erf \left( \frac{\phi_v - \phi_o}{\sqrt{2}\sigma} \right) \right] + \frac{A_o T^2}{2} \left[ 1 - \erf \left( \frac{\phi_v - \phi_o + e \sigma^2/kT}{\sqrt{2}\sigma} \right) \right] \exp \left( \frac{-e}{kT} \left( \phi_o - \sqrt{\frac{eE}{4\pi\varepsilon_o}} - \frac{e \sigma^2}{2kT} \right) \right).
\]

This is the current equation for a cathode with a Gaussian work function distribution, centered at \( \phi_o \), with standard deviation (spread) \( \sigma \).
Appendix B

Derivation of the Moment Equation

In this appendix, the equation for the propagation behavior of the moments of a paraxial beam (Equation 5.3) is derived. The derivation details may also be found in [120] and [121].

First, consider the Fresnel approximation of the Kirchhoff-Huygens diffraction integral [119]:

\[
\psi(x, y, z) = \frac{ik}{2\pi y} \int dx'dz' \left[ \psi(x', 0, z') \exp \left( -i \frac{k [(x' - x)^2 + (z' - z)^2]}{2y} \right) \right].
\]

This general equation describes a paraxial beam at any distance \( y \) in terms of the initial wavefunction.

Next, consider the moments of the wavefunction, which are based on the normalized intensity integrated over the measurement plane at an axial location \( y \):

\[
M_{pq}(y) = \iint x^p z^q A^2_{y}(x, z) dx dz
\]

where the integration is performed over all \( x \) and \( z \). In terms of the wavefunction \( \psi \),
which contains the amplitude $A_y$ and phase, this definition becomes

$$M_{pq}(y) = \iint x^p z^q |\psi(x, y, z)|^2 \, dx \, dz = \iint x^p z^q \left[ \psi(x, y, z) \cdot \psi^*(x, y, z) \right] \, dx \, dz.$$ 

Making the substitution from the Fresnel approximation into the definition of moments, the moments may be written as

$$M_{pq}(y) = \iint dx \, dz \int dx' \, dz' \int dx'' \, dz'' x^p z^q \left( \frac{k^2}{4\pi^2 y^2} \right) \psi(x', 0, z') \cdot \exp\left( -i \frac{k \left[ (x' - x)^2 + (z' - z)^2 \right]}{2y} \right) \psi^*(x'', 0, z'').$$

This may then be rearranged into the following:

$$M_{pq}(y) = \iint dx \, dz \int dx' \, dz' \int dx'' \, dz'' x^p z^q \frac{k^2}{4\pi^2 y^2} \psi(x', 0, z') \psi^*(x'', 0, z'').$$

Rewriting the double integral in $x$ and $z$ using a substitution of variables yields the result:

$$\iint dx \, dz \left( x^p z^q \right) \exp\left( -i \frac{k(x'' - x')^2}{2y} - i \frac{k(z'' - z')^2}{2y} \right)$$

$$= \left( \frac{y}{k} \right)^{p+1} \left( \frac{y}{k} \right)^{q+1} \int d \left( \frac{kx}{y} \right) d \left( \frac{kz}{y} \right) \left( \frac{kx}{y} \right)^p \left( \frac{kz}{y} \right)^q \exp\left( -i \frac{k(x'' - x')^2}{2y} - i \frac{k(z'' - z')^2}{2y} \right).$$

Using the following definition:

$$\delta^{(n)}(x' - x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} (-i\omega)^n \exp[-i\omega (x' - x)] \, d\omega,$$
then performing the double integral gives the result

\[
\int \int = \left( \frac{y}{k} \right)^{p+1} \left( \frac{y}{k} \right)^{q+1} \frac{1}{(-i)^{p} (-i)^{q}} (2\pi)^{2} \delta^{(p)}(x'' - x')\delta^{(q)}(z'' - z').
\]

The moment equation is now:

\[
M_{pq}(y) = \frac{k^{2}}{(2\pi)^{2} y^{2}} \int \int dx'dz' \int \int dx''dz'' \psi(x', 0, z')\psi^*(x'', 0, z'') \cdot \exp \left(-i \frac{k(x'^{2} + z'^{2})}{2y} + i \frac{k(x''^{2} + z''^{2})}{2y} \right) \left( \frac{y}{-ik} \right)^{p} \frac{x}{k} \left( \frac{y}{-ik} \right)^{q} \frac{z}{k}.
\]

(2\pi)^{2} \delta^{(p)}(x'' - x')\delta^{(q)}(z'' - z').

The double integral in \(x''\) and \(z''\) may also be simplified:

\[
\int \int dx''dz'' \psi^*(x'', 0, z'') \exp \left(-i \frac{k(x'^{2} + z'^{2})}{2y} \right) \delta^{(p)}(x'' - x')\delta^{(q)}(z'' - z') = (-1)^{p} (-1)^{q} \frac{\partial^{p+q}}{\partial x^{p} \partial z^{q}} \left[ \psi^*(x', 0, z') \exp \left(-i \frac{k(x'^{2} + z'^{2})}{2y} \right) \right].
\]

So the moment equation becomes

\[
M_{pq}(y) = \int \int dx'dz' \psi(x', 0, z') \exp \left(-i \frac{k(x'^{2} + z'^{2})}{2y} \right) \left( \frac{y}{-ik} \right)^{p} \left( \frac{y}{-ik} \right)^{q} \cdot (-1)^{p+q} \frac{\partial^{p+q}}{\partial x^{p} \partial z^{q}} \left[ \psi^*(x', 0, z') \exp \left(-i \frac{k(x'^{2} + z'^{2})}{2y} \right) \right],
\]

which may be rewritten as

\[
M_{pq}(y) = \left( \frac{-iy}{k} \right)^{p+q} \int \int dx'dz' \psi(x', 0, z') \exp \left(-i \frac{k(x'^{2} + z'^{2})}{2y} \right) \cdot \frac{\partial^{p+q}}{\partial x^{p} \partial z^{q}} \left[ \psi^*(x', 0, z') \exp \left(-i \frac{k(x'^{2} + z'^{2})}{2y} \right) \right].
\]

This is the propagation equation for the moments of a paraxial beam. The moment equation is solely dependent on the propagation distance \(y\) and the initial wavefunction \(\psi(x, 0, z)\), which contains the initial amplitude \(A_{0}(x, z)\) and phase \(\Phi_{0}(x, z)\).

The equation for the linear coefficient \(C^{(1)}_{pq}\) of moment \(M_{pq}\), Equation 5.5, which
measures the slope of the moment at the retrieval plane, is derived from the paraxial approximation. The paraxial equation is stated as:

$$2ik \frac{\partial \psi}{\partial y} = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2},$$

from which two separate equations may be obtained:

$$2ik \frac{\partial \psi^*}{\partial y} = \frac{\partial^2 \psi^*}{\partial x^2} \psi + \frac{\partial^2 \psi^*}{\partial z^2} \psi^*,$$

and

$$-2ik \frac{\partial \psi^*}{\partial y} = \frac{\partial^2 \psi^*}{\partial x^2} \psi + \frac{\partial^2 \psi^*}{\partial z^2} \psi.$$

Subtracting one from the other yields the following result:

$$2ik \frac{\partial |\psi|^2}{\partial y} x^p z^q = \left( \frac{\partial^2 \psi}{\partial x^2} \psi^* - \frac{\partial^2 \psi^*}{\partial x^2} \psi + \frac{\partial^2 \psi}{\partial z^2} \psi^* - \frac{\partial^2 \psi^*}{\partial z^2} \psi \right) x^p z^q.$$

which may be rewritten as

$$2ik \frac{\partial |\psi|^2}{\partial y} x^p z^q = \left( \frac{\partial^2 \psi}{\partial x^2} \psi^* - \frac{\partial^2 \psi^*}{\partial x^2} \psi + \frac{\partial^2 \psi}{\partial z^2} \psi^* - \frac{\partial^2 \psi^*}{\partial z^2} \psi \right) x^p z^q.$$

Now integrating over the measurement plane:

$$2ik \frac{\partial}{\partial y} \int \int x^p z^q |\psi|^2 \, dx \, dz = \int \int dx \, dz \left[ \frac{\partial \psi}{\partial x} \frac{\partial}{\partial x} (\psi^* x^p z^q) + \frac{\partial \psi^*}{\partial x} \frac{\partial}{\partial x} (\psi x^p z^q) - \frac{\partial \psi}{\partial z} \frac{\partial}{\partial z} (\psi^* x^p z^q) + \frac{\partial \psi^*}{\partial z} \frac{\partial}{\partial z} (\psi x^p z^q) \right]$$

Substituting for the wavefunction at the initial plane, $\psi(x, 0, z) = A_0(x, z)e^{i\Phi_0(x, z)}$
and $\psi^*(x, 0, z) = A_0(x, z)e^{-i\Phi_0(x, z)}$, the expression becomes at $y = 0$:

$$2ik \frac{\partial}{\partial y} \iint x^p z^q A_0^2 dx dz = \iint dx dz \left[ -2i \frac{\partial \Phi_0}{\partial x} A_0^2 \frac{\partial}{\partial x} (x^p z^q) - 2i \frac{\partial \Phi_0}{\partial z} A_0^2 \frac{\partial}{\partial z} (x^p z^q) \right].$$

The left hand side contains the definition of moments, $M_{pq}$. Rearranging gives the final result:

$$\frac{\partial}{\partial y} M_{pq} = C_{pq}^{(1)} = -\frac{1}{k} \iint dx dz \left[ \frac{\partial \Phi_0}{\partial x} \frac{\partial}{\partial x} (x^p z^q) + \frac{\partial \Phi_0}{\partial z} \frac{\partial}{\partial z} (x^p z^q) \right] A_0^2.$$

This is the general equation for the slope of moment $M_{pq}$ at the initial plane, or $C_{pq}^{(1)}$. It may also be derived directly from the moment equation.
Appendix C

First- and Second-Order Moments

In this appendix an example is presented of the irradiance moment technique using the first- and second-order moments [118]. After the moments are expanded in terms of the initial phase, the initial phasefront is expanded and substituted into the expressions. Finally, the linear set of equations is obtained after the phasefront is properly truncated.

Deriving explicit expressions for the moments is straightforward. The first- and second-order moments \((p + q) \leq 2\) are as follows (from Equation 5.3):

\[
M_{10}(y) = \langle x \rangle = M_{10}(0) - \frac{y}{k} \iint dx dz \frac{\partial \Phi_0}{\partial x} A_0^2(x, z)
\]

\[
M_{01}(z) = \langle z \rangle = M_{01}(0) - \frac{y}{k} \iint dx dz \frac{\partial \Phi_0}{\partial z} A_0^2(x, z)
\]

\[
M_{20}(y) = \langle x^2 \rangle = M_{20}(0) - 2 \frac{y}{k} \iint dx dz \left[ x \frac{\partial \Phi_0}{\partial x} A_0^2 \right] + \frac{y^2}{k^2} \iint dx dz \left[ \left( \frac{\partial A_0}{\partial x} \right)^2 + A_0^2 \left( \frac{\partial A_0}{\partial x} \right)^2 \right]
\]

\[
M_{11}(y) = \langle x z \rangle = M_{11}(0) - \frac{y}{k} \iint dx dz \left[ x \frac{\partial \Phi_0}{\partial z} A_0^2 + z \frac{\partial \Phi_0}{\partial x} A_0^2 \right] + \frac{y^2}{k^2} \iint dx dz \frac{\partial \Phi_0}{\partial x} \frac{\partial \Phi_0}{\partial z} A_0^2
\]
\[ M_{02}(y) = \langle z^2 \rangle = M_{02}(0) - \frac{2y}{k} \iint dxdz \left[ \frac{\partial \Phi_0}{\partial z} A_0^2 \right] + \frac{y^2}{k^2} \iint dxdz \left[ \left( \frac{\partial A_0}{\partial z} \right)^2 + A_0^2 \left( \frac{\partial \Phi_0}{\partial z} \right)^2 \right]. \]

Note that, by definition, \( M_{00}(0) = 1 \). By generalizing and renaming the coefficients \( C_{pq}^{(m)} \) (Equation 5.4), these five moments are restated as

\[ M_{10}(y) = C_{10}^{(0)} + C_{10}^{(1)} y \]
\[ M_{01}(y) = C_{01}^{(0)} + C_{01}^{(1)} y \]
\[ M_{20}(y) = C_{20}^{(0)} + C_{20}^{(1)} y + C_{20}^{(2)} y^2 \]
\[ M_{11}(y) = C_{11}^{(0)} + C_{11}^{(1)} y + C_{11}^{(2)} y^2 \]
\[ M_{02}(y) = C_{02}^{(0)} + C_{02}^{(1)} y + C_{02}^{(2)} y^2. \]

The moment coefficients required for the irradiance moment algorithm \( C_{pq}^{(m)} \) and \( C_{pq}^{(1)} \) are determined from the data by fitting each moment to its appropriate polynomial.

Continuing our example, the following are expressions for the linear \((m = 1)\) moment coefficients of the moments \( M_{pq} \), where \((p + q) \leq 2\) (also derivable from Equation 5.5):

\[ C_{10}^{(1)} = -\frac{1}{k} \iint dxdz \frac{\partial \Phi_0}{\partial x} A_0^2 \]
\[ C_{01}^{(1)} = -\frac{1}{k} \iint dxdz \frac{\partial \Phi_0}{\partial z} A_0^2 \]
\[ C_{20}^{(1)} = -\frac{2}{k} \iint dxdz \left[ \frac{\partial \Phi_0}{\partial z} A_0^2 \right] \]
\[ C_{11}^{(1)} = -\frac{1}{k} \iint dxdz \left[ x \frac{\partial \Phi_0}{\partial z} A_0^2 + z \frac{\partial \Phi_0}{\partial x} A_0^2 \right] \]
\[ C_{02}^{(1)} = -\frac{2}{k} \iint dxdz \left[ z \frac{\partial \Phi_0}{\partial z} A_0^2 \right]. \]

By substituting the 2-D polynomial expansion for the phase (Equation 5.6), the
linear moment coefficients in our example become

\[
  C_{10}^{(1)} = -\frac{1}{k} \left[ \phi_{10} \int \int dx dz (z^2 A_0^2) + 2 \phi_{20} \int \int dx dz (x A_0^2) + \phi_{11} \int \int dx dz (z A_0^2) + \ldots \right]
\]

\[
  C_{01}^{(1)} = -\frac{1}{k} \left[ \phi_{01} \int \int dx dz (z A_0^2) + \phi_{11} \int \int dx dz (x A_0^2) + 2 \phi_{02} \int \int dx dz (z^2 A_0^2) + \ldots \right]
\]

\[
  C_{20}^{(1)} = -\frac{2}{k} \left[ \phi_{10} \int \int dx dz (z A_0^2) + 2 \phi_{20} \int \int dx dz (x^2 A_0^2) + \phi_{11} \int \int dx dz (xz A_0^2) + \ldots \right]
\]

\[
  C_{11}^{(1)} = -\frac{1}{k} \left[ \phi_{10} \int \int dx dz (z A_0^2) + \phi_{01} \int \int dx dz (x A_0^2) + 2 \phi_{20} \int \int dx dz (x^2 A_0^2) + \phi_{11} \int \int dx dz (xz A_0^2) + \ldots \right]
\]

\[
  C_{02}^{(1)} = -\frac{2}{k} \left[ \phi_{01} \int \int dx dz (z A_0^2) + \phi_{11} \int \int dx dz (xz A_0^2) + 2 \phi_{02} \int \int dx dz (z^2 A_0^2) + \ldots \right].
\]

Recognizing that the integrations in the equations relating \( C_{pq}^{(1)} \) to the initial phase are the moments at the initial plane, the coefficients can be restated as follows:

\[
  C_{10}^{(1)} = -\frac{1}{k} \left[ \phi_{10} + 2 \phi_{20} M_{10}(0) + \phi_{11} M_{01}(0) + \ldots \right]
\]

\[
  C_{01}^{(1)} = -\frac{1}{k} \left[ \phi_{01} + \phi_{11} M_{10}(0) + 2 \phi_{02} M_{01}(0) + \ldots \right]
\]

\[
  C_{20}^{(1)} = -\frac{2}{k} \left[ \phi_{10} M_{10}(0) + 2 \phi_{20} M_{20}(0) + \phi_{11} M_{11}(0) + \ldots \right]
\]

\[
  C_{11}^{(1)} = -\frac{1}{k} \left[ \phi_{10} M_{01}(0) + \phi_{01} M_{10}(0) + 2 \phi_{20} M_{11}(0) \\
  + \phi_{11} (M_{20}(0) + M_{02}(0)) + 2 \phi_{02} M_{11}(0) + \ldots \right]
\]

\[
  C_{02}^{(1)} = -\frac{2}{k} \left[ \phi_{01} M_{01}(0) + \phi_{11} M_{11}(0) + 2 \phi_{02} M_{02}(0) + \ldots \right].
\]

A further simplification may be made by substituting in the coefficients of zero rank in place of the moments at the initial plane, since from Equation 5.4, \( C_{pq}^{(0)} = M_{pq}(0) \). Finally, the phasefront is truncated to the appropriate order, in this case,
\((p + q)_{\text{max}} = (i + j)_{\text{max}} = N = 2\). The set of equations then becomes

\[
C^{(1)}_{10} = -\frac{1}{k} \left[ \phi_{10} + 2\phi_{20}C^{(0)}_{10} + \phi_{11}C^{(0)}_{01} \right]
\]

\[
C^{(1)}_{01} = -\frac{1}{k} \left[ \phi_{01} + \phi_{11}C^{(0)}_{10} + 2\phi_{02}C^{(0)}_{01} \right]
\]

\[
C^{(1)}_{20} = -\frac{2}{k} \left[ \phi_{10}C^{(0)}_{10} + 2\phi_{20}C^{(0)}_{20} + \phi_{11}C^{(0)}_{11} \right]
\]

\[
C^{(1)}_{11} = -\frac{1}{k} \left[ \phi_{10}C^{(0)}_{01} + \phi_{01}C^{(0)}_{10} + 2\phi_{20}C^{(0)}_{11} \right]
\]

\[
+ \phi_{11} \left( C^{(0)}_{20} + C^{(0)}_{02} \right) + 2\phi_{02}C^{(0)}_{11} \right]
\]

\[
C^{(1)}_{02} = -\frac{2}{k} \left[ \phi_{01}C^{(0)}_{01} + \phi_{11}C^{(0)}_{11} + 2\phi_{02}C^{(0)}_{02} \right].
\]

This set of equations is linear and solvable. After the moment coefficients \(C^{(0)}_{pq}\) and \(C^{(1)}_{pq}\) are computed from the data, the phase coefficients \(\phi_{10}, \phi_{01}, \phi_{20}, \phi_{11},\) and \(\phi_{02}\), which describe the initial parabolic phasefront, are then easily obtainable.
Bibliography


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