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EXTERNAL FIELDS IN LEE-WICK THEORY

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1. INTRODUCTION

Quite recently, Lee and Wick ¹⁾ have discussed a class of non-relativistic models with an indefinite metric, showing that one can construct a unitary S matrix of such a theory, provided there are no stable negative states in the model.

More recently, Lee ²⁾ has extended the theory to a class of a relativistic field model with interaction, proving that one can define a unitary S matrix, at least up to second order in the interaction constant.

Field theory with an indefinite metric is not, however, any more strictly local. One has therefore to clarify how the quantities (fields, currents) which are local in the ordinary theory might be described in the new frame. It is indeed obvious that any realistic theory must provide at least some kind of asymptotic description (for "great" space-time regions) of such quantities.

On the other hand, the question of the operative definition of local or pseudolocal quantities in the frame of relativistic quantum field theory is strictly connected with the possibility of introducing in the same frame more or less arbitrary classical (external) fields or currents.

In this connection, it was reported ³⁾ that Pauli and Glaser had proved quite a long time ago that a field theory with an indefinite metric is unstable against the introduction of an arbitrary external field, i.e., that the "bad states", which one has somehow managed to avoid in an unperturbed model, are in general created again by an external arbitrary perturbation.

This difficulty, however, does not appear to be definitely decisive. It is, first of all, obvious that some kind of restrictions on the possibility of introducing an arbitrary classical perturbation must be expected in a theory which is not local, just in connection with the fact that "strictly local quantities" cannot be any more defined.

The interesting point is perhaps to see which are exactly these restrictions and then to see whether they are physically plausible or not. On the other hand, the extension of the ordinary computational rules in the case of external fields and indefinite metric is not so obvious, and unitarity difficulties (as unwanted creation of bad states) might be avoided with a correct procedure. It is the purpose of this paper to introduce the discussion of this question in the frame of the relativistic model suggested by Lee.

We shall begin by considering a classical current, represented by the c numbers $G_\mu(x,t)$. This classical current must be coupled with the non-Hermitian field $\Phi_\mu = A_\mu + iB_\mu$; it cannot be coupled merely with the "ordinary" part of the field (A_μ) because, for instance, difficulties would arise in the definition of the electric charge: the electric charge, as defined by a scattering process, would be different from that defined by the Gauss theorem.

It is then the non-Hermitian part of the interaction $iB_\mu G_\mu(x,t)$ which can give birth to unwanted states. In order to simplify our discussion, we shall not use the (modified) quantum electrodynamics, but the model suggested by Lee in Section 2 of his paper ²⁾.

It has been remarked by T.D. Lee (private discussion) that it is impossible that Pauli's difficulty to the introduction of an indefinite matrix might be very serious, because if a theory satisfies unitarity for all completely quantized systems, it must be so also with an external interaction; an external interaction is indeed an approximate description of an interaction with a quantized system.

This remark is certainly correct (and it will indeed be used especially in the following), but it means that there should exist a method of computation which does not give rise to "unwanted states" even if an external interaction is acting.

The problem is, however, to find this prescription and this is not obvious, because the Lee and Wick theory is, after all, an S matrix theory which cannot be obtained in the usual way as a limiting case of a time depending theory, and on the other hand, an external field depends explicitly on time.

It might therefore seem necessary to generalize all the theory to include the time; in Sections 2 and 3 we shall show, however, that this is not strictly necessary and we shall proceed using merely the S matrix à la Lee and Wick. Only in Section 4 a possible generalization to consider explicitly the time dependence will be introduced. Eventually, in Section 5, we shall discuss the results of our investigation.

2. THE MODEL

We suppose that the field

$$\Phi = A + iB \quad (2.1)$$

is interacting with two "currents" J_1 and J_2 ; we shall call J_1 the "light current" and J_2 the "heavy current".

We suppose there exists a Hamiltonian of the form :

$$H = H_{01} + H_{02} + H_{\Phi} + g \int \Phi J_1 \alpha \vec{x} + g \int \Phi J_2 \alpha \vec{x} \quad (2.2)$$

and we put

$$H_1 = H_{01} + H_{\Phi} + g \int \Phi J_1 \alpha \vec{x} = H_{01} + H_{\Phi} + g V_1 \quad (2.2')$$

H_{01} , H_{02} are the free Hamiltonians of the fields on which J_1 and J_2 are respectively depending. We write :

$$\begin{aligned} \overline{\alpha} &= \text{projection operator on } \alpha) \\ \overline{\alpha} &= \alpha)(\alpha | \eta \end{aligned}$$

$$H_1 = \int E \overline{\alpha}_{E,b} \alpha E \alpha b + \int [W^+ \overline{\alpha}_{\frac{1}{2}W^+,b} + W^- \overline{\alpha}_{\frac{1}{2}W^-,b}] \alpha W \alpha b \quad (2.3)$$

$$\begin{cases} \text{Im } W^+ > 0; \text{ Im } W^- < 0; \quad W = \text{Re } W^\pm \\ (\zeta_{W^\pm, b} | \chi \zeta_{W'^\pm, b'}) = 0; (\zeta_{W^\pm, b} | \chi \zeta_{W'^\mp, b'}) = \delta(W-W') \delta(b-b') \end{cases} \quad (2.3')$$

[b are quantum numbers of a complete set (with the energy) of first integrals of the motion]

$$\begin{cases} H_{02} = \int u \Gamma_{\psi_u} du \\ H_{\Phi} = \int a \Gamma_{g_a} da + \int [W^+ \Gamma_{\chi_{W^+}} + W^- \Gamma_{\chi_{W^-}}] dW \end{cases} \quad (2.4)$$

$$\begin{aligned} H = & \int (E+u) \Gamma_{\mu_{E,u,b}} dE du db + \\ & + \int [(W^+ + u) \Gamma_{\pi_{W^+,u,b}} + (W^- + u) \Gamma_{\pi_{W^-,u,b}}] dW du db \end{aligned} \quad (2.5)$$

and eventually

$$\begin{aligned} \mu_{E,u,b} = & \psi_u \otimes \alpha_{E,b} - \\ - g \int & \psi_{u'} \otimes \alpha_{E',b'} \frac{(\psi_{u'} \otimes \alpha_{E',b'} | \chi V_2 \mu_{E,u,b})}{E' + W' - E - u - i\epsilon} dE' du' db' \\ - g \int & [\psi_{u'} \otimes \zeta_{W^+,b'} \frac{(\psi_{u'} \otimes \zeta_{W^+,b'} | \chi V_2 \mu_{E,u,b})}{W'^+ + u' - E - u} + \\ + & \psi_{u'} \otimes \zeta_{W^-,b'} \frac{(\psi_{u'} \otimes \zeta_{W^-,b'} | \chi V_2 \mu_{E,u,b})}{W'^- + u' - E - u}] dW' du' db' \end{aligned} \quad (2.5')$$

$$\begin{aligned}
 \nu_{E,u,b}) &= \psi_u \otimes \alpha_{E,b}) - \\
 &- g \int \psi_{u'} \otimes \alpha_{E',b'}) \frac{(\psi_{u'} \otimes \alpha_{E',b'}) | \chi V_2 \nu_{E,u,b})}{E' + u' - E - u + i\varepsilon} dE' du' db' \\
 &- g \int [\dots] dw' du' db'
 \end{aligned} \tag{2.5''}$$

$$\begin{aligned}
 \pi_{w^+,u,b}) &= \psi_u \otimes \zeta_{w^+,b}) - \\
 &- g \int \psi_{u'} \otimes \alpha_{E',b'}) \frac{(\psi_{u'} \otimes \alpha_{E',b'}) | \chi V_2 \pi_{w^+,u,b})}{E' + u' - w^+ - u} dE' du' db' \\
 &- g P \int [\dots] dw' du' db'
 \end{aligned} \tag{2.5'''}$$

where

$$\begin{aligned}
 (\psi_{u'} \otimes \alpha_{E',b'}) | \chi V_2 / \mu_{E,u,b}) &= \\
 &= \int (\psi_{u'} \otimes \alpha_{E',b'}) | \chi J_2(\vec{x}) \bar{\phi}(\vec{x}) / \mu_{E,u,b}) d\vec{x}
 \end{aligned} \tag{2.6}$$

and so on, and P means the principal value. (In the following, we shall drop the variables b, b',... for simplicity.)

Let us suppose now that

$$|\operatorname{Im} W^\pm| \geq \bar{\varepsilon} > 0 \quad (2.7)$$

for all states $\sum_{W^\pm}$.

This is of course artificial. A realistic Hamiltonian H_1 will not satisfy such a restriction. In any realistic case one might start with a Hamiltonian H_1 satisfying Eq. (2.7) and then one has to see what happens when $\bar{\varepsilon}$ goes to zero. The point will be discussed in Section 5.

For the moment, we stick to the condition (2.7). Obviously, then Eq. (2.5) represents an in-going state and Eq. (2.5') an out-going state of the total Hamiltonian H .

We require now the transition amplitude a_{E_2, E_1} between the states

$$\psi_1) = \int du c(u) \mu_{E_1, u} \quad (2.8)$$

and

$$\psi_2) = \int du c(u) \nu_{E_2, u} \quad (2.8')$$

with

$$\int c^*(u) c(u) du = 1 \quad (2.8'')$$

We have

$$(2.9)$$

$$a_{E_2, E_1} = \int du du' c^*(u') c(u) (\nu_{E_2, u'} | \eta \mu_{E_1, u})$$

Supposing $E_1 \neq E_2$, we get to first order in g , of course :

$$\begin{aligned} (\psi_{E_2, u'} | \eta \mu_{E_1, u}) &= -g(E_2, u' | \eta V_2 E_1, u) \left[\frac{1}{E_2 + u' - E_1 - u - i\varepsilon} - \right. \\ &\quad \left. - \frac{1}{E_2 + u' - E_1 - u + i\varepsilon} \right] = -2\pi i g(E_2, u' | \eta V_2 E_1, u) \delta(E_2 + u' - E_1 - u) \end{aligned}$$

We have used :

$$(\psi_{u'} \otimes \alpha_{E_2} | \eta V_2 \psi_u \otimes \alpha_{E_1}) \equiv (E_2, u' | \eta V_2 E_1, u) = (E_1, u | \eta V_2 E_2, u')^*$$

and

$$\left(\frac{1}{E_1 + u - E_2 - u' + i\varepsilon} \right)^* = - \frac{1}{E_2 + u' - E_1 - u + i\varepsilon}$$

It follows

$$\begin{aligned} a_{E_2, E_1} &= -g \int du du' C^*(u') C(u) \cdot \int (\psi_{u'} | J_2(\vec{x}) \psi_u) \cdot \\ &\quad \cdot (\alpha_{E_2} | \eta \tilde{\Phi}(\vec{x}) \alpha_{E_1}) d\vec{x} \left(\frac{1}{E_2 + u' - E_1 - u - i\varepsilon} - \frac{1}{E_2 + u' - E_1 - u + i\varepsilon} \right) \end{aligned} \quad (2.9')$$

Notice now that in the states ψ_1) and ψ_2) the "heavy field" remains unchanged [see Eqs. (2.8) and (2.8')]. It is a necessary condition for considering the "heavy field" as a "classical or external source". If we put then :

$$G(\vec{x}, t) = \int du' du C^*(u') C(u) e^{i(u'-u)t} (\psi_{u'} | J_2(\vec{x}) \psi_u) \quad (2.10)$$

$$\begin{cases} g(\vec{x}, t) = \int e^{i\nu t} g(\vec{x}, \nu) d\nu \\ g(\vec{x}, \nu) = \int du c^*(u+\nu) c(u) (\psi_{u+\nu} | \vec{J}(\vec{x}) \psi_u) \end{cases} \quad (2.10')$$

We can write Eq. (2.9') under the following form

$$\begin{aligned} a_{E_2, E_1} = & -ig \left[\int_{-\infty}^0 dt \int d\vec{x} g(\vec{x}, t) e^{i(E_2 - E_1)t + \epsilon t} (\alpha_{E_2} | \vec{J}(\vec{x}) \alpha_{E_1}) + \right. \\ & \left. + \int_0^{\infty} dt \int d\vec{x} g(\vec{x}, t) e^{i(E_2 - E_1)t - \epsilon t} (\alpha_{E_2} | \vec{J}(\vec{x}) \alpha_{E_1}) \right] \end{aligned} \quad (2.9'')$$

Notice that Eq. (2.9'') is equivalent to Eq. (2.9); Eq. (2.9) makes use merely of the S matrix of the "fully quantized" theory. This S matrix, following Lee and Wick, is unitary. In Eq. (2.9'') the unitarity is therefore preserved [up to $O(g^2)$ of course, but the generalization to any order is easy, as will be shown in the next section].

What about "ghost creation" ? If we put, in analogy with Eq. (2.9), for instance

$$a_{w^+, E_1} = \int du' du c^*(u') c(u) (\pi_{w^+, u'} | \mu_{E_1, u}) \quad (2.11)$$

we get $a_{w^+, E_1} = 0$ because $(\pi_{w^+, u'} | \mu_{E_1, u}) = 0$ for any u and u' .

On the other hand, we can introduce $G(x, t)$ [Eq. (2.10)], and following exactly the same procedure as used for getting Eq. (2.9''), we have, up to $O(g^2)$:

$$a_{w^+, E_1} = -g \left\{ i \int_{-\infty}^{\infty} dt \int d\vec{x} g(\vec{x}, t) e^{i(w-E_1)t} (\vec{z}_{w^+} | \gamma \bar{\Phi}(\vec{x}) \alpha_{E_1}) + \right. \\ \left. + \left(i \int_0^{\infty} dt \int d\vec{x} \tilde{g}(\vec{x}, t) e^{i(w^+-E_1)t} (\alpha_{E_1} | \gamma \bar{\Phi}(\vec{x}) \vec{z}_{w^+}) \right)^* \right\} \quad (2.11')$$

where

$$\tilde{g}(\vec{x}, t) = \int g(\vec{x}, \nu) e^{-i\nu t} d\nu \quad (2.10'')$$

The second member of Eq. (2.11') is of course identically equal to zero for all $G(\vec{x}, t)$ which has a Fourier transform in t . We believe that the preceding case can show which is the general procedure for getting correct computational rules for transition probabilities with external field and an indefinite metric.

The physical meaning of this procedure is clear enough : the interaction of the "heavy field" with Φ produces a "dressing" of bare "ghosts" with these fields (and vice versa). Our computational procedure automatically keeps into account the effects of this dressing, even when the "heavy fields" are not introduced explicitly, but are merely represented by a source function $G(x, t)$.

3. COMPUTATIONAL RULES FOR TRANSITION PROBABILITY. GENERALIZATION

In order to generalize to higher order in g , we have to put some restrictions on the functions $c(u)$ and on the matrix elements $(\psi_u | J_2(x) \psi_u) = (u | J_2 u)$. These restrictions are, by no means, required by the indefinite metric, but by the fact that we want to treat the "heavy fields" like an "external field".

These conditions which we are going to impose on $c(u)$ and on $(u' | J_2 u)$ are not necessary but merely sufficient conditions. Let us put

$$(u' | J_2 u) = f(u'-u, u) \quad (3.1)$$

We ask that :

- i) there exists a Δu such that $f(u'-u, u)$ can be considered equal to zero (with negligible error in the final results of our computations) for all $|u'-u| > \Delta u$;
- ii) we can write $c(u)$ instead of $c(u')$ (with negligible error...) for all $|u'-u| < \Delta u$;
- iii) there exists a u_0 and a $\Delta \bar{u} \gg \Delta u$ such that if $|u-u_0| \geq \Delta \bar{u}$ we can write $c(u) = 0$ (with negligible error...);
- iv) if $|u-u_0| < \Delta \bar{u}$ we can write

$$(u' | J_2 u) \simeq f(u'-u, u_0) = g(u'-u) \quad (3.1')$$

(with negligible error...).

Let us put :

$$\begin{aligned} \mu_{E,u}^{(n)} &= \psi_u \otimes \alpha_E - \\ &- g \int \psi_{u'} \otimes \alpha_{E'} \frac{(\psi_{u'} \otimes \alpha_{E'} | \mathcal{V}_2 \mu_{E,u}^{(n-1)})}{E' + u' - E - u - i\epsilon} dE' du' \quad (3.2) \\ &- g \int \left[\psi_{u'} \otimes \beta_{u'} \frac{(\psi_{u'} \otimes \beta_{u'} | \mathcal{V}_2 \mu_{E,u}^{(n-1)})}{u' + u' - E - u} + \dots \right] du' \end{aligned}$$

and so on.

Let us put further

$$a_{\bar{E}_2, \bar{E}_1} = a_{\bar{E}_2, \bar{E}_1}^{(c)} + a_{\bar{E}_2, \bar{E}_1}^{(i)} + a_{\bar{E}_2, \bar{E}_1}^{(z)} + \dots \quad (3.3)$$

where

$$a_{\bar{E}_2, \bar{E}_1}^{(i)} \propto g \dots \quad a_{\bar{E}_2, \bar{E}_1}^{(u)} \propto g^u \dots$$

and

$$\begin{cases} a_{\bar{E}_2, \bar{E}_1}^I = \int c^*(u_2) (\psi_{\bar{E}_2, u_2}^{(u)} | \eta \mu_{\bar{E}_1, u_1}^{(u)}) c(u_1) du_1 du_2 \\ a_{\bar{E}_2, \bar{E}_1}^{II} = \int c^*(u_2) (\psi_{\bar{E}_2, u_2}^{(c)} | \eta \mu_{\bar{E}_1, u_1}^{(z)}) c(u_1) du_1 du_2 \\ a_{\bar{E}_2, \bar{E}_1}^{III} = \int c^*(u_2) (\psi_{\bar{E}_2, u_2}^{(z)} | \eta \mu_{\bar{E}_1, u_1}^{(c)}) c(u_1) du_1 du_2 \end{cases} \quad (3.4)$$

$$a_{\bar{E}_2, \bar{E}_1}^I = a_{\bar{E}_2, \bar{E}_1}^{(c, I)} + a_{\bar{E}_2, \bar{E}_1}^{(i, I)} + a_{\bar{E}_2, \bar{E}_1}^{(z, I)} \quad (3.4')$$

we have obviously

$$a_{\bar{E}_2, \bar{E}_1}^{(z)} = a_{\bar{E}_2, \bar{E}_1}^{(z, I)} + a_{\bar{E}_2, \bar{E}_1}^{(z, II)} + a_{\bar{E}_2, \bar{E}_1}^{(z, III)} \quad (3.5)$$

Now, for instance

$$\begin{aligned}
 a_{\bar{E}_2, \bar{E}_1}^{(2, I)} &= \\
 &= \int C(u_2) \chi(u_1) du_2 du_1 \cdot g^2 \left\{ \int d\bar{E} d\mu \left(\frac{(E, \mu | \gamma V_2 \bar{E}_2, u_2)}{E + \mu - E_2 - u_2 + i\varepsilon} \right)^* \frac{(E, \mu | \gamma V_2 E_1, u_1)}{E + \mu - E_1 - u_1 - i\varepsilon} \right. \\
 &+ \int dW d\mu \left[\left(\frac{(W, \mu | \gamma V_2 E_2, u_2)}{W + \mu - E_2 - u_2} \right)^* \frac{(W, \mu | \gamma V_2 E_1, u_1)}{W + \mu - E_1 - u_1} + \right. \\
 &\left. \left. + \left(\frac{(W, \mu | \gamma V_2 \bar{E}_2, u_2)}{W + \mu - E_2 - u_2} \right)^* \frac{(W, \mu | \gamma V_2 \bar{E}_1, u_1)}{W + \mu - E_1 - u_1} \right] \right\} \quad (3.6)
 \end{aligned}$$

where

$$\begin{aligned}
 (E, \mu | \gamma V_2 E_2, u_2) &= \int d\vec{x} (\mu | J_2(\vec{x}) u_2) (\alpha_E | \gamma \Phi(\vec{x}) \alpha_{E_2}) \\
 (W, \mu | \gamma V_2 E_2, u_2) &= \int d\vec{x} (\mu | J_2(\vec{x}) u_2) (\xi_{W-} | \gamma \Phi(\vec{x}) \alpha_{E_2})
 \end{aligned}$$

and so on.

Let us compute, for instance, the second term of the second member of Eq. (3.6). We have :

$$\begin{aligned}
 & \int C^*(u_2) C(u_1) du_2 du_1 dW du \left(\frac{(W^- u | \gamma \bar{V}_2 \bar{E}_2 u_2)}{W^+ u - E_2 - u_2} \right)^* \frac{(W^+ u | \gamma \bar{V}_2 \bar{E}_1 u_1)}{W^- + u - E_1 - u_1} = \\
 & = \int d\vec{x} d\vec{x}' \int dW \left(\frac{\vec{z}_W^- | \gamma \bar{\Phi}(\vec{x}) \alpha_{E_2}}{W^+ u - E_2 - u_2} \right)^* \left(\frac{\vec{z}_W^+ | \gamma \bar{\Phi}(\vec{x}') \alpha_{E_1}}{W^- + u - E_1 - u_1} \right) \cdot \\
 & \cdot \int dv_2 dv_1 \frac{g^*(v_2) g(v_1)}{(W^+ v_2 - E_2)^* (W^- v_1 - E_1)} \int C^*(u - v_2) C(u - v_1) du
 \end{aligned}$$

where we have performed the change of variables $u, u_1, u_2 \rightarrow u, v_1, v_2$; $v_1 = u - u_1, v_2 = u - u_2$ and we have used Eq. (3.1').

But using now condition ii) and Eq. (2.8''), we can put

$$\int C^*(u - v_2) C(u - v_1) du \simeq 1$$

and eventually we get

$$\begin{aligned}
 & \int C(u_1) C^*(u_2) du_1 du_2 dW du \left(\frac{(W^- u | \gamma \bar{V}_2 \bar{E}_2 u_2)}{W^+ u - E_2 - u_2} \right)^* \frac{(W^+ u | \gamma \bar{V}_2 \bar{E}_1 u_1)}{W^- + u - E_1 - u_1} = \quad (3.7) \\
 & = \int d\vec{x} d\vec{x}' dv_1 dv_2 dW \left(\frac{\vec{z}_W^- | \gamma \bar{\Phi}(\vec{x}) \alpha_{E_2}}{W^+ v_2 - E_2} \right)^* \frac{\vec{z}_W^+ | \gamma \bar{\Phi}(\vec{x}') \alpha_{E_1}}{W^- + v_1 - E_1} (g(v_2))^* g(v_1)
 \end{aligned}$$

On the other hand, using the same approximation, we have :

$$\begin{aligned}
 & \int_0^\infty dt g(x, t) \exp(i(W^+ - E_2)t) = \\
 & = \int_0^\infty dt \int du_1 du_2 C(u_1) C^*(u_2) \exp(i(u_2 - u_1)t + i(W^+ - E_2)t) (u_1 | \bar{J}_2 u_2)
 \end{aligned}$$

$$\begin{aligned}
 &= \int_0^\infty dt \int du_1 \int dv_2 C^*(u_1+v_2) C(u_1) \exp(i(W^+v_2-E_2)t) (u_1+v_2 | J_2 u_1) \approx \\
 &\approx \int du_1 C^*(u_1) C(u_1) \int dv_2 g(v_2) \int_0^\infty dt \exp(i(W^+v_2-E_2)t) = \quad (3.8) \\
 &= - \int dv_2 \frac{g(v_2)}{i(W^+v_2-E_2)}
 \end{aligned}$$

In the same way :

$$\int_{-\infty}^0 g(\vec{x}, t) e^{i(W^-E_1)t} dt = \int \frac{g(v_1)}{i(W^-v_1-E_1)} dv_1$$

It follows that the second member of Eq. (3.7) can also be written as :

$$\begin{aligned}
 &- \left(i \int_0^\infty dt \int d\vec{x} \left(\sum_{W^-} | \chi \Phi(\vec{x}) \alpha_{E_2} \right) g(\vec{x}, t) \exp(i(W^-E_2)t) \right)^* \\
 &\cdot \left(i \int_{-\infty}^0 dt' \int d\vec{x}' \left(\sum_{W^+} | \chi \Phi(\vec{x}') \alpha_{E_1} \right) g(\vec{x}', t') \exp(i(W^+E_1)t') \right)
 \end{aligned}$$

In this way we can evaluate correctly [with approximations allowed by conditions i)-iv)] any transition probability with an external source $G(\vec{x}, t)$. At this point the procedure for generalizing the results of Section 2 to any order should indeed be obvious.

4. CAUSALITY AND LIMITING CONDITIONS ON THE EXTERNAL SOURCE

In a conventional theory, the localization in space time is rigorously introduced through the measure of local quantities, but this implies the use of external sources. We cannot therefore follow from the beginning this procedure in our case without running into a circular argumentation.

We cannot either make a critical use of "free wave packets"; such packets do not even belong to the "good" Hilbert subspace, and it appears essential to keep into account the "ghost dressing" of the physical particle.

The only way out is to postulate a priori a "localization criterion" which might seem reasonable, and to control then the consistency of the criterion by explicit computation. For this purpose we shall again use the model of Section 2, with some convenient specification.

In this model we have two kinds of systems : the light current and the heavy current interacting through the field $\Phi = A+iB$ only. We suppose we know how to solve the Schrödinger problem for the following two cases :

- 1) no heavy current particles present at $t=-\infty$;
- 2) no light current particles present at $t=-\infty$.

Let us put :

$$H_2 = H_{02} + H_\Phi + g \bar{V}_2 \quad (4.1)$$

$$\left\{ \begin{array}{l} H_2 = \int u \bar{f}_u du + \int [z^+ \bar{f}_{z^+} + z^- \bar{f}_{z^-}] dz \\ (f_{z^+} | g f_{z^-}) = \delta(z - z') \end{array} \right. \quad (4.1')$$

We remark now that we have implicitly supposed that all our states belong to a Fock space. Supposing we stop at a finite order of the perturbation expansion, they will be obtained by the application to the vacuum of certain operators which are linear combinations of finite products of creation operators which belong to the fields of the models.

We can also write therefore, e.g.,

$$\gamma_u) = O_{\gamma_u} 0) ; \quad \alpha_E) = O_{\alpha_E} 0) \quad (4.2)$$

and so on, where $0)$ is the vacuum and $O_{\gamma_u}, O_{\alpha_E}$ are the operators which we have mentioned.

Let us introduce the following notation :

$$\gamma_u) \otimes_{op} \alpha_E) = O_{\gamma_u} O_{\alpha_E} 0)$$

Notice that $\otimes_{op}$ is not merely the external product because a creation operator of the field $\bar{\phi}$ might be present both in O_{γ_u} and O_{α_E} . We build now the state :

$$\phi(t) = \int c(u) e^{-iut} du \gamma_u) \otimes_{op} \int a(E) \alpha_E) e^{-iEt} dE \quad (4.3)$$

$c(u)$ is defined in the interval $\bar{u} - \omega$. The analytical prolongation of $c(u)$ in the domain $\text{Re } u \geq \bar{u}$, $|\text{Im } u| \leq \Delta u$ is supposed to be limited. A similar hypothesis holds for $a(E)$. The state defined by Eq. (4.3) will not belong either to the "good" Hilbert subspace because the interaction between the light and heavy currents is not taken into account. Light and heavy currents are, however, already separately "dressed".

Let

$$\begin{aligned} \gamma_u) = & \psi_u) - g \int du' da' \psi_{u'} \otimes g_{a'} \frac{(\psi_{u'} \otimes g_{a'} | \mathcal{V}_2 \gamma_u)}{u' + a' - u - i\varepsilon} - \\ & - g \int du' dw [\dots] \end{aligned} \quad (4.4)$$

and

$$\psi(t) = \int du c(u) dE a(E) \exp(-i(E+u)t) \mu_{E,u} \quad (4.5)$$

$\psi(t)$ is a solution of the Schrödinger problem with the Hamiltonian (2.2). Let us consider

$$N^2(t) = \int du' dE' |(\mu_{E',u'} | \chi(\phi(E) - \psi(t)))|^2 + \quad (4.6)$$

$$+ \int du' dW \{ |\pi_{W,u'} | \chi(\phi(E))|^2 + |\pi_{W,u'} | \chi(\phi(t))|^2 \}$$

Keeping into account the properties of $c(u)$ and $a(E)$, it is not difficult to verify that

$$\lim_{t \rightarrow -\infty} N^2(t) = 0 \quad (4.6')$$

It is illuminating to compute $N^2(t)$ as a function of t : the idea is that if $c(u)$ and $a(E)$ are suitably chosen the light and heavy currents might be approximately "localized" in such a way that for instance they interact appreciably only after a certain time t_1 . This is the control of our device for approximate localization through "dressed packets" like

$$\int c(u) e^{-iut} du \gamma_u$$

and so on.

In order to perform this investigation, we have used a certain specification of the model introduced in Section 2, namely :

$$\bar{J}_1(x) = C(x) D^\dagger(x) + D(x) C^\dagger(x) \quad (4.7)$$

where

$$C(x) = C^-(x) + C^+(x)$$

and $C^-(x)$ and $C^+(x)$ are, respectively, creation and annihilation operators. The quanta of the fields C and D have masses M_C, M_D .

$$\left\{ \begin{array}{l} J_2(x) = N^-(x) L^-(x) L^+(x) N^+(x) \\ [L^\pm(x), N^\pm(x')]_- = [C^\pm(x), D^\pm(x')]_- = \dots = 0 \end{array} \right. \quad (4.7')$$

with masses M_L and M_N . For simplicity, we shall limit ourselves to a one-dimensional case only up to second order in g included.

We suppose that initially (for $t \rightarrow -\infty$) we have one L particle and one N particle [packet $c(u)$] and one C particle only [packet $a(E)$].

The graphs which have then to be considered are of the following type :

Graphs I, II and III are common to $\phi(t)$ and $\psi(t)$. Graph IV belongs to $\psi(t)$ only [interaction of $J_1(x)$ and $J_2(x)$ through ϕ]. It follows that essentially only IV has to be taken into account in order to evaluate $N^2(t)$.

Now, we have two contributions to IV : one coming from A and the other coming from iB. The first one is "causal" : not exactly, of course, because the number of quanta is fixed. The approximation can be however as good as we like if the masses of the "current quanta" are very large. $c(u)$ for instance might be chosen in such a way that the L and M particle packets are practically superposed only for a finite period of time, say between t_1 and t_2 . Only between t_1 and t_2 $J_2(x)$ can then effectively interact with $\phi(\vec{x})$. It is this way that the "causality" of the interaction through A(x) can be verified.

We are, however, interested in the evaluation of the "acausal" contribution to IV due to iB. The computation can obviously be performed using $G(\vec{x}, t)$ [Eq. (2.10)]. We are going to give the results. If we suppose that the superposition of L and N takes place around $t=0$ and at a distance R from the C packet and if we put $\tau = -t = |t|$, we can distinguish three cases :

I	$\tau \gg R$
II	$\tau \ll R$
III	$\tau = R$

If

$$\omega = \sqrt{M^2 + i\gamma M + \frac{k^2}{2}} = M \sqrt{1 + i\alpha + (\frac{k}{M})^2} \quad (4.8)$$

is the energy of a quantum of the B field, if

$$\Delta M = M_D - M_C \geq M > 0 \quad (4.9)$$

and if $a_{B,IV}$ is the amplitude which we have to evaluate, we then have

$$|a_{B,IV}| \leq O\left(\exp\left(-\frac{\Delta M \tau}{2}\right)\right) \quad (4.10)$$

in the case $I^{\wedge}$ and

$$|a_{B,IV}| \leq O\left(\exp\left(-\frac{\Delta M^2 R}{2(\Delta M - M)}\right)\right) \quad (4.10')$$

in case $II^{\wedge}$.

The most interesting case is the third one. Let :

$$\xi = \frac{k}{M} ; \quad \Delta\mu = \frac{\Delta M}{M}$$

where k is the momentum of the exchanged particle.

We have then

$$|a_{B,IV}| \sim \left| \int_{-\infty}^{+\infty} d\xi F(\xi) \frac{\exp(i\tau[\sqrt{1+i\xi+\xi^2} - \xi])}{\sqrt{1+i\xi+\xi^2} - \Delta\mu} \right| \quad (4.12)$$

where $F(\xi)$ is proportional to the Fourier transform of $G(\vec{x}, t)$. The value of the second member of Eq. (4.12) depends essentially on the asymptotic behaviour of $F(\xi)$. If $F(\xi) \sim 1/\xi^n$, it is $O(1/\tau^n)$.

The physical interpretation is obvious : if the amplitude for B quanta exchange between J_1 and the source $G(\vec{x}, t)$ is not decreasing with a sufficient rapidity with the momentum of the exchanged virtual particle, one might observe even macroscopic violations of causality. This of course limits the possibility of assuming arbitrary the source function $G(\vec{x}, t)$; the point will be discussed in the next section.

5. DISCUSSION OF THE RESULTS

We have shown that an external field can be introduced in a quantum model with indefinite metric of the type of Lee and Wick without any trouble for the unitarity, provided one uses a certain computational rule following a procedure which we have sketched in Sections 2 and 3. We have not given a direct formal proof that the unitarity is saved with our procedure. We have, however, proved that the results of the computation are equal inside any wanted approximation to those obtained with suitably chosen entirely quantized systems.

The physical meaning of our computational procedure is particularly clear in Born approximation; it amounts to keep into account the "bare ghost dressing" due to the classical source. In this sense, it is similar (physically, not mathematically) to a charge renormalization for an external field.

In Section 4 we have made an attempt to introduce the discussion about the local observable. We have seen that by using the device of the "dressed wave packets" a reasonable criterion of localization might be introduced by isolating the interaction effect between differently localized currents.

This way, it is implicitly assumed that an acausal effect limits the possibility of real localization. This is after all consistent with the conventional theory of the measure of the electromagnetic field and currents : meaningful measurements are considered indeed only those in which the time dimension of the space time domain of measure is small compared to the space dimension (in the theory à la Bohr and Rosenfeld ⁴⁾) or small compared to the period of the relevant Fourier component (in the extension given by the author ⁵⁾).

It is a feature of our results that "acausality" depends not only on the function $G(\vec{x}, t)$ but also on the state of the light current system interacting with the external source through the field Φ . Essentially in the frame in which the average momentum of the "light system" is zero, any variation of $G(\vec{x}, t)$ on space or time distances of the order of the Compton wavelength or period of the "heavy photon" is physically meaningless.

The possibility of giving a physical meaning to the source functions $G(x,t)$ depends on the state of the system with which the source does interact. Comparison with what happens, even in the ordinary case, when the nature of the physical source for the "external" field is specified, shows, however, that the limitations which we have discussed are not physically unreasonable.

The only general consequence is that the freedom which is supposed to be valid in a local theory is not so in the present case. But this had to be expected.

The preceding conclusions are not qualitatively changed even passing to the limit $\bar{\epsilon} = 0$. If one, for instance, computes a_{E_2, E_1} or more exactly $a_{E_2 b_2, E_1 b_1}(\bar{\epsilon})$ (cf., Sections 2 and 3) for two "artificial" Hamiltonians $H_1(\bar{\epsilon}_1)$ and $H_2(\bar{\epsilon}_2)$ with $\bar{\epsilon}_1 > \bar{\epsilon}_2 > 0$ the limit of $a(\bar{\epsilon}_1) - a(\bar{\epsilon}_2)$ will be zero when $\bar{\epsilon}_1$ goes to zero. This is due to the fact that the "phase space" (resulting from the integration with respect to the variables b) goes generally to zero when $\bar{\epsilon}_2 \leq |\text{Im } W^\pm| \leq \bar{\epsilon}_1$ and $\bar{\epsilon}_1$ goes to zero.

Another question is whether the predictions of the theory are quantitatively compatible with experiment. Particularly with respect to causality, and taking into account the lack of convergence of the perturbative series, one can think that the question is open to doubt; it appears very difficult to overcome such a doubt with the techniques available actually.

On the other hand, the violation of causality which might arise in a theory à la Lee and Wick, if computed with the perturbation theory appears to be deceptively small; one can conclude therefore that there is not, up to now, any definite reason to discard this type of theory with an indefinite metric.

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