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CP, T AND CPT INVARIANCE IN NEUTRAL KAON DECAYS

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A B S T R A C T

A brief survey of the phenomenology (with emphasis on its present status) of CP, T and CPT invariance in neutral kaon decays is given. Existing limits on these invariances of the mass matrix of the $K^0 - \bar{K}^0$ system are stated. Also included are our new results and also those of Dass and Kabir for limits on these invariances for the decay amplitudes, obtained by using existing data, and the available information on the total mass matrix.

The CPT invariant part of amplitudes for decay to the $I = 0$ and $I = 2$, $\pi\pi$ channels is T (and CP) invariant, in the Eberhard phase convention, to about 0.4% and about 4% respectively; the corresponding T invariant part of these amplitudes is CPT (and CP) invariant to about 1% and about 5% respectively. A limit similar to this last one (5%) was already obtained in the literature by making a further approximation. The CP, T and CPT invariant part of the averaged $\Delta S = -\Delta Q$ amplitude in $K_{\ell 3}$ decay is found to be less than about 5% of the corresponding $\Delta S = \Delta Q$ amplitude; the T invariant part of the difference of the averaged $\Delta S = \Delta Q$ and the averaged $\Delta S = -\Delta Q$ amplitudes is CPT (and CP) invariant to about 1%. A limit similar to this one (1%) was also obtained in the literature by making a further assumption about the mass matrix.

These amplitude limits are expected to have about one significant figure accuracy. Due to large experimental errors, the amplitude limits on the other non-invariance parameters for the above channels are not very good. Difficulties in deriving corresponding parameters for the 3π channel are noted.

In certain respects, CPT invariance has been verified experimentally to a poorer accuracy than CP invariance and T invariance.

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1. - INTRODUCTION

The discovery of CP non-invariance in neutral kaon decays has led to extensive investigations on the $K^0 - \bar{K}^0$ complex. The possibility of T and of CPT non-invariance has also been considered. Specific models have been constructed to explain the observed CP non-invariance. Without resorting to specific models, one can describe possible non-invariance under these operations on a phenomenological level. The present paper is devoted to the phenomenology of CP, T and CPT invariance in neutral kaon decays. Since good reviews [e.g., 1-10] on the general phenomenology are available, our emphasis is mainly on the present status of these invariances in neutral kaon decays - in particular, on the accuracy to which these invariances are good. We shall give, as far as data allow, numerical bounds on the relative strength of the parameters which would vanish if these invariances were good. Different parameters of this type would give information on the symmetry properties of different interactions contributing to neutral kaon decays. Such information is obviously useful in understanding the source of the observed CP non-invariance.

On a phenomenological level, the two sources of the non-invariances are the total mass-matrix Λ and the decay amplitudes. There seems to be rather good information on some of the parameters (see Section 4 below) expressing invariances of the total mass matrix, but not so for the amplitudes. The study of the individual amplitudes is important because invariance of the total mass matrix under a certain symmetry operation does not necessarily imply a corresponding invariance of all the interactions contributing to kaon decays. This is because the total mass matrix involves only sums over all physical (and similarly virtual) transitions simultaneously and can, therefore, be sensitive to invariance properties of primarily the interactions responsible for the decay modes which dominate these sums.

In fact, there are already indications [11-15] that the mass matrix is not T invariant, though it is consistent with CPT invariance. This, of course, means that the off-diagonal elements of the total mass matrix in the $K^0 - \bar{K}^0$ representation are unequal, or T (and CP) non-invariant ; but the diagonal ones are consistent with CPT (and CP) invariance (see Section 3 for details). It would help in a phenomenological understanding of the origin of the observed CP non-invariance if one had similar information on the decay amplitudes. We therefore use data and the available information on the mass matrix to deduce limits on parameters which would vanish if the decay amplitudes for different channels were invariant under the CP or T or CPT transformation ; some of the results are from Ref. [16].

The general phenomenology is briefly surveyed ¹⁾ in Section 2 and the requirements which these symmetry transformations impose on the mass matrix and on the decay amplitudes are discussed in Section 3, which also includes a comparison of our phase conventions with the conventional superweak model. The existing information on the limits on the CP, T and CPT invariances of the mass matrix are considered in Section 4.A

and our new results on the corresponding limits for the various decay amplitudes are given in Section 4.B, along with the results ²⁾ of Ref. [16]. The last two Sections are devoted to some discussion and concluding remarks.

2. - STRUCTURE OF THE NEUTRAL KAON SYSTEM : MASS MATRIX AND OTHER NOTATION

The strong interaction eigenstates $|K^0\rangle$ and $|\bar{K}^0\rangle$ do not decay exponentially. We shall take ³⁾ the exponentially decaying eigenstates $|K_S\rangle$ and $|K_L\rangle$:

$$|K_{S,L}\rangle \rightarrow e^{-i\lambda_{S,L}t} |K_{S,L}\rangle, \quad (2.1)$$

$$\lambda_{S,L} = m_{S,L} - \frac{i}{2} \gamma_{S,L} \quad (2.2)$$

to be linear superpositions of the states $|K^0\rangle$ and $|\bar{K}^0\rangle$:

$$\begin{aligned} |K_S\rangle &= p_S |K^0\rangle + q_S |\bar{K}^0\rangle \\ |K_L\rangle &= p_L |K^0\rangle - q_L |\bar{K}^0\rangle, \end{aligned} \quad (2.3)$$

where the complex parameters p_S, q_S, p_L and q_L determine the structure of the exponentially decaying states in terms of the strong interaction eigenstates (and vice versa). Here, $m_{S,L}$ and $\gamma_{S,L}$ are the mass and the decay width of the short-lived (subscript S) and the long-lived (subscript L) kaon and the time t is measured in the rest system of the decaying meson. The normalization conditions

$$|p_S|^2 + |q_S|^2 = |p_L|^2 + |q_L|^2 = 1 \quad (2.4)$$

reduce the number of structure parameters to six, out of which only three are significant because of the arbitrariness in the choice of three phases : the $K^0/\bar{K}^0$ relative phase and the phases of the states $|K_S\rangle$ and $|K_L\rangle$. The structure parameters and the complex quantities $\lambda_{S,L}$ completely determine the total mass matrix Λ which describes the time-evolution of an arbitrary neutral kaon state. For a state prepared initially as $a|K^0\rangle + b|\bar{K}^0\rangle$ where a and b are any complex numbers, the time-development of the wave function

$$\Psi = \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.5)$$

is given by

$$i \frac{d}{dt} \Psi = \Lambda \Psi, \quad (2.6)$$

$$\Lambda = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{pmatrix} \quad (2.7)$$

which defines the total mass matrix Λ in the $K^0 - \bar{K}^0$ representation. The structure (or equivalently, the mass matrix Λ) is determined by not only physical (energy conserving) decays, but also by virtual energy non-conserving decays and by "self-transitions" $K^0 \rightarrow K^0$, $\bar{K}^0 \rightarrow \bar{K}^0$ and $\bar{K}^0 \rightarrow K^0$. The total mass matrix can be decomposed [1,8] into a Hermitian and an anti-Hermitian part :

$$\Lambda = M - i \Gamma \quad (2.8)$$

where M and Γ are separately Hermitian. Unitarity determines the decay matrix Γ in terms of suitable sums over all physical decay amplitudes from $|K^0\rangle$ and $|\bar{K}^0\rangle$ to various possible final states, while M is similarly determined by energy non-conserving transitions from $|K^0\rangle$ and $|\bar{K}^0\rangle$, and by the matrix elements $\langle K^0 | \mathcal{H} | K^0 \rangle$, $\langle \bar{K}^0 | \mathcal{H} | \bar{K}^0 \rangle$ and $\langle \bar{K}^0 | \mathcal{H} | K^0 \rangle$ of the weak interaction Hamiltonian $\mathcal{H}$. Complete expressions for the matrix elements M_{ij} and Γ_{ij} may be found, for example, in Ref. [8].

The overlap $\langle K_L | K_S \rangle$ of the long and short-lived kaons

$$\langle K_L | K_S \rangle = p_L^* p_S - q_L^* q_S \quad (2.9)$$

is an important parameter in the study of the symmetry properties of the structure. Useful information on the overlap comes from the use of the Bell-Steinberger unitarity relation [2]

$$(\lambda_L^* - \lambda_S) \langle K_L | K_S \rangle = i \sum_k \langle k | T | K_L \rangle^* \langle k | T | K_S \rangle \quad (2.10)$$

where the sum is over all physical decay channels k and the decay amplitudes $\langle k | T | K_{L,S} \rangle$ have been normalized as :

$$\gamma_{S,L} = \sum_k |\langle k | T | K_{S,L} \rangle|^2. \quad (2.11)$$

The equations (2.10) and (2.11) are the unitarity relations in the K_S, K_L representation and their content is equivalent to the determination (by unitarity) of the matrix elements of Γ in terms of all the decay amplitudes $\langle k|T|K, \bar{K} \rangle$. The unitarity relations (2.10) and (2.11) can be derived by equating the rate of decrease $-d\mathcal{N}/dt$ (at, say, $t = 0$) of the norm $\mathcal{N}$ of the state $a_S|K_S\rangle + a_L|K_L\rangle$ for arbitrary a_S and a_L :

$$\begin{aligned} a_S|K_S\rangle + a_L|K_L\rangle &\rightarrow a_S e^{-i\lambda_S t} |K_S\rangle + a_L e^{-i\lambda_L t} |K_L\rangle, \\ \mathcal{N} &= \left[|a_S|^2 e^{-\gamma_S t} + |a_L|^2 e^{-\gamma_L t} + \right. \\ &\quad \left. + 2 \operatorname{Re} \left\{ a_L^* a_S \langle K_L|K_S \rangle e^{it(\lambda_L^* - \lambda_S)} \right\} \right], \end{aligned} \quad (2.12)$$

to the corresponding total transition rate (at again, $t = 0$) :

$$\sum_k |a_S \langle k|T|K_S \rangle + a_L \langle k|T|K_L \rangle|^2. \quad (2.13)$$

The overlap and the Bell-Steinberger unitarity relation (2.10) involve only a certain combination of the structure parameters and one has to use some auxiliary information for the remaining parameters, as we shall see in Section 4.

The choice of the three arbitrary phases referred to above defines the convention with respect to which the significant structure parameters are defined. We shall use the Eberhard phase convention [17] which chooses these phases by introducing three real parameters α_S, α_L and θ for describing the structure :

$$\begin{aligned} p_S &= e^{i\theta/2} \cos \alpha_S, \quad q_S = e^{-i\theta/2} \sin \alpha_S, \\ p_L &= e^{-i\theta/2} \cos \alpha_L, \quad q_L = e^{i\theta/2} \sin \alpha_L, \end{aligned} \quad (2.14)$$

where the restrictions on the parameters are :

$$0 \leq \alpha_S, \alpha_L \leq \frac{\pi}{2}; \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}. \quad (2.15)$$

This choice of the $K^0/\bar{K}^0$ phase corresponds to taking

$$\operatorname{Im} (\Lambda_{12}/\Lambda_{21}) = 0. \quad (2.16)$$

We shall use the parameters

$$\begin{aligned}\beta &= \alpha_s - \alpha_L, \\ \gamma &= \frac{\pi}{2} - (\alpha_s + \alpha_L),\end{aligned}\quad (2.17)$$

the overlap being

$$\langle K_L | K_S \rangle = \gamma + i\theta \quad (2.18)$$

where we have neglected quantities (like γ^2 , θ^2 , $\gamma\theta$, β^2 , etc. ; see next Section for details) of second and higher order in parameters which would vanish if CP or/and T or/and CPT invariance holds ; we shall often do so hereafter, also for the parameters describing decay amplitudes. The elements of the total mass matrix Λ become

$$\begin{aligned}\Lambda_{11} &\equiv \frac{1}{2}(\lambda_s + \lambda_L) + \frac{1}{2}[(\lambda_s - \lambda_L)(p_s q_L - p_L q_s)/(p_s q_L + p_L q_s)] \\ &\rightarrow \frac{1}{2}(\lambda_s + \lambda_L) + \frac{1}{2}(\lambda_s - \lambda_L) \cdot (-\beta + i\theta) ; \\ \Lambda_{12} &\equiv (\lambda_s - \lambda_L) p_s p_L / (p_s q_L + p_L q_s) \\ &\rightarrow \frac{1}{2}(\lambda_s - \lambda_L) (1 + \gamma) ; \\ \Lambda_{11} + \Lambda_{22} &\equiv (\lambda_s + \lambda_L) ; \\ \Lambda_{12} / \Lambda_{21} &\equiv (p_s p_L) / (q_s q_L) \\ &\rightarrow (1 + \gamma) / (1 - \gamma) .\end{aligned}\quad (2.19)$$

3. - REQUIREMENTS OF CP, T AND CPT INVARIANCE ON THE STRUCTURE AND ON THE DECAY AMPLITUDES

These can be stated [5,9] in terms of the S matrix as follows :

$$\langle \tilde{\alpha} | S | \tilde{\beta} \rangle = \langle \beta | S | \alpha \rangle , \quad (3.1)$$

$$\langle \alpha^T | S | \beta^T \rangle = \langle \beta | S | \alpha \rangle , \quad (3.2)$$

$$\langle \bar{\beta} | S | \bar{\alpha} \rangle = \langle \beta | S | \alpha \rangle , \quad (3.3)$$

respectively if CPT, T and CP invariances hold. Here the states $\tilde{\alpha}$ and $\tilde{\beta}$ are the CPT conjugates of the states α and β , obtained from α and β by changing particles into antiparticles with reversed spin directions (including helicities) ; the states α^T and β^T are the time-reversed states corresponding to the states α and β .

β , obtained from α and β by reversing the momenta and spin directions (but not helicities) of all particles ; the states $\bar{\alpha}$ and $\bar{\beta}$ are the CP conjugates of the states α and β , obtained from α and β by changing particles into antiparticles with reversed momenta and helicities (but not other spin-directions). Equation (3.2) is just the usual reciprocity relation.

A. - The structure

The requirements of CPT invariance on the structure easily follow from Eq. (3.1) above by choosing both α and β to be the spinless state $|K^0\rangle$, assuming

$$|\bar{K}^0\rangle = CP |K^0\rangle ; \quad (3.4)$$

one gets

$$\langle \bar{K}^0 | S | \bar{K}^0 \rangle = \langle K^0 | S | K^0 \rangle \quad (3.5)$$

which gives, for the pseudo Hamiltonian Λ :

$$\begin{aligned} \Lambda_{11} &= \Lambda_{22} , \text{ or} \\ \Gamma_{11} - \Gamma_{22} &= 0 , \quad M_{11} - M_{22} = 0 , \end{aligned} \quad (3.6)$$

or, equivalently, for the structure parameters

$$q_L p_S = p_L q_S \quad (3.7)$$

which, in the Eberhard convention, gives

$$\theta = 0 , \quad \beta = 0 . \quad (3.8)$$

The requirements of T invariance on the structure follow from Eq. (3.2) above choosing α and β to be respectively the spinless states $|K^0\rangle$ and $|\bar{K}^0\rangle$ at rest (allowing an arbitrary relative phase between $|K^0\rangle$ and $|\bar{K}^0\rangle$) :

$$|\langle K^0 | S | \bar{K}^0 \rangle| = |\langle \bar{K}^0 | S | K^0 \rangle| \quad (3.9)$$

which gives, for the total mass matrix Λ :

$$|\Lambda_{12}| = |\Lambda_{21}| \quad (3.10)$$

or, because Γ and M are Hermitian matrices :

$$\Gamma_{21} = \Gamma_{12}^* \quad ; \quad M_{21} = M_{12}^* ,$$

equation (3.9) gives

$$(\Gamma_{12}^* / \Gamma_{12}) = (M_{12}^* / M_{12}) , \quad (3.11)$$

or, equivalently, for the structure parameters,

$$|q_L q_S| = |p_L p_S| , \quad (3.12)$$

which, in the Eberhard convention, gives

$$\gamma = 0 . \quad (3.13)$$

In fact, the Eberhard convention requires

$$\text{Im } M_{12} \cdot \text{Re } M_{12} + \text{Im } \Gamma_{12} \cdot \text{Re } \Gamma_{12} = 0 , \quad (2.16')$$

so that $\Lambda_{12} = \Lambda_{21}$ and $\text{Im } \Gamma_{12} = 0$, $\text{Im } M_{12} = 0$ for a T invariant structure in the Eberhard convention.

The requirements of CP invariance on the structure follow from Eq. (3.3) above by choosing firstly both α and β to be the spinless state $|K^0\rangle$ at rest :

$$\langle \bar{K}^0 | S | \bar{K}^0 \rangle = \langle K^0 | S | K^0 \rangle \quad (3.14)$$

and secondly, α and β to be respectively the spinless states $|K^0\rangle$ and $|\bar{K}^0\rangle$ at rest :

$$|\langle K^0 | S | \bar{K}^0 \rangle| = |\langle \bar{K}^0 | S | K^0 \rangle| \quad (3.15)$$

where we have used Eq. (3.4) and where Eq. (3.15) allows, like Eq. (3.9), for an arbitrary $K^0/\bar{K}^0$ relative phase.

Since the restrictions in Eq. (3.14) and in Eq. (3.15) are the same as in Eqs. (3.5) and (3.9) respectively, the usual statement that CP invariance of the mass matrix ⁴⁾ requires both its CPT invariance and T invariance follows. CP invariance of the mass matrix Λ therefore requires it to have the general form $A + B \sigma_x$ where A and B are complex numbers and σ_x , the usual Pauli matrix. In particular, one may choose, for a CP invariant structure,

$$p_S = q_S = p_L = q_L = 1/\sqrt{2} . \quad (3.16)$$

As we shall see in Section 4 A, the data indicate that the structure parameters have values close to their CP invariant ones.

B. - The decay amplitudes

The requirements of CP invariance on decay amplitudes follow quite simply from Eq. (3.3) above for any physical decay channel k , by taking α to be the spinless state $|K^0\rangle$ and using Eq. (3.4) :

$$\langle \bar{k} | T | \bar{K}^0 \rangle = \langle k | T | K^0 \rangle \quad (3.17)$$

where $\bar{k}$ is the state CP conjugate to k . The implications of the Eberhard choice of the $K^0/\bar{K}^0$ relative phase on (3.17) would be discussed in Section 3 C below.

The symmetries CPT and T involve an interchange of the initial and final states, in addition to other changes in the states - as stated in Eqs. (3.1) and (3.2). In order to get a useful relation for decay amplitudes, one defines [5]

$$S = S_0 + S_W$$

where S_W is the interaction responsible for weak decays and S_0 is the rest ; one can neglect second and higher orders in S_W . For matrix elements between a state a which would be stable if S_W were zero, and a decay channel b which is an eigenstate of S_0 , the unitarity of the S matrix gives

$$\langle b | S^\dagger S | a \rangle \cong \langle b | S_0^\dagger S_0 + S_0^\dagger S_W + S_W^\dagger S_0 | a \rangle ,$$

or

$$\langle b | S_0^\dagger S_W + S_W^\dagger S_0 | a \rangle = 0 \quad (3.18)$$

because the states a and b are assumed to be connected only through the action of S_W . Using

$$S_0 | b \rangle = e^{2i\delta_b} | b \rangle$$

where δ_b is the eigenphase for the state b , and

$$S_0 | a \rangle = | a \rangle ;$$

Equation (3.18) gives

$$0 = \langle b | S_W | a \rangle e^{-2i\delta_b} + \langle b | S_W^\dagger | a \rangle,$$

or, for the decay amplitudes ($S_W = iT$),

$$\langle b | T | a \rangle = e^{2i\delta_b} \langle a | T | b \rangle^*. \quad (3.19)$$

Here, the phase shift factor on the right-hand side is the modification (due to final state interactions) in the hermiticity relation [9] of the transition amplitude. The relations (3.1) and (3.2) now yield useful restrictions on decay amplitudes.

For CPT invariance, Eq. (3.1) gives

$$\langle \tilde{\alpha} | T | \bar{K}^0 \rangle = \langle K^0 | T | \alpha \rangle;$$

which, using Eq. (3.19), gives, for an eigenstate channel α ,

$$\langle \tilde{\alpha} | T | \bar{K}^0 \rangle = e^{2i\delta_\alpha} \langle \alpha | T | K^0 \rangle^*, \quad (3.20)$$

where we have assumed that no phase is introduced in forming the time-reversed state corresponding to the spinless state $|K^0\rangle$ (and similarly $|\bar{K}^0\rangle$) at rest; and also, we have used Eq. (3.4). A convenient way to satisfy Eq. (3.20) is to take

$$\begin{aligned} \langle \alpha | T | K^0 \rangle &= A_\alpha e^{i\delta_\alpha} \\ \langle \tilde{\alpha} | T | \bar{K}^0 \rangle &= A_\alpha^* e^{i\delta_\alpha}, \end{aligned} \quad (3.21)$$

so that, apart from the final state interaction factors, the amplitudes $\langle \alpha | T | K^0 \rangle$ and $\langle \tilde{\alpha} | T | \bar{K}^0 \rangle$ are complex conjugates of each other. The CPT requirements on amplitudes (or on the structure) are not affected by the Eberhard choice of the $K^0/\bar{K}^0$ relative phase (see Section 3 C below).

For T invariance, Eq. (3.2) gives

$$\langle \alpha^T | T | K^0 \rangle = \langle K^0 | T | \alpha \rangle;$$

which, using Eq. (3.19) above, gives for the eigenstate channel α ,

$$\langle \alpha^T | T | K^0 \rangle = e^{2i\delta_\alpha} \langle \alpha | T | K^0 \rangle^* \quad (3.22)$$

where again we have used our definition of the time-reversed state corresponding to $|K^0\rangle$ at rest. In general, a physical state k is different from its time-reversed state k^T , but the eigenstate α^T obtained from α is essentially identical to it so that one gets

$$\langle \alpha | T | K^0 \rangle = e^{2i\delta_\alpha} \langle \alpha | T | K^0 \rangle^* . \quad (3.23)$$

A corresponding relation should hold also for the $\bar{K}^0$ decay amplitudes :

$$\langle \alpha | T | \bar{K}^0 \rangle = e^{2i\delta_\alpha} \langle \alpha | T | \bar{K}^0 \rangle^* \quad (3.24)$$

Equations (3.23) and (3.24) express the "reality conditions" of the decay amplitudes in the sense that one can choose, for T invariant amplitudes,

$$e^{-i\delta_\alpha} \langle \alpha | T | K^0, \bar{K}^0 \rangle = \text{Real} .$$

The implications of the Eberhard choice of the $K^0/\bar{K}^0$ relative phase for (3.23), (3.24) would be considered in Section 3 C below.

One should note [9] that strong interaction eigenstates are involved in the relations (3.20) for CPT invariance and relations (3.22), (3.23) and (3.24) for T invariance. For physical states, therefore, the moduli on the two sides of such relations need not be equal because of the possible presence of more than one eigenstate in a given physical state.

Tests for CPT, T and CP invariance of both the mass matrix and the decay amplitudes have been considered by various authors. In the following Section, we shall define (and set bounds on) parameters which would vanish if these invariances are valid ²⁾. Before doing so, we compare the Eberhard phase convention and our definition (3.4) of CP with the superweak model [8,18] in which all amplitudes are CP, T and CPT invariant and the only possible non-invariances arise from the partial mass matrix M . The comparison is useful because, in general, if a theory is not invariant under a certain symmetry operation, one cannot uniquely define that operation. In the superweak model, these operations are well-defined for the amplitudes.

C. - The superweak model

In order to bring out the implications of the Eberhard choice of the $K^0/\bar{K}^0$ relative phase, we consider a definition of the superweak model with the following quite common [1,8] definition (which already makes a choice for the $|K_S\rangle$ and $|K_L\rangle$ phases) of structure :

$$\begin{aligned} p_S &= (1+\epsilon_1)/\sqrt{2(1+|\epsilon_1|^2)} , \quad q_S = (1-\epsilon_1) p_S/(1+\epsilon_1), \\ p_L &= (1+\epsilon_2)/\sqrt{2(1+|\epsilon_2|^2)} , \quad q_L = (1-\epsilon_2) p_L/(1+\epsilon_2), \end{aligned} \quad (3.25)$$

where ϵ_1 and ϵ_2 are complex ; the $K^0/\bar{K}^0$ relative phase is still arbitrary and can be fixed, for example, by taking the amplitude

$$\langle I=0, \pi\pi | T | K^0 \text{ or } \bar{K}^0 \rangle e^{-i\delta_0^{\pi\pi}}$$

to be real. As auxiliary parameters ⁵⁾, we use [1]

$$\begin{aligned} \epsilon &= \frac{1}{2} (\epsilon_1 + \epsilon_2) \\ \delta &= \frac{1}{2} (\epsilon_1 - \epsilon_2) \end{aligned} \quad (3.26)$$

which are quite small (see Section 4) and are related to the mass matrix by the relations

$$\begin{aligned} \epsilon(\lambda_S - \lambda_L) &= \text{Im } \Gamma_{12} + i \text{Im } M_{12} , \\ 2i\delta(\lambda_S - \lambda_L) &= (\Gamma_{11} - \Gamma_{22}) + i(M_{11} - M_{22}), \end{aligned} \quad (3.27)$$

to lowest order in ϵ and δ . In fact, $\epsilon = 0$ if T invariance holds for the total mass matrix ; and $\delta = 0$ if CPT invariance holds for the total mass matrix.

With the definition

$$|\bar{K}^0\rangle_S = CP_S |K^0\rangle_S \quad (3.28)$$

where the subscript S means "superweak", one can see that the hypothesis of all amplitudes being CP invariant means that the decay matrix Γ is so :

$$(\Gamma_{11} - \Gamma_{22})_S = 0, \quad \text{Im}(\Gamma_{12})_S = 0. \quad (3.29)$$

This, using Eqs. (3.27) and the empirical relations

$$\begin{aligned} m_L - m_S &\simeq \frac{1}{2} \gamma_S, \\ \gamma_L &\ll \gamma_S, \end{aligned} \quad (3.30)$$

means that the phase of ϵ is $\sim \pi/4$ if the M matrix is T non-invariant [$\text{Im}(M_{12})_S \neq 0$] and the phase of δ is $\sim 3\pi/4$ if the M matrix is CPT non-invariant [$(M_{11} - M_{22})_S \neq 0$]⁶⁾.

It is possible to identify the $|K_S\rangle$ and $|K_L\rangle$ states of this superweak model with those in the Eberhard phase convention. This requires the following correspondence :

$$|K^0\rangle_{\text{ours}} = e^{i\phi} |K^0\rangle_S,$$

$$\begin{aligned}
 |\bar{K}^0\rangle_{\text{ours}} &= e^{-i\phi} |\bar{K}^0\rangle_S, \\
 CP_{\text{ours}} &= e^{-2i\phi} CP_S, \\
 \gamma &= 2 \operatorname{Re} \epsilon, \\
 \theta &= 2 \operatorname{Im} \delta, \\
 \beta &= -2 \operatorname{Re} \delta,
 \end{aligned} \tag{3.31}$$

where the small parameter ϕ is given by

$$\phi \cong \tan \phi \cong \operatorname{Im} \epsilon.$$

This, of course, means that

$$[(\Gamma_{12})_{\text{ours}}/(\Gamma_{12})_S] = [(M_{12})_{\text{ours}}/(M_{12})_S] = e^{-2i\phi}, \tag{3.32}$$

so that

$$\begin{aligned}
 \operatorname{Im}(\Gamma_{12})_{\text{ours}} &\simeq -2\phi \operatorname{Re}(\Gamma_{12})_S + \operatorname{Im}(\Gamma_{12})_S, \\
 &= -2\phi \operatorname{Re}(\Gamma_{12})_S
 \end{aligned} \tag{3.33}$$

Thus, though $\operatorname{Im}(\Gamma_{12})_S = 0$, yet $\operatorname{Im}(\Gamma_{12})_{\text{ours}} \neq 0$ even in the superweak situation, as defined above. This is due to the Eberhard restriction (2.16'), which fixes $\operatorname{Im} \Gamma_{12}$ if $\operatorname{Im} M_{12}$ is known; $\operatorname{Re} \Gamma_{12}$ and $\operatorname{Re} M_{12}$ being determined by $\gamma_S - \gamma_L$ and $m_S - m_L$ respectively. If, in addition, the superweak model is T invariant [$\operatorname{Im}(\Gamma_{12})_S = 0$, $\phi = 0 \rightarrow \epsilon = 0$], one gets $\operatorname{Im}(\Gamma_{12})_{\text{ours}}$ also = 0. On the other hand, the diagonal elements Λ_{11} and Λ_{22} do not undergo any change with respect to this superweak phase convention.

For the CPT invariant part of the decay amplitudes, our parameters (φ_I) which should vanish if T (and CP) invariance holds would turn out to be higher by an amount ϕ than those in a description using these superweak conventions [see Eq. (4.15) below]. The parameters (α_I) which would vanish if CPT (and T) invariance holds for the CP invariant part of the decay amplitudes would not be modified; the same holds for the parameters (β_I) which should vanish if CPT (and CP) invariance holds for the T invariant part of the decay amplitudes. We shall return to this point again when we discuss present numerical limits on CP, T and CPT invariance in Section 4.B.1.

4. - PRESENT STATUS OF CP, T AND CPT INVARIANCE IN NEUTRAL KAON DECAYS

In this Section are given numerical bounds on the relative strength of the parameters which should vanish if these invariances hold in neutral kaon decays. We first consider the structure and then, the decay amplitudes.

A. - The structure

The relevant parameters here are γ , β and θ in the Eberhard representation. Possible non-zero values of β and θ express the extent to which Λ_{11} equals Λ_{22} [see Eqs. (2.19)] and therefore the accuracy to which CPT (and CP) invariance holds for the structure. Similarly, values of γ express the extent to which Λ_{12} equals Λ_{21} [see Eqs. (2.19)] and therefore the accuracy to which T (and CP) invariance holds, in the Eberhard phase convention, for the structure. If CP invariance holds, the overlap $\langle K_L | K_S \rangle$ is zero; if T invariance holds, it is imaginary and if CPT invariance holds, it is real. Bounds on $|\langle K_L | K_S \rangle|$ were obtained by Lee and Wolfenstein [8] by using a weaker form of the unitarity relation (2.10) and were improved by Bell and Steinberger [2] by using a more detailed version (but still not the full phase content) of their unitarity relation (2.10). This was further improved by Kabir [4].

Indications that the overlap $\langle K_L | K_S \rangle$ is consistent with CPT invariance, but not with T invariance, came from evaluations of the relation (2.10) by Kabir [11] and by Ashkin and Kabir [12] who showed that if one assumes complete CPT invariance for the structure and also for the decay amplitudes, data were consistent with the equality in Eq. (2.10); but if the same procedure were repeated for complete T invariance, data were not consistent with relation (2.10). Steinberger [3,7] also made a similar evaluation of the unitarity relation, with the same conclusions. Independent indication for T non-invariance of the structure due to Casella [13] is based on the incompatibility of the available values of $|\eta_{00}/\eta_{+-}|$ [see Eqs. (4.17), (4.18) for definition] with (i) the known $\pi\pi$ scattering phase shift difference ($\delta_2 - \delta_0$) where the subscripts refer to the total isospin of the $\pi\pi$ state, and (ii) the expected phase of the relevant structure parameter $[\epsilon_2$ of Eq. (3.25) above] if the structure were T invariant and (iii) the known values of η_{+-} . Casella assumed the $|\Delta I| = \frac{1}{2}$ rule; his argument would produce the same result if one used the available result for the phase of η_{00} instead of the $\pi\pi$ phase shift information on ($\delta_2 - \delta_0$).

These arguments do indicate T non-invariance and CPT invariance for $\langle K_L | K_S \rangle$, but do not set numerical bounds on the real and imaginary parts of $\langle K_L | K_S \rangle$. Assuming CPT invariance and the $\Delta S = \Delta Q$ rule, the real part of the overlap $\langle K_L | K_S \rangle$ is determined by the charge-asymmetry in the $\pi\pi\gamma$ decay of K_L . Wolfenstein [20] gave a numerical bound for the imaginary part of the overlap $\langle K_L | K_S \rangle$ in the approximation of saturating the unitarity relation (2.10) by the 2π channel. A more complete evaluation of the relation (2.10) was done by Schubert et al. [15] to get numerical values

for the structure parameters. They analyzed the contribution of the $\pi\ell\nu$ channels without assuming CPT or T or CP invariance (but assuming μ -e universality); the 2π channel was also evaluated using the available data and their η_{00} . It has been assumed that the contributions of decay channels other than 2π , $\pi\ell\nu$ and 3π can be neglected ⁷⁾ in a numerical evaluation of the unitarity relation (2.10). However, the 3π channel presents some problem because the amplitude ratio

$$\eta_{3\pi^0} = \left[\frac{\langle 3\pi^0 | T | K_L \rangle}{\langle 3\pi^0 | T | K_S \rangle} \right]^{-1} \quad (4.1)$$

is essentially unknown, though values for the corresponding quantity η_{+-0} for the $\pi^+\pi^-\pi^0$ channel are known (with large errors). Schubert et al. [15] assumed dominance of the 3π channel by the totally symmetric $I = 1$ state in which case $\eta_{3\pi^0} = \eta_{+-0}$. Their results are again consistent with CPT invariance, but not with T invariance for the structure.

In fact, the present indications of T non-invariance rest rather crucially on assumptions about $\eta_{3\pi^0}$. Some assumption or the other has been made about the 3π channel in almost all the present proofs of T non-invariance of the structure. For example, $|\eta_{3\pi^0}| \leq 1$ and $|\eta_{+-0}| \leq 1$ was assumed in Refs. [1,12,2] and $\eta_{3\pi^0} = \eta_{+-0}$ in Ref. [15]. While it seems very plausible to expect the relative strength of CP non-invariance to be small (and therefore, $\eta_{3\pi^0}$ to be a small number), a really "assumption independent" proof for T non-invariance would need an experimental number for $\eta_{3\pi^0}$ and also a precise knowledge of the rates for decay modes other than 2π , 3π and $\pi\ell\nu$, to rule out the possibility of large contributions from unknown or poorly known modes. In fact, Steinberger [7] has observed that one would have a proof of T non-invariance if $|\eta_{3\pi^0}| < 2$, the argument being based, again, on an evaluation of the unitarity relation (2.10). In principle, it is possible that $\eta_{3\pi^0}$ could be large because the only established CP non-invariance in the non-leptonic interactions is for the parity violating mode $K \rightarrow 2\pi$, while the $K \rightarrow 3\pi$ decay is parity conserving. This would be the case if CP non-invariance of amplitudes resides in parity conserving interactions; the corresponding effect in $K \rightarrow 2\pi$ amplitudes would then be very small. In order to avoid the effects of structure, one may then look for possible CP non-invariant effects in $K^\pm \rightarrow 3\pi$ decays.

For numerical bounds on γ and θ which are respectively [see Eq. (2.18)] the real and imaginary parts of the overlap $\langle K_L | K_S \rangle$, we shall use the results of Schubert et al. [15]. If one allows a) for $1.4 < 10^3 |\eta_{00}| < 4.2$ (which covers most of the available values of $|\eta_{00}|$) and b) for $|\eta_{3\pi^0}| \lesssim 1.5$, the tentative number quoted in Ref. [3], in the absence of any established value, the results of Refs. [14,15] yield ⁸⁾

$$10^3 |\theta| \lesssim 2.7 \quad (4.2)$$

$$0.7 \lesssim 10^3 \gamma \lesssim 5 \quad (4.3)$$

The upper limits in (4.2) and (4.3) would be about 1.7 and 4 respectively if one uses $\eta_{3\pi^0} = \eta_{+-0}$, as done in Refs. [14,15] ; and the lower limit in (4.3) would then be about 1.7. The indications for T non-invariance rest on the lower limit in (4.3) being non-zero. In our derivation of the numerical bounds on CP, T and CPT invariance of decay amplitudes (see also [16]), we shall not use the lower limit in (4.3) at all. Very precise values of the upper limits in (4.2) and (4.3) would also not matter in any crucial sense for the amplitude limits ; for example, 3 is a good enough limit in (4.2). The limits in (4.2) and (4.3) arise by interpreting a quantity quoted as having the value $(x \pm y)$ to lie between $(x - y)$ and $(x + y)$. Numbers similar to those in (4.2) and (4.3) can also be derived by using the results of Refs. [3,11,20].

The remaining structure parameter β does not occur explicitly in the unitarity relation (2.10). If one inverts Eqs. (2.3), one gets [5]

$$\begin{aligned} |K^0\rangle &= (q_L |K_S\rangle + q_S |K_L\rangle) / (p_S q_L + p_L q_S), \\ |\bar{K}^0\rangle &= (p_L |K_S\rangle - p_S |K_L\rangle) / (p_S q_L + p_L q_S), \end{aligned} \quad (4.4)$$

so that the ratio of the amplitudes of the $|K_S\rangle$ in $|K^0\rangle$ and $|\bar{K}^0\rangle$ states is (q_L/p_L) and that of the intensities is

$$|q_L/p_L|^2 \cong 1 - 2(\gamma + \beta). \quad (4.5)$$

Similarly, the ratio of the intensities of the $|K_L\rangle$ component in $|K^0\rangle$ and $|\bar{K}^0\rangle$ states is

$$|q_S/p_S|^2 \cong 1 - 2(\gamma - \beta), \quad (4.6)$$

so that one could [16] determine β by using (4.3) and determining the ratio of the $|K_S\rangle$ (or $|K_L\rangle$) intensities in $|K^0\rangle$ and $|\bar{K}^0\rangle$ states. Assuming charge conjugation invariance for the interactions responsible for the processes

$$\begin{aligned} \bar{p} p &\rightarrow K^0 K^- \pi^+, \text{ and} \\ \bar{p} p &\rightarrow \bar{K}^0 K^+ \pi^-, \end{aligned} \quad (4.7)$$

the ratio of the strength of $|K_S\rangle$ decays from the two reactions measure $(\gamma + \beta)$. This gives [4,17,22]

$$10^2 |\gamma + \beta| \lesssim 1.6 \quad (4.8)$$

which, when combined with the limits for γ in (4.3), gives

$$10^2 |\beta| \lesssim 2.1. \quad (4.9)$$

It would be very desirable to improve this limit on β by more accurate determinations of $(\gamma \pm \beta)$ using (4.5) or (4.6) or by other methods [16] because the corresponding limits on θ and $\frac{1}{2}\gamma$ in (4.2) and (4.3) are better by about an order of magnitude.

The physical meaning of the limits (4.2) and (4.9) using Eqs. (2.19) and (3.30) is that $|\text{Im}(\Lambda_{11} - \Lambda_{22})|$ and $|\text{Re}(\Lambda_{11} - \Lambda_{22})|$ are both bounded at present by the upper limit ⁹⁾ $\sim 0.01 \gamma_S$. In other words, the sum of the physical transitions (in Γ) could be almost as strongly CPT (and CP) non-invariant, as the corresponding sum of the energy non-conserving ones (plus the "self-transition" contributions) in M :

$$\begin{aligned} |\Gamma_{11} - \Gamma_{22}| &\lesssim 0.01 \gamma_S, \\ |M_{11} - M_{22}| &\lesssim 0.01 \gamma_S. \end{aligned} \quad (4.10)$$

These limits, of course, allow for CPT invariance of the mass matrix ($\Lambda_{11} = \Lambda_{22}$). If γ_S is taken as a measure of the $|\Delta S| = 1$ weak interactions, the limits (4.10) mean that the present accuracy for CPT (and CP) invariance is [4] about one percent for the $|\Delta S| = 1$ weak interactions contributing to γ_S (it would be dominantly the non-leptonic parity violating decay into the 2π channel). Expressed [1,4] in terms of the accuracy of CPT (and CP) invariance of strong interactions, this number is about 1 part in 10^{16} because $\gamma_S \sim 10^{-14}$ of the kaon mass ; the corresponding number for electromagnetic interactions is about 1 part in 10^{14} because of the usual factor of α . This is a rather strong limit on possible CPT (and CP) non-invariant contributions in strong and electromagnetic interactions. This evidence for CPT invariance, one should remark, is simultaneous also for the CP invariance of the same interaction.

Similarly, the limits in (4.3) using Eqs. (2.19) and (3.30) mean that the real and imaginary parts of Λ_{12} and Λ_{21} are equal to within $\sim 2.5 \cdot 10^{-3} \gamma_S$:

$$0.4 \gamma_S \lesssim 10^3 \text{Re}(\Lambda_{21} - \Lambda_{12}) \simeq 10^3 \text{Im}(\Lambda_{21} - \Lambda_{12}) \lesssim 2.5 \gamma_S. \quad (4.11)$$

In other words,

$$0.2 \gamma_S \lesssim 10^3 \text{Im} \Gamma_{21} \simeq 10^3 \text{Im} M_{21} \lesssim 1.3 \gamma_S \quad (4.12)$$

where, in fact, the near equality

$$\begin{aligned} \text{Im}(\Lambda_{21} - \Lambda_{12}) &\simeq \text{Re}(\Lambda_{21} - \Lambda_{12}) \\ \text{and} \quad \text{Im} M_{21} &\simeq \text{Im} \Gamma_{21} \end{aligned} \quad (4.13)$$

for the sum of the real and of the virtual transitions is only a property of the Eberhard phase convention ¹⁰⁾, using the experimental facts of (3.30).

Because of the poorer limit on β than on γ and θ , the above-given limits mean that the bound [see (4.10)] for CPT invariance of the mass matrix is worse (by a factor of about 4) than the upper bound [see (4.11)] for T invariance ; though the present evidence is consistent with CPT invariance (lower bound zero) but not T invariance [lower bound non-zero, apart from the $\eta_{3\pi^0}$ uncertainty and the ambiguity of footnote 7) mentioned above]. As we shall see in the following subsection, the upper limits for T invariance turn out to be better than those for CPT invariance also for the $\pi\pi$ amplitudes (especially for the $I = 0$ state) for the same reason.

One may also note that the superweak model (where the partial mass matrix M is the only source of possible non-invariances) would require [see the remarks following (3.30)]

$$\text{Re } \delta \simeq - \text{Im } \delta$$

if the M matrix is not CPT invariant ($M_{11} \neq M_{22}$). The limits on $|\text{Im } \delta|$ and $|\text{Re } \delta|$ implied by Eq. (3.31) and the limits given in this subsection differ by about an order of magnitude so that there is a lot of scope for improvement [16] of the limit on $\text{Re } \delta$ to eliminate the possibility that the superweak model might be the CPT non-invariant one.

B. - The decay amplitudes

CP, T and CPT invariances put different restrictions on a decay amplitude. One also distinguishes between channels which transform into themselves under CP (or CPT) transformation from those which do not.

B.1. - Self-conjugate channels

The important ones to consider are the non-leptonic decay modes - the parity violating $\pi\pi$ channel and the parity conserving 3π channel.

The $\pi\pi$ channel

In Wolfenstein's notation [20], we write the transition matrix elements into $I = 0$ and 2 eigenstates as

$$\begin{aligned} \langle I | T | K \rangle &= a_I e^{i\delta_I} (1 + i\varphi_I + i\alpha_I + \beta_I), \\ \langle I | T | \bar{K} \rangle &= a_I e^{i\delta_I} (1 - i\varphi_I + i\alpha_I - \beta_I), \end{aligned} \quad (4.14)$$

where Eq. (3.4) is assumed, a_I is real, δ_I is the $\pi\pi$ scattering phase shift in the S wave state of isospin I, and the real parameters φ_I , α_I and β_I phenomenologically represent the relative strengths of the amplitudes with the following transformation properties (as discussed in Section 3.B) :

$$\begin{aligned}\varphi_I &: CPT^\checkmark, \quad \cancel{CP}, \quad T; \\ \alpha_I &: \cancel{CPT}, \quad CP^\checkmark, \quad T; \\ \beta_I &: \cancel{CPT}, \quad \cancel{CP}, \quad T^\checkmark.\end{aligned}$$

Due to the $K^0/\bar{K}^0$ relative phase choice of the Eberhard convention, our φ_I 's [similarly the φ_f and φ_g of Eq. (4.26)] are higher than their corresponding values in a theory using the conventions of the superweak model of Section 3.C by an amount ϕ :

$$(\varphi_I)_{\text{ours}} = (\varphi_I)_{\text{theirs}} + \phi \quad (4.15)$$

so that in the superweak situation when all amplitudes are CP, T and CPT invariant,

$$(\varphi_I)_S = 0$$

and

$$(\varphi_I)_{\text{ours}} = \phi. \quad (4.16)$$

Thus all our φ_I 's should have the value ϕ in the superweak situation, in which case, using Eq. (3.30), the phase of ϵ is about $\pi/4$ and therefore, the numerical value of ϕ is about half that of the Eberhard parameter γ , using Eq. (3.31).

Our phase convention does not change the definitions of the parameters α_I and β_I with respect to their superweak definitions.

We shall now quote limits on these parameters α_I , β_I and φ_I , obtained by using data on the $\pi\pi$ decay modes. The amplitude ratios

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | T | K_L \rangle}{\langle \pi^+\pi^- | T | K_S \rangle} \quad (4.17)$$

and

$$\eta_{00} = \frac{\langle \pi^0\pi^0 | T | K_L \rangle}{\langle \pi^0\pi^0 | T | K_S \rangle} \quad (4.18)$$

become, in our notation,

$$\begin{aligned}\eta_{+-} &= \frac{1}{2}(\gamma + \beta - i\theta) + [(i\varphi_0 + \beta_0) + X_2(i\varphi_2 + \beta_2)] / (1 + X_2), \\ \eta_{00} &= \frac{1}{2}(\gamma + \beta - i\theta) + [(i\varphi_0 + \beta_0) - 2X_2(i\varphi_2 + \beta_2)] / (1 - 2X_2), \\ X_2 &= \frac{a_2}{\sqrt{2} a_0} e^{i(\delta_2 - \delta_0)}.\end{aligned}\quad (4.19)$$

The X_2 terms in the denominator were neglected in Ref. [20]. Inverting these equations, one gets

$$\begin{aligned}\begin{pmatrix} \varphi_0 \\ \beta_0 \end{pmatrix} &= \frac{\text{Im}}{\text{Re}} \left[\frac{(2\eta_{+-} + \eta_{00})}{3} + \frac{2}{3} X_2 (\eta_{+-} - \eta_{00}) - \frac{1}{2} (\gamma + \beta - i\theta) \right], \\ \begin{pmatrix} \varphi_2 \\ \beta_2 \end{pmatrix} &= \frac{\text{Im}}{\text{Re}} \left[\frac{(\eta_{+-} + 2\eta_{00})}{3} + \frac{(\eta_{+-} - \eta_{00})}{3X_2} - \frac{1}{2} (\gamma + \beta - i\theta) \right].\end{aligned}\quad (4.20)$$

In order to set limits on φ_0 , β_0 , φ_2 and β_2 , one needs a knowledge of η_{+-} , η_{00} and X_2 , in addition to the limits (4.2) and (4.8). We take

$$\begin{aligned}10^3 \eta_{+-} &= (1.92 \pm 0.05) e^{i(44 \pm 5)^\circ}, \\ 10^3 \eta_{00} &= (2.8 \pm 1.4) e^{i(51 \pm 30)^\circ}, \\ 10^2 X_2 &= (3.2 \pm 1) e^{-i(40 \pm 20)^\circ},\end{aligned}\quad (4.21)$$

where η_{+-} is taken from Ref. [24]; the phase of η_{00} is taken from Ref. [15] and $|\eta_{00}|$ is taken as some "average" of all the values given in Ref. [14]. For $|X_2|$, we take the value [1] obtained by assuming ¹¹⁾ $|\Delta I| < 5/2$ and by using the ratios of $K^+ \rightarrow \pi^+ \pi^0$ and $K_S \rightarrow \pi \pi$ rates; the error is supposed to take into account a $|\Delta I| = 5/2$ amplitude up to about half as strong as the $|\Delta I| = 3/2$ one. The phase of X_2 is taken from Ref. [25]. None of the ambiguous quantities (for example $|\eta_{00}|$ and $\delta_2 - \delta_0$) will be used in any crucial sense, and only the most unfavourable values will be used in getting final numbers. For example, we shall take $10^2 |X_2|$ to lie between 2.2 and 4.2; if the worst limit arises from the largest (smallest) value of $|X_2|$, we shall use the number 4.2 (2.2) in getting the limits quoted below. Using the above data ¹²⁾ and the limits (4.2) and (4.8), one gets (see [16] also)

$$-0.007 \lesssim \beta_0 \lesssim 0.01, \quad (4.22.a)$$

$$-0.0005 \lesssim \varphi_0 \lesssim 0.004, \quad (4.22.b)$$

$$-0.04 \lesssim \beta_2 \lesssim 0.05, \quad (4.22.c)$$

$$-0.04 \lesssim \varphi_2 \lesssim 0.02, \quad (4.22.d)$$

where, in fact, the limits obtained by using the central values of Eqs. (4.21) above are much better : for example, one gets $10^2 |\beta_2| < 1.2$ and $10^3 |\varphi_2| < 9$. The dependence of (4.22.a,b) on X_2 is very slight, but that of the limits for φ_2 and β_2 on X_2 is rather strong : for φ_2 , it is almost directly proportional to $1/X_2$, while for β_2 , about 85% of the upper limit (4.22.c) arises from a term which is directly proportional to $1/X_2$. The other terms contributing to β_2 were neglected in Ref. [20]. The limits (4.22.c,d) are worse than those in (4.22.a,b) because $|X_2| \ll 1$. Using the definitions (4.14), the limits (4.22) mean that for the effective parity violating non-leptonic decay amplitudes, the T invariant part is CPT (and CP) invariant to $\sim 1\%$ for the $I = 0, \pi\pi$ final state and to $\sim 5\%$ for the $I = 2$ state. The corresponding CPT invariant parts are T (and CP) invariant to $\sim 0.4\%$ for decay to the $I = 0$, and to $\sim 4\%$ for the $I = 2$, $\pi\pi$ final state, in our phase convention.

The mass matrix limits of the previous subsection can teach one mostly about the dominant terms in the sums contributing to the various elements of the decay matrix Γ (and similarly, the matrix M). The fact that the $I = 0, 2\pi$ channel is the most dominant one reflects itself in the accuracy ($\sim 1\%$) [see the limit (4.10) above] of CPT (and CP) invariance for the total decay matrix Γ being about the same as the corresponding accuracy (4.22.a) for the $\pi\pi, I = 0$ channel. Also, the parameters γ and φ_0 have similar limits. In this context, one may understand that the small limit obtained in Ref. [15] for the parameter $\text{Re } \tilde{\delta}$ (which should vanish if CPT or CP invariance holds) which, in our notation, is

$$\text{Re } \tilde{\delta} = -\left[\frac{\beta}{2} + \beta_0 \right], \quad (4.23)$$

is quite consistent with our limits (4.9) and (4.22.a) above if β and $2\beta_0$ are nearly equal (and opposite) to each other, within a few percent.

As a passing remark, one notes that the possibility $\eta_{+-} = \eta_{00}$ implies $\varphi_0 = \varphi_2$ and $\beta_0 = \beta_2$. The superweak situation would require, in addition, $\varphi_0 = \phi$, $\beta_0 = 0$.

The $\pi^+\pi^-/\pi^0\pi^0$ branching ratio for K_S decay

For the parameters α_0 and α_2 , one has to consider [20] a CP invariant effect. The charged to neutral $K_S \rightarrow \pi\pi$ branching ratio offers such a possibility. This ratio, in our notation, becomes

$$\frac{1}{2} \frac{\Gamma(K_S \rightarrow \pi^+\pi^-)}{\Gamma(K_S \rightarrow \pi^0\pi^0)} = 1 + \frac{6 \text{Re } X_2 - 3|X_2|^2 - 6(\alpha_2 - \alpha_0) \text{Im } X_2}{|1 - 2X_2|^2 + 4(\alpha_2 - \alpha_0) \text{Im } X_2}, \quad (4.24.a)$$

$$\rightarrow \simeq 1 + \frac{6 \operatorname{Re} X_2 - 6 (\alpha_2 - \alpha_0) \operatorname{Im} X_2}{1 - 4 \operatorname{Re} X_2} \quad (4.24.b)$$

Using the world [26] average (2.196 ± 0.065) for this branching ratio and taking X_2 from Eq. (4.21), Eq. (4.24.b) gives

$$\alpha_2 - \alpha_0 \simeq -0.5 \pm 0.6 \quad (4.25)$$

Inclusion of phase space corrections [1] or of electromagnetic effects [1,27] or use of the exact form (4.24.a) does not significantly alter the rather poor limit (4.25). For the detection of the $(\alpha_2 - \alpha_0)$ term in Eq. (4.24), one definitely needs at least two isospin amplitudes ($\operatorname{Im} X_2 \neq 0$).

The $\pi\pi\pi$ channel

The $\pi\pi\pi$ final state is dominantly CP = -1 and one may hope that the amplitude ratio $\eta_{3\pi^0}$ of Eq. (4.1) above (and similarly η_{+-0}) could be used to determine amplitude parameters similar to the φ_I, α_I and β_I [Eq. (4.14)] of the $\pi\pi$ case; also, the structure parameters that appear in $\eta_{3\pi^0}$ would be $(\gamma - \beta + i\theta)$ as compared to $(\gamma + \beta - i\theta)$ for η_{+-} and η_{00} . However, there are basic difficulties in extracting these parameters from $\eta_{3\pi^0}$ and η_{+-0} , quite apart from the facts 1) that very poor information is available on η_{+-0} , and almost none at all for $\eta_{3\pi^0}$; and 2) that $3\pi^0$ is the only final state of pure CP = -1; and extraction of CP non-invariance parameters for the $\pi^+\pi^-\pi^0$ state is very difficult because of the presence of CP = +1 components in addition.

The point is that we do not know how to take into account the final state interactions in a model-independent way. Total isospin, parity and total angular momentum are not sufficient to describe the quantum numbers characterizing a $\pi\pi\pi$ eigenstate, and we do not know what this formal set of quantum numbers for even the pure CP = -1, $3\pi^0$ state in general is. Defining any invariance parameters like α_I, φ_I and β_I , Eqs. (4.14), then becomes difficult in general. We shall, therefore, not attempt to derive any amplitude limits for the 3π decays.

B.2. - Non-self conjugate channels

The most important channels here are the $\pi^+\ell^-\bar{\nu}$ and $\pi^-\ell^+\nu$ with $\ell = e, \mu$. These channels offer a chance to study limits on the $\Delta S = -\Delta Q$ amplitudes in addition to those on CP, T and CPT invariance. The invariance restrictions, in general, apply to a given final state configuration [20] with all momenta and polarizations fully specified. For obvious reasons, data on these amplitudes are available only after averaging over the whole Dalitz plot. Furthermore, due to the presence of spin, CP and CPT conjugates of a $\pi\ell\nu$ state are, in general, different. This is quite apart

from the effect of reversing the momenta by the operation T ; however, in the case of a planar decay like $\pi l \nu$, a 180° rotation about the normal to the scattering plane is enough to bring the momenta to their original values. In the derivation of the limits given below, we shall have to neglect such detailed effects.

In Wolfenstein's [20] notation (see also Refs. [1,28]),

$$\begin{aligned}\bar{f} &\equiv \langle l^+ \pi^- \nu | T | K^0 \rangle = f (1 + i\varphi_f + i\alpha_f + \beta_f) , \\ F &\equiv \langle l^- \pi^+ \bar{\nu} | T | \bar{K}^0 \rangle = f (1 - i\varphi_f + i\alpha_f - \beta_f) , \\ g &\equiv \langle l^+ \pi^- \nu | T | \bar{K}^0 \rangle = f (X + i\varphi_g + i\alpha_g + \beta_g) , \\ G &\equiv \langle l^- \pi^+ \bar{\nu} | T | K^0 \rangle = f (X - i\varphi_g + i\alpha_g - \beta_g) ,\end{aligned}\tag{4.26}$$

the different effective amplitudes are described in terms of the real parameters $f, X, \varphi_f, \varphi_g, \alpha_f, \alpha_g, \beta_f$ and β_g . While the invariance properties of the parameters $\varphi_f, \varphi_g, \alpha_f, \alpha_g, \beta_f$ and β_g are, respectively, the same as those for φ_I, α_I and β_I of Eqs. (4.14), the real parameter X is now a measure of the relative strength of the CP, T and CPT invariant part of the $\Delta S = -\Delta Q$ amplitude with respect to the corresponding $\Delta S = \Delta Q$ amplitude. The parameter f denotes the CP, T and CPT invariant part of the $\Delta S = \Delta Q$ amplitudes. We would like to repeat our remark [see Eq. (4.15)] about the implications of the Eberhard choice of the $K^0/\bar{K}^0$ relative phase on the meaning of our φ parameters. One should also note that not all the Wolfenstein parameters introduced in Eq. (4.26) are experimentally accessible. The only measurable amplitude phases are those of the bilinear combinations F^*G and $\bar{f}^*g$, so that one can study the implications of different symmetry operations on only the relative phases of F and G , and of $\bar{f}$ and g ; the absolute phases of F and G (and similarly of $\bar{f}$ and g) are not measurable. Without any loss of generality, therefore, one could choose F and $\bar{f}$ to be real or $\varphi_f = \alpha_f = 0$, because the only measurable combinations involving φ_f and α_f are $(\varphi_g - X\varphi_f)$ and $(\alpha_g - X\alpha_f)$; this would also reduce the number of parameters needed in a phenomenological analysis of K_{l3} data by two. We shall use the symbols

$$\begin{aligned}\varphi_G &= \varphi_g - X\varphi_f \quad \text{and} \\ \alpha_G &= \alpha_g - X\alpha_f.\end{aligned}\tag{4.27}$$

The parameter $\text{Im } x$ of usual CPT invariant ($\beta_f = 0, \beta_g = 0, \alpha_G = 0$) analyses corresponds to φ_G . Our purpose is to quote limits on the parameters φ, α, β and X using the mass matrix limits of Section 4.A and the available data (see also [16]). We first express the relevant experimental quantities in terms of the parametrization of Eq. (4.26).

If the fractional number of K^0 ($\bar{K}^0$) mesons decaying per unit time into any channel k is

$$R_{\pm}^k(t) = a_{\pm}^k e^{-Y_s t} + b_{\pm}^k e^{-Y_l t} + 2 \operatorname{Re} [c_{\pm}^k e^{it(\lambda_l^* - \lambda_s)}], \quad (4.28)$$

the subscripts + and - referring to initial K^0 and $\bar{K}^0$ states respectively, one can get information about invariance parameters by considering the coefficients a, b and c for both the ℓ^+ and ℓ^- channels. Using Eq. (4.4), one gets [1,29]

$$\begin{aligned} [a_+^k / a_-^k] &\simeq 1 - 2(\gamma + \beta), \\ [b_+^k / b_-^k] &\simeq 1 - 2(\gamma - \beta), \\ [c_+^k / c_-^k] &\simeq -(1 - 2\gamma + 2i\theta); \end{aligned} \quad (4.29)$$

and, therefore, we shall consider only positive subscripts. The superscripts + and - refer to ℓ^+ and ℓ^- channels, respectively. One gets

$$\begin{aligned} 4 \begin{pmatrix} a_+^+ \\ b_+^+ \end{pmatrix} &= f^2 [(1 \mp 2\beta + 2\beta_f) + X^2(1 - 2\gamma + 2\beta_f) + 2(\beta_f - X\beta_f)(X \pm 1) \pm \\ &\quad \pm 2X(1 - \gamma \mp \beta + 2\beta_f)], \\ 4 \begin{pmatrix} a_+^- \\ b_+^- \end{pmatrix} &= f^2 [X^2(1 \mp 2\beta + 2\beta_f) + 2(\beta_f - X\beta_f)(X \pm 1) + (1 - 2\gamma + 2\beta_f) - \\ &\quad - 4(1 \pm X)(\beta_f \pm \beta_g) \pm 2X(1 - \gamma \mp \beta + 2\beta_f)], \\ 4 \operatorname{Im} c_+^+ &= f^2 [2\theta + 2(\alpha_G + \varphi_G)], \\ 4 \operatorname{Im} c_+^- &= f^2 [2\theta X^2 - 2(\alpha_G - \varphi_G)], \\ 4 \operatorname{Re} c_+^+ &= f^2 [1 + 2(\beta_f - X\beta_g) + 2\beta X - (1 - 2\gamma)X^2], \\ 4 \operatorname{Re} c_+^- &= f^2 [X^2 + 2X(\beta - \beta_g) - (1 - 2\gamma - 2\beta_f)]. \end{aligned} \quad (4.30)$$

Charge asymmetry in $K_L \rightarrow \pi \ell \nu$ decay

Using the data [3,30] on the K_L charge asymmetry $\mathcal{A}$,

$$\mathcal{A} = \frac{b_+^+ - b_+^-}{b_+^+ + b_+^-}, \quad (4.31)$$

and the limits of Section 4 A, one gets ¹³⁾ [16] for the electron and muon channels respectively

$$\begin{aligned} 10^2 |(\beta_f - \beta_g) \cdot (1 - X)|_{\mu, e} &\lesssim 1.1, 1.0 \\ \text{or } |\beta_f - \beta_g|_{\mu, e} &\lesssim 0.011, 0.010 \end{aligned} \quad (4.32)$$

if one neglects second and higher orders in the non-invariance parameters, and also [26] neglects X as compared to 1. Because of the definitions (4.26), the limit (4.32) says that the T invariant part of the difference of the $\Delta S = \Delta Q$ and $\Delta S = -\Delta Q$ averaged amplitudes in K_{L3} decay is CPT (and CP) invariant to an accuracy of $\sim 1\%$. If the $\Delta S = \Delta Q$ rule holds, $\beta_g = 0$ and the limit (4.32) refers to the $\Delta S = \Delta Q$ amplitude only.

Charge asymmetry behind a thick regenerator

If one starts with a coherent mixture

$$|\psi\rangle = |K_L\rangle + |\rho| \cdot e^{i\phi_\rho} |K_S\rangle \quad (4.33)$$

of $|K_S\rangle$ and $|K_L\rangle$ states instead of a pure $|K_L\rangle$, the time-dependent asymmetry can be worked out in terms of $|\rho| e^{i\phi_\rho}$, Eqs. (4.30), and the structure parameters. For the charge asymmetry

$$\delta(t) = \frac{\text{No: of } l^+ \text{ Decays} - \text{No: of } l^- \text{ Decays}}{\text{No: of } l^+ \text{ Decays} + \text{No: of } l^- \text{ Decays}} \quad (4.34)$$

behind a thick regenerator such that the invariance parameters $|\gamma + \beta|$, $|\beta_f - \beta_g| \ll \ll |\rho| \ll 1$, one gets

$$\delta(t) \cong 2|\rho| \left\{ \cos(\phi_\rho + t(m_L - m_S)) \cdot [(1 - 2\beta_f) \cdot (1 - X^2)] - \sin(\phi_\rho + t(m_L - m_S)) \cdot 2\alpha_G \right\} \left[\frac{e^{-t(\gamma_S - \gamma_L)/2}}{(1 - 2\beta_f) \cdot (1 - X)^2} \right] \cdot \quad (4.35)$$

If one neglects the $\sin(\phi_\rho + t(m_L - m_S))$ term because its relative strength is of first order in CPT non-invariance and also in T non-invariance, one gets

$$\frac{1 - X^2}{(1 - X)^2} = \frac{1 + X}{1 - X} = 0.96 \pm 0.05 \quad (4.36)$$

from the " $\Delta S - \Delta Q$ factor" measurement [3] of Ref. [31]. This gives, using the smallness [26] of X ,

$$2X \cong -0.04 \pm 0.05 \quad (4.37)$$

implying that the CP, T and CPT invariant part of the $\Delta S = -\Delta Q$ amplitude is less than $\sim 5\%$ as strong as the corresponding $\Delta S = \Delta Q$ amplitude.

Analysis of Schubert et al.

Some further limits on the non-invariance parameters can be deduced from the analysis of the K_{e3} data of Webber et al. [32], done by Schubert et al. [15]. Their equations give, in our notation,

$$\frac{\text{Re } c_-^- + \text{Re } c_-^+}{\text{Re } c_-^- - \text{Re } c_-^+} = \frac{0.46 \pm 0.40}{1.92 \pm 0.10} ,$$

$$\frac{\text{Im } c_-^- + \text{Im } c_-^+}{\text{Re } c_-^- - \text{Re } c_-^+} = \frac{0.17 \pm 0.64}{1.92 \pm 0.10} ,$$

$$\frac{\text{Im } c_-^+ - \text{Im } c_-^-}{\text{Re } c_-^- - \text{Re } c_-^+} = \frac{0.0 \pm 0.33}{1.92 \pm 0.10} ,$$

and

$$\frac{a_-^+ - a_-^-}{\text{Re } c_-^- - \text{Re } c_-^+} = \frac{-0.88 \pm 1.12}{1.92 \pm 0.10} , \quad (4.38)$$

where the last equation has been derived by using their analysis. Equations (4.29) and (4.30) help one to express $a_-^\pm$ and $c_-^\pm$ in terms of the various non-invariance parameters. Using the limits of Section 4 A, and the smallness of X , Eqs. (4.38) lead respectively to the following limits

$$\beta_g X - \beta_f \simeq 0.12 \pm 0.11 , \quad (4.39.a)$$

$$\varphi_G \simeq -0.045 \pm 0.18 , \quad (4.39.b)$$

$$\alpha_G \simeq 0.0 \pm 0.09 , \quad (4.39.c)$$

$$\beta_g + \beta_f \simeq 0.25 \pm 0.3 . \quad (4.39.d)$$

Unfortunately, the errors on the right-hand side of Eqs. (4.39) are too large to draw definite conclusions. For example, Eqs. (4.39.b,c) are consistent with T invariance ($\alpha_G = 0$, $\varphi_G = 0$), CP invariance ($\varphi_G = 0$) and CPT invariance ($\alpha_G = 0$). Better limits on φ_G come from other CPT invariant analyses of $K_{\ell 3}$ decays. A recent compilation [26] gives the average value as

$$\varphi_G = -0.037 \pm 0.043 . \quad (4.40)$$

The need for future experimental analyses without assuming any invariance is obvious for obtaining better limits and to be able to decide in favour of one or the other symmetry operation.

Some other pieces of information which could lead to limits for decay amplitudes are available. Since the experimental errors are generally large, we shall not derive limits on the relevant invariance parameters, but only mention some of these data : the transverse polarization of the muon in $\pi\mu\nu$ decay of K_L and the charge asymmetry in the $\pi^+\pi^-\pi^0$ decay of the K_L are two interesting ones.

5. - DISCUSSION

After a brief survey of the relevant general aspects of neutral kaon phenomenology (in Section 2) and of the requirements of CP, T and CPT invariance (in Section 3), we obtained limits for these invariances in neutral kaon decays in Section 4 for A) the mass matrix, and B) the amplitudes for decay into the $\pi\pi$, the $\pi\mu\nu$ and the $\pi e\nu$ channels. The results were given in terms of phenomenological parameters which would vanish if one (or more) of these three invariances hold. These limits are stated in equations (or inequalities) (4.2), (4.3), (4.8), (4.9), (4.22), (4.25), (4.32), (4.37), (4.39). In general, the better amplitude limits arose by using the parameters (η_{+-} , η_{00} and K_L charge asymmetry) having small experimental values.

The existing limits ¹⁴⁾ for the structure meant that the contribution of the $|\Delta S| = 1$ weak interactions to the total mass matrix Δ in the $K^0 - \bar{K}^0$ representation is CPT (and CP) invariant to an accuracy of $\sim 1\%$; the corresponding numbers for the lower and upper limits referring to T (and CP) non-invariance are $\sim 0.04\%$ and $\sim 0.25\%$ respectively. At present, the limits on the invariance parameters for the sums of virtual and of physical transitions are about the same (for CPT and similarly, in the Eberhard convention, for T) using the empirical facts $2(m_L - m_S) \simeq Y_S$ and $Y_S \gg Y_L$.

The T invariant part of the $I = 0$, $\pi\pi$ decay amplitude is found to be CPT (and CP) invariant [16] to an accuracy of $\sim 1\%$ and the CPT invariant part of this amplitude is T (and CP) invariant to an accuracy of $\sim 0.4\%$ in the Eberhard phase convention. The corresponding numbers for the $I = 2$, $\pi\pi$ channel are, respectively, $\sim 5\%$ and $\sim 4\%$. The T invariant part of the difference of the $\Delta S = \Delta Q$ and the $\Delta S = -\Delta Q$ averaged amplitudes for the $\pi e \nu$ (and similarly $\pi \mu \nu$) channel is found to be CPT (and CP) invariant [16] to an accuracy of $\sim 1\%$. The CP, T and CPT invariant part of the averaged $\Delta S = -\Delta Q$ amplitude is found to have a relative strength of at most about 5% with respect to the corresponding $\Delta S = \Delta Q$ amplitude. One can similarly interpret the other limits, but the large errors do not allow very useful statements.

One can expect the amplitude limits given above to have about one significant figure accuracy. In fact, as stated at the appropriate places, these limits should be conservative estimates of their true values based on the present data. In particular, the present indications that $|\eta_{00}|$ might equal $|\eta_{+-}|$ would considerably improve the limits for the $\pi\pi$ channel, as stated in footnote 12).

The physical importance of the different limits has been discussed by many authors (see, for example, Refs. [1,8,20,29,33]). The total mass matrix Λ gets contributions [1,8] from both physical (energy-conserving) transitions (in Γ) and virtual (energy non-conserving) ones along with "self-transitions" (in M). In order to know whether it is the reactive effect of the physical transitions (the Γ matrix) or the M matrix (or both) that is non-invariant, one needs complete information on the invariance parameters of the matrix Λ . At present, the limit on β is much poorer than that on the other CPT (and CP) parameter θ ; in fact, the number for β dominates the limit $0.01 Y_S$ given in (4.10) for $(\Lambda_{11}-\Lambda_{22})$, the difference of the diagonal matrix elements relating to CPT (and CP) invariance. Even if one completely knows the decay matrix Γ , it would not reveal very much information about the weakly coupled channels like $\pi\pi\pi$ because of dominance by the $\pi\pi$ channel. One should look at the individual decay amplitudes in order to study the invariance properties of the minor channels; for example, it would be hard to get a limit as good as $\sim 1\%$ (obtained above by considering the K_L charge asymmetry) for the $\pi l \nu$ channel at present by considering only the mass matrix Λ . In fact, especially among the minor channels, there could be compensatory cancellations (between, for example, $\pi e \nu$ and $\pi \mu \nu$) to produce a sum (in Λ) which is invariant under a certain symmetry operation, though the individual contributions are not.

Of course, in superweak models [8,18] one has CP non-invariance due to either $M_{11} \neq M_{22}$ (CPT non-invariance) or $\text{Im}(M_{12})_S \neq 0$ (T non-invariance) or both. All individual amplitudes are CP, T and CPT invariant - so is the decay matrix Γ ($\Gamma_{11} = \Gamma_{22}$, $\text{Im}(\Gamma_{12})_S = 0$). In such a situation ¹⁵⁾, the true limits would be much smaller than those quoted above in Section 4 B.

The amplitude limits contain more information than summed contributions in Γ also from another point of view: while CP invariance of the matrix Γ (or M) implies its T and CPT invariance, it is not necessarily so for amplitudes. In the notation of Eqs. (4.14), (4.26), there are the amplitude parameters α of which non-zero values are consistent with CP invariance, but not with CPT - or with T - invariance. These amplitudes would not contribute to non-invariance of the mass matrix Λ in the lowest order, while the other two types, of course, do: β 's to the diagonal elements and φ 's to the off-diagonal ones. With the available data, the limits on the parameters α for the 2π channel and for the $\pi l \nu$ channels are rather poor. The difficulty of detecting CPT non-invariance without simultaneous CP non-invariance has been emphasized before (see for example Ref. [33]); this corresponds to differentiating between T non-invariant effects which are CPT invariant (the parameters φ) from those which are CP invariant (the parameters α).

One should worry about possible changes in the above amplitude limits due to isospin non-invariance of the strong interactions, especially for the $\pi\pi$ channel. If isospin invariance is not good for $\pi\pi$ scattering, one should reformulate $K \rightarrow 2\pi$ decays in terms of the true eigenstates instead of the $I = 0$ and $I = 2$ states. If, however, the departures from isospin invariance are small, the isospin eigenstates would be close to the true eigenstates and the amplitude limits quoted above would not be significantly affected because the rather large errors that have been allowed in $|X_2|$ and $(\delta_2 - \delta_0)$ could cover the effective changes in the Clebsch-Gordan coefficients and in the phase shifts.

For some channels, the structure limits contribute significantly to the amplitude limits of the previous Section ; for example, about 80% of the CPT (and CP) limit (4.32) for the $\pi l \nu$ channel arises from the limit (4.8) for the mass matrix. For the corresponding channels ¹⁶⁾ in charged kaon decays, one should then expect quite small CPT (and/or CP) non-invariant asymmetries between K^+ and K^- decays because of the absence of the structure effects in the $K^\pm$ case. In the superweak situation where the only CP non-invariance is in the partial mass matrix M , there should be no such asymmetry.

For the total mass matrix Λ , if the M matrix is the only source of non-invariance, one should find the upper limits on invariance parameters for all amplitudes to be very small, apart from the effect (ϕ) of our choice for the $K^0/\bar{K}^0$ relative phase. The phases of the structure parameters ϵ and δ (of Section 3 C) would be respectively $\sim \pi/4$ and $\sim 3\pi/4$ in that situation. If, on the other hand, the decay matrix Γ is making the structure non-invariant, this should show up in the lower limit on invariance parameters for at least some amplitude(s) being correspondingly large. If the only non-invariance is in virtual transitions or self-transitions (the M matrix), one would be in the happy situation [8] where CP, T and CPT invariance would be good everywhere except in neutral kaon decays.

One may note that it is difficult to pin down completely the origin of a non-invariance on the basis of only the phenomenological limits derived from effective amplitudes or from the mass matrix Λ in the sense that one would not know whether the blame should be on the strong or the electromagnetic or the weak interactions contributing to a given amplitude or a given element of Λ . To sort this out, one should study also the appropriate strong and electromagnetic interaction processes.

5. - CONCLUDING REMARKS

Non-invariance under the discrete symmetries CP, T and CPT can appear in two ways in neutral kaon decays : in the structure (i.e., the relation of K_L and K_S to K^0 and $\bar{K}^0$), or in the decay amplitudes. Because any of these symmetries is a combination of the other two, it is not possible to violate only one at a time ; at least two must be violated. The only established ¹⁷⁾ violation is in the structure, by a term which may be classified as

$$(\cancel{CP}, \mp, CPT^{\checkmark}) \quad (6.1)$$

i.e., as respecting CPT invariance, but one that should vanish if CP invariance and/or T invariance holds. This is consistent, in particular, with the superweak model of Wolfenstein [18]. Observations on η_{+-} and η_{00} and on the charge asymmetry in the $\pi l \nu$ decays of the long-lived kaon are also consistent with this model.

But it would be quite premature to conclude that this is the end of the discrete - symmetry - violation story. For example, the superweak result $\eta_{+-} = \eta_{00}$ (for which there is experimental indication) can arise even if the decay amplitudes do not obey the superweak requirements. This result requires only the equality ¹⁸⁾ of the behaviour of the decay amplitudes into the $I = 0$ and the $I = 2$ channels under the CPT (and CP) transformation for the T invariant part ; and similarly, under the T (and CP) transformation for the CPT invariant part. Also, the common phase of η_{+-} and η_{00} could be arranged to have the superweak value. On the other hand, the superweak model would require the decay amplitudes to be CP, T and CPT invariant. In order to eliminate the possibility of conspiracy between the $I = 0$ and the $I = 2$ channels to produce the result $\eta_{+-} = \eta_{00}$ even though the individual amplitudes do not obey the superweak model, one should improve the limits on the parameters (β_I and φ_I), especially for the $I = 2$, $\pi\pi$ channel. This requires a better number for η_{00} , a better knowledge of the relative strength of the CP, T and CPT invariant part of the $I = 2$ amplitude, a more precise value for the relevant $\pi\pi$ phase shifts and also a better limit on the structure parameter $\gamma + \beta$ (see below for experiments).

In particular, the superweak model would forbid the presence of amplitudes with the character

$$(CP^{\checkmark}, \mp, \cancel{CPT}) \quad (6.2)$$

i.e., respecting CP invariance, but which should vanish if one or both of the other two symmetries hold. We have seen that the bounds on the amplitude parameters (α_I) which represent amplitudes of the type (6.2) are very poor. This has its origin in the fact that the K_L system is nicely designed to show up CP violations, because CP conserving terms largely cancel or appear only in higher orders of smallness. It is for the same

reason that the K_L system is poorly adapted to the detection of CP invariant terms like (6.2), and we have no comparably delicate way of seeing such amplitudes. These remarks are especially relevant for the $\pi\pi$ channel.

For exactly the same reason, it is not correct to think of CPT symmetry as being more firmly established empirically than CP and T ; it is not. It follows from the limit (4.25) that there could be present in $K_S \rightarrow \pi\pi$ decay, an amplitude of the type (6.2) (which should vanish if CPT invariance holds) having a strength some two orders of magnitude larger than the fairly well-established amplitude of the type (6.1) [which should vanish if T (and CP) invariance holds] in $K_L \rightarrow \pi\pi$ decay. Moreover, the limit (4.22.a) shows that there could be an amplitude of the type

$$(\text{CP}, T, \text{CPT}) \quad (6.3)$$

which is much stronger than the corresponding CPT conserving amplitude of the type (6.1). The situation is not so clear for the other decay channels. For the structure also, the upper limit on contributions of the type (6.1) is better than on contributions of the type (6.3), though there are indications that the lower limit is non-zero only in the case of the former type. The structure is not sensitive to contributions of the type (6.2). So, one can conclude that although CP non-invariance is well established, and there are strong indications that T invariance is not good for the structure, and we have no indication against CPT invariance, yet CPT invariance has been verified experimentally to a poorer accuracy than the other two symmetries. In this sense, CPT invariance is less well-established than CP and T invariances.

It is therefore important to improve the present limits. In particular, the present upper limit on the structure parameter β referring to CPT invariance is worse than the corresponding bounds on the other CPT structure parameter (θ) and the T structure parameter ($\frac{1}{2}\gamma$) by about an order of magnitude. This parameter β contributes significantly also to the present limits for amplitudes of the type (6.3). A possible method to improve on β is to improve the accuracy for the measurement of the relative intensity of the K_S component in K^0 and $\bar{K}^0$ states through the processes

$$\begin{aligned} \bar{p}p &\rightarrow K^+ \bar{K}^0 \pi^-, \text{ and} \\ \bar{p}p &\rightarrow K^- K^0 \pi^+, \end{aligned}$$

the K_S component being detected by its $\pi^+\pi^-$ mode. The present experiments give this relative intensity to be 1.00 ± 0.03 ; one should aim at about an order of magnitude improvement (or more). This measurement gives the combination $(\gamma + \beta)$, as in Eq. (4.5). If one could similarly measure the relative intensity of the K_L component in K^0 and $\bar{K}^0$ states, one gets $(\gamma - \beta)$, as in Eq. (4.6). Combining the two, one gets β . Since one does not have a decay channel where the K_L is known to dominate strongly over the K_S , the measurement of the K_L component may be harder. One may

therefore combine the K_S measurement which gives $(\gamma + \beta)$ with the information on γ from evaluations of the unitarity relation ; unfortunately, this latter step involves assumptions about contributions to the unitarity relation from unknown and poorly known channels.

In fact, these measurements of the K_S (and similarly K_L) components in K^0 and $\bar{K}^0$ states can also be done by comparing the time distributions for decay into any channel k ; the ratio of the term characterized by the time dependence $e^{-\gamma_S t}$ gives

$$a_+^k / a_-^k \cong 1 - 2(\gamma + \beta) \quad (6.4)$$

where $a_{\pm}^k$ are the coefficients of this term for beams that were originally K^0 and $\bar{K}^0$ respectively. Similarly, the ratio for the terms with the time dependence $e^{-\gamma_L t}$ gives

$$b_+^k / b_-^k \cong 1 - 2(\gamma - \beta), \quad (6.5)$$

as mentioned in Eq. (4.29). If the $K_S \rightarrow k$ decay dominates $K_L \rightarrow k$ decay, Eq. (6.4) is more useful ; if $K_L \rightarrow k$ dominates, Eq. (6.5) is more useful. One may similarly utilize the interference term in various ways. For example, for a channel like $\pi^+ \pi^-$ where $K_S \rightarrow k$ is dominant, one may determine β by determining the ratio of the coefficient c^k of the interference term to the coefficient a^k , as defined in Eq. (4.28) :

$$\frac{|c_+^k / a_+^k|^2}{|c_-^k / a_-^k|^2} \cong 1 + 4\beta.$$

For a channel like $\pi \ell \nu$ where the K_S and K_L rates are comparable, one has a chance of extracting the structure parameters by using Eq. (4.29) in many more ways.

An analysis of the time distribution of decay into the different charge modes of the $\pi \ell \nu$ channel without assuming CPT or CP or T invariance or the $\Delta S = \Delta Q$ rule cannot only give information on the structure parameters $(\gamma, \beta$ and $\theta)$, but can also improve on rather poor present limits on the amplitude parameters for this channel. Such an analysis is highly desirable ; it can also teach one about $\mu - e$ universality ; the relevant formulae for this analysis are given in Section 4.B.2.

Improvement of the present limits on the amplitude parameters (α_G for the $\pi \ell \nu$ channel and α_I for the $\pi \pi$ channel) of the type (6.2) should be quite easy for the $\pi \ell \nu$ channel if the general analysis just mentioned is carried out. Improvement of the present limit (4.25) for the $\pi \pi$ case would require a better

knowledge of the relevant $\pi\pi$ phase shift combination from strong interaction studies, a better knowledge of the relative strength of the CP, T and CPT invariant part of the $I = 2$ amplitude and, finally, a better number for the charged to neutral branching ratio in $K_S \rightarrow \pi\pi$ decay.

Experiments for modes other than $\pi\pi$ and $\pi\ell\nu$ in order to derive amplitude limits may be rather hard, but good measurements of the absolute decay rates into various channels would be very useful for a more reliable evaluation of the Bell-Steinberger unitarity relation.

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FOOTNOTES

- 1) Our treatment of Section 2 and Section 3 is based on the reviews quoted above, especially [1,2,4,5,9].
- 2) Some of these limits have been dealt with more fully in Ref. [16].
- 3) Our description follows that of Refs. [4,5] for this Section.
- 4) This is not true, in general, of decay amplitudes because of the possibility of CP invariant amplitudes which are neither CPT invariant, nor T invariant, as we shall see in Sections 4 and 5 below.
- 5) The Eberhard phase convention chooses the $K^0/\bar{K}^0$ relative phase by taking $\text{Im } \epsilon = 0$.
- 6) This is why the T invariant, but CPT non-invariant, theories of CP non-invariance (assuming the M matrix to be the only source of non-invariance) gave rise to a phase ($\sim 3\pi/4$) value in disagreement with the observed ($\sim \pi/4$) phase of η_{+-} [see Eq. (4.17) for definition]. One could have the phase value ($\sim \pi/4$) in a CPT non-invariant theory by taking $\Gamma_{11} \neq \Gamma_{22}$ to be the only source of CP non-invariance ($\epsilon = 0, M_{11} = M_{22}$). However, this would imply a corresponding amount of inequality between the magnitudes of the decay amplitudes from the K^0 and the $\bar{K}^0$ for the dominant channel ($\pi\pi, I = 0$). A way to avoid this has been considered in Ref. [19] by modifying the conventional unitarity relations (2.10) and (2.11) and putting the blame on the "possibility that the elementary particle system is not closed". This type of a modification of the unitarity relation seems to us less attractive than the Wolfenstein superweak model [18] which also produces the same phase for η_{+-} .
- 7) The contribution of a given channel to the unitarity relation (2.10) depends on the amount of CP non-invariance in that channel and on the K_L and the K_S decay rates into that channel. Contributions at a few percent level from channels other than $\pi\pi, \pi\pi\pi$ and $\pi\ell\nu$ are not ruled out by the present accuracy of the known experimental rates. However, it is quite permissible that decay rates into these "other" channels may be small enough to justify the omission of their contributions.
- 8) If one takes $|\eta_{00}| = |\eta_{+-}|$, the upper limits in (4.2) and (4.3) become about 2.2 and 3.9 respectively; and the lower limit in (4.3) becomes about 0.9.
- 9) In fact, this number arises dominantly from the contribution of the parameter β ; the corresponding contribution of the parameter θ is lower by about an order of magnitude.

- 10) This convention differs from that of Wu and Yang [23] in the choice of the $K^0/\bar{K}^0$ relative phase. Their choice corresponds to taking [see Eq. (4.14) below] $\varphi_0 + \alpha_0 = 0$, while Schubert et al. [15] take effectively $\varphi_0 = 0$. Our choice corresponds to taking $\text{Im } \epsilon$ of Ref. [15] to be zero.
- 11) In fact, the strength of the $|\Delta I| = 5/2$ amplitude can be expected to be $\sim 20\%$ of the observed $|\Delta I| = 3/2$ amplitude just from virtual electromagnetic effects alone [1].
- 12) In the approximation of 2π saturation of the unitarity sum, one can relate β_0 to $(\Gamma_{11} - \Gamma_{22})$. In that case, one expects $|\beta_0| \lesssim 10^{-3}$ in similarity to the case of $K^\pm$ decays [20]. Direct use of data gives the poorer limits (4.9) for β and the limit (4.22.a) for β_0 . In Wolfenstein's [20] notation, $(\frac{1}{2}\delta_K + \beta_0)$ and $(\frac{1}{2}\delta_K - \beta_0)$ represent our structure parameters θ and β respectively. With the limits (4.2) and (4.9), the present data imply $\frac{1}{2}\delta_K \simeq -\beta_0$, each being limited by the magnitude ~ 0.01 . The limits on β_2 and on $(\beta_g - \beta_f)$ (see Section 4.B.2) were considered and numbers similar to ours obtained also by Wolfenstein. However, he retained only the term proportional to $1/X_2$ in the expression [see Eq. (4.20)] for β_2 ; he also took $|\beta_0| \lesssim 10^{-3}$ in the derivation of his number for $(\beta_g - \beta_f)$ [see (4.32) below].

As implied above, our φ_0 should be equal to $\text{Im } \epsilon$ of Refs. [14, 15]. Our limit on φ_0 is worse because of our limit (4.2) on θ and of the larger ranges of $|\eta_{00}|$ (and $|\eta_{3\pi 0}|$) that we have allowed; see Section 4 A for further details.

If one takes $|\eta_{00}| = |\eta_{+-}|$, the limits (4.22) become :

$$\begin{aligned} -0.007 &\lesssim \beta_0 \lesssim 0.01 \\ -0.00009 &\lesssim \varphi_0 \lesssim 0.003 \\ -0.021 &\lesssim \beta_2 \lesssim 0.03 \\ -0.005 &\lesssim \varphi_2 \lesssim 0.014 \end{aligned}$$

These limits, as expected, are better than the ones in (4.22).

- 13) A very similar limit was obtained in Ref. [20]; see the footnote 12).
- 14) We are referring to (4.10) and (4.11) here.
- 15) Because of the Eberhard phase convention, our M_{12} and Γ_{12} are not the same as those with the superweak definitions. See Section 3 C for further details.
- 16) The most important channel ($I = 0, \pi\pi$) of this type is not relevant to $K^\pm$ decays.

- 17) "Established" on the assumptions that $|\eta_{3\pi^0}| \lesssim 2$ and that the decay modes other than $\pi\pi$, $\pi\ell\nu$ and 3π do not show gross CP non-invariant effects.
- 18) In the notation of Eq. (4.14), this means $\varphi_0 = \varphi_2$ and $\beta_0 = \beta_2$. One should have $\beta_0 = 0$, $\beta_2 = 0$, $\varphi_0 = \varphi_2 = \phi$ in the superweak situation.