Inflation, dark matter and dark energy in the string landscape

Andrew R. Liddle and L. Arturo Ureña-López

1 Astronomy Centre, University of Sussex, Brighton BN1 9QH, United Kingdom
2 Instituto de Física de la Universidad de Guanajuato, C.P. 37150, León, Guanajuato, México

(Dated: October 9, 2006)

We consider the conditions needed to unify the description of dark matter, dark energy and inflation in the context of the string landscape. We find that incomplete decay of the inflaton field gives the possibility that a single field is responsible for all three phenomena. By contrast, unifying dark matter and dark energy into a single field, separate from the inflaton, appears rather difficult.

PACS numbers: 98.80.-k, 98.80.Cq

INTRODUCTION

It has become clear recently that potentials of the form $V_0 + \frac{1}{2}m^2\phi^2$, where $V_0$ has the small value needed to explain the observed dark energy density, are plausibly motivated by a combination of the string landscape picture and the anthropic principle, and are not necessarily hopelessly fine-tuned as previously thought. The gist of the argument is that string theory contains a huge number of possible configurations of differing vacuum energy, which might be exhaustively explored throughout the very large scale Universe, for instance via a self-reproducing inflationary cosmology mechanism. The selection effect that we must live in a region of the Universe capable of forming stars and galaxies then enforces that we live in a region where $V_0$ is atypically small, but non-zero. With some caveats, this picture gives an impressive probabilistic prediction of the order of magnitude of the dark energy density.

Potentials of the form above are of interest as they offer the possibility of a unified description of various features of the Universe for which scalar fields have been invoked, specifically inflation, dark matter, and dark energy. The main ingredients to do this are already in the literature, though they have not been explicitly connected. In their work on post-inflationary preheating, Kofman et al. remarked that the inflaton decay might be incomplete, with the residue having the capability of acting as dark matter. Separately, Linde has noted that with $m \simeq 10^{-6}m_P$, the above potentials unify standard chaotic inflation (during which $V_0$ is utterly negligible) with dark energy. The precise form of the non-constant part of the potential is not of course crucial to this argument; any of the normal inflationary potentials will achieve the same once $V_0$ is added.

In this paper we wish to consider the possible additional unification of cold dark matter (CDM) with dark energy using such potentials. While usual particle dark matter candidates such as weakly-interacting massive particles (WIMPs) correspond to incoherent distributions of individual particles, it has long been known that an alternative CDM candidate is a coherently oscillating scalar field, the archetypal example being axion dark matter. Provided the potential is of quadratic form about its minimum, such a field behaves on average like pressureless matter, and is indistinguishable from traditional CDM candidates provided the oscillation period is much shorter than any other dynamical scale in the problem (true unless the field is super-light). Furthermore, it is known that linear perturbations in such a coherent field mimic those of a pressureless fluid, and that non-linear top-hat collapse proceeds in the same way. Such coherent scalar fields are therefore a well-developed alternative to the WIMP paradigm.

We do not aim to make any specific proposals for how such unified scenarios might arise from fundamental theories, but rather wish to explore what conditions would have to be met in order for such scenarios to be compatible with observations. We explore two types of scenario:

1. Unification of inflation, dark matter and dark energy into the same scalar field $\phi$.
2. Unification of dark matter and dark energy into a single scalar field $\phi$, with inflation provided by a separate scalar field $\psi$.

The main conditions that will concern us is whether a complete history of the Universe from inflation onwards can be constructed, with the fields taking plausible values, and whether perturbations can be generated that are compatible with the observation that isocurvature perturbations, if present at all, are subdominant to adiabatic ones.

There have been many attempts to use scalar fields to unify combinations of inflation, dark matter, and dark energy. For instance, Ref. proposed a tachyon-type scalar-field Lagrangian, in which the scalar fluid can be broken up into dark matter and dark energy components. A k-essence unification of dark matter and dark energy was given in Ref. Staying instead with the canonical Lagrangian, Ref. introduced a complex scalar field...
field with a mixed potential made of quadratic and exponential terms, which then mimic dark matter and dark energy, respectively, at the scales of interest. Unification scenarios featuring inflation include quintessential inflation (unifying inflation+dark energy) \[12\], inflaton+dark matter in the braneworld scenario \[13\], and braneworld inflaton+dark matter+cosmological constant from multiple fields in a type IIB supergravity theory \[14\]. Our proposal is a simpler one than any of those listed above, with Ref. \[14\] being the closest.

Scalar fields in quadratic potentials have a generic evolution. Initially, while \( m \ll H \) (where \( H \) is the Hubble parameter), the scalar field is frozen by the friction of the expanding Universe and remains constant, corresponding to a constant energy density. If at that time the field is the dominant energy density in the Universe it will drive inflation. Once the scalar field begins to oscillate, and its time-averaged evolution has density \( \rho_\phi \) falling as \( 1/a^3 \), exactly as CDM \[7\]. The normalization of the density is determined by the initial amplitude of the scalar field oscillations, as follows.

In order to recover the standard dark matter scenario, the scalar mass should satisfy \( m \gg H_{\text{eq}} \), where \( H_{\text{eq}} \) is the Hubble parameter at the time of radiation and matter equality, so the oscillations of the field begin well within the radiation-dominated era. If we denote by \( t_* \) the time at which the scalar mass equals the Hubble parameter, \( m = H_* \), then \( m^2 = 8\pi \rho_{\text{m}}/3m_{\text{Pl}}^2 \), where \( \rho_{\text{m}} \) is energy density of relativistic matter and \( m_{\text{Pl}} \) is the Planck mass. The photon density is related to the total radiation density by \( \rho_{\text{r}} = (g/2) \rho_\gamma \), where \( g \) is the total number of relativistic particle degrees of freedom \[7\].

The averaged scalar field energy density is given by \( \rho_\phi = \frac{1}{2} m^2 \phi^2 \Delta^2 / a^3 \) for \( t > t_* \); here \( \phi_* \) is the initial scalar field amplitude at \( t_* \). We define the scalar field dark matter mass per photon as \( \xi_\text{dm} = \rho_\phi / n_* \). This quantity is constant for \( t_* < t \) apart from changes in the number of relativistic species; we assume expansion at constant entropy implying that \( \xi / g_\text{S} \) remains constant where \( g_\text{S} \) is the entropic degrees of freedom, usually very similar to \( g \) \[7\]. Using ‘0’ to indicate present values, the present scalar field dark matter mass per photon is then

\[
\xi_{\text{dm},0} \simeq 4 \frac{g_{\text{S},0}^{1/4}}{g^{1/2}} \left( \frac{m}{m_{\text{Pl}}} \right)^{1/2} \frac{\phi_*^2}{m_{\text{Pl}}^2} m_{\text{Pl}}. \tag{1}
\]

The measured value of the current dark matter mass per photon is \( \xi_{\text{dm},0} = 2.2 \times 10^{-28} m_{\text{Pl}} \) using values from WMAP3 \[12\], which for typical values \( g_* \simeq 100 \), \( g_{\text{S},0} = 3.9 \) then gives the following constraint

\[
\left( \frac{m}{m_{\text{Pl}}} \right)^{1/2} \frac{\phi_*^2}{m_{\text{Pl}}^2} \simeq 4 \times 10^{-29}. \tag{2}
\]

A lower limit can be placed on \( m \) from structure formation. The linearly-perturbed scalar field equation resembles a damped and forced harmonic oscillator. For scales with a comoving wavenumber \( k < a(t)m \), where \( a(t) \) is the scale factor, the field perturbation is in resonance with the force term. In this case, the field’s density contrast grows as that of CDM \[8\]. However, for \( k > a(t)m \) the perturbed field is out of phase with the force term, and then the perturbations are suppressed relative to the standard CDM case. The largest scale at which suppression occurs corresponds to the smallest scale factor; in our case that scale is \( k_\star = a_\star m \). Assuming the same conditions that led to Eq. \(1\), together with the restriction \( k_\star > 1 \text{Mpc}^{-1} \), we get the lower bound \( m/m_{\text{Pl}} > 7 \times 10^{-52} \), i.e. \( m > 10^{-23} \text{eV} \). Provided the field is significantly more massive than this, it will behave indistinguishably from standard CDM. If instead it more or less saturates this bound, the Compton wavelength of the particles may become comparable to astrophysical scales with observable consequences (see Refs. \[14, 17\] and references therein).

**TRIPLE UNIFICATION: INFLATION, DARK MATTER AND DARK ENERGY FROM A SINGLE FIELD**

In this section, we explore the conditions needed to unify all three phenomena — dark matter, dark energy, and inflation — into a single field. For simplicity we will assume that the quadratic form of the potential holds for all relevant \( \phi \) values, though other choices can be made.

The advantage of the single-field unified scenario is that the only perturbations generated during inflation are adiabatic, as that is the only type that a single field can support. Obtaining the correct amplitude of scalar primordial perturbations requires \( m/m_{\text{Pl}} \simeq 10^{-6} \), and the spectral index is independent of \( m \) and a good fit to WMAP3 data \[15\]. However at the end of inflation \( \phi \) is still of order \( m_{\text{Pl}} \). By contrast, Eq. \(2\) requires an initial amplitude for the dark matter oscillations of the order of \( \phi_* \simeq 10^{-1} m_{\text{Pl}} \). The main requirement for a working scenario therefore is a drastic but incomplete reduction of the amplitude of the inflaton oscillations during reheating, reducing the energy density of the inflaton field by a factor of about \( 10^{26} \). This is necessary to permit a long radiation-dominated epoch.

Such an incomplete decay indicates that the reheating mechanism should be via inflaton annihilations, rather than decays. This is, for instance, guaranteed to be true if the reflection symmetry of the inflaton potential is not spontaneously broken, as then only quadratic interaction terms are permitted. In such circumstances it is generally true that there will be some residual inflaton density left over, because once the density becomes low enough the particles are no longer able to ‘find’ each other to annihilate \[8\]. The question is whether a mechanism can be found which reduces the inflaton density by the amount required by the considerations above.

The two main paradigms for conversion of the inflaton into other matter are preheating (coherent multi-particle decays) and reheating (single particle decays), which may happen in sequence. Some mechanisms for the decay of the inflaton have been proposed in the literature, see for instance Refs. [4, 8, 13, 15, 21, 22] and references therein. The relevance of (p)reheating to unification scenarios has been discussed in Ref. [22].

The conventional reheating mechanism, corresponding to single-particle decays, adds a constant decay width $\Gamma_{\phi}$ to the inflaton equation of motion [7, 18] in the form

$$\rho_{\phi} + 3H\dot{\phi}^2 = -\Gamma_{\phi}\phi^2.$$  \hspace{1cm} (3)

Eq. (3) implies the usual exponential decay law for scalar particles which are linearly coupled to other bosons and fermions [7, 22]. It proceeds once $\Gamma_{\phi} \gg H$, and leads to complete decay of the scalar field. Conventional reheating can play an important role in reducing the inflaton energy density as required by the triple unification scenario, but needs modification to prevent the decay being complete.

By contrast, preheating offers a mechanism for rapid but incomplete decay of the inflaton field, provided the inflaton is coupled to another scalar field $\chi$ through simple four-legs interactions of the form $g^2\phi^2\chi^2$, where $g$ is the coupling constant. That this could make the inflaton field a dark matter candidate was first noted by Kofman et al. [5], though they did not evaluate in detail the conditions needed to realize this. Further analysis of the scenario can be found in Refs. [12, 20, 21, 24].

The conclusion of that work is that the decay is indeed incomplete, with preheating coming to an end once the amplitude of the inflaton oscillations becomes smaller than $m/g$ [3, 10]. While this does give a large reduction provided $g$ is not too small, the amplitude required for CDM, Eq. (4), is $\phi_* \sim 10^{-7}m$, and hence we would need $g \sim 10^7$. Such a non-perturbatively large coupling is unattractive, even if supersymmetry is invoked to cancel radiative corrections [20]. A further problem [20] is that the density of $\chi$-particles produced may be less than that of incoherent inflaton particles, which prevents generation of a satisfactory radiation-dominated era.

The efficiency of preheating is enhanced if one includes a linear (three-leg) coupling between the inflaton field and other bosonic and fermionic fields [14, 20]. This however makes the inflaton field decay completely [21], contrary to our aim.

In conclusion, preheating sets the precedent of incomplete inflaton decay, but existing models do not satisfy the conditions needed by the triple unification scenario; quadratic interactions give too little decay and linear ones too much. It may therefore be necessary to exploit annihilations via perturbative interactions. As a simple toy model, consider Eq. (4) but with the decay width now allowed to depend on the scalar field density; for instance $\Gamma_{\phi} \propto \rho_{\phi}$ corresponds to two-body annihilations (‘decay’ rate proportional to the local density). This alone is insufficient as the annihilations would be important during inflation; a viable form would be

$$\Gamma_{\phi} = \frac{\rho_{\phi}/\rho_c}{1 + \rho_{\phi}/\rho_c} \cdot (4)$$

which makes a smooth transition from single particle decays to two-body annihilations as $\rho_{\phi}$ reduces. With suitable tuning of the constants $\Gamma_0$ and $\rho_c$, a viable scenario can be constructed, though this form of the decay width is not motivated by any fundamental considerations.

To end this section, we note that it is by no means essential for the potential to take the form $V_0 + \frac{1}{2}m^2\phi^2$ all the way up to the $\phi$ values responsible for inflation; see for instance Refs. [17, 23]. The unification of dark matter and dark energy only requires that it is of this form for very small $\phi$. Indeed, in the context of the string landscape, and bearing in mind the spectral index measurements from WMAP3, it may be more natural that inflation takes place near a maximum of the potential [20], perhaps with initial conditions fixed by the topological inflation mechanism [27]. Clearly it would be interesting to explore incomplete decay mechanisms for a range of inflationary potentials.

UNIFICATION OF DARK MATTER AND DARK ENERGY INTO A SINGLE FIELD

In this section we consider what appears to be a less ambitious scenario, where only dark matter and dark energy are unified by the $\phi$ field, with some other field responsible for inflation. This has some similarities to the curvaton scenario, but is more restrictive since the $\phi$ field is the dark matter, rather than decaying into the dark matter. The mass of the field is now not directly determined by the perturbation normalization, and instead should have a sufficiently small value to avoid interfering with the inflaton, $m^2\phi_*^2 \ll m_{\text{pl}}^2H^2$. In fact, this scenario proves rather hard to achieve.

The first problem encountered by such scenarios would be to explain the small value of $\phi_*$ required by Eq. (2), since there appears no reason why the field should be so close to its minimum. Furthermore, this initial condition must not be spoiled by quantum fluctuations induced in $\phi$ during inflation, which are of order $H/2\pi$ per e-folding. Indeed these fluctuations must be small enough that the primordial CDM perturbation does not exceed the observed $10^{-5}$ value. This imposes the tight condition

$$\frac{\delta\phi}{\phi_*} \lesssim \frac{H}{2\pi\phi_*} \lesssim 10^{-5}.$$  \hspace{1cm} (5)

Inflation generates the correct amplitude of perturbations provided $H/m_{\text{pl}} \simeq 10^{-4}\epsilon^{1/2}$, where $\epsilon < 1$ is the slow-roll parameter for the inflaton. Since the observable
perturbations may come from $\phi$ rather than the inflaton, this is an upper limit on $H$. The combination of these constraints with Eq. 2 gives the powerful limit $m \lesssim \epsilon^{-2} \times 5 \times 10^{-30} \text{eV}$. For a viable scenario satisfying the lower mass limit from structure formation quoted earlier, this forces $\epsilon$, and hence the inflationary energy scale, to be very low, and even then the scalar mass is forced to be extremely light. Additionally, the appropriate initial value of $\phi_i$ must arise by accident (this is also a feature of the curvaton scenario).

Even if these circumstances are satisfied, there is a further problem that perturbations in the CDM arise from separate fluctuations to those in the baryon–photon fluid. They are therefore of isocurvature form, which is highly disfavoured by data if the adiabatic perturbations are negligible. This issue has typically been ignored in previous attempts to unify dark matter and dark energy. Perhaps the scenario can be saved by allowing the inflaton perturbations to be non-negligible, thus giving a mixture of adiabatic and (partially correlated) isocurvature perturbations, but previous studies are not encouraging. Another possible escape would be if the $\phi$ field is at least partly excited by the inflaton decay, giving it an adiabatic perturbation (c.f. the mention of trace decoupled CDM in curvaton decays in Ref. 24). Note this must be a coherent excitation generating a universal mean value $\bar{\phi}$ about which small perturbations are superimposed via the adiabatic perturbations, requiring a breaking of the potential’s reflection symmetry in the interactions between the inflaton and $\phi$.

CONCLUSIONS

We have examined the possibility that ideas coming from the string landscape can unify various key aspects of cosmology into a single field, specifically inflation, dark matter and dark energy. We have not been able to be very specific in terms of particle physics models, but we have investigated the general conditions necessary to bring about such a unification. Curiously, scenarios unifying all three phenomena appear to be easier to realize than those which keep a separate inflaton. The key ingredient required to make such scenarios a reality is partial, nearly complete, decay of the inflaton into the baryon–radiation fluid, so that the residual decoupled component can survive as dark matter and dark energy. Preheating scenarios may offer such a possibility.

A.R.L. was supported by PPARC and L.A.U.-L. by CONACYT (42748, 46195, 47641), CONCYTEG 05-16-K117-032, DINPO and PROMEP-UGTO-CA-3. A visit to Sussex by L.A.U.-L., funded by the Royal Society and the Academia Mexicana de Ciencias, initiated this work. We thank Ed Copeland, James Lidsey, Tonatiuh Matos, Cédric Palud, and David Wands for discussions.