Earth Matter Effects in Detection of Supernova Neutrinos

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Abstract

We calculated the matter effect, including both the Earth and supernova, on the detection of neutrinos from type II supernovae at the proposed Daya Bay reactor neutrino experiment. It is found that apart from the dependence on the flip probability $P_H$ inside the supernova and the mass hierarchy of neutrinos, the amount of the Earth matter effect depends on the direction of the incoming supernova neutrinos, and reaches the biggest value when the incident angle of neutrinos is around $93^\circ$. In the reaction channel $\bar{\nu}_e + p \rightarrow e^+ + n$ the Earth matter effect can be as big as about 12%. For other detection processes the amount of the Earth matter effect is a few per cent.

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I. INTRODUCTION

One of the most spectacular cosmic events is a supernova (SN) explosion which ends the existence of a giant star. There are two types of such explosions: types I and II. In the case of type I supernovae the explosion gives out gigantic fireworks display, while in the type II supernovae intensive neutrinos are emitted for a very short period of time followed by an intensive electromagnetic radiation. Supernovae are a very good natural laboratory to study various fundamental issues of physics and astrophysics.

The type II SN explosion is caused by the mechanism of core collapse. The total energy release is approximately the gravitational binding energy of the core. It generates intensive neutrinos which take away about 99% of the total energy. The explosion itself consumes about 1% of the total energy. The neutrino emission happens several hours prior to the brightening of the SN which only consumes 0.1% of the total energy. The observation of the SN neutrinos can serve as an early warning for the optical emission of a type II SN.

The first observations of the type II SN were made by Kamiokande II, IMB, and Baksan in 1987, with totally 24 neutrino events observed in a short period of about 13 seconds [1]. Although this number of events is too small for a quantitative study of neutrinos from supernova explosion, they are definitely very valuable for providing the first signal of cosmic neutrinos and initiating astrophysics and particle physics studies of supernova neutrinos.

The Daya Bay underground neutrino laboratory has been proposed and R&D is now being carried out. It is located in south China close to Hong Kong and provides an excellent site for measuring the neutrino mixing angle $\theta_{13}$ by using 4 reactor cores in two clusters, each core with a thermal power of 2.9 GW and therefore 11.6 GW total power. Furthermore, two more cores with an additional thermal power of 5.8 GW are expected to be online in 2011. Since the site is close to a group of hills, the cosmic ray background to the neutrino signals is vastly reduced. While the main purpose of the Daya Bay experiment is to measure a neutrino mixing angle, it can be used to detect possible neutrino events from a SN explosion and serves as a part of the Supernovae Early Warning System (SNEWS) [2]. Hence the theoretical prediction for the detection of SN neutrinos at Daya Bay is very desirable.

In Ref. [3] the authors calculated the expected neutrino events from a type II SN explosion in Borexino. For a typical SN at a distance of 10 kpc a burst of around 110 events would appear in the detector of Borexino. In this calculation, matter effects are ignored. Later Whisnant and Young [4] considered the detection of type II SN neutrinos in the Daya Bay experiment with the matter effect inside the supernova being included. This leads to the modified results which depend on the flip probability of neutrinos while passing through resonance regions inside a supernova.

In the realistic case, the neutrinos produced from a SN explosion will likely go through some portion of the Earth before reaching the detector. Therefore, in order to give a more accurate prediction for the number of signal events, the Earth matter effect should also be taken into account. The purpose of the present paper is to calculate the Earth matter effect and therefore takes into account all matter effects, both the Earth and Supernova, on the detection of type II SN neutrinos in the Daya Bay experiment.

II. THE FORMALISM

During a SN explosion, neutrinos are produced in two bursts. In the first burst which lasts for only a few milliseconds, electron neutrinos are generated via the inverse $\beta$-decay
process \( e^- + p \rightarrow \nu_e + n \) which leads to a neutron rich star. In the second burst which is longer (\( O(10) \) seconds) neutrinos of all flavors (\( \nu_\alpha \) and \( \bar{\nu}_\alpha \) with \( \alpha \) being \( e, \mu, \tau \)) are produced via the \( e^+e^- \) annihilation \( e^+ + e^- \rightarrow \nu_\alpha + \bar{\nu}_\alpha \), the electron neutrino antineutrino annihilation \( \nu_e + \bar{\nu}_e \rightarrow \nu_\alpha + \bar{\nu}_\alpha \), and nucleon-nucleon bremsstrahlung \( N + N \rightarrow N + N + \nu_\alpha + \bar{\nu}_\alpha \) processes. While these neutrinos generated in the SN explosion travel out of the SN, they interact with the high density core of the SN. Neutrinos \( \nu_x (\nu_x = \nu_\mu, \nu_\tau, \bar{\nu}_\mu, \bar{\nu}_\tau) \) decouple first from deep inside the core due to their lesser interactions as they only interact via the neutral current. Then \( \bar{\nu}_e \) decouples. The last one to decouple is \( \nu_e \) due to its interaction with the neutron which has a higher density than the proton. The typical temperatures of these neutrinos are in the following ranges:

\[
T_{\nu_e} = 3 - 4 \text{ MeV}, \quad T_{\bar{\nu}_e} = 5 - 6 \text{ MeV}, \quad T_{\nu_x} = 7 - 9 \text{ MeV}.
\] (1)

For the neutrino of flavor \( \alpha \), the time-integrated neutrino energy spectrum can be described by the Fermi-Dirac distribution,

\[
F^{(0)}_\alpha(E) = \frac{N^{(0)}_\alpha}{F_2 T_\alpha^3} \frac{E^2}{\exp(E/T_\alpha) + 1},
\] (2)

where \( E \) is the energy of the neutrino and \( T_\alpha \) the temperature as given in Eq. (1), \( N^{(0)}_\alpha \) is the total number of the neutrino of flavor \( \alpha \), and \( F_j \), where \( j \) is an integer, is defined by

\[
F_j = \int_0^\infty \frac{x^j}{\exp(x) + 1} dx.
\] (3)

The average neutrino energy is then obtained from the distribution function Eq. (2)

\[
\langle E^{(0)}_\alpha \rangle = \frac{F_3}{F_2} T_\alpha.
\] (4)

The total number of the neutrino \( \nu_\alpha \) is

\[
N^{(0)}_\alpha = \frac{L^{(0)}_\alpha}{\langle E^{(0)}_\alpha \rangle},
\] (5)

where \( L^{(0)}_\alpha \) is the luminosity of \( \nu_\alpha \). Then the energy spectrum function can be rewritten as

\[
F^{(0)}_\alpha(E) = \frac{L^{(0)}_\alpha}{F_3 T_\alpha^4} \frac{E^2}{\exp(E/T_\alpha) + 1}.
\] (6)

In the simplest argument the luminosity of each flavor is the same. Hence

\[
L^{(0)}_\alpha = \frac{0.99}{6} E^{(0)}_{SN}.
\] (7)

where \( E^{(0)}_{SN} \) is the total energy released during the SN explosion.

When the SN neutrinos of each flavor are produced they are approximately the effective mass eigenstates due to the extremely high matter density environment. While they propagate outward to the surface of the SN they could experience level crossing (neutrinos jump from one mass eigenstate to another) in the MSW resonance regions. There are two MSW
resonance regions which are determined by the two pairs of parameters \( (\Delta m^2_{31}, \sin^2 2\theta_{13}) \) which is referred to as the high resonance region and \( (\Delta m^2_{21}, \sin^2 2\theta_{12}) \) the low resonance region, where \( \Delta m^2_{kj} = m^2_k - m^2_j \), i.e., the mass square difference of the k and j neutrinos, and \( \theta_{jk} \) the corresponding mixing angle. Let us denote the probability that the neutrinos jump from one mass eigenstate to another at the high (low) resonance layer by \( P_H(P_L) \). The large mixing angle (LMA) solution of the solar neutrino constrains the flip probability at the low resonance region to be zero \( (P_L = 0) \). \( P_H \) is a function of the mixing angle \( \sin \theta_{13} \) and \( \Delta m^2_{31} \). Since the value of the mixing angle \( \sin^2 2\theta_{13} \) is still not determined, in numerical calculation we let \( P_H \) vary between 0 and 1. The level crossing diagrams are different for normal and inverted mass hierarchies \( [5] \). This leads to different forms for neutrino flux of mass eigenstates at the surface of SN.

When the neutrinos arrive at the Earth after travelling through the cosmic distance all the oscillation factors are averaged out and there is no coherence between different mass eigenstates. Hence the neutrino fluxes reaching the Earth are the following: For the normal hierarchy \( (\Delta m^2_{31} > 0) \),

\[
F^{(N)}_{\nu_e} = F^{(0)}_{\nu_e} + (|U_{e2}|^2 P_H + |U_{e3}|^2 (1 - P_H))(F^{(0)}_{\nu_e} - F^{(0)}_{\nu_x}),
\]

\[
F^{(N)}_{\nu_x} = F^{(0)}_{\nu_x} + |U_{e1}|^2 (F^{(0)}_{\nu_e} - F^{(0)}_{\nu_x}),
\]

\[
2F^{(N)}_{\nu_x} = F^{(0)}_{\nu_e} + F^{(0)}_{\nu_x} - (|U_{e2}|^2 P_H + |U_{e3}|^2 (1 - P_H))(F^{(0)}_{\nu_e} - F^{(0)}_{\nu_x}),
\]

\[
2F^{(N)}_{\nu_e} = F^{(0)}_{\nu_e} + F^{(0)}_{\nu_x} - |U_{e1}|^2 (F^{(0)}_{\nu_e} - F^{(0)}_{\nu_x}),
\]

and for the inverted hierarchy \( (\Delta m^2_{31} < 0) \),

\[
F^{(I)}_{\nu_e} = F^{(0)}_{\nu_e} + |U_{e2}|^2 (F^{(0)}_{\nu_e} - F^{(0)}_{\nu_x}),
\]

\[
F^{(I)}_{\nu_x} = F^{(0)}_{\nu_x} + (|U_{e1}|^2 P_H + |U_{e3}|^2 (1 - P_H))(F^{(0)}_{\nu_e} - F^{(0)}_{\nu_x}),
\]

\[
2F^{(I)}_{\nu_x} = F^{(0)}_{\nu_e} + F^{(0)}_{\nu_x} - |U_{e2}|^2 (F^{(0)}_{\nu_e} - F^{(0)}_{\nu_x}),
\]

\[
2F^{(I)}_{\nu_e} = F^{(0)}_{\nu_e} + F^{(0)}_{\nu_x} - (|U_{e1}|^2 P_H + |U_{e3}|^2 (1 - P_H))(F^{(0)}_{\nu_e} - F^{(0)}_{\nu_x}),
\]

where \( U_{ei}(i = 1, 2, 3) \) are the elements of the neutrino mixing matrix.

Due to the smallness of the mixing angle \( \theta_{13} \), we ignore the terms proportional to \( \sin \theta_{13} \), then Eqs. \( 8 \) and \( 9 \) can be simplified as follows: For the normal hierarchy,

\[
F^{(N)}_{\nu_e} = P_H \sin^2 \theta_{12} F^{(0)}_{\nu_e} + (1 - P_H \sin^2 \theta_{12}) F^{(0)}_{\nu_x},
\]

\[
F^{(N)}_{\nu_x} = \cos^2 \theta_{12} F^{(0)}_{\nu_e} + \sin^2 \theta_{12} F^{(0)}_{\nu_x},
\]

\[
2F^{(N)}_{\nu_x} = (1 - P_H \sin^2 \theta_{12}) F^{(0)}_{\nu_e} + (1 + P_H \sin^2 \theta_{12}) F^{(0)}_{\nu_x},
\]

\[
2F^{(N)}_{\nu_e} = \sin^2 \theta_{12} F^{(0)}_{\nu_e} + (1 + \cos^2 \theta_{12}) F^{(0)}_{\nu_x},
\]

and for the inverted hierarchy \( (\Delta m^2_{31} < 0) \),

\[
F^{(I)}_{\nu_e} = \sin^2 \theta_{12} F^{(0)}_{\nu_e} + \cos^2 \theta_{12} F^{(0)}_{\nu_x},
\]

\[
F^{(I)}_{\nu_x} = P_H \cos^2 \theta_{12} F^{(0)}_{\nu_e} + (1 - P_H \cos^2 \theta_{12}) F^{(0)}_{\nu_x},
\]

\[
2F^{(I)}_{\nu_x} = \cos^2 \theta_{12} F^{(0)}_{\nu_e} + (1 + \sin^2 \theta_{12}) F^{(0)}_{\nu_x},
\]

\[
2F^{(I)}_{\nu_e} = (1 - P_H \cos^2 \theta_{12}) F^{(0)}_{\nu_e} + (1 + P_H \cos^2 \theta_{12}) F^{(0)}_{\nu_x}.
\]

If the SN matter effect is neglected, the neutrinos, which are produced in the core of the SN and propagate through cosmic distance to reach Earth, are subject to the vacuum
oscillation only. In this case when the Earth matter effect is also neglected the neutrino fluxes at the detector are given by:

\[
F_{\nu_e}^{(V)} = (1 - \frac{1}{2} \sin^2 2\theta_{12}) F_{\nu_e}^{(0)} + \frac{1}{2} \sin^2 2\theta_{12} F_{\nu_x}^{(0)},
\]

\[
F_{\bar{\nu}_e}^{(V)} = (1 - \frac{1}{2} \sin^2 2\theta_{12}) F_{\bar{\nu}_e}^{(0)} + \frac{1}{2} \sin^2 2\theta_{12} F_{\nu_x}^{(0)},
\]

\[
2 F_{\nu_x}^{(V)} = \frac{1}{2} \sin^2 2\theta_{12} F_{\nu_e}^{(0)} + (2 - \frac{1}{2} \sin^2 2\theta_{12}) F_{\nu_x}^{(0)},
\]

\[
2 F_{\bar{\nu}_x}^{(V)} = \frac{1}{2} \sin^2 2\theta_{12} F_{\bar{\nu}_e}^{(0)} + (2 - \frac{1}{2} \sin^2 2\theta_{12}) F_{\nu_x}^{(0)}.
\]

(12)

FIG. 1: Illustration of the path of the SN neutrinos reaching the detector in the Earth. D is the location of the detector, θ is the incident angle of neutrinos, and O is the centre of the Earth.

In Eqs. (10) and (11) the Earth matter effect is neglected. In reality, neutrinos from an SN may likely travel through a significant portion of the Earth before reaching the detector and are therefore subject to the matter effect. The Earth matter effect can be parameterized in terms of the distance between the neutrino and the Earth center and the total distance travelled in the Earth before reaching the detector. The distance that the neutrino travels in the Earth depends on their incident orientation. Suppose a neutrino reaches the detector with the incident angle θ (see Fig. 1), then the distance the neutrino travels through the Earth is

\[
L = (-R + h)\cos\theta + \sqrt{R^2 - (R - h)^2 \sin^2 \theta},
\]

(13)

where \( h \) is the depth of the detector in and \( R \) the radius of the Earth. The distance of the neutrino to the Earth center can be obtained as follows: Let \( x \) be the distance that the neutrino travels into the Earth, which is shown as the segment AB in Fig. 1. The distance of the neutrino to the center of the Earth \( \tilde{x} \) is given by

\[
\tilde{x} = \sqrt{(R - h)^2 + (L - x)^2 + 2(R - h)(L - x)\cos\theta}.
\]

(14)
The Earth matter effects on the detection of SN neutrinos have been discussed extensively in literature \[1, 2, 3, 11, 12, 13, 14, 15]. As mentioned before, after travelling the cosmic distance to reach the Earth, the arriving neutrinos are definite mass eigenstates which, then, oscillate in going through the Earth matter. Let $P_{\nu e}$ be the probability that a neutrino mass eigenstate $\nu_i$ enters the surface of the Earth and arrives at the detector as an electron neutrino $\nu_e$. Then the flux of $\nu_e$ at the detector, denoted as $F_{\nu_e}^D$, can be written as

$$F_{\nu_e}^D = \sum_i P_{\nu ei} F_i,$$ \hspace{1cm} (15)

where $F_i$ is the flux of $\nu_i$ at the Earth surface, in either the normal or inverted hierarchy. $P_{\nu e}$ obey the unitary condition $\sum_i P_{\nu ei} = 1$. In the case where the Earth matter effect is ignored, $P_{\nu e} = |U_{\nu e}|^2$, then one recovers the expressions of Eqs. (8)-(11). After some straightforward derivations with the terms proportional to $\sin \theta_{13}$ being ignored, one obtains, in the case of the normal hierarchy,

$$P_{\nu e}^{D(N)} = P_{2e} P_H F_{\nu_e}^{(0)} + (1 - P_{2e} P_H) F_{\nu_e}^{(0)},$$

$$P_{\bar{\nu}_e}^{D(N)} = (1 - P_{2e}) F_{\bar{\nu}_e}^{(0)} + P_{2e} F_{\nu_e}^{(0)},$$

$$2P_{\nu e}^{D(N)} = (1 - P_{2e} P_H) F_{\nu_e}^{(0)} + (1 + P_{2e} P_H) F_{\nu_e}^{(0)},$$

$$2P_{\bar{\nu}_e}^{D(N)} = P_{2e} F_{\bar{\nu}_e}^{(0)} + (2 - P_{2e}) F_{\nu_e}^{(0)},$$ \hspace{1cm} (16)

and in the case of the inverted hierarchy,

$$P_{\nu e}^{D(I)} = P_{2e} F_{\nu_e}^{(0)} + (1 - P_{2e}) F_{\nu_e}^{(0)},$$

$$P_{\bar{\nu}_e}^{D(I)} = P_H (1 - P_{2e}) F_{\bar{\nu}_e}^{(0)} + (1 + P_{2e} P_H - P_H) F_{\nu_e}^{(0)},$$

$$2P_{\nu e}^{D(I)} = (1 - P_{2e}) F_{\nu_e}^{(0)} + (1 + P_{2e}) F_{\nu_e}^{(0)},$$

$$2P_{\bar{\nu}_e}^{D(I)} = (1 + P_{2e} P_H - P_H) F_{\bar{\nu}_e}^{(0)} + (1 + P_H - P_{2e} P_H) F_{\nu_e}^{(0)}.$$ \hspace{1cm} (17)

The probability $P_{\nu e}$ ($i = 1, 2, 3$) has been calculated in Refs. \[3\] and \[10\]. In the Earth the potential which an electron neutrino experiences is

$$V(x) = \sqrt{2} G_F N_e(x),$$ \hspace{1cm} (18)

where $G_F$ is the Fermi constant and $N_e(x)$ is the Earth electron number density. Let us introduce a small parameter

$$\epsilon(x) = \frac{2EV(x)}{\Delta m^2_{21}},$$ \hspace{1cm} (19)

For a typical neutrino energy $E = 10$ MeV, $\epsilon$ is less than 0.13. Neglecting contributions of $O(\epsilon^2)$, $P_{2e}$ can be expressed as the following \[6\]:

$$P_{2e} = \sin^2 \theta_{12} + \frac{1}{2} \sin^2 2\theta_{12} \int_{x_0}^{x_f} dx V(x) \sin m^2 \phi_{x-a}^{m},$$ \hspace{1cm} (20)

where $\theta_{12} \approx 33^\circ$ \[16\] and $\phi_{a-b}^{m}$ is defined as

$$\phi_{a-b}^{m} \equiv \int_a^b dx \Delta m(x),$$ \hspace{1cm} (21)
where
\[ \Delta m(x) \equiv \frac{\Delta m^2_{21}}{2E} \sqrt{(\cos 2\theta_{12} - \epsilon(x))^2 + \sin^2 2\theta_{12}}. \] (22)

Let the matter density inside the Earth be \( \rho(x) \) then the nucleon number density is
\[ 2\rho(x)/(m_p + m_n), \] which, for matters of equal number of protons and neutrons, leads to the following density of electrons inside the Earth:
\[ N_e(x) = \rho(x)/(m_p + m_n). \] (23)

In our calculations of the Earth matter effect we choose a simple model for \( \rho(x) \) which is called the mantle-core-mantle density profile given in [17] and [18]. In this model a step function is used to describe the Earth matter effect: \( \rho = 12 \text{g/cm}^3 \) for the core, and \( \rho = 5 \text{g/cm}^3 \) for the mantle. The core radius and the thickness of the mantle are each half of the Earth radius.

In the next section we will use the formulas given in this section to calculate numerically the neutrino fluxes at the detector, taking into account the Earth and supernova matter effects, and then apply the results to various physical channels through which neutrinos can be detected.

III. NUMERICAL RESULTS

In the following we calculate the predicted numbers of events that can be observed through various reaction channels at the Daya Bay experiment. This will be done by integrating, over the neutrino energy \( E_\nu \), the product of the target number \( N_T \), the cross section of each channel, and the neutrino flux function \( F_\alpha^{D}(E_\nu)/4\pi D^2 \) given in Eqs. (16) and (17) [4]:
\[ N(i) = N_T \int dE_\nu \sigma(i)(E_\nu) \frac{1}{4\pi D^2} F_\alpha^{D}(E_\nu), \] (24)
where \( D \) is the distance between the SN and the Earth and the index \( i \) represents different channels through which the SN neutrinos are observed.

The target material includes electrons, protons and carbon nuclei since the chemical composition of the expected liquid scintillator, mesitylene or pseudocumene, is \( C_9H_{12} \) which is the same as that of Borexino [3]. The cross section for each reaction channel can be found in Ref. [3]. For neutrino-electron elastic scatterings via charged and neutral currents, \( \nu_\alpha(\bar{\nu}_\alpha) + e^- \rightarrow \nu_\alpha(\bar{\nu}_\alpha) + e^- \), the cross sections are:
\[ \sigma(\nu_ee) = 9.2 \times 10^{-45}E_\nu(\text{MeV}) \text{ cm}^2, \]
\[ \sigma(\bar{\nu}_ee) = 3.83 \times 10^{-45}E_\nu(\text{MeV}) \text{ cm}^2, \]
\[ \sigma(\nu_\mu\tau e) = 1.57 \times 10^{-45}E_\nu(\text{MeV}) \text{ cm}^2, \]
\[ \sigma(\bar{\nu}_\mu\tau e) = 1.29 \times 10^{-45}E_\nu(\text{MeV}) \text{ cm}^2. \] (25)

The inverse beta decay \( \bar{\nu}_e + p \rightarrow e^+ + n \) has the most number of events due to its large cross section and low threshold of the neutrino energy.
\[ \sigma(\bar{\nu}_ep) = 9.5 \times 10^{-44}(E_\nu(\text{MeV}) - 1.29)^2 \text{ cm}^2, \]
\[ E_{\text{th}} = 1.80 \text{ MeV}. \] (26)
For the neutrinos and $^{12}C$ system, there are two charged-current and three neutral-current reactions. Their effective cross sections are obtained by scaling the experimentally measured energy values from muon decay at rest to the energy scale for supernova neutrinos. In this way, one obtains the following effective cross section [3]:

- Charged-current capture of $\bar{\nu}_e$: $\bar{\nu}_e + ^{12}C \rightarrow ^{12}B + e^+ + \bar{\nu}_e,$
  $$\langle \sigma(^{12}C(\bar{\nu}_e, e^+)^{12}B) \rangle = 1.87 \times 10^{-42} \text{ cm}^2; \quad (27)$$

- Charged-current capture of $\nu_e$: $\nu_e + ^{12}C \rightarrow ^{12}N + e^-,$ $^{12}N \rightarrow ^{12}C + e^+ + \nu_e,$
  $$\langle \sigma(^{12}C(\nu_e, e^-)^{12}N) \rangle = 1.85 \times 10^{-43} \text{ cm}^2; \quad (28)$$

- Neutral-current inelastic scattering of $\nu_\alpha$ or $\bar{\nu}_\alpha$: $\nu + ^{12}C \rightarrow ^{12}C^* \rightarrow ^{12}C + \gamma,$
  $$\langle \sigma(^{12}C(\nu_x, e^-)^{12}C) \rangle = 3.73 \times 10^{-42} \text{ cm}^2, \quad x = \mu, \tau. \quad (31)$$

These reactions will be referred to collectively as neutrino-carbon scatterings.

There will be several detectors located in the near and far sites. We take the total detector mass to be 300 tons which is the same as that of Borexino [3]. Then the total numbers of target electrons, protons, and $^{12}C$ are

- $N_T^{(e)} = 9.94 \times 10^{31}$
- $N_T^{(p)} = 1.82 \times 10^{31}$
- $N_T^{(C)} = 1.36 \times 10^{31}$

In the numerical calculations we take the total energy release of SN as

- $E_{SN}^{(0)} = 3 \times 10^{53} \text{ erg} = 1.97 \times 10^{50} \text{ MeV}$, 

and the distance of SN as

- $D = 10 \text{ kpc} = 3.09 \times 10^{22} \text{ cm}$.

Other parameters are taken as follows:

- $h = 0.4 \text{ km}$, $R = 6400 \text{ km}$, $\Delta m_{21}^2 = 7.1 \times 10^{-5} \text{ eV}^2$, $\theta_{12} = 32.5^\circ$.

The numerical results for the event numbers for various reaction channels are obtained from Eqs. (24)-(31), (16) and (17). The results are shown in Figs. 2, 3, and 4. It can be seen from these figures that the Earth matter effect depends on the incident angle of the neutrino, the mass hierarchy, and the flip probability $P_H$. When the incident angle of the SN neutrino is smaller than a value $\theta_0$, where $\theta_0 \sim 90^\circ$, the Earth matter effect can be ignored for all the relevant reactions that concern us here.

The inverse beta decay $\bar{\nu}_e + p \rightarrow e^+ + n$ has the largest Earth matter effect among all the channels. The maximum Earth matter effect for the inverse beta decay appears at around $\theta \sim 93.6^\circ$ in the cases of $P_H = 0,1$ (normal hierarchy) and $P_H = 1$ (inverted hierarchy) and the effect is 12.0%. When the incident angle become larger than about 106°, the Earth matter effect becomes about 6.7% and almost independent of the incident angle.
For the neutrino-electron and neutrino-carbon scattering channels the behavior is quite similar to that of the inverse beta decay. For neutrino-electron elastic scattering, the maximum Earth matter effect appears at $\theta \sim 92.7^\circ$ and the amount could be as large as 3.75% in the case of $P_H = 1$ for both normal and inverted hierarchies, 3.11% in the case of $P_H = 0$ for the inverted hierarchy, and 0.66% in the case of $P_H = 0$ for the normal hierarchy. When the incident angle become larger than about $100^\circ$, the Earth matter effect is independent of the incident angle and the amount is about 2.16% for $P_H = 1$ (normal and inverted hierarchies), 1.76% for $P_H = 0$ (inverted hierarchy), and 0.38% for $P_H = 0$ (normal hierarchy), respectively.

It should be noted that there are complications in dealing with reactions of neutrinos with $^{12}C$. The effective cross sections in Eqs. (27-31) are given for supernova neutrinos without oscillations. It is pointed out in Ref. [3] that the effective cross sections are affected when neutrino oscillations are taken into account. For instance, the oscillation of higher energy $\nu_\mu$ into $\nu_e$ results in an increased event rate since the expected $\nu_e$ energies are just at or below the charged-current reaction threshold. This leads to an increase by a factor of 35 for the cross section $^{12}C(\nu_e e^-)^{12}N$ if one average over a $\nu_e$ distribution with $T = 8$ MeV rather than 3.5 MeV. After numerical calculations for the reactions of neutrinos with $^{12}C$, we find that the Earth matter effects become the largest when $\theta \sim 91.8^\circ$ and the maximum amount is 4.00% in the case of $P_H = 1$ for both normal and inverted hierarchies, 1.65% in the case of $P_H = 0$ for the inverted hierarchy, and 2.00% in the case of $P_H = 0$ for the normal hierarchy. Again when the incident angle become larger than about $100^\circ$, the Earth matter effect is almost independent of the incident angle and the amount is about 2.87% for $P_H = 1$ (normal and inverted hierarchies), 1.21% for $P_H = 0$ (inverted hierarchy), and 1.36% for $P_H = 0$ (normal hierarchy), respectively.

The above results can be understood from the fact that the oscillation behavior is determined by the phase factor $\Delta m^2_{21} (eV^2) L(m)/E(\text{MeV})$, where $L$ is the distance in the Earth that the neutrino travels. Equation (13) gives the distance the SN neutrinos travel in the Earth. It can be seen from this equation that when $\theta < \theta_0$ this distance is smaller than 10 km and hence the amount of Earth matter effect is very small. In this case $P_{2e}$ in Eq. (20) and $F^D_{\nu_e}$ in Eq. (15) can be replaced by their values in the vacuum $|U_{e2}|^2$ and $F_{\nu_e}$, respectively. When $\theta$ is larger than $90^\circ$ this distance becomes bigger than 100 km, the Earth matter effect becomes large and reaches a maximum for $\theta \sim 91^\circ - 94^\circ$. When $\theta$ is larger than $100^\circ$, the distance that neutrinos travel in the Earth is greater than 2000 km, then there could be many oscillations in $F^D_{\nu_e}$. This leads to averaging the Earth matter effect which is smaller than the maximum value. As an example, the flux of the SN neutrino $\bar{\nu}_e$ as a function of neutrino energy in various cases are plotted in Fig. 5. It can be seen from Figs. 2 to 5 that the event numbers detected at the detector depend on the incident angle of neutrinos, the SN matter effects ($P_H$), and the mass hierarchy of neutrinos. By measuring these event numbers we can obtain information on the SN and the Earth matter effect, and the neutrino mass hierarchy.
FIG. 2: The event number of the reaction $\bar{\nu}_e + p \rightarrow e^+ + n$ as a function of the incident angle of the SN neutrino. The solid curve corresponds to $P_H = 0, 1$ (normal hierarchy) and $P_H = 1$ (inverted hierarchy). The dashed curve corresponds to $P_H = 0$ (inverted hierarchy).

IV. SUMMARY

We have calculated matter effects, including both the supernova and the Earth, on the detection of type II supernovae neutrinos at the proposed Daya Bay experiment. It is found that the amount of the matter effect depends on the neutrino incident angle, the neutrino mass hierarchy, and the flip probability $P_H$. When the incident angle of the SN neutrinos is smaller than a definite value $\theta_0$, the Earth matter effect is negligible due to the small distance that the neutrino traverses in the Earth. The Earth matter effect becomes manifested and reaches a maximum value for $\theta \sim 91^\circ - 94^\circ$. When $\theta$ is greater than $100^\circ$, the Earth matter effect becomes insensitive to the neutrino incident angle and the effect is smaller than the maximum value. There is a window in $\theta$, $\theta \sim 90^\circ - 100^\circ$, corresponding to the neutrino travelling distance of 500 km to 2000 km, in which the neutrino event numbers can vary significantly as a function of $\theta$. This phenomenon can be understood by the fact that the Earth matter effect is controlled by $\Delta m^2_{21}$ and the variation of the neutrino number is significant for $\Delta m^2_{21}(eV^2)L(m)/E_\nu(\text{MeV}) \sim$ a few.

In the reaction channel $\bar{\nu}_e + p \rightarrow e^+ + n$ the amount of the Earth matter effect is the largest and the maximum amount is about 12%. For the neutrino-electron and neutrino-carbon scattering channels, the amount of the Earth matter effect is a few per cent at most. By measuring the event numbers in various channels we can obtain information on the Earth matter effect, the matter effect inside the SN, and the neutrino mass hierarchy.
FIG. 3: The total event number of neutrino-electron elastic scattering via charged and neutral currents $\nu_\alpha(\bar{\nu}_\alpha) + e^- \to \nu_\alpha(\bar{\nu}_\alpha) + e^-$ as a function of the incident angle of the SN neutrinos. The solid curve corresponds to $P_H = 1$ (normal and inverted hierarchies), the dashed curve corresponds to $P_H = 0$ (normal hierarchy), and the dotted curve corresponds to $P_H = 0$ (inverted hierarchy), respectively.

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Fig. 4: The total event number of the reactions of neutrinos with $^{12}C$ as a function of the incident angle of the SN neutrinos. The solid curve corresponds to $P_H = 1$ (normal and inverted hierarchies), the dashed curve corresponds to $P_H = 0$ (normal hierarchy), and the dotted curve corresponds to $P_H = 0$ (inverted hierarchy), respectively.

FIG. 5: The flux of the SN $\bar{\nu}_e$ at the detector multiplied by $10^{-8}$. The solid curve represents the cases where supernova and Earth matter effects are included for $\theta = 180^\circ$, $P_H = 0, 1$ (normal hierarchy) or $P_H = 0$ (inverted hierarchy). The dashed curve represents the case where supernova and Earth matter effects are included for $\theta = 180^\circ$, $P_H = 0$ (inverted hierarchy) or the case where the supernova matter effect is included but the Earth matter effect is not for $P_H = 0$ (inverted hierarchy). The dotted curve represents the cases where the supernova matter effect is included but the Earth matter effect is not for $P_H = 0, 1$ (normal hierarchy) or $P_H = 1$ (inverted hierarchy). The dot-dashed (dot-dot-dashed) curve represents the flux given by Eq. (6) (Eq. (12)) where the neutrinos do not oscillate at all (oscillate as in vacuum).