CM-P00058730

Ref.TH.1340-CERN

MANY PARTICLE PRODUCTION AND REGGE BEHAVIOUR *)

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*) Expanded English write up of an invited talk presented at the Evian Meeting of the French Physical Society, May 1971.

Also to be used as a partial contribution to Erice Lecture Notes (1971), and Les Houches Lecture Notes (1971).

Ref.TH.1340-CERN

1 July 1971

1. INTRODUCTION

Forwards

At present, many-particle processes are gathering an increasing interest among high-energy physicists. As probing energies are expanding beyond the 30 GeV range of the CERN PS and the Brookhaven AGS, one has, indeed, seriously to tackle the fact that reactions which cannot be considered as two-body or quasi two-body processes are by far the dominant lot. On pure phenomenological grounds we may reasonably hope to grasp some key relations which should eventually lead to an understanding of the leading mechanisms at work being now helped in this endeavour by a wealth of Track Chamber and counter data in the 5 to 30 GeV range which are becoming available. One should also stress that we have by now gained enough confidence in our parametrization of quasi two-body phenomena as to try to extend this understanding to more complicated processes. In any case, we are bound to witness a pronounced drift in the emphasis of strong interaction phenomenology toward production processes. It should be stressed though that, if so far the preoccupation of a minority group among particle physicists, many-particle phenomena have received a sustained interest during the past decade and, indeed, production processes, as approximatively described in terms of multi-Regge exchange, have been the core of recent bootstrap approaches ¹⁾. The prominent role of low-energy resonances, which reduces many a multiparticle reaction to a quasi two-body process has, however, and fruitfully so, attached so far most of the theoretical effort to the analysis of a priori simpler two-body reactions ²⁾. It is, indeed, this way that the powerful concept of duality ³⁾ has recently been introduced into hadron physics. It now becomes of much importance in our understanding of a priori more complicated production reactions.

The organization of this paper is the following. The introduction is continued with a short survey of some key empirical features of many-particle processes as ascertained from the analysis of machine energy reactions and cosmic ray events. Section 2 tries

to demystify the still prominent role of Jacobians in this branch of particle physics. We show how a specific property gets a priori quite different a phrasing and apparent rational when spelt out in terms of different sets of variables. In Section 3, we indicate that, even though a precise theoretical understanding is still lacking, many key features appear to be remarkably model-independent (provided, of course, that one limits oneself to "reasonable" models). This leads to some specific properties which already gather enough confidence as to be systematically tried against existing data or used as a guide in our immediate quest for new data. In Section 4, we consider more specifically the predictions which can be made according to Regge models and show how available results compare to them. A case study based on recent CERN data is presented. Possible connections between the prominent properties of the electron missing mass results and a similar behaviour in hadron collisions are discussed in Section 5. Finally, Section 6 deals with the special properties attached to Pomeranchon (and maybe multi-Pomeranchon) exchange in production processes and the available phenomenological clues at our disposal.

The main features of many-particle processes have been already stressed in several review papers ⁴⁾. For the sake of completeness let us recall some prominent facts. The first feature is that total cross-sections appear to approach constant values with increasing energies when the number of secondaries increases if much less than the available energy would permit. The next feature is the smallness of the transverse momentum of the secondaries. It appears to be limited to values of the order of 0.4 GeV/c. This is illustrated on Fig. 1 which shows, as an example, the transverse momentum distribution of the π^- 's produced in pp collisions at 28.5 GeV/c and in K^+p collisions at 12 GeV/c ⁵⁾. The longitudinal momentum distribution is on the contrary widely spread and exhibits a leading particle effect, the incident particles retaining most of their centre-of-mass momentum. All these features are easily seen on a Peyrou plot whereby the longitudinal and transverse momentum of a final particle, as measured in several registered events, are displayed.

As an example, Fig. 2a shows the Peyrou plot drawn for the reaction $K^-p \rightarrow K^-p \pi^+ \pi^-$ at 10 GeV/c ⁶⁾. As expected the leading particle effect becomes less pronounced as the multiplicity increases and Fig. 2b shows the variation observed in the longitudinal momentum distribution of a "leading" secondary for larger and larger multiplicities. The leading particle effect is gradually washed out.

Another prominent feature is the slow increase of the number of secondaries with increasing energy. It now appears that a very good parametrization is obtained in terms of a logarithmic rise. The charged multiplicity as obtained from machine and recent cosmic ray results ⁷⁾ can be written as

$$\langle n_c \rangle = (2.04 \pm 0.19) + (1.41 \pm 0.2) \log W_0 \quad (1)$$

where W_0 is the available energy. This is illustrated on Fig. 3. As far as we may know the total multiplicity seems to follow the more easily measured charged one ⁸⁾. Let us stress, though it may look obvious, that such a logarithmic increase is very slow indeed ! The charged multiplicity which is known to be between 4 and 5 at 16 GeV should increase to only 7 to 8 at ISR energy. The CERN ISR correspond to an equivalent incident energy of 1700 GeV. The probability distribution of the number of secondaries at a given energy has been fitted with variable success with formula of various level in sophistication. It seems that a Poisson formula reproduces the observed distribution fairly well ⁹⁾. The type of fit thus obtained is illustrated by Fig. 4. As a last prominent fact one should mention the abundance of pions. At machine energy the number of secondary kaons is only 5 to 10 % that of the pions, when the number of antiprotons represents only 1 % of the number of pions. These ratios, however, change as expected with increasing energy. The kaon to pion ratio seems to rise to 20 % at standard cosmic ray energies.

Exclusive and Inclusive Descriptions

A description of a multiparticle reaction aims at being complete or "exclusive". All final particles are then tackled by the analysis which has to face energy behaviour in the various subchannels as well as mass and angular distributions. This appears at present as a paramount task except for very low multiplicities (three final particles) for which dual models have recently given most interesting guiding lines. Our present purpose is, however, to discuss large multiplicity configurations and the application of dual models the so-called B_5 phenomenology - to three-body final states which have been already extensively reviewed ¹⁰⁾ - will not be discussed here. With many-particle reactions one cannot, at present, be ambitious enough as to proceed in an exclusive way but several interesting pieces of information can already be obtained and discussed in an "inclusive" way, whereby the distribution of one (or a very few) final particle(s) is analyzed, everything else being summed over ¹¹⁾.

The main pieces of information are single particle spectra [the spectrum of the π^- (say) produced in K^+p collisions as an example] which correspond to the standard beam surveys performed at any new machine. Next come correlations among secondaries or the analysis of discrepancies between two (say) particle distributions and the product of the pertinent single-particle distributions. It is the aim of this paper to focus on such a type of analysis.

2. FROM JACOBIANS TO DYNAMICS

In a long quest for simple and general statements amenable to multiparticle phenomena many a set of variables has been tried. It may, therefore, at first sight, often appear difficult to decide between a specific dynamical property and a specific feature of the Jacobian associated with a particular choice of variables. The direction of the secondaries (angular distributions) has had

for obvious experimental reasons a prominent role. It turns out to be cumbersome when speaking dynamics and will not be considered here. However, in order to get some acquaintance with various possible sets of variables let us consider two impinging particles, a target (T) and a projectile (P), and switch from the reference frame attached to either T or P to the centre-of-mass frame. The momentum of a secondary, as measured in one of these three different frames, will be respectively denoted by q^T , q^P and q . The three-momentum $\underline{q}$ is decomposed into a longitudinal momentum q_L and a transverse momentum $q_\perp$, which is the same in the three frames here considered and, we know, small. The energy of the secondary is denoted by ω , with $\omega^2 = \underline{q}^2 + \mu^2$ where μ is the mass. The invariant phase space volume element is $d^3\underline{q}/\omega$ and the key and invariant quantity for the single particle distribution is therefore

$$\omega \frac{dN}{d^3q} = \omega \frac{dN}{\pi dq_L dq_\perp^2} = f(q_L, q_\perp, \omega) \quad (2)$$

where $W = \sqrt{s}$ is the production reaction energy.

The Peyrou plot displays the single-particle distribution weighted by the particle energy. Dynamical features are, however, more readily to be seen on f as defined by (2). The centre-of-mass energy squared is denoted by s and the "scaling" variable $x = (2q_L/\sqrt{s})$ will appear as of key importance ^{11),12),13)}.

The Fragmentation Region

These definitions being given, if one considers relatively large values of q_L ($q_L \gg (q^2 + \mu^2)^{\frac{1}{2}}$) and a negative x (the secondary goes in the direction of the target as seen in the

centre-of-mass frame), one easily derives that

$$q_L^T = \frac{q_L m_T}{2\omega_T} - \omega_T \frac{q_\perp^2 + \mu^2}{2q_L m_T}$$

where m_T and ω_T are the target mass and centre-of-mass energy.
Writing

$$x = \frac{q_L}{\omega_T} \approx \frac{2q_L}{\sqrt{s}}$$

one gets

$$q_L^T = \frac{1}{2} \left(x m_T - \frac{q_\perp^2 + \mu^2}{x m_T} \right) \quad (3a)$$

Considering now positive x values (the secondary goes in the direction of the projectile as seen in the centre-of-mass frame), one obtains the same relation but in terms of the projectile frame variables

$$q_L^P = \frac{1}{2} \left(x m_P - \frac{q_\perp^2 + \mu^2}{x m_P} \right) \quad (3b)$$

Considering fixed longitudinal and transverse momenta in the target (projectile) frame is, therefore, the same as considering fixed negative (positive) x values when describing the secondary distribution in the centre of mass system.

As advocated by Yang and his coworkers ¹⁴⁾ the physical picture for a very high energy reaction should be most easily phrased either in terms of the target or in terms of the projectile which both receive a tremendous energy and, as a result, break up into several fragments. The key variables should, therefore, be the energies of the secondaries as measured in the target frame (target fragments) or in the projectile rest frame (projectile fragments). According to (3) we see that this corresponds to using the scaled variable x , when one considers the centre-of-mass frame which may be of a priori little physical significance for a very high energy collision whereupon two semi-transparent bodies go through each other at the speed of light ! Yang, furthermore, advocates a limiting fragmentation ¹⁴⁾, according to which the cross-section for the production of a fixed final state tends to a limit at infinite energy. A fixed final state here means that the fragments of the target particles have fixed momenta in the target (in most cases laboratory) frame, whereas the fragments of the projectile have fixed momenta in the projectile frame or, as readily obtained through a Lorentz transformation, longitudinal momenta which are fixed fractions of the projectile momentum. This may be looked at as resulting from a diffraction dissociation of both target and projectile, or, Reggewise, in terms of Pomeron exchange but with a vanishingly small slope for the effective leading vacuum trajectory ¹⁵⁾. It follows that the secondary distribution defined in either the target or projectile frames and both written in an invariant way as (2) should approach constant as s increases when $q_{\perp}$ is fixed and q_L^T (or q_L^P) is also held fixed. The constant depends of course on $q_{\perp}^T$ and q_L^T . Clearly, separating between target and projectile fragments is possible if $q_{\perp}$ is large enough or either q_L^T or q_L^P small enough. This is straightforward for values of $|x|$ which are sufficiently large though, as the energy increases asymptotically, the fragmentation picture can be formally extended down to $|x| \approx 0$.

At present machine energy one should focus on either relatively slow (target fragmentation) or fast (projectile fragmentation) secondaries as seen in the laboratory: $q(q^P) \lesssim 1 \text{ GeV}/c$ say.

As it is obvious from (3), a limiting distribution in either of these variables implies a limiting distribution in x . As a result $(d\sigma)/(dx dq_{\perp}^2)$ at fixed $q_{\perp}^2$ should be asymptotically independent of the incident energy. f as defined in (2) should not depend separately on $q_{\perp}$, q_L and $\sqrt{s}$ but only on $q_{\perp}$ and $x = 2(q_L)/(\sqrt{s})$. How fast this asymptotic regime sets in remains yet an open question at present.

The Rapidity Distribution

In order to study the low x region, one introduces the rapidity description. One defines ¹⁶⁾

$$x = q_x ; \quad y = q_y ; \quad z = \log \left(\frac{q_L + \omega}{m} \right) \quad (4)$$

where

$$m = (q_{\perp}^2 + \mu^2)^{1/2} = (\omega^2 - q_L^2)^{1/2}$$

z is called the rapidity. It follows that

$$\omega + q_L = m e^z \quad \text{and} \quad \omega - q_L = m e^{-z}$$

or

$$q_L = m \operatorname{Sh} z ; \quad \omega = m \operatorname{Ch} z$$

One then has

$$dq_L = m \operatorname{Ch} z \, dz = \omega \, dz$$

so that the invariant phase space integration volume reads

$$\frac{d^3q}{\omega} = dx dy dz = d^3r \quad (5)$$

A constant invariant production amplitude leads, therefore, to a uniform rapidity distribution. Using laboratory variables, the rapidity of the target and projectile particles are easily found to be

$$y_T = 0 \quad \text{and} \quad y_P = \log \frac{s}{m_P m_T} \quad (6)$$

keeping but the leading terms in the second argument. In terms of centre-of-mass variables we would take the two rapidities respectively equal to

$$\log \frac{\sqrt{s}}{m_P} \quad \text{and} \quad - \log \frac{\sqrt{s}}{m_T}$$

a particle at rest in the centre of mass having then rapidity zero.

As it is well known, a Lorentz transformation along the longitudinal momentum corresponds to a translation in rapidity. Conservation of longitudinal momentum and energy is written as

$$\begin{aligned} m_P e^{-y_P} + m_T e^{-y_T} - \sum_i m_i e^{-y_i} &= 0 \\ m_P e^{y_P} + m_T e^{y_T} - \sum_i m_i e^{y_i} &= 0 \end{aligned} \quad (7)$$

where the sum extends over all secondaries. It follows that the rapidity of any secondary must be found between

$$y_{\min} = y_T - \log \frac{m_T}{m_i} \quad \text{and} \quad y_{\max} = y_P + \log \frac{m_P}{m_i}$$

Asymptotically a rapidity plot should, therefore, show all secondary points between a "target wall" and a "projectile wall" translated from it by $\log s/m_T m_P$. This is shown on Fig. 5a, where the rapidity zero is arbitrarily attached to the centre of mass, and where we have normalized the rapidity scale by $\log s/m_P m_T$. z and x are related by

$$z = A_9 \operatorname{sh} \left(\frac{x \sqrt{s}}{m} \right) \quad (8)$$

and, though obvious, it should be stressed that for small x , a tiny variation of x will correspond to a significant variation of z ¹⁷⁾. On the contrary, when x gets close to 1 (or -1) a significant variation of x gives but a smooth logarithmic variation of the rapidity. This is illustrated by the corresponding x plot drawn on Fig. 5b ¹⁸⁾.

A rapidity plot, therefore, expands the small x region at the expense of the larger x one. In particular, a secondary with a small fixed centre-of-mass longitudinal momentum corresponds to a fixed point on the rapidity plot, whereas its representative point on the x plot goes to 0 as $s^{-\frac{1}{2}}$. Such are the "wee" secondaries of Feynman ¹¹⁾. To the contrary, a secondary clearly associated with limiting fragmentation (x somewhat close to 1 or -1) corresponds to a well-defined ratio either between the secondary energy and the target energy or between the secondary energy and the projectile energy. This corresponds to specific distances from the edges on the rapidity plot.

The invariant mass M of two secondaries of mass μ_1 and μ_2 , taken as relatively small as compared to M , is given by

$$\begin{aligned} M^2 &= (q_1 + q_2)^2 = \mu_1^2 + \mu_2^2 + 2\omega_1\omega_2 - 2q_{1L}q_{2L} - 2q_{1\perp}q_{2\perp} \\ &\simeq 2\mu_1\mu_2 (\operatorname{ch} z_1 \operatorname{ch} z_2 - \operatorname{sh} z_1 \operatorname{sh} z_2) \\ &= 2\mu_1\mu_2 \operatorname{ch} (z_1 - z_2) \end{aligned} \quad (9)$$

Notwithstanding small transverse momenta two secondaries issued from a resonance have a fixed interval on the rapidity plot. A uniform rapidity distribution is, therefore, invariant against sequential decays. At the same time, points in the neighbourhood of the two edges are expected to show strong correlations with either the target or the projectile motion. This, indeed, corresponds to the fragmentation regions. Even though this may at present appear as somewhat arbitrary, we may assume that, as the energy increases, the fragmentation regions proper will extend only down to some specific absolute values of x . What is meant by this is that, when a secondary laboratory momentum becomes very large, though much smaller than the projectile momentum (or much smaller than the projectile momentum though still very large), it is pointless though still formally possible to try to label it as a target (or projectile) fragment. Any correlation with either of them is a priori useless looking for. As it is stressed later, this is at least what is expected in a multiperipheral model where, due to the small momentum transfer between neighbouring secondaries in the multiperipheral chain, two particles with a very large centre-of-mass energy ratio are predicted to be uncorrelated as distant in the chain. In between two yet vaguely defined points in rapidity one could find, as advocated by Feynman¹¹⁾, secondaries equally distributed in rapidity and uncorrelated with either the target or the projectile. This is called the pionization phenomenon. Getting large enough rapidity intervals may, however, require very high energies, indeed.

Such secondaries would all correspond to low values of x and the constancy of the rapidity distribution implies that f , as defined in (2), is there independent of x , $(d\sigma)/(dx)$ behaving as x^{-1} as $x \rightarrow 0$ ¹⁹⁾. Again the distribution scale ! Thanks to pionization the x distribution would then not sharply dip around $x = 0$ as the energy increases asymptotically.

The Missing Mass Distribution

In analogy with deep inelastic electron scattering^{20),21)} one may consider the missing mass and momentum transfer associated with a particular secondary. This is tempting to do in an inclusive reaction such as $pp \rightarrow (p)$ or $K^+p \rightarrow (K^0)$ in which the incident particle has, roughly speaking, a chance to "preserve its identity" while exciting the target. (We, heretofore, denote by $AB \rightarrow (C)$ a reaction in which AB collide to give C and anything else.) Nevertheless, as stressed in Section 5, simple connections between electron and hadron deep inelastic scattering are, if any, difficult to get at present machine energy. As later discussed, however, such a distribution might be quite useful for low momentum transfers between the projectile (target) and the selected secondary. Writing the missing mass squared M^2 and the momentum transfer squared t in terms of laboratory variables^{*)}, one finds

$$t = (q - q_p)^2 = \mu^2 + m_p^2 - 2\omega\omega_p + 2q_L q_p \quad (10)$$

$$M^2 = t + m_T^2 + 2m_T(\omega_p - \omega)$$

with

$$\omega^2 = q_L^2 + q_\perp^2 + \mu^2$$

At fixed t , the large values of M^2 correspond to the lower values of ω (or small values of x) one has $M^2 = t + m_T^2 + s(1-x)$. The lowest values of t correspond to x close to 1 ($q_\perp/q_p \rightarrow 1$ at high energy). One then finds that $-t/q_\perp^2 \rightarrow 1$. One has

$$t = m_p^2(1-x) + \mu^2 \frac{x-1}{x} - \frac{q_\perp^2}{x}$$

*) We, therefore, omit the target superscript and use a subscript to designate the target and the projectile.

One then readily finds that

$$\frac{d\sigma}{dt dM^2} = \frac{\omega}{2m_T q_F} \frac{d\sigma}{dq_L dq_L^2} = \frac{\pi}{2m_T q_F} \frac{\omega d\sigma}{d^3 q}$$

which asymptotically gives

$$\frac{d\sigma}{dt dM^2} = \frac{1}{s} \omega \frac{d\sigma}{dq_L dq_L^2} = \frac{1}{s} \times \frac{d\sigma}{dx dq_L^2} \quad (11)$$

For large s and M^2 and t fixed, s/M^2 is a scaled variable. At present machine energy the distribution (11) is a priori useful for low values of $|t|$ and relatively low values of M^2 . The high missing mass spectrum carries many projectile fragments which, when included in the missing mass to a specific one of them together with the target fragments, contribute very large values to it and this for simple kinematical reasons.

For fixed x and q_L^2 (and therefore t) the previously stressed scaling property implies that $s(d\sigma)/(dt dM^2)$ should be a function of s/M^2 only. As an example we may consider a standard quasi two-body reaction with production of a heavy state of mass M (Fig. 6). At large s the Regge argument should be written as s/M^2 ²²⁾ and the asymptotic Regge amplitude reads

$$A = \beta(t) \gamma(t, M^2) \left(\frac{s}{M^2} \right)^{\alpha(t)} \quad (12)$$

where α is the leading Regge trajectory which may be exchanged in the production process. We have factorized the residue function with the M^2 dependence in one of the terms only. One readily obtains

$$\omega \frac{d\sigma}{d^3q} = \frac{1}{s} \beta^2(t) \gamma^2(t, M^2) \left(\frac{s}{M^2} \right)^{2\alpha(t)} \quad (13)$$

scaling implies that $\gamma^2(t, M^2)$ is a linear function of M^2 at fixed Reggeon mass t . This is the case if the Reggeon target total cross-section tends to a constant at high energy (large M^2), a property which fits with our present understanding. Such a behaviour seems highly reliable when the exchanged trajectory belongs to the leading nonets of vector and tensor trajectories²³⁾. It still carries a question mark when it is the Pomeron since a leading linear M^2 behaviour would then call for a triple Pomeron coupling which is known to be small²⁴⁾. We shall discuss later the extension of (13) to lower values of M^2 .

The Scaling Property

Summing up, we have seen that scaling may be suggested from a priori different physical pictures and rationales when different sets of variables are considered: a diffraction picture when one focusses on the target (or projectile) frame, the search for a property which does not depend on which fragments actually come first (resonances and then pions) when one considers the rapidity distribution, or, finally, Regge behaviour when the differential cross-section is written in terms of missing mass and momentum transfer to one particular secondary. We are then led to abstract it from our present models as a key property of inclusive reactions, and we shall see later how it comes out naturally from asymptotic Regge behaviour. It should be stressed though that it comes out as an asymptotic property. One should be prepared to acknowledge deviations at low energies. Furthermore, the scaling variable $x = 2q_{\perp}/\sqrt{s}$ is unambiguously defined only at high energy. At lower energy it may

turn out, as it was the case in quite a different context for electron inelastic scattering ¹³⁾, that a different scaling variable, which would involve masses in a specific way, might already reveal scaling when x as so far defined would not. One may as well obviously use $x = q_{\perp}/q_{\max}$ where $q_{\max}$ is the maximum value of the centre-of-mass momentum and search for more sophisticated scaling variables is open.

Turning to experiment we have still to face rather meagre information. However, much work is at present going on along this line and we may expect to know more soon. In order to illustrate what is now available, we may look at Fig. 7 which gives the x distribution of the π^- observed in $K^+p(\pi^-)$ at 12 GeV/c and compare it to the same distribution found for the backward π^- in $\pi^-p(\pi^-)$ at 19.2 GeV/c ⁵⁾. This does not speak for scaling proper but, nevertheless, shows that the same trend appears as a conspicuous common feature in two different reactions when both are analyzed in terms of x . Figure 8 is much more relevant. It shows a comparison between the K^0x distributions observed in the reaction $K^+p(K^0)$ at 5 and 8.2 GeV/c ²⁵⁾. An "optimistic eye" may already see their evidence for scaling. Needless to say that data at higher energies (we may expect soon some at 12 and at 16 GeV/c) would be highly welcome. Finally, Fig. 9 shows the ISR results recently reported ²⁶⁾ on $pp(\pi^+)$ at 500 GeV (15 GeV ISR) and 1100 GeV (22 GeV ISR) presented together with data at 12 GeV/c. The results are still preliminary and still carry important error bars. Nevertheless, the scaling property seems to hold the test very well !

In search for a still more simple behaviour, one may check whether the invariant distribution factorizes or not in terms of its x and $q_{\perp}^2$ dependence

$$\omega \frac{d\sigma}{d^3q} = f(q_{\perp}^2, x) = F(x) G(q_{\perp}^2) \quad (14)$$

This is suggested by some theoretical models and worth searching for anyway. The answer seems to be no. Figure 10 shows the x distribution observed for different values of $q_{\perp}^2$ in the $K^+p(\pi^-)$ reaction ⁵⁾. The x distribution becomes flatter as $q_{\perp}^2$ increases.

3. GENERAL DYNAMICAL PROPERTIES

The Multiperipheral Model

Besides diffraction dissociation to which we have readily to grant part of the production process, the most reliable and sophisticated model at hands is the multiperipheral model ¹⁶⁾ in its modern version of multi-Regge exchange ^{27),28),29)}. Each version has its specific features. It seems, however, that the key property is an increasing lack of correlation between secondaries as the ratio of their centre-of-mass energy increases down to zero and as their centre-of-mass momenta get opposite signs (or actually as their distance increases on a rapidity plot). The leading multiperipheral contribution to the production amplitude, as drawn on Fig. 11, corresponds to large subenergies and small momentum transfers, conditions which may well require the grouping of several secondaries into clusters considered as a single particle. If ω_i and μ_i respectively stand for the energy and mass of any of the secondaries (particle or cluster) one can write each subenergy squared (referring to the neighbouring particles i and $i+1$) as ²²⁾

$$s_{i,i+1} = (\omega_i + \omega_{i+1}) \left(\frac{\mu_i^2}{\omega_i} + \frac{\mu_{i+1}^2}{\omega_{i+1}} \right) \quad (15)$$

In order to have them all large, one needs $(\omega_i / \omega_{i+1}) \gg 1$. One may then write

$$s_{i,i+1} \approx \mu_{i+1}^2 \left(\frac{\omega_i}{\omega_{i+1}} \right) \quad (16)$$

When one also finds that

$$\frac{\omega_i}{\omega_{i+1}} \approx \frac{s}{s_i s'_i} \langle q_{\perp}^2 \rangle \quad (17)$$

s_i and s'_i are respectively the invariant mass squared of the group of secondaries including particles 1 to i and $i+1$ to n as they appear on the multiperipheral chain drawn on Fig. 11

$\langle q_{\perp}^2 \rangle$ stands for some average value (small) of the transversed momentum squared. For two clusters of particles with s/s_i , s/s'_i and s/t all large $|t|$ is equal to $q_{\perp}^2$. In order to preserve best the low momentum transfer and high subenergy conditions, the ratios between secondary centre-of-mass energies must increase as one moves down the multiperipheral chain and produced particles are expected to lose correlations as they originate from more distant vertices. If the chain is long enough the central secondaries, which therefore have but relatively low centre-of-mass energies, should show a distribution independent of x , the variable which measures their connection to the target or projectile. Generally speaking one expects scaling at very large energies since the key variables are the ratios between centre-of-mass energies (16) which increase as one moves down the multiperipheral chain. The secondary distribution should now even be independent of x as x becomes small enough. Pionization should eventually show up though one may, of course, well need very high energies in order to observe it.

Such general properties were, indeed, abstracted already a decade ago ¹⁶⁾ from more specific multiperipheral models as they were recognized to be much more general than the models themselves. The key feature, stressed by Wilson ¹⁶⁾, is, indeed, the loss of correlation between secondaries as the ratio of their centre-of-mass energies becomes very large. On a rapidity plot this corresponds to a uniform distribution except for the edges where, up to (or down to) fixed distances, correlations are expected to prevail and typical fragmentation effects should be observed.

Following Feynman one may then consider a rapidity three-dimensional representation whereby each secondary is represented by its coordinates x , y and z . All points are to be found within a tube with fuzzy but rather well-defined boundaries ($\log s/m_T m_P$ and $\langle q_{\perp}^2 \rangle$ giving the longitudinal and transverse dimensions). They should look asymptotically as gas molecules with a well-defined longitudinal density except for edge effects and a well-defined gradient in their transverse density. Correlations are, of course, expected between neighbouring molecules but they should get smeared out if one considers large enough rapidity intervals. As previously considered, this is, in particular, the case for resonance production. Let us again stress that $\log s$ increases slowly, indeed, and the edges of the gas container could well still be too close at energies where we can probe at present. $\log s$ increases only by a factor two as one moves from top PS energy to top ISR energy !

As shown by Fig. 7 there is not any sign of pionization at 20 GeV (a flattening of the x distribution at small x in between two bumps associated with fragmentation) but this could still be observed at top ISR energies (Fig. 9).

Multiplicity of Secondaries

Another key property of the multiperipheral model (also more general than the specific models which can be considered) is the maximum logarithmic increase in the number of secondaries. This is obvious in the Feynman gas picture, the density being constant and the longitudinal dimension of the tube increasing as $\log s$, the number of secondaries produced through pionization should increase as $\log s$ while those produced through fragmentation should remain finite in number. Nevertheless, it is worth deriving it at an earlier stage of the analysis since the $\log s$ increase here results from the actual occurrence pionization. As an illustration let us get it in three different though not fully independent ways. Taking specifically a multiperipheral model the production cross-section for n particles σ_n is proportional to the n^{th} power

of the coupling constant. Scaling up the coupling constant by $\sqrt{\lambda}$, the total cross-section should scale as ³⁰⁾

$$\sigma_{\text{tot}}(\lambda) = \sum_n \lambda^n \sigma_n \quad (18)$$

The average multiplicity which reads

$$\langle n \rangle = \frac{1}{\sigma_{\text{tot}}} \sum_n n \sigma_n \quad (19)$$

is then written as

$$\lim_{\lambda \rightarrow 1} \frac{\lambda}{\sigma_{\text{tot}}(\lambda)} \left(\frac{\partial}{\partial \lambda} \sigma_{\text{tot}}(\lambda) \right)$$

We now assume that $\sigma_{\text{tot}}(\lambda)$ has an asymptotic Regge behaviour with Regge trajectories changing with λ

$$\sigma_{\text{tot}}(\lambda) = \beta(\lambda) s^{\alpha(\lambda)-1} \quad (20)$$

and, of course,

$$\lim_{\lambda \rightarrow 1} \alpha(\lambda) = 1$$

$\langle n \rangle$ is then readily found to increase as $\log s$.

We now retain but the fact that the centre-of-mass energy ratio between two neighbouring clusters must be at least R in

order to have them rightfully separated in a multiperipheral amplitude ²²⁾. This is a much more general property of multiperipheral models. We then readily find that the number n of clusters can increase at most at a rate such that

$$R^{\frac{n}{2}} \sim \sqrt{s} \quad (21)$$

The number of clusters, and therefore of secondaries, cannot, therefore, increase faster than $\log s$ and will do so if the pionization process, indeed, develops, the individualized secondaries on the multiperipheral chain getting as numerous as they can be.

Finally, integrating the single particle distribution we should find the total cross-section multiplied by the average number of secondaries as obvious from the definition of cross-sections according to fluxes. One then writes

$$\langle n \rangle \sigma_{tot} = \int f(x, q_{\perp}^2) \frac{\pi dx dq_{\perp}^2}{\sqrt{x^2 + \frac{4}{s}(q_{\perp}^2 + \mu^2)}} \quad (22)$$

Provided now that $f(x, q_{\perp}^2)$ goes smoothing to a finite limit when x goes to zero which is a very general property of multiperipheral models. The multiplicity is asymptotically dominated by the small x values and depends logarithmically on a lower cut-off which decreases as $s^{-\frac{1}{2}}$. If again low centre-of-mass energy secondaries are produced the multiplicity should increase logarithmically while the total cross-section remains constant.

Diffraction and Multiperipheralism

It should be stressed though that such a logarithmic increase comes out as a higher limit, obtained provided that a certain (for instance, multiperipheral) mechanism shows up. As already mentioned, this logarithmic behaviour appears to be met by experiment.

Multiperipheral production though it includes fragmentation by no means excludes an independent diffractive process more readily associated with the fragmentation picture. In a diffraction process the cross-section for each final configuration should tend towards a fixed finite value (scaling property). As a result cross-sections for relatively low multiplicities (and values of x relatively close to 1) should remain constant and this despite the leading role which could be taken up by pionization at asymptotic energies with vanishing cross-sections for low multiplicities and fast centre-of-mass secondaries.

The persistence of small (charged) multiplicities as the average multiplicity increases is most interesting to check. At extremely high energy one could, as stressed by Wilson, expect a two-bump distribution¹⁶⁾ as the diffraction contribution stays constant with a maximum at very low multiplicity and as the multiperipheral pionization contribution increases in size and displaces its maximum at $\log s$; this is depicted on Fig. 12. This would provide evidence for two separate mechanisms. Nevertheless, and as already stressed, the ISR energy is probably too low for such an effect to show up even if its prediction is, indeed, correct. Present experimental information seems to indicate a persistence of low (charged) multiplicities at increasing energy. This is shown on Fig. 13 which combines machine and cosmic-ray results.

In the fragmentation picture large multiplicities are expected to correspond but to small cross-sections since the cross-section for each multiplicity is fixed. Indeed, σ_n should decrease faster than n^{-1} so that the total cross-section remains

finite, and probably decreases still much faster. The same applies for the cross-section with small x values (or large laboratory or projectile frame energy) since a fixed and important fraction of the cross-section is already provided by secondary configuration with $|x|$ in the neighbourhood of 1. The diffraction cross-section should decrease faster than $(q^T)^{-1}$ and this should asymptotically lead to a dip in the x distribution at $x = 0$. It remains though that multiplicities could well increase with energy. If σ_n decreases only as n^{-2} say, the average multiplicity obviously increases as $\log s$. In the Pomeron exchange picture limiting fragmentation implies a vanishingly small slope for the Pomeron, a high energy property which is not excluded by the recent ISR results³¹⁾.

Concluding it should be stressed that the two extreme dynamical pictures respectively associated with diffraction dissociation (and readily leading to limiting fragmentation) and with multi-peripheral production (and readily leading to pionization) may still have a very large overlap in their predictions at presently available energies. The scaling property common to both should be carefully checked. The behaviour of f in the neighbourhood of $x = 0$ could through its flattening or absence of dip provide evidence for pionization while the constancy of low multiplicity cross-sections should provide evidence for limiting fragmentation. With these guiding lines in mind there is an obvious need for more experimental information.

The next step is to ascertain correlations or absence of correlations between secondaries. A typical such experiment would be to compare the double distribution of a fast and a slow secondary, a π^+ and a π^- say, at the ISR with the product of the two single particle distributions simultaneously measured. As previously stressed, such correlations could disappear with increasing rapidity interval.

In connection with the secondary spectrum it should be stressed that the pion distribution in pp collisions is

kinematically concentrated away from $|x| = 1$ while the proton distribution goes all the way to 1. This is most readily seen in terms of rapidity. Most secondaries will have rapidity below the one of the projectile. This means that

$$q_L < \frac{m}{M} q_P \approx \frac{q_P}{3}$$

when m , which includes the transverse momentum (4), corresponds typically to 300 MeV. The pion centre-of-mass spectrum is, therefore, expected to fall quickly above $|x| = 0.3$, though vanishingly extending to 1. The secondary proton spectrum should, on the other hand, extend healthily to $|x| = 1$.

Factorization

The multiperipheral model implies a factorization property which results from the factorization of residue functions. The distribution of a secondary with a quite different rapidity as that of the projectile (target) should show a distribution independent (up to a global common factor) on the nature of the projectile (target). Such a property is, of course, also expected in the fragmentation picture. As an example, the slow laboratory π^- obtained in pp and π^-p collisions should show proportional laboratory distributions.

$$f(q_L^T, q_\perp^2)_{\pi-p} \propto f(q_L^T, q_\perp^2)_{pp} \quad (23)$$

$$f_{\pi-p} = C f_{pp}$$

With scaling the distribution at fixed q_L^T should not depend on s but only on q_L^T . Notwithstanding scaling, what matters is the rapidity of the projectile once that of the secondary has been fixed. One could then even better compare the proton

induced distribution with that obtained with pions with 1/7 of the energy in the centre of mass (Batavia pions and ISR protons, say).

In the multiperipheral model, if one further assumes that the extreme vertex involves Pomeron coupling, the distributions are expected to scale according to the total cross-sections $C = (\sigma_{\text{tot}}(\pi^- p)) / (\sigma_{\text{tot}}(pp))$ or rather their asymptotic values.

4. REGGE APPROACH TO INCLUSIVE CROSS-SECTIONS

Mueller Analysis

A very important step in the understanding of inclusive reactions is the association of the inclusive differential cross-section for production of 1,2,... observed particles with a particular discontinuity of the 3,4,... body amplitude at zero momentum transfer³²⁾. This generalizes the well-known optical theorem whereby the total cross-section is given in terms of the imaginary part of the forward elastic amplitude: $\sigma_{\text{tot}} \sim \frac{1}{s} \text{Im } A(s,0)$.

The cross-section for production of one particle together with n-1 others reads

$$\omega \frac{d\sigma}{d^3q} = \frac{\pi}{4pW} \int \delta(p_A + p_B - q - Q_n) | \langle n-1, Q_n | j_c(0) | p_A p_B \rangle |^2 \prod_{i=1}^{n-1} \frac{d^3k_i}{(2\pi)^3 2\omega_i} \quad (24)$$

where the invariant pW , the Møller factor written here as the product of the centre-of-mass momentum and energy, goes asymptotically to $s/4$. $Q_n = \sum_i k_i$ and j_c stand for the current associated with the observed particle. p_A and p_B are the momenta of the colliding particles. Summing over all possible n one obtains the inclusive $A+B(C)$ cross-section. The invariant quantity

$$A = \sum_n \pi \int \delta(P_A + P_B - q - Q_n) | \langle P_A P_B | d_c(0) | n-1, Q_n \rangle |^2 \prod_{i=1}^{n-1} \frac{d^3 k_i}{(2\pi)^3 2\omega_i}$$

is rewritten as

$$A = \pi \int d^4 x e^{-iqx} \langle P_A P_B | d_c(x) d_c(0) | P_A P_B \rangle \\ \equiv A(s, P_A \cdot q, P_B \cdot q) \quad (25)$$

Using momentum conservation and completeness A depends on s , $p_A \cdot q$ and $p_B \cdot q$ and could be associated to a part of the absorptive part of the ABC elastic amplitude at zero momentum transfer though continuation from positive C to negative $\bar{C}$ energy. Even though very little is yet known about it, one may reasonably postulate Regge asymptotic behaviour for large values of s and large values of either $(p_A \cdot q)$ or $(p_B \cdot q)$ or large values of both. This has been proposed by Mueller in a recent paper ³²⁾. The Regge analysis proceeds in the following way ³³⁾.

When both $p_A \cdot q$ and $p_B \cdot q$ are large - this corresponds to a slow centre-of-mass secondary - it is easier to work in the secondary rest frame. To this end, one writes

$$q = \mu (1, 0, 0, 0) \\ P_A = m_A (\cosh \varphi_A, \sinh \varphi_A \cos \varphi, \sinh \varphi_A \sin \varphi, 0) \\ P_B = m_B (\cosh \varphi_B, -\sinh \varphi_B, 0, 0) \quad (26)$$

where φ_A and φ_B are the rapidity of the two colliding particles and φ the angle with which they collide in the secondary particle rest frame.

$$P_A \cdot q = m_A \mu \operatorname{ch} \varphi_A \quad ; \quad P_B \cdot q = \mu m_B \operatorname{ch} \varphi_B$$

$$P_A P_B = m_A m_B (\operatorname{ch} \varphi_A \operatorname{ch} \varphi_B + \operatorname{sh} \varphi_A \operatorname{sh} \varphi_B \cos \varphi) \approx \frac{s}{2}$$

With leading double Regge pole behaviour assumed to hold for large $p_A p_B$, $p_A q$ and $p_B q$, one then has ³³⁾

$$A \approx (\operatorname{ch} \varphi_A)^{\alpha_1(0)} (\operatorname{ch} \varphi_B)^{\alpha_2(0)} \beta(\varphi) \quad (27)$$

A simple calculation shows that

$$1 + \cos \varphi = \frac{2\mu^2}{q_\perp^2 + \mu^2}$$

when $q_\perp$ is the centre-of-mass transverse momentum. The approximation thus reached for A reads

$$A \approx R(q_\perp^2) (P_A \cdot q)^{\alpha_1(0)} (P_B \cdot q)^{\alpha_2(0)} \quad (28)$$

Using now centre-of-mass variables and keeping only leading terms one gets

$$P_A \cdot q = P(\omega - q_L) \quad ; \quad P_B \cdot q = P(\omega + q_L)$$

and

$$(P_A \cdot q)(P_B \cdot q) = \frac{s}{4} (q_\perp^2 + \mu^2) \quad (29)$$

If the leading Regge singularity is the Pomeron, one readily combines (24), (28) and (29) to find

$$\omega \frac{d\sigma}{d^3q} = \frac{1}{4} (q_{\perp}^2 + \mu^2) R(q_{\perp}^2) = f(q_{\perp}^2) \quad (30)$$

which corresponds to the pionization limit ! The invariant distribution depends only on $q_{\perp}^2$ and neither on s nor on x .

It should be stressed though that double Regge pole behaviour has been assumed for an object which is but part of the absorptive part of an elastic three-body amplitude. If the leading singularity is instead a Regge cut the limit thus obtained could be spoiled by possible powers of $\log s$. Furthermore, if double Pomeron residue functions, which, as later discussed, appear to be small for production amplitudes, would be also small for the specific discontinuity here considered, we would expect the limit thus obtained to be somewhat irrelevant. Pionization would exist but at a very low level ! As a final remark it is tempting to factorize the residue function and thus derive the factorization properties previously mentioned, with scaling according to total cross-sections. This is readily obtained but assumes a Regge pole behaviour which is probably less reliable than the asymptotic behaviour (28). On the other hand, if the factorization property actually holds one may derive that the inclusive reactions will not depend on the projectile or target polarization as no helicity flip is then allowed at zero transfer ³⁴⁾. As a final remark the term next to this asymptotic one is expected to be a mixed Pomeron Regge term. The limiting distribution should then be approached as $s^{-\frac{1}{4}}$ ³⁵⁾.

More reliable is probably the single Pomeron limit which one has to consider when either $(p_A q)$ or $(p_B q)$ is large but not both. As seen now, this corresponds to the fragmentation region ³²⁾. The natural system is then the rest frame of the target

$\angle(p_A, q)$ large. One then writes

$$\begin{aligned} P_B &= m_B (1, 0, 0, 0) \\ P_A &= m_A (\cosh \varphi, \sinh \varphi, 0, 0) \\ q &= \mu (\cosh \xi, -\sinh \xi \cos \varphi, -\sinh \xi \sin \varphi, 0) \end{aligned} \quad (31)$$

φ is now the angle between the projectile and secondary directions as seen in the target frame

$$\begin{aligned} P_A \cdot q &= \mu m_A (\cosh \varphi \cosh \xi + \sinh \varphi \sinh \xi \cos \varphi) \\ P_B \cdot q &= \mu m_B \cosh \xi \\ P_A \cdot P_B &= m_A m_B \cosh \varphi \approx \frac{s}{4} \end{aligned}$$

$\cosh \varphi$ is large and $\cosh \xi$ is held fixed. Leading Regge behaviour then gives

$$s^{\alpha(0)} R \left(P_B \cdot q ; \frac{P_A \cdot q}{P_A \cdot P_B} \right) \quad (32)$$

since the residue depends only on φ and ξ . At large energy $\sinh \varphi \rightarrow 1$, the φ dependence drops out of the argument of R which becomes a function of q_L^T and $q_{\perp}^T{}^2$ only. With $\alpha(0) = 1$ the combination of (24) and (32) gives the limiting fragmentation limit, with an over-all scaling of distributions obtained with different projectiles according to the total cross-section.

As the total cross-section $A+B \rightarrow (0)$ with complicated subamplitudes is simply related by the optical theorem to the absorptive part of the two-body elastic amplitude $AB \rightarrow AB$ at zero transfer (Fig. 14a,b), the inclusive cross-section $A+B \rightarrow (C)$ with complicated subamplitudes is related to a term in the absorptive part of the three-body elastic amplitude $A+B+\bar{C} \rightarrow A+B+\bar{C}$ at zero transfer (Fig. 14a,b), or rather an $AB\bar{C}$ discontinuity of the ABC elastic amplitude. In both cases, one may consider a single Pomeron behaviour symbolically represented on Fig. 14c. A Regge asymptotic behaviour (Fig. 14c) has, however, to be considered for the discontinuity or a fraction of the discontinuity of the amplitude (Fig. 14), and not for the full amplitude.

Next to the leading Pomeron contribution, one should find standard Regge terms ($\rho, \omega, f^0, A_2, \dots$). The limiting distribution is, therefore, expected to be approached as $s^{-\frac{1}{2}}$. Figure 15 shows a comparison of the $K^+p(\pi^-)$ reaction at 12 GeV/c and the $pp(\pi^-)$ reaction at 29 GeV/c in the proton fragmentation region respectively scaled down by the K^+p and pp total cross-sections. The agreement is remarkable. It would be of great interest to compare them to data at different energies. This already suggests that limiting fragmentation distributions, with scaling according to total cross-sections, may already be seen at present machine energies. Non-Pomeron Regge terms may be assumed to contribute still to 10 to 20 % of the cross-section, and are, therefore, difficult to be seen with certainty on the present data. It may be argued, however, that in such cases asymptotia should be reached already at relatively low energies.

Approaching the Limiting Distribution

As a result of duality, leading Regge contributions should cancel out their imaginary part in channels where no conspicuous resonances are present, which is the case for exotic channels³⁶⁾. The imaginary part results then only from the smooth Pomeron contribution. It is almost constant with energy in contradistinction

with the decreasing Regge behaviour met in non-exotic channels. As a result, exotic total cross-sections become practically constant at relatively low energy and one may expect a similar type of property for inclusive cross-sections if some channels have exotic quantum numbers. From the previous discussion the $AB\bar{C}$ channel should be exotic ³⁷⁾. It turns out that the condition is not enough and that the AB channel ³⁸⁾ should also have exotic quantum numbers.

If T_x stands for the production amplitude $A+B \rightarrow C+X$, the inclusive cross-section is given by

$$\omega \frac{d\sigma}{d^3q} = \pi \frac{1}{s} \sum_x |T_x|^2 = \pi \frac{1}{s} \sum_x T_x(s^+, s'^+) T_x(s^-, s'^-) \quad (33)$$

where s' is the $AB\bar{C}$ subenergy squared, and $s^\pm(s'^\pm)$ indicates which limit is taken in the cut s and s' planes. The kinematics of the inclusive reaction are such that there are the two channels where resonances may show up in the physical region (Fig. 14). This is, for instance, not the case for the $B\bar{C}$ subenergy squared which is below its physical threshold and actually corresponds to a momentum transfer. It is for this reason that such variables have not been explicitly introduced in the argument of T_x . We rewrite now (33) as ³⁸⁾

$$\begin{aligned} \frac{s}{\pi} \omega \frac{d\sigma}{d^3q} &= \sum_x T_x(s^+, s'^+) T_x(s^+, s'^-) \\ &+ \sum_x \left(T_x(s^-, s'^-) - T_x(s^+, s'^-) \right) T_x(s^+, s'^+) \end{aligned} \quad (34)$$

The first term on the right-hand side of (34) is the $ABC\bar{C}$ discontinuity of the ABC elastic amplitude as defined in the AB physical sheet when the real s axis is approached from above. The second term is the sum of terms proportional to the AB discontinuities of the production amplitudes as defined on the different unphysical sheet reached through crossing the different $ABC\bar{C}$ branch lines. One may then apply duality to these discontinuities, expecting Regge behaviour unless all discontinuities correspond to exotic channels, a special case, where the asymptotic limit should be reached at already relatively low energy.

Proceeding in a more general way the $ABC\bar{C}$ discontinuity which has to be calculated, and which corresponds to Fig. 16, can be written as ³⁹⁾

$$\Delta = A(++-) - A(+--) \quad (35)$$

The signs indicate the limits as they have to be respectively taken in the AB , $ABC\bar{C}$ and $A'B'$ channels. A is the three-body elastic amplitude $ABC \rightarrow A'B'C'$ (the initial and final particles are the same but their momenta could be different). The other variables are spacelike in the kinematic region of interest and do not have to be specified explicitly. Δ is somewhat ill-defined as an $ABC\bar{C}$ discontinuity since it has an AB discontinuity unless one has an explicit model which specifies how the discontinuity should be taken. In order to be able to write it without ambiguity we transform it into a sum of complex but well-defined quantities as

$$\begin{aligned} \Delta = & \left(A(+++) - A(+--+)) \right) - \left(A(+++) - A(++-) \right) \\ & + \left(A(+--+)) - A(+--) \right) \end{aligned} \quad (36)$$

The first bracket is the $AB\bar{C}$ discontinuity with AB and $A'B'$ both defined on the upper side of their cut. The second bracket is the $A'B'$ discontinuity calculated for $AB\bar{C}$ on the upper side of the $AB\bar{C}$ cut, whereas the third bracket is again the $A'B'$ discontinuity but now calculated with $AB\bar{C}$ on the lower side of the $AB\bar{C}$ cut. The last two brackets can be looked at as the double $AB\bar{C} A'B'$ discontinuity.

Since all discontinuities are now spelt out, we may use duality to state that if the $AB\bar{C}$ channel is exotic the first bracket should be independent of energy (the discontinuity should be given by the Pomeron contribution only) but that $AB(A'B')$ exotic is also needed in order to have the second two brackets constant. Absence of resonances is translated by constancy of discontinuities in the two relevant channels.

Were we using a dual model $AB\bar{C}$ exotic would imply no discontinuity in the $AB\bar{C}$ subenergy and the two last brackets would cancel themselves out even if AB were not exotic. However, if there is an $AB\bar{C}$ discontinuity, as it should be due to Pomeron exchange, the last two brackets will not cancel. Their contribution is, nevertheless, supposed to reduce to a constant term if AB is an exotic channel.

The two cases considered $K^+p(\pi^-)$ and $pp(\pi^-)$ meet this requirement ⁴⁰⁾. In the reaction $K^-p(\pi^+)$ we expect, on the contrary, that the inclusive cross-section will only slowly approach its limiting distribution in the 10-20 GeV/c region ($AB\bar{C}$ and AB are both non-exotic). This should also be the case for $K^-n(\pi^+)$ where $AB\bar{C}$ is exotic but AB is not.

It would be misleading to consider the ABC amplitude as the sum of a dual term with Regge behaviour (a B_6 amplitude, say) together with a limiting term. This disregards mixed Regge Pomeron terms, as the one shown on Fig. 17, which contributes to the Regge behaviour. Such terms are, however, not present if AB is exotic. Similar conclusions are reached associating the Pomeron with a dual loop ⁴¹⁾.

Concluding Regge behaviour can be used in a simple way in the phenomenology of inclusive reactions and duality leads to very interesting predictions which wait for checks.

Triple Reggeon Couplings

Considering the three-body amplitude of Fig. 14c in the projectile fragmentation region ($p_A q$) small we may focus on the low momentum transfer domain (momentum transfer between particles A and C) and approximate amplitude in terms of leading Regge exchange (Fig. 18). For large missing mass $M^2 = (p_A + p_B - q)^2$ one may further write the lower blob in terms of leading Regge exchange (Fig. 18), and associate the inclusive cross-section with a triple Reggeon term.

A straightforward calculation gives ^{22), 23)}

$$\omega \frac{d\sigma}{d^3q} = \frac{1}{2|\underline{p}_A| m_B} \left(\frac{s}{M^2} \right)^{2K(t)} (M^2)^{\bar{\alpha}(0)} \quad (37)$$

where p_A is the projectile momentum in the laboratory and α and $\bar{\alpha}$ stand for the leading trajectories respectively coupled at the $A\bar{C}$ and $B\bar{B}$ vertices. A high energy

$$2|\underline{p}_A| m_B \approx s \quad (38)$$

Using a simple Regge formula, it is, however, irrelevant to try to improve the asymptotic argument (s/M^2) . Relation (35) is reliable provided (s/M^2) and M^2 are large and $|t|$ is small.

Relation (35) may seem to imply some typical Regge behaviour in s when a leading trajectory can be exchanged in the $A\bar{C}$ channel,

that is, whenever $A\bar{C}$ is not exotic. One may be tempted to conclude that $A\bar{C}$ (and $B\bar{C}$ as it follows from a mirror argument made for the target vertex) should also be exotic so that the limiting distribution is already reached at relatively low energy. It should be stressed though that the Regge argument is here a scaled variable (s/M^2 is independent of s !) and that the non-exoticity of $A\bar{C}$ (or $B\bar{C}$) allows non-vanishing triple Reggeon terms which contribute to the strength of the production amplitude but are not relevant at describing the energy behaviour.

If M^2 is not very large, $\bar{\alpha}(0)$ should be taken as an effective intercept combining the Pomeron P and the vector-tensor trajectories R with $\frac{1}{2} < \bar{\alpha}(0) < 1$. Using duality this can even be extended down to the resonance region, the P and R trajectories respectively, interpolating through the "background" and resonances contributions. If $\alpha(t)$ is itself the Pomeron trajectory, the inclusive cross-section as obtained from (35) involves a double Pomeron coupling PPR which is usually believed to be small and a triple Pomeron coupling PPP known to yield if large to theoretical difficulties ²⁴⁾.

If, however, $\alpha(t)$ is one of the leading vector-tensor trajectories, (35) is proportional to the RRP and RRR Reggeon couplings which nothing restrict yet. We have thus a triple Regge expression for (13) and the analysis of inclusive reactions in the low $|t|$ high s/M^2 limit should thus give very interesting information about triple Reggeon couplings.

One may also easily obtain (35) considering the inelastic contribution to the imaginary part of the elastic AB amplitude. At high energy this is written in terms of multiperipheral contributions and since the momentum transfer between A and C is kept small, the line associated with particle C , when present, is always drawn on top of the ladder diagram (Fig. 17a). The inclusive cross-section is then readily obtained not performing the phase space integration over the momentum of particle C (Fig. 17b). This gives relation (13) and, with Regge behaviour for the lower blob,

one obtains relation (35). Relation (35) has some obvious and very interesting implications. For the whole M range, where it is applicable, the s dependence should be the same. The resonances and the "background" seen on the missing mass spectrum at fixed t should go down with energy at the same pace. Once $\alpha(t)$ is determined by the energy behaviour the missing mass spectrum is predicted and, as $\alpha(t)$ decreases with $|t|$, one expects it to rise the more sharply with M the larger $|t|$ is.

An a priori favourable case should be charge exchange inclusive reactions such as $K^+p(K^0)$ where $\alpha(t)$ should be the ρ - A_2 trajectory. As already said when discussing scaling, data exist at present at 5 and 8.2 GeV/c and some preliminary data are also already available at 16 GeV/c ⁴²⁾. As shown on Fig. 19, the missing mass spectra drawn for four different t intervals show the qualitative features which are predicted. The value of $\alpha(t)$ which can be determined from two different missing mass intervals agrees reasonably well with the ρ - A_2 trajectory as known from the analysis of two-body reactions. As expected at such relatively low energies, the low missing mass determination is better. This is displayed on Fig. 20. The value of $\bar{\alpha}(0)$ which can then be obtained from the missing mass behaviour using such values of $\alpha(t)$ in (35) is shown on Fig. 20. Again the agreement is encouraging since $\bar{\alpha}(0)$ is expected close to 1 in between $\frac{1}{2}$ and 1.

This analysis first calls for more data but, nevertheless, shows that a triple Regge approach is a priori very interesting for charge exchange inclusive reactions at relatively low values of the momentum transfer ⁴³⁾. Using exchange degeneracy one predicts the same behaviour for the $K^-p(\bar{K}^0)$ cross-section in the low $|t|$ region. This should be checked. Small momentum transfer between the target and a baryon secondary $\bar{K}^-p(\Lambda)$, for instance ⁷ should also be considered.

It would also be interesting to study pion exchange as well as vector-tensor Regge exchange. As the same analysis also applies to the $K^+p(K^*)$ reaction, one could separate out the natural and unnatural parity exchange contribution considering $(\rho_{11} + \rho_{1-1})^f$

and $\sigma_{00} f$ [and $f(e_{11}-e_{1-1})$] instead of the inclusive K^* production reaction f . As α_π is lower in the second case, one expects the missing mass spectrum to increase more sharply with M , when the cross-section should decrease faster with s .

5. IS THERE A DEEP INELASTIC HADRON-HADRON SCATTERING ?

It is a remarkable fact that the total inelastic electron-nucleon scattering cross-section does not decrease with $|t|$ as the cross-section for any specific reaction does (elastic scattering or specific N^* production, say). As it is well known, the cross-section for inelastic electron-proton scattering is obtained in terms of the tensor ^{20),44)}

$$W_{\mu\nu} = W_1 \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + W_2 \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) \quad (39)$$

where the strength functions W_1 and W_2 , which a priori depend upon $\nu = (p \cdot q)$ and q^2 , are, indeed, such that W_1 and νW_2 depend only on $\omega = (\nu)/(q^2)$ ²⁰⁾. q^2 is the momentum transfer squared of the electron, or the intermediate photon mass. ν/m is the energy transferred to the target nucleon. In present vernacular W_1 and νW_2 scale ! This is an asymptotic property ⁴⁵⁾ found to hold at already relatively small values of ν and q^2 ⁴⁶⁾.

The inelastic differential cross-section written in terms of laboratory variables is

$$\frac{d\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{(q^2)^2} \left(2W \sin^2 \frac{\theta}{2} + W_2 \cos^2 \frac{\theta}{2} \right)$$

(40)

The missing mass relates q^2 to ν

$$M^2 = 2\nu + m^2 + q^2 \quad (41)$$

Each resonance corresponds, therefore, to a value of ω which moves down as $|q^2|$ increases. At the same time, its contribution to W_1 and W_2 is expected to drop very sharply as all measured form factors do. The large missing mass contributions remain, however, as large at large $|q^2|$ as the lower missing mass (prominent resonances) contributions at lower values of $|q^2|$. One may expect similar features to show up in hadron-hadron interactions. It should be stressed though that they could be due to the fact that the tensor $W_{\mu\nu}$ is built from current matrix elements when strength functions relevant to hadron-hadron collisions could have a very different structure. Nevertheless, it has been much advocated that the behaviour of the differential cross-section in pp scattering could be associated to the rapid drop-off observed for electromagnetic form factors. One may see this ¹⁴⁾ as the result that a nucleon should fragment the same way whether hit by an electron or a nucleon. The t (or q^2) distribution associated with a one-nucleon final state should, therefore, be given by the form factor squared. For pp elastic scattering, fragmentation, applied to both vertices, gives

$$\left(\frac{d\sigma}{dt}\right)_{pp} = \left(\frac{d\sigma}{dt}\right)_{t=0} G^4(t) \quad (42)$$

This should correspond to the asymptotic s behaviour and be valid already at small $|t|$ values. One may also argue on the basis of the success of (42) at very large transfers ⁴⁷⁾ that, besides diffraction and Regge exchange there is some effective vector-vector interaction which would show up only at very large transfer values and high energy where diffraction (first condition) and Regge exchange

(second condition) would be too small as to be relevant. This is illustrated by Fig. 21⁴⁷⁾, which shows the differential cross-section as a function of energy for different t values. It tends to values which agree with the fourth power of the form factor. An effective vector-vector interaction yields a constant energy behaviour, it automatically introduces form factors in a factorized way and thus lead to (42) but now for large values of $|t|$. The only form factor to be relevant is therefore the magnetic form factor $G_M(t)$, as opposed to the Dirac form factor $F_1(t)$ which would in contradistinction be relevant at low $|t|$ values. Instead of (42) one then writes

$$\left(\frac{d\sigma}{dt}\right)_{pp} = \left(\frac{d\sigma}{dt}\right)_{t=0} \frac{G_M^4(t)}{\mu_p^4} \quad (43)$$

where μ_p is the proton magnetic moment and where, for practical purposes, a dipole formula normalized to 1 at $t = 0$ can be used in order to parametrize G_M at large $|t|$.

According to both high-energy pictures, either some limiting fragmentation or relevance of current-current matrix elements, extension to inelastic processes cannot be avoided. As a result the strength functions W_1 and W_2 should replace the form factor squared in (42) or (43) and give a total inelastic cross-section with but a much slower decrease in $|t|$ and scaling properties. Furthermore, with the second picture (current matrix elements used at both vertices) one cannot avoid factorization properties.

The cross-section for inelastic proton-proton collision is then written as²¹⁾

$$\frac{d\sigma}{dm_1^2 dm_2^2 dt} = C \frac{m^2}{s^2} W_{\mu\nu}^{(1)} W_{\mu\nu}^{(2)} \quad (44)$$

It corresponds to the process of Fig. 22 whereby two protons collide into two complicated systems with invariant masses m_1 and m_2 . The secondaries are assumed to be well enough separated that any important final state effect is already taken care of in the definition of $W_{\mu\nu}$.

According to the effective vector-vector interaction picture C is furthermore given by

$$C = \frac{1}{k_p^4} \left(\frac{d\sigma}{dt} \right)_{t=0} \quad (45)$$

The corresponding inelastic cross-sections can be easily calculated when using the fact that the transverse cross-section is much larger than the longitudinal one, which reads $W_1 \approx (\nu)/(q^2) (\nu W_2)$. This gives

$$\frac{d\sigma}{dm_1^2 dm_2^2 dt} = C (\nu_1 W_2^{(1)}) (\nu_2 W_2^{(2)}) \left\{ \frac{1}{(t+m_1^2)(t+m_2^2)} - \frac{1}{st} + \frac{(t+m_1^2)(t+m_2^2)}{2s^2 t^2} \right\} \quad (46)$$

If one takes the value $\nu W_2 \approx 0.3$ known from experiment ⁴⁶⁾ to be true over wide enough a range of W values as to suggest an asymptotic constant value and integrate this cross-section over the three variables assuming the limiting behaviour to be true for arbitrary large values of m_1^2 , m_2^2 and t , one obtains a total cross-section increasing as s which should be unacceptable. If, however, one limits oneself to the range at present accessible at machine energy, thus assuming conservative cut-off of the order of 20 GeV² in t , m_1^2 and m_2^2 for the validity of (44) with constant

γW_2 , one finds a cross-section of the order of 1 mb which is compatible with present experimental information. Had we taken this picture at relatively low $|t|$ values, using F_1 instead of G_M or (42) instead of (43). The calculated cross-section would have been much too large ! At large $|t|$ values, however, production of two high mass fire-balls with a cross-section given by (44) and (43) is consistent with present information. Such objects could perhaps be looked for, without too many difficulties, at the ISR. At accelerators one should rather focus on quasi-elastic events whereby the target nucleon only is fragmented, Fig. 22b. One of the strength functions appearing in (44) is then replaced by G_M^2 and one readily finds

$$\begin{aligned} \frac{d\sigma}{dH^2 dt} &= \frac{s + M^2 - m^2}{2s\sqrt{s - 4m^2}} \frac{d^2\sigma}{dp_L dp_\perp^2} \\ &= C G_M^2(t) (\gamma W_2) \frac{1}{2s^2} \frac{(s - m^2)^2 + (s - t - M^2 - m^2)^2}{t + M^2 - m^2} \end{aligned} \quad (47)$$

where p_L and $p_\perp$ are the longitudinal and transverse momentum of the outgoing proton. Large missing mass could be observed at large transfers and it would be most interesting to see if the strength function which can be formally calculated from the data using (47) agrees with the one obtained from electron scattering results. Relation (47) supposes, however, that it is possible to separate out the proton which has merely excited the target without being itself affected. This is, formally speaking, impossible. Nevertheless, one may bias a sample in order to favour an amplitude such as the one of Fig. 22b which, if alone, would yield the cross-section given by (47). One should play on the fact that an excited proton loses on the average 1/7 of its energy to an emitted pion and keep only those events where an outgoing proton has a laboratory energy which is not less by more than a 7th of the value of the incident energy. At PS energy this limits the analysis of values

of $\sqrt{s}$ of the order of 4 GeV maximum which does not lead us very deep in the inelastic region ⁴⁸⁾. At Serpukhov and, of course, at Batavia such an analysis could be performed with most favourable bias imposed. In any case, a comparison between the strength function thus determined and the one measured at SLAC should not be attempted unless it is found to have reached an energy-independent value ⁴⁹⁾.

As a last remark, the proton-proton and neutron-neutron deep inelastic cross-sections are expected to be the same. The effective vector-vector interaction should behave as an isoscalar. Combining (42) and (43) one sees that the proton and neutron strength functions should be in the ratio of the square of the magnetic moments, this at least for large $|t|$ or relatively small ω value at fixed missing mass. This seems in agreement with experiment ⁴⁶⁾.

Feynman Scaling and Bjorken Scaling

Another parallel between the different scaling properties found for hadron production cross-sections (Feynman scaling) and electron production cross-sections (Bjorken scaling) can be drawn if one imposes Feynman scaling to the deep inelastic electron-proton cross-section. This is a specific inclusive reaction in which the electron is selected as the observed final particle. Considering the final electron as a fragment of the initial electron, we associate with the inclusive cross-section the graph of Fig. 23 which, at high energy, should be approximated in terms of Pomeron exchange. We merely use the fact that asymptotically Pomeron exchange should dominate Compton scattering. Following the Mueller's approach described in the preceding section, one obtains scaling, the cross-section depending only on x and on the momentum transfer.

The inclusive cross-section can be written in terms of (37). Neglecting for the sake of the argument the irrelevant complications brought by the spin of the electron one readily writes

$$\omega \frac{d\sigma}{d^3q} = \frac{A}{s} \frac{1}{t^2} \sum_{\mu\nu} (p+q)_\mu (p+q)_\nu W_{\mu\nu} \quad (48)$$

where p and q are the initial and final momentum of the electron. $A = 4\pi\alpha^2$. We have written the asymptotic term s instead of the correct $2pm$ factor (using laboratory variables). Combining (37) and (48) and using current conservation, one gets ⁵⁰⁾

$$\omega \frac{d\sigma}{d^3q} = \frac{A}{s} \frac{1}{t^2} \left\{ (p+q)^2 W_1 + \left(\frac{P(p+q)}{m^2} \right)^2 \frac{W_2}{m^2} \right\} \quad (49)$$

where P is the target momentum. One then respectively approximates $(p+q)^2$ as $-t$ and $P \cdot (p+q)$ as $s - (M^2 - m^2 - t)/(2) = s - \gamma$.

Using again the smallness of the longitudinal cross-section, as compared to the transverse cross-section, one finds $t W_1 \approx (\gamma^2)/(m^2) W_2$ and one may rewrite (49) as

$$\omega \frac{d\sigma}{d^3q} = \frac{A}{t^2 m^2} (\gamma W_2) \left(\frac{s}{\gamma} - 2 \right)$$

or

$$\frac{d\sigma}{dt dM^2} = \frac{2\pi A}{t^2 m^2} \frac{(\gamma W_2) \left(1 - \frac{M^2 - m^2 - t}{s} \right)}{(M^2 - m^2 - t)}$$

Neglecting the electron mass in front of $q_{\perp}^2$, one writes

$$M^2 - m^2 - t = 2\nu = s(1-x) \quad \text{and} \quad t = -\frac{q_{\perp}^2}{x}$$

Hence

$$\omega \frac{d\sigma}{d^3q_{\perp}} = \frac{A}{(q_{\perp}^2)^2 m^2} (\gamma W_2) \frac{2x^3}{x-1} \quad (50)$$

At fixed x and $q_{\perp}^2$ (and hence fixed t) the inclusive cross-section which obeys Feynman's scaling is independent of s . γW_2 is therefore independent of ν at fixed t and, obeying Bjorken's scaling⁴⁵⁾, it must be independent of ω . However, this holds at large s only so that Pomeron exchange is good enough an approximation and, therefore, at large ν (or large ω for fixed t). We therefore expect γW_2 to tend towards a constant for large values of ω . This constant with only background (and no resonance) contribution, using the standard duality correspondence, should, however, be somewhat lower than the value determined at SLAC which includes some resonance contribution.

Instead of using the Mueller argument of Section 4 we may take benefit of the one-proton exchange approximation and start directly from the missing mass analysis of Section 2. The same calculation works through with $K(t)$ now replaced by the photon Kronecker delta term at $\alpha = 1$. $\delta^2(q^2, M^2)$ is written as $M^2 \sigma_{\gamma p}(M^2) \delta_{op}(q^2)$ where $\sigma_{\gamma p}$ is the photon-proton total cross-section.

As previously mentioned, Feynman scaling imposes a constant value for the $\sigma_{\gamma p}$ cross-section, a property which should hold (Pomeron exchange) at sufficiently large energy⁵¹⁾. As it is well known, the strength functions W_1 and W_2 are related to the photon-nucleon total cross-section through the following relations

$$\begin{aligned}
 W_1 &= \frac{2\gamma - q^2}{8\pi^2 m \alpha} \sigma_T \\
 W_2 &= \frac{2\gamma - q^2}{8\pi^2 m \alpha} \frac{m^2 q^2}{m^2 q^2 + \gamma^2} (\sigma_T + \sigma_S)
 \end{aligned} \tag{51}$$

where σ_T and σ_S are the total cross-sections for transversely and longitudinally polarized photons. Bjorken scaling can be imposed but this requires a photon-photon Pomeron residue function behaving as $1/q^2$ in order to compensate for the extra q^2 readily needed (at large W) in (51). As a result, $\sigma_0(q^2)$ must be linear in q^2 . γW_2 and W_1 are then found to behave respectively as a constant and as W for large values of W .

For lower values of γ (and of W at fixed q^2), we expect a relevant resonance contribution to the cross-section. They may also satisfy scaling¹³⁾, but the photon-photon P' and A_2 residue functions, which according to duality should be a key factor in the Regge term which provides a correct interpolation to the resonance contribution, should now behave as $(q^2)^{-\frac{1}{2}}$ rather than $(q^2)^{-1}$. As a result, the resonance effect in deep inelastic scattering is expected to disappear with increasing $|q^2|$ but the relevant effective Regge coupling (P', A_2) should decrease slower than the one associated with diffraction (P).

The same analysis can be extended to deep inelastic neutrino scattering with the intermediate W meson used as a convenient, though unnecessary label, for the Kronecker delta term of the Regge picture.

6. POMERANCHON EXCHANGE

Single and Multi-Pomeranchon Exchange

Pomeranchon exchange is the Regge summary of asymptotic elastic amplitudes. A few properties of the Pomeranchon, considered as a Regge pole, indeed summarize many features common to all elastic cross-sections. Nevertheless, what exactly the Pomeranchon is in the J plane is still an open question ²⁹⁾. In the present paper, Pomeranchon exchange has been considered at two different levels. Its contribution to production amplitude through diffractive dissociation was said to lead to the presence of some specific non-asymptotically vanishing cross-sections, attached to processes, whereby target and projectile may be transformed into different objects with the same isospin and strangeness. Such processes may be associated with limiting fragmentation, a picture of course not limited to them. Pomeranchon exchange has also been used in order to approximate a discontinuity of a three-body elastic amplitude introduced as an intermediary step in the calculation of inclusive cross-sections. This leads us to consider also double-Pomeranchon exchange as an approximation yielding pionization and, as easily generalized, multi-Pomeranchon exchange for pionization when several secondaries are simultaneously observed. The question of multi-Pomeranchon exchange has been much debated in the recent literature, Refs. 52), 53), and it is generally advocated as involving but small couplings ²⁴⁾. It should be stressed that the discontinuity value giving (30) (the pionization cross-section) could be much different in over-all magnitude to a double-Pomeranchon exchange amplitude as the one corresponding to Fig. 24a, since a continuation from positive to negative energy is involved if one wishes to consider the ABC elastic amplitude. Nevertheless, it could well be that the smallness of multi-Pomeranchon coupling should have to be in the long run associated with the weakness of pionization. In any case, it would be extremely interesting to have information about the practical presence or absence of such a term. If the Pomeranchon has a zero slope, this would lead to fixed asymptotic cross-sections for

low multiplicity reactions with secondaries which are fast in both the target and projectile rest frame. As emphasized by Wilson ¹⁶⁾, this would not fit simply with either fragmentation or pionization as defined so far. A mixed Pomeron-Regge exchange, as given by Fig. 24b, which we have at present no reason to expect small, would, to the contrary, yield in any case but a vanishing cross-section at infinite energy. It has, indeed, a power-law decrease as opposed to the logarithmic decrease expected from double-Pomeron exchange with a non-zero slope.

In practice, an a priori convenient way to confirm (or infirm) double-Pomeron exchange would be to look at two pion production in pp collision (say) selecting them as to be relatively slow in the centre-of-mass frame. At relatively low energies one should see an important ρ signal associated with the process of Fig. 24b, but as the energy increases (at the ISR or at NAL) one should see the signal disappearing (no double-Pomeron exchange) or fattening itself into an ϵ^0 (as expected from double-Pomeron exchange).

A reason for relatively shy Pomeron exchange contributions to production processes is that the neighbouring particle sub-energies required for the multiperipheral mechanism to appear is low enough ($\lesssim 1$ GeV) that diffraction is not yet important. The success of simple dual models (no dual loop) in meson-meson scattering suggests that diffraction could even be ignored to a good approximation for meson production, as the two by two subenergies associated with a multiperipheral process could all remain small enough. This in turns explains the neat success of simple dual models when applied to production reactions. In other words, the multiplicity increases in such a way at present machine energy that the most probable reactions exhibit subenergies which are too small as to show conspicuous diffractive effects. However, if one focusses on high energy and low multiplicity processes, one should see some typical Pomeron exchange effect. This leads us therefore to discuss some exclusive reactions, an example of which is the $pp \rightarrow pp \pi^+ \pi^-$ reaction already mentioned.

The Longitudinal Phase Space Method

Several small multiplicity reactions are already well enough studied so that it is possible to determine where in the multi-dimensional phase space the reaction amplitude is important or small. As a matter of fact, since transverse momenta are small and relatively insensitive in value to the other variables, one may focus on the longitudinal momentum distribution of the different secondaries⁵⁴⁾. The centre-of-mass longitudinal and transverse momenta add up to zero and since the transverse momenta are relatively very small at high energy the sum of the moduli of the longitudinal momenta should add up to the centre-of-mass energy

$$\sum_{i=1}^n q_L^i = 0 \quad ; \quad \sum_{i=1}^n |q_L^i| \approx \sqrt{s} \quad (52)$$

There are, therefore, only $n-2$ independent (large) variables. In the simplest case of a three-body final state reaction, the representative points should therefore all lie on a curve and if they are defined in a triangular plot whereby the three co-ordinates give the three final state longitudinal momenta, they should roughly be at the boundary of an hexagon. Each event can therefore be characterized by an angle ω . This is illustrated by Figs. 25a and 25b. The distances to the diagonals specify the longitudinal momenta of the three particles. The non-zero value of the transverse momenta and masses of the secondaries gives, of course, the blurring effect observed experimentally. The concentration of events near the kinematical border of the plot is the more pronounced, relative to the actual dimensions of the plot the higher the energy is.

If one takes as a case study the reaction $\pi^+ p \rightarrow \pi^+ \pi^0 p$ at 8 GeV/c⁵⁵⁾, there are obvious and conspicuous effects associated with the production of ρ and Δ . One may then exclude them for a further analysis aimed at studying diffraction. Figure 26, taken from Ref. 55), gives, for instance, some angular longitudinal phase space (LPS) distribution obtained with ρ (K^*) and Δ included

and excluded. The angular range has been limited to the interval 60° - 240° since there are only very few events with the proton (defined as having longitudinal momentum q_z on Fig. 25a) moving forward in the centre-of-mass system. LPS is not uniform and one may further divide the observed distribution by the invariant phase space in order to directly get information about the invariant matrix element squared. The invariant LPS has obviously maxima where the pion has low centre-of-mass energy. Division by phase space thus mostly deplete the distribution around $\omega = 120^\circ$ and $\omega = 180^\circ$. One is therefore left with a marked broad bump localized in the interval of 90° - 120° . This corresponds to the proton and the π^0 coasting together while the π^+ flies with large momentum in the opposite direction. Proton and π^0 have close rapidities and their centre-of-mass energy is minimum. This is what would be expected from Pomeron exchange and the particular set of events, present in the 90° - 120° bump, thus separated out of the whole lot is the best to study specific properties of Pomeron exchange, whereas others at the 180° bump before subtraction of the ρ and Δ events, would show dominant meson exchange features.

One may then focus on this region of LPS to ascertain certain features associated with Pomeron exchange such as energy behaviour, cross-section ratio pointing at the exchange of the quantum numbers of the vacuum, which satisfied for this particular LPS region should not hold for other regions.

If the number of secondaries increase, the LPS dimensions increase accordingly and a planar description is no longer possible. It remains though that one learns a lot in looking at the weighted densities in different regions defined according to the values of the longitudinal momenta or Feynman's x 's⁵⁶⁾. In particular, one may separate events with relatively large x , where pions are more readily associated with diffraction fragments, from events with small x values which could be candidates for the study of pionization (energy independence, absence of ρ in two pion systems ...). How the two densities vary with energy should eventually lead to ascertain or rule out the pionization mechanism.

7. CONCLUSIONS AND OUTLOOK

We do not have a Regge pole theory for hadron processes. We only have Regge models which even though we cannot, at present, control the remainders give us relatively well-defined leading terms which, within a 20 % accuracy, say, appear as very suitable for the description of quasi-two-body reaction. At such an accuracy many subtle and, let us face it, not yet understood polarization results should not be considered. At the same time, energy behaviour should be predictable and it is gratifying that the latest Serpukhov results ⁵⁷⁾ with a rising of the K^+p total cross-section yielding a smooth behaviour for the difference of the K^-p and K^+p cross-section, cast away doubt raised by the relative flatness of the K^-p total cross-section above 30 GeV.

If efforts to get beyond such an approximative picture have not yet been rewarded, the study of inclusive reaction now offers a whole new range of applications for our approximate though a priori reliable Regge models. We may hope to test all gross features thus predictable in a yet unexplored domain. The possible application of Regge phenomenology with its weakness but also its actual sophistication, to the partial information which can be relatively easily made available out of complicated many particle processes, should be an important step forward in our understanding of strong interaction processes. If, as seen at several instances, this seems to be already rewarded by some success it should be stressed that experimental information is still scanty. If we may hide ignorance behind a satisfactory fit of results with 20 % accuracy error bars, there are many pieces of information for which data should be met with scrutiny. In particular, it is very important to test the same inclusive reactions at different energies and this for different values of $q_{\perp}$. This obviously calls for a greater variety in the reaction studied and for more statistics on what is available at present [as an example among many $K^+p(\pi^-)$ and $K^-p(\pi^-)$ at different energies]. Correlations between secondaries or differences between two-particle distributions and the pertinent product of two

single particle distributions will then be a most important piece of information (as an example among many $\pi^+ \pi^-$ correlations at ISR energies). One should not forget, however, that if we can at present venture predictions and hope to learn much from such inclusive (and partial) information the discovery of possible regularity patterns in exclusive many-particle reactions remains of key importance.

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FIGURE CAPTIONS

Figure 1 : Transverse momentum distribution for the π^- produced in pp collisions at 28.5 GeV/c and K^+p collisions at 12 GeV/c. Both distributions are normalized according to the total cross-sections and show a remarkable similarity.

Figure 2 : Momentum distributions.

2a - Peyrou plot for the reaction $K^-p \rightarrow K^-p \pi^+ \pi^-$ at 10 GeV/c. The leading role of the K^- and p are clearly seen.

2b - Longitudinal momentum distributions observed in K^-p collisions at 10 GeV/c and π^+p collisions at 8 GeV/c. The leading particle effect disappears with larger and larger multiplicities.

Figure 3 : 3a - Energy behaviour of the charge multiplicity.
A logarithmic increase is supported by experiment.
3b - Energy increase of the larger multiplicity cross-sections as observed in π^-p collisions.

Figure 4 : Distribution of charged secondaries in π^-p collisions at 16 GeV/c.

Figure 5 : Rapidity plots. Rapidity domain available in a reaction. The total length is scaled to unity (reduced rapidity), and correspondence between the Feynman scaling variable x and the reduced rapidity z.

Figure 6 : Production process with quasi-elastic scattering and Reggeon exchange.

- Figure 7 : π^- distributions observed in K^+p collisions at 12 GeV/c and π^-p collisions at 19.2 GeV/c (solid line). In order to avoid leading particle effects only the backward π^- distribution is drawn.
- Figure 8 : K^0 distributions observed in 5 GeV/c (full dots) and 8.2 GeV/c (open dots) collisions. The transverse momentum distributions are summed over. Resonance formation is seen close to $x = 1$.
- Figure 9 : π^+ distributions obtained in pp collisions at 12, 500 and 1100 GeV. The transverse momentum is kept fixed at 0.16 (GeV/c)^2 .
- Figure 10 : π^- longitudinal momentum distributions observed in K^+p collisions at 12 GeV/c for different values of the transverse momentum.
- Figure 11 : Multiperipheral production amplitudes s_i and s'_i are the invariant mass squared of the group of secondaries including particles 1 to i and i+1 to n respectively.
- Figure 12 : Futuristic view of charged particle distribution. At sufficiently (extremely by present standards) high energies the multiperipheral maximum with average value $\log s$ could separate itself out from a low multiplicity maximum associated with fragmentation.
- Figure 13 : Cross-sections for different multiplicities as observed at machine energy and in cosmic rays, see also Fig. 3b.
- Figure 14 : Optical theorem for two-body amplitude and relation between an inclusive cross-section and a three-body discontinuity.

Figure 14 : 14a - Cross-section (total cross-section and inclusive cross-section).

14b - Discontinuity of the two-body (three-body) elastic amplitude; it is related to σ_T .

14c - Leading Regge approximation (single Regge limit for the inclusive reaction).

Figure 15 : π^- distributions observed in K^+p collisions at 12 GeV/c and in pp collisions at 29 GeV/c. This emphasizes the proton fragmentation region (low laboratory momentum pions). The distributions are scaled according to the total cross-sections.

Figure 16 : Specific ABC discontinuity of the ABC elastic amplitude associated with the inclusive cross-section. This ABC discontinuity has itself an AB discontinuity and the proper limits are specified by the signs in the two blobs.

Figure 17 : Mixed Regge-Pomeranchon exchange.

Figure 18 : Reggeon exchange approximation.

18a - Two-Reggeon exchange.

18b - Triple-Reggeon coupling.

Figure 19 : Missing mass spectra observed for four different t intervals

$$0.02 < |t| < 0.2 \quad ; \quad 0.2 < |t| < 0.4$$

and

$$0.4 < |t| < 0.6 \quad ; \quad 0.6 < |t| < 1$$

in the reaction $K^+p(K^0)$ at 5 GeV/c. The larger $|t|$ is the sharper is the rise of the missing mass spectrum.

Figure 20 : Regge trajectory determination from the $K^+p(K^0)$ reaction.

20a - Charge exchange trajectory determined from two different mass ranges; open dots ($1 < M < 1.7$), full dots ($1.7 < M < 2.2$). As expected the agreement with the known $\rho-A_2$ trajectory gets worse as the missing mass increases.

20b - $\alpha(0)$ as determined from the missing mass data once the value of $\alpha(t)$ is obtained from the energy behaviour. Since the data show a scaling property the intercept should be found compatible with 1.

Figure 21 : pp differential cross-section as a function of energy for different momentum transfers. Limits seem to be reached. They seem to be in the ratio of the fourth power of the electromagnetic form factor.

Figure 22 : 22a - Double deep inelastic scattering.

22b - Quasi-elastic scattering.

Are such processes meaningful at large transfers ?

Figure 23 : The final electron seen as a fragment of the initial electron in deep inelastic scattering. The picture, of course, holds for any value of s and t .

Figure 24 : 24a - Pomeron Regge exchange in $pp \rightarrow pp \pi^+ \pi^-$. Whether or not the ρ will leave way to the ϵ , as the energy increases, is a very important question.

24b - Double-Pomeron exchange and

Figure 25 : Longitudinal phase space distribution.

25a - Definition: the hexagon corresponds to the limiting shape at infinite energy.

25b - An illustration of longitudinal phase space distribution $\pi^+ p \rightarrow p \pi^+ \pi^0$ at 8 GeV/c, see Refs. 54) and 55).

Figure 26 : Longitudinal phase space angular distributions observed in several reactions.

- Figure 1 -

$\bar{K}p \rightarrow p K^- \pi^+ \pi^-$ at 10 GeV/c

- Figure 2a -

- Figure 2b -

- Figure 3a -

π^-p TOPOLOGIC CROSS SECTIONS

- Figure 3b -

- Figure 4 -

- Figure 5 -

- Figure 6 -

- Figure 7 -

$K^+p(K^0)$

- Figure 8 -

- Figure 9 -

$K^+ p (\pi^-) \ 12 \text{ GeV}/c$

- Figure 10 -

- Figure 11 -

- Figure 12 -

- Figure 13 -

total cross section

inclusive cross section

(a)

(b)

(c)

fragmentation of B

- Figure 15 -

- Figure 16 -

- Figure 17 -

- Figure 18 -

- Figure 19 -

- Figure 20 -

- Figure 21 -

- Figure 22 -

- Figure 23 -

- Figure 24 -

- Figure 25a -

$\pi^+ p \rightarrow p \pi^+ \pi^0$

AT 8 GeV/c

- Figure 25b -

DISTRIBUTION OF THE ANGLE ALONG THE HEXAGON

- Figure 26 -