Selective truncations of an optical state using projection synthesis

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Selective truncation of Fock-state expansion of an optical field can be achieved using projection synthesis. The process removes predetermined Fock states from the input field by conditional measurement and teleportation. We present a scheme based on multiport interferometry to perform projection synthesis. This scheme can be used both as a generalized quantum scissors device, which filters out Fock states with photon numbers higher than a predetermined value, and also as a quantum punching device, which selectively removes specific Fock states making holes in the Fock-state expansion of the input field.

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I. INTRODUCTION

Recent theoretical and experimental works have prompted increasing interest in quantum state engineering using linear optics. It has been shown that linear optics can be used for efficient quantum computation, entanglement manipulation and generation of nonclassical optical states. Linear-optical schemes require single-photon generation and detection, beam splitters (BSs) and phase shifters (PSs). Parametric down-conversion process is exploited to build triggered single-photon source, and avalanche photodiodes are used as photodetectors to discriminate between the absence and presence of photons. Therefore, such schemes are experimentally realizable with the present level of optics technology.

In this paper, we study a linear-optical scheme for quantum state engineering using projection synthesis. Our main interest is to employ the scheme to perform the following transformation

$$|\psi\rangle = \sum_{n=0}^{\infty} \gamma_n |n\rangle \rightarrow |\phi(d)\rangle = \mathcal{N} \sum_{n=0}^{d-1} \gamma_n |n\rangle,$$

(1)

where the unknown input optical state $|\psi\rangle$ is truncated to obtain the state $|\phi(d)\rangle$, which is a finite superposition of $d$ states (for a review see [8]). In Eq. (1), $\mathcal{N}$ is the normalization constant, which will be dropped from equations in the following, thus the sign $\sim$ will be used instead of equality to denote that the state should be normalized. This transformation is achieved by conditional measurement and teleportation. This process was originally described by Pegg, Phillips and Barnett to obtain a superposition state of $d = 2$ of the form $|\phi(2)\rangle \sim \gamma_0 |0\rangle + \gamma_1 |1\rangle$ by truncating a coherent state $|\psi\rangle = |\alpha\rangle$, and was named as the quantum scissors device (QSD) [2,3]. Later we have worked out a theoretical treatment of the QSD by proposing an experimentally realizable scheme and discussing how arbitrary superposition states of $d = 2$ can be generated by this simple scheme [10,11]. The first experiment was performed by Babichev et al. [12]. An extension of the original QSD scheme to $d = 3$ was proposed Koniorczyk et al. by a simple modification of the original QSD scheme [13]. The original QSD scheme is an interesting one because it finds its direct application as a basic element of single-rail version of the linear-optical quantum computer. Moreover, it is not only a truncation scheme but also a communication scheme for superposition states of arbitrary $d$.

The drawback of the original QSD scheme is that it enables generation of truncated states up to $d = 3$. In this paper, we extend the results of [13] to describe an application of a modified version of the multiport Mach-Zehnder interferometer in the configuration of Zeilinger et al. [15], which has been experimentally demonstrated [16,17]. The important difference between the original multiport interferometer and the modified version discussed here is the elimination of the apex BS so that the direct path from the input field to the output field is eliminated. This is crucial for the truncation scheme as we want the process to be done via teleportation.

In the following, we will introduce the generalized QSD scheme based on multiport interferometer and give some examples of the possible truncated states. Then we will discuss how the same scheme can be used as a quantum punching device, which eliminates selectively some Fock states from the superposition state and makes holes in the Fock-state expansion by proper choices of conditional measurement and input states.

II. MULTIPORT INTERFEROMETER AS QUANTUM SCISSORS DEVICE

A schematic diagram of the eight-port Mach-Zehnder interferometer in the configuration of Zeilinger et al. and
the generalized QSD is given in Fig. 1. In a special case, the original Pegg-Phillips-Barnett scheme of QSD can be considered as a six-port interferometer presented in Fig. 2. The beam splitter shown with the dotted lines corresponds to the apex BS that is to be removed from the interferometer to obtain the generalized QSD scheme. If we define the $N$-mode input state as $|\Psi\rangle$ then the $N$-mode output will be $|\Phi\rangle = \hat{U} |\Psi\rangle$, where $\hat{U}$ is the unitary operator describing the evolution of the input state in the interferometer. Denoting the annihilation operators at the input and output ports as column vectors $\mathbf{\hat{a}} = [\hat{a}_1; \hat{a}_2; \ldots; \hat{a}_N]$ and $\mathbf{\hat{b}} = [\hat{b}_1; \hat{b}_2; \ldots; \hat{b}_N]$, respectively, we obtain $\mathbf{\hat{b}} = \hat{U}^\dagger \mathbf{\hat{a}} \hat{U} = S \mathbf{\hat{a}}$, where $S = P_6 P_5 P_4 P_3 P_2 P_1 = [S_{j, k}]$ is the scattering matrix obtained by multiplying the scattering matrices of the beam splitters, $B_i$ and phase shifters $P_i$, used in the scheme from the input to the output. We assume $B_i$ to be described by a real $2 \times 2$ matrix $[t_i, r_i; -r_i, t_i]$ embedded in a $4 \times 4$ matrix, where $t_i^2$ and $r_i^2$ being the BS transmittance and reflectance, respectively. Internal phase shifts of BSs can formally be included using external phase shifters described by parameters $\xi_i$. For simplicity, we analyze the system without $P_6$, i.e. assuming $\xi_6 = 0$.

Now, considering that at the input port we have the Fock state $|\Psi\rangle = |n_1, \ldots, n_N\rangle$, the output state is found as

$$|\Phi\rangle = \hat{U} |\Psi\rangle = \frac{1}{\sqrt{n_1! \cdots n_N!}} \sum_{j=1}^{N} \nu \prod_{i=1}^{\nu} S_{j, x_i} \hat{a}_{j, i} |0\rangle,$$

where $S_{j, x_i}$ are the elements of the unitary scattering matrix $S$, $\nu = \sum_{i=1}^{\nu} n_i$ is the total number of photons, and $\sum_j$ stands for the multiple sum over $j, j_2, \ldots, j_\nu$. Moreover, $x_i = j$ for $\sum_{i=1}^{j-1} n_i < l \leq \sum_{i=1}^{j} n_i$ and $j = 1, \ldots, N$.

### III. SELECTIVE STATE TRUNCATIONS

#### A. Quantum scissors device

In a truncation scheme, we are interested in obtaining a superposition state by truncating an input optical state, which is usually a coherent state. Therefore, in the generalized QSD scheme, based on the multiport interferometer shown in Fig. 1, we consider the state $|\nu\rangle$ as one of the inputs. In that case, for the eight-port interferometer we can write the total input state as $|\Psi\rangle = |n_1\rangle |n_2\rangle |n_3\rangle |\psi\rangle$. Now assume that the detectors at the output ports detect $N_2$, $N_3$ and $N_4$ photons whose sum is the total number of photons input into the interferometer, and satisfies the relation $N_2 + N_3 + N_4 = n_1 + n_2 + n_3 = d - 1$. This means that we project the total output state $|\Phi\rangle_{1, 2, 3, 4}$ onto the detected states $|N_2\rangle |N_3\rangle |N_4\rangle$. Then the state at the first output mode becomes

$$|\phi\rangle = 2^{d-1} \langle N_2 | 3 \langle N_3 | 4 \langle N_4 | \Phi\rangle = \sum_{n=0}^{d-1} c_n^{(d)} |n\rangle$$

where $c_n^{(d)} = \langle n, N_2, N_3, N_4 | \hat{U} |n_1, n_2, n_3, n\rangle$ depends on the beam-splitter transmittances $T \equiv [t_1, t_2, t_3, t_4]$ and phase shifts $\xi \equiv [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5]$. Then our task is to find $T$ and $\xi$ according to the desired state at the output in such a way that the fidelity of the output state to the desired state is maximized.

It is seen from Figs. 1 and 2 that eliminating the third modes at the input and output, and removing the components on the path from the third input to the third output, the eight-port interferometer becomes the original QSD (six-port) when $|n_1\rangle |n_2\rangle |1\rangle = |1\rangle |0\rangle$ and $|N_2\rangle |N_4\rangle = |1\rangle |2\rangle$. In this case, the optimized solution with the highest probability of successful truncation,
that is corresponding to the output state

$$|\phi^{(2)}\rangle \sim \gamma_0|0\rangle + \gamma_1|1\rangle,$$

(4)

becomes $T = [t_1^2 = 1/2, t_2^2 = 1/2]$ and $\xi = [\xi_4 = \pi]$. In the same way Koniorczyk’s QSD is obtained in the same six-port interferometer with $|n_1\rangle_1|n_2\rangle_2 = |1\rangle_1|1\rangle_2$ and $|N_2\rangle_2|N_4\rangle_4 = |1\rangle_2|1\rangle_4$. Then we find that there are four solutions for the successful truncation with the highest probability to obtain the state

$$|\phi^{(3)}\rangle \sim \gamma_0|0\rangle + \gamma_1|1\rangle + \gamma_2|2\rangle.$$  

(5)

These solutions are $T_1 = [t_1^2 = t_2^2 = (3 - \sqrt{3})/6]$, $T_2 = [t_1^2 = t_2^2 = (3 + \sqrt{3})/6]$ if $\xi = [\xi_4 = 0]$, and $T_3 = [t_1^2 = (3 - \sqrt{3})/6, t_2^2 = (3 + \sqrt{3})/6]$. $T_4 = [t_1^2 = (3 - \sqrt{3})/6, t_2^2 = (3 + \sqrt{3})/6]$ if $\xi = [\xi_4 = \pi]$. The first two solutions were given by Koniorczyk et al. $^{12}$, but the rest have been found by us.

For the generalized QSD with the modified eight-port interferometer, we are more interested in the device to act as a QSD with a simple solution than the optimality of the solutions. We find that an input coherent state at the fourth-mode of the input can be truncated to give the output state

$$|\phi^{(4)}\rangle \sim \gamma_0|0\rangle + \gamma_1|1\rangle + \gamma_2|2\rangle + \gamma_3|3\rangle.$$  

(6)

by inputting single-photon states at $|n_1\rangle_1|n_2\rangle_2|n_3\rangle_3 = |1\rangle_1|1\rangle_2|1\rangle_3$ and by the conditional measurement $N_2 = N_3 = N_4 = 1$. We find a number of solutions for transmittances and phase shifts in the QSD, for which the input state is truncated to form (6). One simple solution is given by $T = [1/3, 1/4, 1/3, 1/2]$ with $\xi = [0, 0, 0, \pi/2]$.

Consequently, various output states with desired coefficients can be obtained by proper choices of $T$ and $\xi$ provided that the total number of photons detected at the output detectors equal to the total number of input photons. For example, by inputting $|n_1\rangle_1|n_2\rangle_2|n_3\rangle_3 = |1\rangle_1|2\rangle_2|1\rangle_3$ and detecting $|N_2\rangle_2|N_3\rangle_3|N_4\rangle_4 = |1\rangle_2|2\rangle_3|1\rangle_4$, a truncated output state in the form

$$|\phi^{(5)}\rangle \sim \gamma_0|0\rangle + \gamma_1|1\rangle + \gamma_2|2\rangle + \gamma_3|3\rangle + \gamma_4|4\rangle$$  

(7)

can be obtained by choosing the BS and PS parameters by $T = [0.305, 0.388, 1, 0.817, 0.184]$ with $\xi = [0, 0, 0, \pi, 0]$. For larger dimensional output states, it is difficult to obtain analytical solutions, therefore solutions are found by numerical analysis on condition that the fidelity of the output state to the desired one is the highest.

B. Quantum punching device

Here, we consider the cases where the output state obtained by truncating the input state $|\psi\rangle$ has some of its Fock states removed. Let us assume that $|k_1\rangle, |k_2\rangle, \ldots$ are removed, then the output state is written as

$$|\phi^{(d)}_{\text{punch}}\rangle \sim \sum_{n \neq k_1, k_2, \ldots}^{d-1} \gamma_n|n\rangle.$$  

(8)

We call this kind of process, which makes holes in the Fock-state expansion, the quantum punching device (QPD). We have observed that by choosing proper BSs and PSs we can achieve this kind of state engineering using the multiport interferometer. For example in the state $|\phi^{(4)}\rangle$, given by (6), we can punch out (or remove) the state $|2\rangle$ to get

$$|\phi^{(4)}_{\text{punch} \cdot 2}\rangle \sim \gamma_0|0\rangle + \gamma_1|1\rangle + \gamma_3|3\rangle,$$

(9)

by choosing $T = [(7 + \sqrt{21})/14, 1/3, 1/2, (5 - \sqrt{5})/10]$ with $\xi = 0$ (i.e., with all phase shifts equal to zero). Hereafter, we assume the input Fock states $|n_i\rangle_i = |1\rangle_i$ ($i = 1, 2, 3$) and all measurement outcomes equal to one. In the same way, we can obtain the state

$$|\phi^{(4)}_{\text{punch} \cdot 123}\rangle \sim \gamma_1|1\rangle + \gamma_2|2\rangle + \gamma_3|3\rangle,$$

(10)

for $T = [(7 + \sqrt{21})/14, 1/3, 1/2, (2 - \sqrt{2})/4]$ with $\xi = 0$. We have observed that superpositions of any two Fock states

$$|\phi^{(4)}_{\text{punch} \cdot k\ell}\rangle \sim \gamma_k|k\rangle + \gamma_\ell|\ell\rangle$$

(11)

can be obtained as special cases of the truncation process, e.g., for $\xi = 0$ and the transmittances given by

$$|\phi^{(4)}_{\text{punch} \cdot 0\cdot 2}\rangle \text{ for } T = [1, 1/2, 1, 1/2],$$  

$$|\phi^{(4)}_{\text{punch} \cdot 1\cdot 2}\rangle \text{ for } T = [5/2, 1, T, 1/2],$$  

$$|\phi^{(4)}_{\text{punch} \cdot 0\cdot 0}\rangle \text{ for } T = [T', 1, T', T''],$$

(12)

where $T = (3 - \sqrt{3})/3, T' = (1 - \sqrt{5/\sqrt{3}})/2$, and $T'' = (1 + 3\sqrt{3/\sqrt{5}})/2$. It is interesting to see that one can synthesize two and three photon Fock states in the $|\phi^{(4)}\rangle$ process by choosing $T = [1, 1/2, 1/3, 1/2, 1]$ and $T = [1/2, 1/2, 1, 1/2, 1/2, 1/2, 1/2]$ respectively, and assuming $\xi = 0$ except $\xi_5 = \pi/2$ in the latter case.

It must be noted we have given only some specific examples, which guarantee the desired output state but the solutions are usually not optimized for the success probability.

IV. CONCLUSION

We have shown that the original Peggs-Phillips-Barnett scheme of QSD can be generalized using multiport Mach-Zehnder interferometers in the configuration of Zeilinger et al. $^{15}$. The original QSD scheme can be represented...
as a six-port interferometer. The multiport interferometer approach can help us not only to truncate a coherent state to obtain a superposition state up to an arbitrary Fock state but it also enables selective truncation of a given state and selective removal of Fock-state components from it. As it was the case in the original QSD, the generalized one also produces the desired output state with very high fidelity when the input state to be truncated is a weak coherent state. A hard problem we face in this scheme is the optimization of the solutions to obtain the highest probability of truncation when $d$ is high.

In the present study, we did not focus in optimizing our solutions but in showing that the scheme is working as a truncation or punching device. The effects of imperfections (such as non-ideal photon counting and non-ideal single-photon source) in the scheme are currently being investigated.

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