CALCULATIONS FOR BREAKDOWN INDUCED BY "LARGE DEFECTS"
IN SUPERCONDUCTING NIOBIUM CAVITIES

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1. INTRODUCTION

Significant progress in understanding the phenomenon of breakdown in superconducting cavities was made when it was determined that this effect takes place within a localized, point-like, region of the cavity surface rather than over a large fraction of the superconducting areas. By measuring the temperature of the entire outer wall of a superconducting cavity the stored energy of the cavity has been found to be dissipated within a small area of the cavity surface [1]. A reasonable hypothesis therefore is that a spot on the cavity surface makes a transition to the normal state. This can happen if (a) the temperature of the spot exceeds T_c , the critical temperature or (b) if the applied magnetic field exceeds the local critical field at the spot. We refer to possibility (a) as a thermal transition and (b) as a magnetic transition. Two kinds of experiments show that it is the first possibility. The same thermometers that are used to locate the quench most often show temperature increases at field levels below the quench. At these spots therefore, temperatures are rising and could eventually cross T_c . In the second type of experiment [2] the applied magnetic field is increased at the spot without increasing the total power dissipated in the spot. This experiment is accomplished by mixing two cavity modes and has conclusively shown that the breakdown field level is determined by the total applied power and not by the total applied field. A magnetic transition therefore does not take place.

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These observations lead to a phenomenological model for breakdown. The temperature of the "good" superconductor around a defect is increased due to enhanced losses at the defect. As the field level, H , is increased to its maximum value, $H_{\max}$, this temperature crosses T_c and the region of enhanced losses grows unstably. This is a simple picture of a purely thermal effect so that the quantity of power dissipated at the defect as well as the efficiency with which this power can be transported away from the defect could influence the maximum achievable field level. The factors that influence the production of heat in the defect are the defect size and resistance. The power dissipated in the neighbouring superconductor is determined by the surface resistance of superconducting Niobium which depends on rf frequency and temperature and could also include a residual resistance. The heat transport factors are the thermal conductivity of the Niobium material, the wall thickness and the heat transfer coefficient at the Niobium-Helium interface. The latter depends on the bath temperature, the state of the liquid helium, nucleate boiling, superfluid or subcooled, and, at low temperatures ($T < 2$ K) on the detailed nature of the outer wall surface.

A computer program has been written to model the thermal effect described. (The program is similar to one described in an earlier report [3]). It incorporates all the heat production and heat transport factors. The temperature of the defect and vicinity is calculated for increasing rf field levels until the defect grows unstably, determining the breakdown field level. Calculations are performed for a variety of circumstances to explore the relative influence of the various heat production and heat transport factors.

2. PROGRAM DESCRIPTION

A finite cylindrical disk of thickness d and radius R bounded by liquid helium on the lower surface and by a uniform rf magnetic field H on the top surface is assumed. Liquid helium is also assumed to contact the outer rim of the disk. The disk is divided into a series of $N_R \times N_Z$ ring-like mesh elements of radius $I \Delta r$ ($I = 1, 2, 3 \dots N_R$), thickness ΔZ and width ΔR (the cylindrical co-ordinates are r and z). There are N_Z rings of each radius.

Depending on the particular case studied, the values of ΔR and ΔZ have been chosen between 0.05 and 0.2 mm, N_R between 40 and 240, N_Z between 10 and 40. In a typical run for a defect of 0.2 mm radius on the surface of a 2 mm thick disk $DR = DZ = 0.1$ mm, $N_R = 80$ and $N_Z = 20$ were chosen. The first two elements were chosen as defect elements. The power dissipated in a mesh element of area A at the rf surface is given by $1/2 R_S(T) H^2 A$ where $R_S(T)$ is given by the BCS surface resistance everywhere except at a defect located at the center of the disk. A convenient analytic expression [4] for the BCS surface resistance is chosen

$$R_S(T) = 1.61 \times 10^{-4} f_n^2 \frac{1}{T} \ln \left(\frac{16T}{f_n} \right) \exp \left[-17.2 \frac{g(T)}{T} \right]$$

where $f_n = f(\text{Ghz})/2.856$, f is the microwave frequency in Ghz, $g(T) = \Delta(T)/\Delta(0)$ is the reduced gap function and is approximated by an analytic function [5]

$$g(T) = [\cos \pi t^2 / 2]^{1/2}$$

where $t = T/9.2$, the reduced temperature. The surface resistance is plotted in fig. 1 for 2.856 Ghz. In the presence of the rf magnetic field H , the transition temperature is given by

$$T_c = 9.2 \sqrt{1 - \frac{H}{2000}} \text{ (Gauss) } .$$

It is assumed that the relevant rf critical field is the thermodynamic field of ~ 2000 Gauss. When a surface mesh element reaches this critical temperature the resistance of that element is increased to the normal state value of 8×10^{-3} ohms^(*).

At the lower surface of the disk and at the outer rim which are in contact with liquid helium the boundary condition

$$K(T) \frac{\partial T}{\partial z} (r, z) = Q (\Delta T)$$

is chosen, where $K(T)$ is the temperature dependent thermal conductivity of

(*) This value has been determined by measuring the unloaded quality factor, Q_0 , of a 2856 Mhz niobium cavity at ~ 10 K.

niobium, and Q the heat flux density (i.e. the power flow per unit area) from the outer niobium surface to the helium bath for a temperature difference ΔT between the niobium outer surface temperature and the helium bath temperature T_{bath} . For a major fraction of this study we restrict ourselves to using the thermal conductivity $K(T)$ of unannealed commercial niobium shown in fig. 2 [6]. The residual resistivity ratio (RRR) for this material was 35. It is believed that this curve is most representative of the material being extensively used in cavity fabrication at CERN. In sect. 5.5 we explore the influence of different thermal conductivity functions on the breakdown field level.

Depending on the helium bath condition the heat flux density Q is chosen as follows:

(a) For superfluid helium ($T < 2.18$ K)

$$Q = H_K(T) \Delta T$$

where $H_K(T)$ is chosen as the measured [7] Kapitza conductivity function for unannealed niobium

$$H_K(T) = 0.017 f T_{\text{bath}}^{3.62} \text{ (watts/cm}^2\text{-K)}$$

here

$$f = 1 + 1.5 \frac{\Delta T}{T_{\text{bath}}} + \left(\frac{\Delta T}{T_{\text{bath}}} \right)^2 + \frac{1}{4} \left(\frac{\Delta T}{T_{\text{bath}}} \right)^3$$

derived from a T^* phonon radiation law.

In sect. 5.5 when we explore the influence of improved thermal conductivity by annealing we use for annealed niobium [7]

$$H_K = 0.02 f T_{\text{bath}}^{4.65} \text{ (watts/cm}^2\text{-K)}.$$

(b) For nucleate boiling helium ($2.18 \text{ K} < T < 4.2 \text{ K}$) $Q(\Delta T)$ is chosen by fitting the curve determined experimentally in ref. [8] i.e.

$$\log_{10} Q = 1.45 \log_{10} (\Delta T) + 0.09 .$$

- (c) For subcooled helium used extensively for thermometry diagnostics of cavity losses we use [9]

$$Q = (7.813 \Delta T)^{1.333} \left(\frac{T_{\text{bath}}}{2.5} \right)^{1.6} \text{ mwatts/cm}^2.$$

A composite graph of the various heat transfer coefficients is shown in fig. 3.

3. CALCULATION PROCEDURE

The object of the program is to solve for the equilibrium temperature distribution over the mesh elements given the rf surface magnetic field. An initial temperature distribution is input (for example, all temperatures equal to T_{bath}). In the steady state the temperature of each mesh element can be related to the temperature of its nearest neighbours by equating the heat produced at the element to the net heat flux away from the element. One complete iteration consists of solving the appropriate equation (Appendix 1) for the new temperature of each mesh element. After a specified number of iterations, the heat flow into the rf surface is compared to the total heat flow out of the sides and bottom of the disk. If these heat flow values are not equal, the program is continued for an additional specified number of iterations.

Generally the disk diameter is chosen to be sufficiently large that a major fraction of the total heat input flows out from the lower surface. In this case the radial truncation of the disk does not perceptibly influence the temperature of the rf surface in the vicinity of the defect. Frequent checks that these conditions are fulfilled have been made by increasing the diameter of the disk. Fig. 4 compares the rf surface temperature solutions close to breakdown for an 8 mm diameter disk with that of a 16 mm diameter disk, the defect radius being 0.1 mm in both cases.

During the iteration procedure, another quantity that keeps track of the approach to convergence is

$$\text{error} = \sum_{i,j} T_n^{i,j} - T_{n-1}^{i,j}$$

where i, j are mesh element labels and n and $n-1$ are two successive iteration numbers. The steady state solution is determined when "error" is less than some specified permissible value (e.g. 2×10^{-3}) and when the heat balance is less than $\sim 1\%$ of the total heat input.

At each stage of the calculation the heat flows through the mesh elements in contact with liquid helium and are calculated and if any are found to exceed a film boiling limit of 1 watt/cm^2 the value of the heat transfer coefficient, $Q/\Delta T$ (or H_K) is lowered by the amount necessary to stay below the limit. Before the start of each new iteration, the surface resistance of each surface mesh element is recalculated using the temperature obtained at the end of the previous iteration. Values of thermal conductivity and heat transfer coefficients are similarly readjusted at each iteration.

The approach to convergence can be considerably accelerated by using the over-relaxation method. After the n th iteration we use

$$T_n = T^* \Omega + (1-\Omega) T_{n-1} \quad \text{where } 1 \leq \Omega < 2$$

and where T^* is the solution of the mesh equation obtained during the n th iteration. When $\Omega = 1.0$, we obtain the standard relaxation method.

4. DEFECT FREE CASE

For the defect free case the boundary condition at the outer rim (i.e. liquid helium contact) is inadequate as it permits lateral heat flow. To handle this case the problem is reduced to one dimension by setting all lateral thermal conductivities (i.e. K_1 and K_2 in Appendix 1) to zero. Only heat flow normal to the rf surface is then possible and just a few (e.g. $N_R = 5$) radial mesh elements can be chosen to solve the problem. With this approach the defect free case can be solved by using essentially the same code and the solutions are found significantly more rapidly due to the reduced mesh.

Difficulties with using the program are encountered when a defect becomes very small $\ll$ (0.1 mm) or has a very low surface resistance so that the power dissipated within the defect is only somewhat larger than the superconducting neighbourhood. In this case it is neither reasonable to cut off the lateral heat flow or to truncate the disk radially. A variable mesh would be one approach to circumvent this problem but is not adopted here. Another approach to treat very small radius defects would be to keep the defect size fixed at values that can be treated by the current program (e.g. $\sim$ 0.1 mm) but to proportionately reduce the defect resistance to produce the equivalent power dissipated in a micro-defect with higher resistance. This technique (adopted previously [3]) would yield the breakdown field level as well as the qualitative dependencies on certain parameters (e.g. wall thickness, bath temperature, thermal conductivity etc.) but would mask the dependencies on defect size. The very small defect case is therefore not treated in this study; only the results of the zero defect case will be discussed as a limiting case.

5. LARGE DEFECT BEHAVIOUR

Based on the results of calculations with the program under a variety of circumstances we find it is useful to divide defects into two broad categories: (1) large defects and (2) small defects including the zero defect limiting case. In the first category belong defects that breakdown below $\sim$ 400 Gauss ($\sim$ 10 MV/m accelerating field). Since most cavities usually show breakdown below 400 Gauss this is the more interesting category of defects and as such we plan to concentrate our discussion on these. For large defects the power input into the defect is very much (typically $>$ 100 times) larger than the power input into the neighbouring superconductor because the defects have a large area or a large surface resistance (i.e. comparable to normal state niobium). The approach to T_c is qualitatively different in the two categories and so is the influence of the various heat production and transport factors. For large defects the temperature of the defect periphery rises smoothly with increasing field level until it crosses T_c . This temperature rise is determined mainly by the power flow into the defect and the local thermal conductivity of the niobium. For small and zero defects the temperature of

the superconductor rises very slowly, almost imperceptibly until a threshold near $H_{\max}$ above which the temperature increase is very steep. This behaviour reflects the exponential nature of the BCS surface resistance. Fig. 5 depicts this dissimilarity by contrasting the temperature rise outside a 0.2 mm radius defect with that of the defect free case. Even a 50 μ m radius defect shows the behaviour characteristic of large defects.

We now discuss in detail how the breakdown field level for large defects is influenced by various heat production and heat transport factors listed in sect. 1. Wherever appropriate we shall compare the behaviour of large defects with the limiting defect free case.

5.1 Dependence on RF frequency and residual resistance

Since the power dissipated in the neighbouring superconductor does not play a significant role in the case of large defects, the residual resistance of the superconductor and the rf operating frequency play a minor role in determining the breakdown field level below ~ 400 Gauss. This is illustrated with results of calculations between 500 and 3000 MHz in fig. 6 for 0.05 mm and 0.2 mm radius defects. In both cases identical results for breakdown field have been obtained after adding a residual resistance of 2×10^{-7} ohms (corresponding to a typical Q_0 of 1.45×10^9). This is not surprising since the resistance of the hot zone around the defect ($\sim 10^{-5}$ Ohms) is still much larger than the residual resistance.

In the zero defect case the superconducting surface resistance determines the behaviour so that $H_{\max}$ decreases with increasing frequency and increasing residual resistance. This is illustrated in fig. 7.

5.2 Dependence on defect size and resistance

An interesting and useful relationship derived from studies with the program is that for large defects the maximum power tolerable depends linearly on the radius of the defect. This linearity is illustrated in fig. 8 by results of calculations for two different sets of conditions

found in typical experiments, e.g. rf frequency = 2856 MHz, $T_{\text{bath}} = 1.8$ K and $f = 500$ MHz, $T_{\text{bath}} = 4.2$ K. The coefficient of the linear dependence, ~ 200 milliwatts/mm radius, is almost identical in the two cases as will be evident from the discussion in the following subsections. An important consequence of this relationship is that if, during a cryogenic measurement of a superconducting cavity, it is possible to measure the power dissipated by the defect just below H_{max} , then from the linear coefficient given by the program one can determine the size of the defect (radius = r_D). An experimental technique to measure this dissipated power using thermometry has already been demonstrated [9]. Once the size of the defect is known then from the measured dissipated power ($= 1/2 R_D H_{\text{max}}^2 \pi r_D^2$) and from the measured H_{max} one can also estimate the surface resistance of the defect. Thus a large amount of information characterizing the defect can be obtained from cryogenic measurements alone. As defects get smaller in size and can tolerate smaller values of maximum power it may become more difficult to measure their size by thermometry alone although it would still be possible to locate them easily from the thermometer signal during quench.

Another important consequence that can be derived from the linear relationship

$$P_{\text{max}} = \frac{1}{2} R_D (\pi r_D^2) H_{\text{max}}^2 = \alpha r_D$$

is that

$$H_{\text{max}} = \frac{\alpha}{\sqrt{r} \sqrt{R_D}} .$$

This functional dependence of maximum field strength versus defect radius is plotted in fig. 9 for a defect resistance equal to the resistance of normal conducting Nb ($= 8 \times 10^{-3} \Omega$). A scale for the size of typical defects in cavities which breakdown between 100 and 400 Gauss is established. It is interesting to note from this behaviour that to increase field levels by a factor of 2 it is necessary to reduce defect sizes by a factor of 4. To reliably achieve field levels above 5 MVm it is necessary to be free of defects larger than 0.05 mm radius.

5.3 Influence of conditions at outer wall: bath temperature, nucleate boiling helium, superfluid helium, subcooled helium

To first order the breakdown field level for large defects is independent of bath temperature and the nature of the cooling bath. Fig. 10(a) illustrates the temperature of the rf surface for different bath temperatures. The heat transfer coefficients used correspond to nucleate boiling helium. In all cases the field levels chosen are close to the breakdown level.

Identical temperatures are obtained for the outer periphery of the defect. At the outer wall substantially different temperatures are obtained as shown in the companion fig. 10(b). The overall temperature dependence expected from 1.8 to 4.2 K is illustrated in fig. 11 for defect sizes ranging from 0.2 mm to 0.05 mm radius. As defects get smaller and reach progressively higher fields the temperature dependence grows slightly stronger due to the increasing importance of the role played by the superconducting surface resistance. Nevertheless for the smallest defect considered here only a 13.5% increase in field is expected between 4.2 and 1.8 K. In the zero defect limit the bath temperature dependence is much stronger as shown in fig. 12. At higher bath temperature the thermal instability sets in at a significantly lower field level because the starting point on the exponential BCS resistance curve is higher.

We now examine more closely the influence on the breakdown field level of the heat transfer coefficient at the outer wall. Compare in fig. 13(a) the rf surface temperature profiles when the helium bath is in the nucleate boiling region with the rf surface temperature when the bath is in the subcooled state. As shown earlier (fig. 3) the heat transfer coefficients for the two states differ by almost two orders of magnitude, a property that is used to advantage to improve the detection ability of hot spots by the cavity in subcooled helium during diagnostic experiments. In spite of the large decrease in the cooling efficiency the temperature of the defect and vicinity is essentially unchanged in going from nucleate boiling to subcooled. This temperature is largely governed by the defect power and the local thermal conductivity. However, the temperature at the outer wall below the defect shows an enormous difference (fig. 13(b)) which allows the thermometry

diagnostic system to easily pick out the hot spot even before breakdown. A comparison of the breakdown field level in nucleate boiling helium with that in the subcooled state for the 0.2 mm radius defect shows essentially no change and this supports the experimental observation on the non-influence of the breakdown field level on the state of helium: subcooled, nucleate boiling or superfluid.

5.4 Influence of wall thickness (0.5 mm to 3 mm)

We have seen that lowering the heat transfer coefficient at the outer wall has very little effect on the breakdown field level. Bringing the outer wall closer to the rf surface has also little effect, to first order, on the rf surface temperature near the defect. In the area below the defect the heat flux density in the vertical direction is increased and the lateral heat flow is reduced. Increased normal heat flow out to the helium is possible only by an increased outer wall temperature and by an accompanying increase in the inner wall temperature. However the temperature increment at the inner wall is not substantial leading to only a slight decrease in breakdown field level. Figs 14(a), 14(b) and 14(c) illustrate these effects for wall thickness between 1 and 3 mm and for a 0.1 mm radius defect operating close to the breakdown field level.

For very small thicknesses (comparable to the defect diameter) and sufficient power levels to the defect the heat flux at the outer wall can approach the film boiling limit, so that cooling can be effectively lost for the part of the surface directly below the defect bringing the defect closer to breakdown. For 0.5 mm wall thickness shown in figs 15(a) and 15(b) this limit is approached with a 0.1 mm radius defect near breakdown. For smaller (higher field) defects on 0.5 mm thick Nb the film boiling limit will not be reached because the maximum power tolerable by the defect will be progressively reduced as discussed in sect. 5.2.

Far away from the defect we begin to approach the expected behaviour for a defect free case. In this region the heat flow is predominantly normal to the rf surface. Reducing the wall thickness has no effect on the uni-directional heat flow and therefore does not change the outer wall temperature. A fixed outer wall temperature leads to a lower inner wall temperature for a thinner wall, given the same power flow in both cases.

The temperature at the inner wall far away from the defect and in the defect free case is lower for a thinner wall. Breakdown field values for defect free cavities can therefore be expected to increase with decreasing wall thickness as shown in fig. 16.

5.5 Influence of thermal conductivity

Of all the heat transport parameters described so far thermal conductivity is the only one that exerts a large influence on breakdown field levels. Fig. 17 shows how the rf surface temperature near a defect is reduced by increasing the thermal conductivity by a factor of 2. To make these calculations we have simply scaled the unannealed Nb conductivities curve shown in fig. 2. The resulting increase in breakdown field levels are shown in fig. 18. Roughly speaking we expect the improvement in maximum field to scale as $\sqrt{K}$.

To accomplish the improvements indicated here it is important to increase thermal conductivities in the high temperature region ($T > 4$ K) because as the breakdown levels are approached the temperature of the defect vicinity approaches 9 K. Improving the low temperature (< 4 K) conductivities also has an effect but not as substantial. For example when commercial grade Nb is annealed at 1800°C for a long time large crystal growth (grain size $\sim$ cm) increases the phonon conductivity (fig. 19) and adds a "phonon peak" to the conductivity curve. At 2 K almost 2 orders of magnitude enhancement is observed. We have tagged on such a phonon peak to the conductivity curve of unannealed Nb and performed calculations. Breakdown field levels with and without a phonon peak are plotted in fig. 20 for various defect sizes. At best a 30-40% improvement can be expected and less if the overall crystal sizes are smaller (e.g. a few mm). On the other hand, the high temperature conductivity of niobium can also be improved by improving the electron conductivity [10]. By reducing the impurity content of commercial grade Nb (RRR $\sim$ 35) to the extent necessary to give an RRR value of $\sim$ 300 the conductivity shown in fig. 19 has been obtained [12]. Using this curve calculated breakdown field levels are shown for various defect sizes in fig. 21. Now defects can tolerate large amounts of power without breaking down resulting in large field improvement possibilities. Although no thermal runaway will occur there

will be a Q_0 decrease depending on the number, size of defects and chosen operating field level. Clearly it would not be economical to operate a cavity at the allowed maximum field levels indicated in fig. 21 if there are more than one or two defects of 0.4 mm radius. Another problem with pushing large amounts of power tolerable by large defects is that film boiling can be reached. For 2 mm wall thickness, operating at field levels inside the shaded region of fig. 21 implies film boiling. This effect can be avoided by increasing the wall thickness permitting the heat to spread out laterally over a large area before reaching the bath. In the worst case of fig. 21, with a defect of 0.4 mm radius it is indeed possible to operate close to the maximum permissible field level without film boiling if the wall thickness is increased up to 6 mm. At the lower end of the scale with a defect radius of 0.05 mm no film boiling is reached with 2 mm thickness even at the maximum field of ~ 1000 Gauss.

6. CONCLUSIONS

The influence of various heat production and heat transport factors on the breakdown field level $H_{\max}$ of superconducting cavities is examined using a mesh calculation approach. For large defects ($H_{\max} < 400$ Gauss) the field level is found to be essentially independent of rf frequency, residual resistance, bath temperature, wall thickness and state of liquid helium (nucleate boiling, superfluid or subcooled) but strongly dependent on the size and resistance of defects and on the temperature dependent thermal conductivity of the material. The computer model predicts that the maximum power tolerable by a defect depends linearly on the radius of the defect. These conclusions can be summarized as $H_{\max} \sim \sqrt{K(T)/rR_D}$. A measurement of the power dissipated at a defect below quench (by using calibrated thermometers for example) can lead to an estimate of the size of the defect using the results of the calculations. This information coupled with the breakdown field level measurement allows in addition an estimate of the surface resistance of defects. Thus it is possible to characterize defects from cryogenic measurements together with model calculation presented here.

Substantial increases in operating field levels are possible even with modest improvements in the purity of Nb assuming that large numbers of very large defects (radius > 0.2 mm) can be easily avoided to prevent Q loading. With small quantities of Nb RRR's of 10^3 to 10^4 have been shown possible [11]. However even RRR's of a few 100 can be very effective and perhaps this may be possible on a large scale. For electron storage ring application a factor of 2 improvement in routine operating field levels could be possible by a very modest increase in conductivity of a factor of 4 corresponding to an RRR value of ~ 150 .

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APPENDIX

DETAILS OF THE CALCULATION METHOD

1A HEAT FLOW EQUATIONS

We derive the general heat flow equation for a mesh element of temperature T_0 with nearest neighbours of temperature T_1 , T_2 , T_3 , and T_4 (see fig. 22(b)). The conductivity from each nearest neighbour to T_0 is K_1 , K_2 , K_3 , and K_4 respectively. The heat flow from nearest neighbour T_1 to T_0 , is given by

$$T_1 \rightarrow T_0: Q_1 = (T_1 - T_0) 2\pi \left(r - \frac{1}{2} \Delta r\right) \Delta Z K_1 / \Delta r.$$

Similarly, the heat flows from T_2 , T_3 , and T_4 , to T_0 are

$$T_2 \rightarrow T_0: Q_2 = (T_2 - T_0) 2\pi r \Delta r K_2 / \Delta Z$$

$$T_3 \rightarrow T_0: Q_3 = (T_3 - T_0) 2\pi \left(r + \frac{1}{2} \Delta r\right) \Delta z K_3 / \Delta r$$

$$T_4 \rightarrow T_0: Q_4 = (T_4 - T_0) 2\pi r \Delta r K_4 / \Delta Z.$$

If the heat produced in the ring T_0 is Q_0 , then in the steady state

$$\sum_{i=1}^4 Q_i + Q_0 = 0.$$

This equation can be inverted to solve for T_0 in terms of T_1 , T_2 , T_3 , and T_4 .

Every mesh element can be classified as belonging to one of nine different types of mesh elements depending on the boundary conditions (see fig. 22(a)).

- (1) This is an outer ring element so $K_3 = H_K \Delta r$ at the Helium Interface. On the side of T_2 , there is a vacuum and therefore no heat transfer, so $K_2 = 0$. This element can be a potential heat source since it is located at the rf surface ($Q_0 \neq 0$). Finally for a uniform material $K_1, K_4 = K$.

- (2) Interior mesh element. $K_1, K_2, K_3, K_4 = K$; no heat production, $Q_0 = 0$.
- (3) Same as (1) except $Q_0 = 0$ and $K_2 = K (\neq 0)$.
- (4) Same as (3) except $K_4 = H_K \Delta Z$ at the helium interface.
- (5) Nb-He interfaces, $K_4 = H_K \Delta Z$, $K_3 = H_K \Delta r$. Also, $K_1, K_2 = K$ and $Q_0 = 0$.
- (6) Axial element of disk: $T_1 = T_0$, $K_1, K_2, K_3, K_4 = K$, $Q_0 = 0$.
- (7) Same as (1) except $K_3 = K$.
- (8) Defect element. $Q_0 \neq 0$. Axial element $T_1 = T_0$. No heat conduction to vacuum so $K_2 = 0$. $K_3, K_4 = K$.
- (9) Same as (5) except $T_1 = T_0$.

With these restrictions and the general heat flow equation, nine separate equations are developed and the appropriate equation for each mesh element is solved for the temperature of the element.

2A CONVERGENCE AND PROGRAM TESTS

According to ref. 12, if a system of equations is irreducible

$$a_{11}x_1 \dots a_{1n}x_n = b_1$$

$$a_{n1}x_1 \dots a_{nn}x_n = b_n$$

and if I:

$$|a_{ii}| \geq \sum_{j=1}^n |a_{ij}| - |a_{ii}| \text{ for all } i$$

and II:

$$|a_{ii}| > \sum_{j=1}^n |a_{ij}| - |a_{ii}| \text{ for at least one } i$$

then a Gauss-Seidel iterative method for solution of these equations converges.

Since each ring has a unique set of nearest neighbours (greater than 2) the equations derived above form an irreducible set. Furthermore, it is easy to verify that conditions I and II are met so that the method converges in principle.

Several checks are made on the program. In the first, the rf magnetic field is set to zero (no heat sources) and an initial guess different from the bath temperature is chosen. A final solution where all temperatures are equal to the bath temperature is readily obtained. Another check consists of evaluating the heat flow out of the sides and bottom of any disk contained within the problem disk. At convergence this value is always found to be equal to the total heat input at the rf surface.

REFERENCES

- [1] C.M. Lyneis et al., Proc. of the 1972 Proton Linear Acc. Conference (Los Alamos) (1972) p. 98;
H. Padamsee et al., CLNS-340, Cornell University (1976);
Ph. Bernard et al., Nucl. Instr. and Meth. 190 (1981) 257.
- [2] H. Padamsee et al., IEEE Trans. Mag-17, 947 (1981).
- [3] A. Deniz and H. Padamsee, CLNS-79/434 (1979).
- [4] P.B. Wilson, SLAC-TN-70-35 (1970).
- [5] T.P. Sheehan, Phys. Rev. 149 (1966) 368.
- [6] M. Blin, CERN (priv. comm.)
- [7] K. Mittag, Cryogenics 73 (1973) 94.
- [8] C. Johannes, Proc. of 3rd Int. Cryog. Engr. conf., ICEC3, p.97 (1970).
- [9] R. Romyn and W. Weingarten, CERN/EF/RF 81-4 (1981).
- [10] K. Krafft, CLNS 80/468, Cornell University (1980).
- [11] W. Wasserbach, Phys. Stat. Sol. B84 (1977) 205.
- [12] "Numerical methods and Fortran Programming", D.D. McCracken and W.S. Don, John Wiley and Sons, Inc., 260.

FIGURE CAPTIONS

- Fig. 1 Temperature dependence of the superconducting microwave surface resistance of Nb at 2856 MHz. The analytic expression from which this curve was calculated is given in ref. [4].
- Fig. 2 Measured Thermal Conductivity for unannealed commercial grade Nb. This curve was measured at CERN using material that is believed to be representative of that used in cavity fabrication.
- Fig. 3 Comparison of heat transfer coefficients for various conditions of the cooling bath: superfluid, nucleate boiling and subcooled helium.
- Fig. 4 Comparison of solutions obtained from the program using different disk radii. In this case, typical for a "large" defect, truncating the disk at 4 mm radius is sufficient as increasing the diameter by a factor of 2 has essentially no effect on the r.f. surface temperature.
- Fig. 5 Temperature rise outside an 0.2 mm radius "large" defect compared with the temperature rise of a defect free r.f. surface. In the large defect case T_c is approached gradually since the temperature is determined mainly by the power level into the defect and the local thermal conductivity. In the second case a sharp thermal instability is caused by the exponential surface resistance of the superconductor.
- Fig. 6 Frequency dependence of the breakdown field for "large" defects. Two defect sizes, 0.05 mm and 0.2 mm radius are illustrated. Adding a residual resistance of 2×10^{-7} ohms has no effect on the curves.
- Fig. 7 Frequency dependence of breakdown field level for the defect free case. Adding a residual resistance of 2×10^{-7} ohms sharply lowers the maximum field level.

FIGURE CAPTIONS (Cont'd)

- Fig. 8 Defect size dependence of the maximum tolerable defect power for two distinct experimental conditions: $f_{r.f.} = 500$ MHz, $T_{bath} = 4.2$ K and $f_{r.f.} = 2856$ MHz, $T_{bath} = 1.8$ K.
- Fig. 9 Defect size dependence of the breakdown field level assuming a defect resistance comparable to that of normal state Nb (8×10^{-3} ohms).
- Fig. 10 Influence of helium bath temperature on (a) r.f. surface temperature and (b) outer wall temperature. A typical defect is assumed to be sitting on the r.f. surface.
- Fig. 11 Influence of the helium bath temperature on the breakdown field of "large" defects, several defect sizes are illustrated.
- Fig. 12 Influence of the helium bath temperature on the breakdown field level for the defect free case.
- Fig. 13 Influence on the state of the helium bath on (a) r.f. surface temperature and (b) the outer wall temperature. The nucleate boiling state is compared with the subcooled state for a typical defect. The heat transfer coefficients for the two states differ by almost 2 orders of magnitude but the temperature outside the defect, governed by the local thermal conductivity is largely unaffected.
- Fig. 14 Influence of cavity wall thickness on (a) the r.f. surface temperature and (b) the outer wall temperature for a typical "large defect". Wall thickness between 1 and 3 mm are considered. (c) Breakdown field level as a function of wall thickness for a typical defect.
- Fig. 15 (a) Rf surface temperature and (b) outer wall temperature for 0.5 mm wall cavities. Film boiling below the defect is reached before breakdown is reached for an 0.1 mm radius defect.

FIGURE CAPTIONS (Cont'd)

- Fig. 16 Wall thickness dependance of the breakdown field for a defect free cavity.
- Fig. 17 Influence of thermal conductivity on the r.f. surface temperature around a typical defect. Increasing the conductivity by a factor of 2 significantly reduces the temperature outside the defect allowing higher field levels.
- Fig. 18 Influence of thermal conductivity on the breakdown field level for "large" defects.
- Fig. 19 Comparison of the thermal conductivity of unannealed commercial Nb with expected conductivity due to firing at 1800°C (additional phonon peak) and with the conductivity of pure Nb (RRR = 300) measured in ref. [11].
- Fig. 20 Improvement in the breakdown field expected from high temperature firing commercial grade Nb. Heat treatment at 1800°C results in a phonon peak which was included in $K(T)$ for this calculation.
- Fig. 21 Effect on the breakdown field of reducing the impurity content of commercial grade Nb to improve the RRR to 300. Operating at field levels inside the shaded region would lead to film boiling immediately below the defect area for 2 mm wall thickness. Film boiling can be avoided and operation in the shaded region made possible by increasing the thickness of the Nb.
- Fig. 22 (a) Layout of mash for calculation. The number define the type of mash element. Each type has a separate operation associated with it which is derived from the general mash equation by applying the conditions peculiar to that element.
- (b) A general mash element and its nearest neighbours.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig.10

Fig.11

Fig.12

Fig.13

Fig.14

Fig.14a)

Fig.15

Fig.16

Fig.17

Fig.18

Fig.19

Fig.20

Fig.21

Fig.22