Performance of the LHCb RICH Photon Detectors and Tagging Systematics for CP Violation Studies

Laura Somerville
The Queen’s College

Thesis submitted in partial fulfilment of the requirements for the Degree of Doctor of Philosophy in the University of Oxford
Hilary Term 2006
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Abstract

The LHCb experiment, currently under construction at CERN, is designed to perform high precision CP violation measurements in the $B$-meson system. Two Ring Imaging Cherenkov (RICH) detectors provide charged particle identification, and these utilise the novel pixel Hybrid Photon Detectors (HPDs) to detect the Cherenkov photons. A programme was designed and implemented to ensure quality control at each stage of the photon detector production process. A detailed study of the HPD anodes was carried out, including accelerated ageing tests required to demonstrate their robustness over the lifetime of the LHCb experiment. A RICH demonstrator detector with an aerogel radiator was tested in a particle beam and the data were analysed to determine the Cherenkov angle resolution and photon yield. The results were compared with expectations for the detector, taken from a Monte Carlo simulation.

The tagging of neutral $B$ mesons, to find their flavour at production, is essential for many CP asymmetry measurements. Biases in the tagging process lead to systematic errors which must be understood and quantified. The tagging performance is measured in a self-tagging control channel, then used to estimate the tagging performance in the CP channel of interest. The effects of the triggering biases on the tagging were studied for event samples generated using the LHCb simulation software, in two control channels $B_s^0 \rightarrow D_s \pi$ and $B_d^0 \rightarrow J/\Psi (\mu \mu) K^{*0}$, as well as in the four CP channels $B_s^0 \rightarrow D_s K$, $B_s^0 \rightarrow J/\Psi (\mu \mu) \phi$, $B_d^0 \rightarrow J/\Psi (\mu \mu) K_s$ and $B_d^0 \rightarrow \pi \pi$. A strategy was developed to estimate the mistag fraction in the CP channels from the mistag fraction measured in the control channels. Mistag fractions of between 30-40% are expected, dependent on the specific decay channel and trigger class.
Preface

The LHCb experiment, currently under construction at CERN, is designed to perform high precision CP violation measurements in the $B$ system. The research presented in this thesis contributes to several aspects of the LHCb project:

- The understanding of the effect of triggering and selection biases on the measurement of CP asymmetries, in particular how the determination of the flavour of the $B$ meson at production affects the systematic error on the CP measurement.

- The development of the LHCb RICH detectors, which are essential in providing particle identification information for the experiment.

The structure of this thesis is as follows:

**Chapter 1** introduces the formalism to describe CP violation in the Standard Model and the mixing of neutral $B$ mesons. The phenomenology of CP violation measurements is also introduced, for the four decay channels studied in this thesis. These are $B^0_s \to D_s K$, $B^0_s \to J/\Psi(\mu\mu)\Phi$, $B^0_s \to J/\Psi(\mu\mu)K_s$, and $B^0 \to \pi\pi$. The requirement to determine the flavour of the neutral $B$ meson at production, known as *tagging* the $B^0$, is discussed, including the effect any wrong tag assignment has on the systematic error of the asymmetry measurement.

**Chapter 2** gives an overview of the LHCb detector design, in particular the LHCb RICH detectors, as well as the LHCb software for simulation, reconstruct-
Chapters 3, 4 and 5 present the research carried out by the author.

Chapters 3 and 4 describe work carried out as part of the development of the LHCb RICH photon detectors, namely the pixel Hybrid Photon Detectors, HPDs. In Chapter 3 the design and implementation of a programme to ensure quality control at each stage of the HPD production process is presented. A detailed study of the HPD anodes is also recorded, including a programme of accelerated ageing tests required to demonstrate the performance of the HPDs over the lifetime of the LHCb experiment. In Chapter 4 the analysis of data collected from a beam test of a LHCb RICH demonstrator detector is presented. In particular, the author was responsible for carrying out Cherenkov angle resolution studies on the data, as well as a study of the photoelectron yield.

Chapter 5 concerns the tagging of neutral B mesons and the measurement of the flavour tagging performance in real data. The tagging algorithms can be calibrated on a control channel, where the flavour of the $B^0$ is given by the charge of one of the final state particles. To translate this measurement to an estimate of the mistag fraction in another channel where the CP asymmetry measurement is to be made requires an understanding of how the different trigger and selection biases in each channel affect the tagging. From event samples generated using the LHCb simulation software the effect of the triggering and selection on the tagging performance in two control channels $B^0_s \to D_s \pi$ and $B^0_d \to J/\Psi(\mu\mu)K^{*0}$ was studied, as well as in the four CP channels listed above. A strategy was developed to allow the mistag fraction in the CP channels to be estimated from the mistag fraction measured in the control channels.

Chapter 6 presents conclusions and summarises the results discussed in the previous chapters.
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Chapter 1

CP Violation

Introduction

The Standard Model of particle physics is a relativistic quantum field theory which describes quarks and leptons, the elementary particles of which all matter is composed, and their interaction via the exchange of gauge bosons. The theory has been extremely effective at describing experimental observations, and making predictions which have been confirmed by experiments.

Nevertheless the Standard Model leaves unanswered a number of outstanding questions. It does not include a theory of gravity. The origin of mass in the Standard Model is described by the Higgs theory, but the Higgs boson has yet to be discovered. In addition, a large number of parameters in the theory are not fundamental, but must be measured experimentally.

Free parameters in the theory include the phases which govern the breaking of the CP symmetry, so that the extent of CP violation in the Standard Model is not predicted. The observed asymmetry between matter and anti-matter in our universe requires large CP-violating effects [Sak67]. Only through experiments
can it be understood if this CP violation can be accommodated by the current theory, or if it is generated by new physics beyond the Standard Model.

1.1 Symmetry in Particle Physics

Symmetry arises when a physical system is invariant under a particular transformation. Translation and rotation in space are examples of continuous symmetries, where the parameters of the operators in the symmetry group have an infinite number of values. The invariance of the laws of physics under translation and rotation leads to the conservation of linear and angular momentum.

The interactions of the Standard Model arise from the requirement that the theory is invariant under local gauge transformations [AH96]. Invariance under a local complex phase transformation, $U(1)$ symmetry, leads to the electromagnetic interaction and the conservation of charge. The electroweak theory is derived by combining this with invariance under a local, non-Abelian $SU(2)$ group of isospin transformations. In the electroweak theory the weak hypercharge current is conserved, and the interactions are mediated by three massive and one massless vector gauge bosons. The theory of the strong interaction, QCD, follows from invariance under the $SU(3)$ group of local colour transformations, indicating the colour current is conserved.

For discrete symmetries the operators of the group can take only a finite number of values. This is true for the transformations of charge conjugation, parity and time reversal. Charge conjugation, with the operator $C$, replaces each particle with its antiparticle by reversing the sign of all internal quantum numbers in the system. The parity operator, $P$, reverses the sign of all spatial coordinates; equivalent to mirror reflection followed by a 180° rotation. Time
reversal, operator $T$, transforms $t$ to $-t$ and is a anti-unitary operator, with $T|f(t,x) > \rightarrow |f^*(-t,x) >$.

Discrete symmetries can be broken. For example, charge conjugation symmetry is only preserved for particles which are their own antiparticles. However the combined transformation CPT is a fundamental symmetry of all quantum field theories. The symmetries $C$ and $P$ are preserved in the gravitational, weak and strong interactions, whereas the weak interaction strongly violates $C$ and $P$ symmetry. The charged $W$ boson only couples to the left-handed electron, $e^-_L$, and not to the P-conjugate right-handed electron, $e^-_R$, or the C-conjugate left-handed positron, $e^+_L$. The weak interaction does in general preserve the combined CP symmetry, hence the charged $W$ boson has the same coupling to the CP-conjugate of the left-handed electron, the right handed positron $e^+_R$.

It was thought that the CP symmetry was universal, until CP violation was discovered in rare decays of neutral kaons in 1964 [C+64]. More recently, the BaBar [A+01b] and Belle [A+01a] $B$-factory experiments have observed CP violation in decays of neutral $B_d$ mesons. Many asymmetries in the $B$ meson system are expected to be large and these experiments provide excellent laboratories for CP violation studies, although the $B$-factories operate at the $\Upsilon(4S)$-resonance, with a mass of 10.58 GeV, and therefore can only study the decays of $B_d$ mesons.

At LHCb all the $b$ species can be studied and in particular LHCb will provide the world’s best measurements of CP symmetry violation in the $B_s$ system. This will allow precision tests to be made of the Standard Model predictions for CP violation, and the possibility to search for new physics beyond the Standard Model.
1.2 CP Violation in the Standard Model

CP violation in the strong interaction is permitted in the Standard Model, but it is not a necessary component of the theory. In accordance with the experimental observations, CP violation in the Standard Model is only possible via the weak interaction.

The quarks participate in the weak interaction as linear combinations of mass eigenstates, allowing mixing between the generations. Each generation of massive fermions is composed of a doublet of left-handed particles, weak isospin \( +\frac{1}{2} \) and \( -\frac{1}{2} \), and two right-handed singlet particles, weak isospin 0. The weak interaction couples only to the left-handed component of a particle. With three generations of quarks, the doublets which are eigenstates of the weak interaction are

\[
\begin{align*}
\begin{pmatrix}
  u \\
  d
\end{pmatrix}_L,
\begin{pmatrix}
  c \\
  s
\end{pmatrix}_L,
\begin{pmatrix}
  t \\
  b
\end{pmatrix}_L
\end{align*}
\]

where \( \tilde{d}, \tilde{s} \) and \( \tilde{b} \) are linear combinations of the mass eigenstates, defined by

\[
\begin{pmatrix}
  \tilde{d} \\
  \tilde{s} \\
  \tilde{b}
\end{pmatrix} = V_{\text{CKM}} \begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix},
\]

The 3 x 3 unitary matrix \( V_{\text{CKM}} \) is the Cabibbo-Kobayashi-Maskawa (CKM)
matrix. It enters into the charged weak current, $j^\mu$:

$$j^\mu = (\overline{\tau} \gamma^\mu (1 - \gamma^5) \tau) V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = (\overline{\tau} \gamma^\mu V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \cdot.$$ (1.3)

So, the CKM matrix element enters only for processes in which a W boson is exchanged. The elements of the CKM matrix are then

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \quad (1.4)$$

Such a 3 x 3 unitary matrix can have nine independent parameters, counting the real and imaginary parts of a complex element as two parameters. Since there can be six fermions involved in the charged weak current, there are five relative phase transformations, leaving four independent parameters. The four independent parameters consist of three mixing angles between the three generations of quarks, $\theta_{12}, \theta_{13}$ and $\theta_{23}$, and a single complex phase, $\delta_{13}$.

The CKM matrix can be explicitly parameterised as shown in Equation 1.5, where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$.

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}. \quad (1.5)$$

CP symmetry holds if all the elements in the CKM matrix are real, and so the complex phase $\delta_{13}$ introduces CP violation in the Standard Model. Since there is
only this single source of CP violation, the Standard Model is strongly predictive for CP asymmetries.

A useful perturbative parameterisation of the CKM matrix, the Wolfenstein parameterisation, is given in Equation 1.6, for the four parameters $(\lambda, A, \rho, \eta) \,[\text{Wol83}].$ The expansion parameter, $\lambda$, equal to the sine of the Cabibbo angle, has a value $|V_{us}| = 0.22$ and the expansion is given for terms up to the order $\lambda^5.$ The CP violating phase is $\eta.$

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \frac{1}{2} \lambda^2 - \frac{1}{8} \lambda^4 & \lambda & A \lambda^3 (\rho - i\eta) \\ -\lambda + \frac{1}{2} A^2 \lambda^5 [1 - 2 (\rho + i\eta)] & 1 - \frac{1}{2} \lambda^2 - \frac{1}{8} \lambda^4 (1 + 4 A^2) & A \lambda^2 \\ A \lambda^3 [1 - (1 - \frac{1}{2} \lambda^2) (\rho + i\eta)] & -A \lambda^2 + \frac{1}{2} A A \lambda^4 [1 - 2 (\rho + i\eta)] & 1 - \frac{1}{2} A^2 \lambda^4 \end{pmatrix}$$

(1.6)

The Wolfenstein parameterisation highlights the hierarchy within the CKM matrix, and therefore within the mixing between the quark generations. The coupling between the first and second generations is $\mathcal{O}(\lambda),$ and between the second and third generations it is $\mathcal{O}(\lambda^2),$ suggesting a relative suppression factor of $\sim \lambda$ between flavour changing decays which are otherwise equivalent. From the imaginary terms in Equation 1.6 which are proportional to $\eta$ CP violating effects will be seen in decays involving the transitions $b \to u$ and $t \to d$ to order $\lambda^3,$ and smaller CP violating effects, of order $\lambda^4,$ will be seen in the transition $t \to s.$

The unitarity of the CKM matrix leads to six orthogonality conditions between any pair of columns or any pairs of rows of the matrix. Each orthogonality condition requires the sum of three complex numbers to vanish, and so can be represented as a triangle of three vectors in the complex plane. For studies of CP
violation in the decays of B mesons the most useful unitarity relations include one or more couplings involving the b quark, and ideally have sides with lengths that are the same order in λ. The openness of such a triangle implies large CP asymmetries in B decays. Hence, the interesting relations are those given in Equations 1.7 and 1.8.

\[
V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \tag{1.7}
\]

\[
V_{tb}V_{ub}^* + V_{ts}V_{us}^* + V_{td}V_{ud}^* = 0 \tag{1.8}
\]

Equation 1.7 relates to observables in the decays of \(B_d\) mesons, and Equation 1.8 to \(B_s\) meson decays; up to \(O(\lambda^3)\) in the Wolfenstein parameterisation, the two triangles coincide, and so the triangle relating to Equation 1.7 is sometimes called The Unitarity Triangle in the literature.

The triangles representing the two unitarity conditions are shown in Figure 1.1, where the sides of each have been divided by \(|V_{cb}^*V_{cb}|\) so that the base of the upper triangle extends from the origin of the Argand diagram to the point (1,0). The angles \(\alpha, \beta, \gamma, \delta\gamma\) are defined as:

\[
\alpha \equiv \arg \left( -\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right), \quad \beta \equiv \arg \left( -\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right), \quad \gamma \equiv \arg \left( -\frac{V_{us}V_{ts}^*}{V_{cd}V_{cb}^*} \right), \quad \delta\gamma \equiv \arg \left( -\frac{V_{us}^*V_{ts}}{V_{cd}V_{cb}^*} \right) \tag{1.9}
\]

and for the lower triangle, the angle \(\gamma' = \gamma - \delta\gamma\).

It is now possible to relate the complex phases of the CKM matrix in the Wolfenstein parameterisation to the convention-independent description in terms of the unitarity triangles. For example, taking the expression for the angle \(\delta\gamma\) from Equation 1.9, and considering the contribution of each CKM matrix element
Figure 1.1: The two important unitarity triangles for CP violation in the decays of B mesons.
as given in Equation 1.6, it can be seen that \( V_{ud}, V_{cd} \) and \( V_{cb} \) are real up to order \( \lambda^4 \). The only complex contribution is from \( V_{ts} \), so that \( \delta \gamma = \arg(V_{ts}) \). Up to order \( \lambda^4 \), dividing each entry of the matrix by its modulus gives the complex argument of each element:

\[
\begin{pmatrix}
1 & 1 & e^{-i\gamma} \\
1 & 1 & 1 \\
e^{-i\beta} & e^{i\delta \gamma} & 1
\end{pmatrix} + \mathcal{O}(\lambda^5). \tag{1.10}
\]

So, up to \( \mathcal{O}(\lambda^4) \), the angles \( \gamma, \beta \) and \( \delta \gamma \) are the only non-zero phases in the CKM matrix. As a result, only decays involving \( b \to u,d \to t \) or \( s \to t \) transitions can violate CP. The relation of these phases to observables in the decays of \( B^0_d \) and \( B^0_s \) mesons will be discussed in Section 1.5 below.

### 1.3 Mixing of Neutral B Mesons

\( B^0_d \) and \( B^0_s \) mesons are produced at the LHC from the hadronisation of quarks with definite flavour:

\[
B^0_q = (\bar{b}q) \quad \overline{B^0_q} = (b\bar{q}) \quad \text{where} \quad q = d, s. \tag{1.11}
\]

Since flavour is not conserved in the weak interaction, mixing between \( B^0_q \) and \( \overline{B^0_q} \) mesons is possible. To leading order, in the Standard Model, the mixing is dominated by the box diagrams shown in Figure 1.2. In these diagrams one or both \( t \) quarks can be replaced by \( u \) or \( c \) quarks, but since the amplitude is proportional to the mass in the loop the \( t \) quark contribution dominates.

The flavour eigenstates are not equivalent to the mass eigenstates. Instead the mass eigenstates are formed from a mixture of the flavour eigenstates, so that
Figure 1.2: The box diagrams governing mixing between $B^0_q$ and $\bar{B}^0_q$ mesons.

the composition of such a mass eigenstate, $\psi$, can be written as

$$|\psi(t)\rangle = a(t)|B_q\rangle + b(t)|\bar{B}_q\rangle,$$

and the time evolution of this state is given by the Schrödinger equation:

$$i\frac{d}{dt} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \mathbf{H} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}.$$

The Hamiltonian can be written:

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \mathbf{M} - \frac{i}{2} \mathbf{\Gamma} = \begin{bmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{bmatrix} - \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{bmatrix},$$

where the Hermitian matrices $\mathbf{M}$ and $\mathbf{\Gamma}$ are defined as

$$\mathbf{M} = \frac{1}{2}(\mathbf{H} + \mathbf{H}^\dagger) \quad \mathbf{\Gamma} = i(\mathbf{H} + \mathbf{H}^\dagger).$$

Invariance under the combined CPT transformation requires the mass and integrated lifetime of the $B^0_q$ and its antiparticle, $\bar{B}^0_q$, to be equal. Therefore:

$$\langle B_q|a(t)|B_q\rangle = \langle \bar{B}_q|a(t)|\bar{B}_q\rangle$$
and so the diagonal elements of $\mathbf{H}$ are the same. As a consequence, $M_{11} = M_{22} \equiv M$, and $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$. Hence the mass eigenstates of this Hamiltonian are mixtures of the flavour eigenstates in Equation 1.11. They are characterised by their masses, and are defined as the heavy, $B_H$, and light, $B_L$, states:

$$
|B_L > = p|B_q > + q|\overline{B_q} > \\
|B_H > = p|B_q > - q|\overline{B_q} > .
$$ (1.17)

The normalisation condition is

$$
|p|^2 + |q|^2 = 1.
$$ (1.18)

The eigenvalues of $|B_{H,L} >$, $\epsilon_{H,L}$, are found by solving the characteristic equation:

$$
|\mathbf{H} - \mathbf{I}| = 0.
$$ (1.19)

where $\mathbf{I}$ is the 2x2 identity matrix and $\mathbf{H}$ is the Hamiltonian defined in Equation 1.14. This yields the eigenvalues:

$$
\epsilon_{H,L} = M - \frac{i}{2} \Gamma \pm \sqrt{(M_{12} - \frac{i}{2} \Gamma_{12})(M^*_{12} + \frac{i}{2} \Gamma^*_1)}  \\
= (M \pm \frac{\Delta M}{2}) - \frac{i}{2}(\Gamma \pm \frac{\Delta \Gamma}{2})  \\
= M_{H,L} - \frac{i}{2} \Gamma_{H,L}
$$ (1.20)

where $M_{H,L}$ are the masses of the two eigenstates, and $\Gamma_{H,L}$ are the widths. The difference in the masses and widths are $\Delta M = M_H - M_L$, and $\Delta \Gamma = \Gamma_H - \Gamma_L$. 

From the eigenvector equation,

\[
\mathbf{H} \left( \begin{array}{c} p \\ \pm q \end{array} \right) = e_{H,L} \left( \begin{array}{c} p \\ \pm q \end{array} \right)
\]  
\tag{1.21}

the relation between \( p \) and \( q \) in Equation 1.17 can be calculated:

\[
\frac{q}{p} = \frac{M_{12} - i \frac{1}{2} \Gamma_{2}}{\sqrt{M_{12} - \frac{1}{2} \Gamma_{12}}} = \frac{\Delta M - i \frac{\Delta \Gamma}{2}}{2(M_{12} - \frac{1}{2} \Gamma)}
\]  
\tag{1.22}

Now, the time evolution of the mass eigenstates \( B_H \) and \( B_L \) is given by

\[
|B_H(t)\rangle = e^{-i(M_H - \frac{1}{2} \Gamma_H)t}(p|B_q\rangle - q|\overline{B_q}\rangle)
\]
\[
|B_L(t)\rangle = e^{-i(M_L - \frac{1}{2} \Gamma_L)t}(p|B_q\rangle + q|\overline{B_q}\rangle)
\]  
\tag{1.23}

and, from Equation 1.17, the relation between the flavour and mass eigenstates can be written as

\[
|B_q\rangle = \frac{1}{2p}[|B_H\rangle + |B_L\rangle]
\]
\[
|\overline{B_q}\rangle = \frac{1}{2q}[|B_H\rangle - |B_L\rangle].
\]  
\tag{1.24}

Thus Equation 1.25 describes the evolution of the initial particle and anti-particles states of definite flavour, \( B_q \) and \( \overline{B_q} \).

\[
|B_q(t)\rangle =
\frac{1}{2p}\left[ e^{-i(M_H - \frac{1}{2} \Gamma_H)t}(p|B_q\rangle - q|\overline{B_q}\rangle) + e^{-i(M_L - \frac{1}{2} \Gamma_L)t}(p|B_q\rangle + q|\overline{B_q}\rangle) \right]
\]
\[
|\overline{B_q}(t)\rangle =
\]
\[ \frac{1}{2q} \left[ -e^{-i(M_n - \frac{1}{2} \Gamma) t} (p|B_q > - q|\overline{B}_q >) + e^{-i(M_E - \frac{1}{2} \Gamma) t} (p|B_q > + q|\overline{B}_q >) \right]. \]

(1.25)

The time varying parts of these equations can be separated out by defining

\[ f_{\pm}(t) = \frac{1}{2} \left[ e^{-i(M_n - \frac{1}{2} \Gamma) t} \pm e^{-i(M_E - \frac{1}{2} \Gamma) t} \right] \]

(1.26)

then, as a result, Equation 1.25 can be written:

\[ |B_q(t) > = f_{+}(t)|B_q > + f_{-}(t)\frac{q}{p}|\overline{B}_q > \]
\[ |\overline{B}_q(t) > = f_{-}(t)\frac{p}{q}|B_q > + f_{+}(t)|\overline{B}_q > \]

(1.27)

Using these equations it is possible to write the probabilities for finding a $B_q$ or a $\overline{B}_q$, given the initial state was a $B_q$, after a time $t$ as

\[ |< B_q|B_q(t) >|^2 = |f_{+}(t)|^2 \]
\[ |< B_q|\overline{B}_q(t) >|^2 = |f_{-}(t)\frac{q}{p}|^2, \]

(1.28)

and similarly for an initial $\overline{B}_q$

\[ |< B_q|\overline{B}_q(t) >|^2 = |f_{-}(t)\frac{p}{q}|^2 \]
\[ |< \overline{B}_q|\overline{B}_q(t) >|^2 = |f_{+}(t)|^2. \]

(1.29)

Hence, the flavour states $B_q$ and $\overline{B}_q$ remain unchanged over time, or oscillate into each other, with a time-dependent probability proportional to $|f_{+}(t)|^2$ and
\[ |f_+(t)|^2 = \frac{1}{2} e^{-\bar{\Gamma} t} \left[ \cosh \left( \frac{\Delta \Gamma}{2} t \right) \pm \cos (\Delta M t) \right] \]  

(1.30)

where \( \bar{\Gamma} = (\Gamma_H + \Gamma_L)/2 \). The oscillation parameter, \( x \), defined in Equation 1.31, is important in determining the frequency of the oscillations.

\[ x = \frac{\Delta M}{\bar{\Gamma}} \]  

(1.31)

The two mesons, \( B_q^0 \) and \( \overline{B}_q^0 \), can in principle have different differential lifetimes. For the \( B_d^0 \) meson, channels common to both \( B_d^0 \) and \( \overline{B}_d^0 \) are favoured in terms of the CKM matrix element involved for the \( B_d^0 \) case, but doubly suppressed for the \( \overline{B}_d^0 \) case, or vice versa [BS00]. Thus, overall, cancellations between these two classes of common channels lead to the assumption that, for the difference in width:

\[ \frac{\Delta \Gamma_d}{\bar{\Gamma}_d} = \mathcal{O}(10^{-3}). \]  

(1.32)

For \( B_s^0 \) mesons, the lifetime difference is considerably larger. The most recent measurements indicate

\[ \frac{\Delta \Gamma_s}{\bar{\Gamma}_s} < 0.54 \]  

(1.33)

at the 95\% confidence level [E+04].

The oscillation parameter has been measured for both the \( B_d^0 \) and \( B_s^0 \) systems [E+04]:

\[ x_d = 0.771 \pm 0.0012 \]

\[ x_s > 20.6 \] at the 95\% confidence level.  

(1.34)
Consequently for both $B_d^0$ and $B_s^0$ meson decays $\Delta M \ll \Delta \Gamma$, hence $\Gamma_{12} \ll M_{12}$ and therefore, to a good approximation,

$$|p| = |q|$$

(1.35)

for both the $B_d^0$ and $B_s^0$ systems.

### 1.4 The Phenomenology of CP Violation

The possible CP violating effects in $B$ meson decays can be classified in three categories, following the detailed treatment given in Ref. [HQ98].

- CP violation in the mixing between the neutral meson states, when the mass eigenstates are not CP eigenstates, known as *indirect* CP violation.
- CP violation in the decay, which occurs when the amplitude for a decay and its CP conjugate process have different magnitudes, known as *direct* CP violation.
- CP violation in the interference of mixing and decay.

The first type of CP violation, CP violation in the mixing, occurs when CP violation enters the time evolution described in Equation 1.13. This implies a difference in the rates $B_q \rightarrow \overline{B}_q$ and $\overline{B}_q \rightarrow B_q$, which requires the magnitude of the off-diagonal elements of the Hamiltonian to be different:

$$\left| M_{12} - \frac{i}{2} \Gamma_{12} \right| \neq \left| M_{12}^* - \frac{i}{2} \Gamma_{12}^* \right|$$

(1.36)
due to a phase difference between $M_{12}$ and $\Gamma_{12}$. From Equation 1.22, clearly CP violation in the mixing requires:

$$\left| \frac{q}{p} \right| \neq 1. \quad (1.37)$$

For the $B$ meson system, both $B_d^0$ and $B_s^0$, experimental measurements indicate that $\Gamma_{12} \ll M_{12}$ is a good approximation, and hence $|p| = |q|$. Therefore CP violation in the mixing of $B$ mesons is expected to be a small effect, with the parameter $1 - |\frac{q}{p}|^2$ of order $(10^{-3})$ for the $B_d^0$ system and $(10^{-4})$ for the $B_s^0$ system [E+04].

The second type, CP violation in the decay, can occur due to CP-violating interference between terms in the decay amplitude, causing a difference between the rate for $B_q \rightarrow F$ and the CP conjugate process $\overline{B_q} \rightarrow \overline{F}$. If the decay amplitudes are written as $A_F \equiv \langle F|H_{weak}|B_q \rangle$, and $\overline{A_F} \equiv \langle \overline{F}|H_{weak}|\overline{B_q} \rangle$, then

$$A_F = \sum_i A_i e^{i\phi_i} e^{i\delta_i} , \quad \overline{A_F} = \sum_i A_i e^{-i\phi_i} e^{i\delta_i} \quad (1.38)$$

where the two types of phases in the amplitudes are $\phi_i$, the weak phases, and $\delta_i$, the strong phases. The weak phases are the phases which occur in the CKM matrix, and hence are due to complex parameters in the Lagrangian of the weak interaction. These appear in complex conjugate form in the CP conjugate amplitude, so with a different sign in $A_F$ and $\overline{A_F}$. The strong phases are due to final state interactions, in scattering or decay amplitudes, even when the Lagrangian is real. Since they appear in $A_F$ and $\overline{A_F}$ with the same sign, they do not violate CP.
When CP is conserved, all the weak phases are equal, $\phi_i = \phi_j$, and so
\[
\left| \frac{A_F}{A_F} \right| = 1. \tag{1.39}
\]

Conversely, if it is possible for the meson to decay by several mechanisms with different amplitudes and, crucially, different weak and strong phases, interference between the decay amplitudes causes a difference in the rates for $B_q \to F$ and $\bar{B}_q \to \bar{F}$. Then,
\[
\left| \frac{A_F}{A_F} \right| \neq 1 \tag{1.40}
\]

the condition for CP violation in the decay. This type of CP violation is most easily measured from the decays of charged mesons, where mixing is not possible. It has been observed in the decay of $K$ mesons [B+02]. Several charged $B$ decays will be studied at LHCb, most notably $B^\pm \to D K^\pm$, as discussed in Ref. [B+00].

For the neutral $B$ meson system, the third type of CP violation is potentially the most interesting. When a $B_q^0$ and $\bar{B}_q^0$ can both decay to the same final state, $F$, then CP violation can occur as a result of interplay between the mixing and decay amplitudes. By calculating these decay rates and mixing amplitudes, then observing them experimentally, it is possible to measure the free theoretical parameters in the CKM matrix.

From Equation 1.27, the decay amplitude for a neutral $B_q$ meson decaying to a final state $F$, $B_q \to F$, is
\[
< F | H_{\text{weak}} | B_q(t) > = f_+(t) < F | H_{\text{weak}} | B_q > + \frac{q}{p} f_-(t) < F | H_{\text{weak}} | \bar{B}_q > \tag{1.41}
\]
where $H_{\text{weak}}$ is the Hamiltonian of the weak interaction, and $<F|H_{\text{weak}}|B_q>$ and $<F|H_{\text{weak}}|\overline{B}_q>$ are time-independent amplitudes. Writing these as

$$A_F \equiv <F|H_{\text{weak}}|B_q> \quad \overline{A}_F \equiv <F|H_{\text{weak}}|\overline{B}_q>$$

(1.42)

then Equation 1.27 can be written as

$$<F|H_{\text{weak}}|B_q(t)> = A_F \left(1 + \frac{q}{p} \frac{\overline{A}_F}{A_F} \right) e^{-iM_xt} e^{-i\Gamma_t t} + A_F \left(1 - \frac{q}{p} \frac{\overline{A}_F}{A_F} \right) e^{-i(Mt + \Gamma_t t).}$$

(1.43)

It is useful to define

$$\eta \equiv \frac{q}{p} \frac{\overline{A}_F}{A_F}$$

(1.44)

to simplify this expression. Then, the decay rate for $B_q \rightarrow F$ is proportional to the squared matrix element, $|<F|H_{\text{weak}}|B_q(t)>|^2$, Equation 1.45, neglecting the phase-space factor and some constants.

$$B_q \rightarrow F: \quad R_F(t) \propto \frac{1}{2} e^{-\Gamma_t} |A_F|^2 [I_+(t) + I_-(t)]$$

(1.45)

Here, $I_+(t)$ and $I_-(t)$ are defined as

$$I_+(t) = (1 + |\eta|^2) \cosh \left(\frac{\Delta \Gamma t}{2}\right) + 2 \text{Re}(\eta) \sinh \left(\frac{\Delta \Gamma t}{2}\right)$$

$$I_-(t) = (1 - |\eta|^2) \cos(\Delta Mt) - 2 \sin(\Delta Mt) \text{Im}(\eta)$$

(1.46)

Similarly, the decay rate for $\overline{B}_q \rightarrow F$ is proportional to the squared matrix element, $|<F|H_{\text{weak}}|\overline{B}_q(t)>|^2$. The matrix element can be written as

$$<F|H_{\text{weak}}|\overline{B}_q(t)> \propto f_-(t) \frac{p}{q} <F|H_{\text{weak}}|B_q> + f_+(t) <F|H_{\text{weak}}|\overline{B}_q>$$
\[ = \frac{p}{q} \left( f_\rightarrow(t) < F|H_{weak}|B_q > + f_\rightarrow(t) \frac{q}{p} < F|H_{weak}|Bq > \right) \]

(1.47)

and so the decay rate is given by

\[ \overline{B}_q \rightarrow F : \quad \overline{R}_F(t) \propto \frac{1}{2} e^{-\sqrt{\eta} \left| \frac{p}{q} \right|^2 |A_F|^2 [I_+(t) - I_-(t)]} \quad (1.48) \]

The rates for the decays \( B_q \rightarrow \overline{F} \), and \( \overline{B}_q \rightarrow \overline{F} \), where \( \overline{F} \) is the CP conjugate state to \( F \), can be calculated in the same way. The definition

\[ \overline{\eta} \equiv \frac{p A_F}{q A_{\overline{F}}} \quad (1.49) \]

is required, where \( A_F \equiv < \overline{F}|H_{weak}|B_q > \) and \( A_{\overline{F}} \equiv < \overline{F}|H_{weak}|Bq > \), as is the definition of \( \overline{T}(t) \):

\[ \overline{T}_+(t) = (1 + |\overline{\eta}|^2) \cosh \left( \frac{\Delta \Gamma t}{2} \right) + 2 \text{Re}(\overline{\eta}) \sinh \left( \frac{\Delta \Gamma t}{2} \right) \]
\[ \overline{T}_-(t) = (1 - |\overline{\eta}|^2) \cos (\Delta M t) - 2 \sin (\Delta M t) \text{Im}(\overline{\eta}) \quad (1.50) \]

The resulting decay rates are proportional to the squared matrix elements:

\[ \overline{B}_q \rightarrow F : \quad \overline{R}_F(t) \propto \frac{1}{2} e^{-\sqrt{\overline{\eta}} \left| \frac{q}{p} \right|^2 |A_F|^2 \overline{T}_+(t) + \overline{T}_-(t)} \]
\[ B_q \rightarrow \overline{F} : \quad R_{\overline{F}}(t) \propto \frac{1}{2} e^{-\sqrt{\overline{\eta}} \left| \frac{q}{p} \right|^2 |A_{\overline{F}}|^2 \overline{T}_+(t) - \overline{T}_-(t)} \quad (1.51) \]

The complex ratios \( \eta \) and \( \overline{\eta} \), defined in Equations 1.44 and 1.49, are independent of phase convention, and physically meaningful for describing the CP violating effects. For CP violation in the interference between mixing and decay,
the condition

\[ \eta \neq \pm 1 \]  \hspace{1cm} (1.52) 

is required. The existence of CP violation in the mixing or CP violation in the decay is sufficient to satisfy Equation 1.52, since \(|\eta| \neq 1\). However, for the \(B_d\) and \(B_s\) meson systems, to a good approximation, \(|q/p| = 1\), so no significant CP violation in the mixing is expected. If \(|\overline{A}_F/A_F| = 1\) as well, then CP is conserved both in the mixing and the decay, but it possible to have CP violation in the interference between mixing and decay when:

\[ |\eta| = 1, \quad Im(\eta) \neq 0 \]  \hspace{1cm} (1.53) 

so that \(\eta\) is a pure phase. This is the most theoretically clean situation for extracting the values of the CKM parameters from experimental measurements, as discussed in the next section.

### 1.5 Measuring CP Violation

To measure CP violation, the time-dependent decay rate asymmetries \(A_F\), Equation 1.54, and \(\overline{A}_F\), Equation 1.55 are defined, as functions of the decay rates given in Equations 1.45, 1.48 and 1.51.

\[ A_F = \frac{R_F(t) - \overline{R}_F(t)}{R_F(t) + \overline{R}_F(t)} \]  \hspace{1cm} (1.54) 

\[ \overline{A}_F = \frac{\overline{R}_F(t) - R_F(t)}{\overline{R}_F(t) + R_F(t)} \]  \hspace{1cm} (1.55) 

These asymmetries are measured from the time-dependent decay rates of neutral \(B_q^0\) and \(\overline{B}_q^0\) mesons to the same final state, \(F\), and the CP conjugate state \(\overline{F}\).
For the decays of $B$ mesons, the approximation $|p| = |q|$ is valid, so that the time-dependent decay rate asymmetries become

\[
A_F = \frac{I_-(t)}{I_+(t)} \tag{1.56}
\]

\[
\bar{A}_F = \frac{\bar{T}_-(t)}{\bar{T}_+(t)} \tag{1.57}
\]

where $I_+(t)$, $I_-(t)$ were defined in Equation 1.46, and $\bar{T}_+(t)$, $\bar{T}_-(t)$ were defined in Equation 1.50.

### 1.5.1 Decays to CP Eigenstates

When the neutral $B^0_q$ and $\bar{B}^0_q$ mesons decay to the same final CP eigenstate, then $\eta = \bar{\eta}$, and the decay rates of Equation 1.51 are equivalent to the two decay rates of Equation 1.45 and Equation 1.48. In this case the time-dependent asymmetry is

\[
A_F = \frac{(1 - |\eta|^2)\cos(\Delta M t) - 2\sin(\Delta M t)Im(\eta)}{(1 + |\eta|^2)\cosh(\frac{\Delta M t}{2}) + 2Re(\eta)\sinh(\frac{\Delta M t}{2})}. \tag{1.58}
\]

This expression holds for three decays studied in this thesis, $B^0_d \to J/\Psi K^0_s$, $B^0_d \to \pi^+\pi^-$, and $B^0_s \to J/\Psi \Phi$, which are discussed in detail below. The time-dependent asymmetry caused by CP violation in the decay is proportional to $\cos(\Delta M t)$, and the asymmetry due to mixing-induced CP violation is proportional to $\sin(\Delta M t)$.

#### $\beta$ from $B^0_d \to J/\Psi K^0_s$

The time-dependent decay asymmetry of the channel $B^0_d \to J/\Psi K^0_s$ allows a measurement of the CP violating parameter $\beta$. In the $B^0_d$ system, $\Delta \Gamma$ is small,
Figure 1.3: The tree-level diagram for $B_d^0 \rightarrow J/\Psi K_s^0$ decays.

Figure 1.4: The leading order penguin diagram for $B_d^0 \rightarrow J/\Psi K_s^0$ decays. The label ‘q’ indicates a u, c, or t quark, however the t quark contribution dominates.

so the expression for the asymmetry simplifies to:

$$A_F = \frac{(1 - |\eta|^2) \cos (\Delta M t) - 2 \sin (\Delta M t) Im(\eta)}{(1 + |\eta|^2)}$$

(1.59)

The Feynman diagram for this decay is shown, at tree level, in Figure 1.3. The CKM matrix elements which contribute at each vertex are indicated. With reference to Equation 1.10, the phase of each CKM element is zero. The corresponding leading order penguin diagram, Figure 1.4, has the same overall phase. The non-zero phase, and therefore CP-violating, contribution from the $b \rightarrow u$ transition is negligible, as it is both doubly Cabibbo suppressed and loop mass
Figure 1.5: Diagram illustrating the phase difference between the two interfering decay paths for the $B^0_d \to J/\Psi K^0_s$ decay.

suppressed. As a result there is no CP-violation in the decay, $|A_F/A_F| = 1$, hence $|\eta| = 1$, and the term proportional to $\cos(\Delta M t)$ vanishes.

The non-zero phase in the $B^0_d \to J/\Psi K^0_s$ decay, and thus the possible CP-violation, enters via the CKM matrix elements which contribute to the mixing between the neutral B mesons. From Figure 1.2, two $V_{td}$ terms contribute to the phase of $q/p$. Therefore although $|p| = |q|$ is a good approximation, and hence CP violation in the mixing is not significant, CP violation can occur in the interference between mixing and decay. So for this decay the parameter $\eta$, defined in Equation 1.44, is the phase difference between the two interfering decay paths. It is given by Equation 1.60, where the minus sign is required because $J/\Psi K^0_s$ is a CP-odd state.

$$\eta = \frac{q}{p} \frac{\bar{A}_F}{A_F} = q - 1 = -e^{-2i\beta}, \text{ hence } |\eta| = 1. \quad (1.60)$$

The CKM matrix parameter $\beta$ enters this phase difference between the two interfering decay paths as illustrated in Figure 1.5. The experimentally measurable asymmetry is then given by Equation 1.61, allowing a measurement of the pa-
rameter $\beta$.

$$A_x = -\sin(\Delta M t) \sin(2\beta).$$  (1.61)

The $B$-factory experiments, BaBar [A$^+02b$] and Belle [A$^+03a$], have measured the $B^0_d \rightarrow J/\Psi K^0_s$ asymmetry, contributing to the current world average of $\sin(2\beta) = 0.726 \pm 0.037$ [Hea05]. This is in good agreement with the Standard Model prediction. The LHCb contribution to this measurement will equal the BaBar and Belle combined precision in one year, thanks to both the large statistics and the excellent mass and time of flight resolution.

After one year of data taking the three LHC experiments (LHCb, ATLAS and CMS) will reach a combined uncertainty of 0.011 on the $B^0_d \rightarrow J/\Psi K^0_s$ asymmetry measurement, allowing not only a precise measurement of $\beta$, but the possibility to measure any small contribution from CP violation in the decay [Kop02]. Since in the Standard Model this is expected to be very small, an observation of this time-dependent asymmetry with a contribution proportional to $\cos(\Delta M t)$ would be an indication of new physics.

$\gamma$ (and $\alpha$) from $B^0_d \rightarrow \pi\pi$

The decay of a $B^0_d$ to the CP eigenstate $\pi^+\pi^-$ requires the quark transition $b \rightarrow u$, as shown in Figure 1.6, and therefore involves the CKM parameter $\gamma$.

If the decay is governed by this single diagram then, as for the $B^0_d \rightarrow J/\Psi K^0_s$ decay, CP violation can only occur in the interference between mixing and decay. The phase difference between the two interfering decay paths is indicated in
1.5 Measuring CP Violation

Figure 1.6: The tree level diagram for $B_d^0 \to \pi^+\pi^-$ decays.

Figure 1.7: Diagram illustrating the overall phase difference between the two interfering decay paths for the $B_d^0 \to \pi^+\pi^-$ decay.

Figure 1.7, giving $\eta$

$$\eta = \frac{q}{p} \frac{A_F}{A_F} = \frac{q}{p} 1 = e^{-2i(\beta + \gamma)}, \quad \text{hence } |\eta| = 1.$$ \hspace{1cm} (1.62)

and the measurable asymmetry is simply

$$A_F = -\sin (\Delta M t) \sin (2(\beta + \gamma)).$$ \hspace{1cm} (1.63)

However, the contribution from the penguin diagram, Figure 1.8, is not negligible. This mechanism contributes with different weak and strong phases to those in the tree-level diagram and as a result there is CP violation in the decay,
meaning that the decay rates for $B^0_d \to \pi^+\pi^-$ and $\overline{B}^0_d \to \pi^+\pi^-$ are different. Consequently, $A_F/A_F \neq 1$, and Equation 1.63 is no longer correct; there is also a contribution proportional to $\cos(\Delta M t)$.

The penguin contribution dilutes the measurement of the CP asymmetry by an unknown factor. The CKM angles $\beta$ and $\gamma$, and hence $\alpha = \pi - \beta - \gamma$, can be extracted if the penguin contribution is measured from other, flavour-symmetry related decays [Fle00]. For example, a strategy using $B_s \to KK$ decays to understand the penguin contribution will yield a precision on $\gamma$ of $\sim 5^\circ$ after one year of LHCb data-taking [Bar05].

$\delta \gamma$ from $B^0_s \to J/\Psi \Phi$

The final state of the $B^0_s \to J/\Psi \Phi$ decay is a mixture of two CP eigenstates, which can be separated by an angular analysis of the decay products of the $J/\Psi$ and $\Phi$ [KKPS90]. This decay is governed by a single tree-level diagram, as the penguin contribution is expected to be negligible. Hence, it is the $B^0_s$ equivalent to the $B^0_d \to J/\Psi K^0_s$ decay, giving access to higher order terms in the Wolfenstein parameterisation of the CKM matrix.

The tree level Feynman diagram, Figure 1.9, shows that the overall phase of the CKM matrix elements which contribute to each vertex is zero. The non-
Figure 1.9: The tree-level diagram for $B_s^0 \rightarrow J/\Psi \Phi$ decays.

Figure 1.10: Diagram illustrating the overall phase difference between the two interfering decay paths for the $B_s^0 \rightarrow J/\Psi \Phi$ decay.

zero phase enters in the interference between mixing and decay, as shown in Figure 1.10 which shows the overall phase difference between the two interfering decay paths. In this case $\eta$ is given by

$$
\eta = \frac{q \overline{A}}{p A} = \frac{q}{p} = \frac{1 - r}{1 + r} e^{2\eta \gamma}
$$

(1.64)

where $r$ is the ratio of the decay amplitudes to the CP-even and CP-odd states. Neglecting the penguin contribution, the CP-violation in the interference between mixing and decay can be measured from the asymmetry, as for the $B_d^0 \rightarrow J/\Psi K_s^0$ decay.

However, in the $B_s^0$ system $\Delta \Gamma$ cannot be neglected, as indicated in Equation 1.33, and so the complete expression for the time-dependent asymmetry is
required, Equation 1.65.

\[ A_F = \frac{(1 - |\eta|^2)\cos(\Delta M t) - 2\sin(\Delta M t)Im(\eta)}{(1 + |\eta|^2\cosh(\frac{\Delta \Gamma}{2}) + 2Re(\eta)\sinh(\frac{\Delta \Gamma}{2})} \]  

(1.65)

The ability of the LHCb experiment to make precision measurements of the \( B_s \) system distinguishes it from the \( B \)-factory experiments. From a full characterisation of \( B_s^0 \) mixing, measuring \( \Delta M \) and \( \Delta \Gamma \), a measurement of \( \delta \gamma \) in this channel will be possible, with a precision of \( 2^\circ \) after one year [Reo03].

### 1.5.2 Decays to Non-CP Eigenstates

Measurements of the CKM matrix parameters are also possible in decays to final states which are not CP-eigenstates, if the \( B_q \) and \( \bar{B}_q \) can decay to the same final state. All four decay rates \( B \to F, \bar{B} \to F \), as well as \( B \to \bar{F} \) and \( \bar{B} \to F \) need to be studied. The experimental measurement is then of two time-dependent decay rates asymmetries:

\[ A_F = \frac{(1 - |\eta|^2)\cos(\Delta M t) - 2\sin(\Delta M t)Im(\eta)}{(1 + |\eta|^2\cosh(\frac{\Delta \Gamma}{2}) + 2Re(\eta)\sinh(\frac{\Delta \Gamma}{2})} \]  

(1.66)

\[ \overline{A}_F = \frac{(1 - |\eta|^2)\cos(\Delta M t) - 2\sin(\Delta M t)Im(\eta)}{(1 + |\eta|^2\cosh(\frac{\Delta \Gamma}{2}) + 2Re(\eta)\sinh(\frac{\Delta \Gamma}{2})} \]  

(1.67)

One such decay, studied in this thesis, is \( B_s \to D_s K \).

**\( \delta \gamma \) and \( \gamma \) from \( B_s \to D_s K \).**

Both \( B_s^0 \) and \( \bar{B}_s^0 \) can decay to the same final state, \( D_s^- K^+ \), or the charge conjugate state. The single tree diagram for each decay is shown in Figure 1.11, and the two branching fractions are of the same order. There is no significant
1.5 Measuring CP Violation

Figure 1.11: Two tree-level diagrams for $B_s \rightarrow D_s K$ decay.

Figure 1.12: Diagram illustrating the overall phase difference between the two interfering decay paths for the $B_s \rightarrow D_s K$ decay.

penguin contribution expected. CP violation in the interference between mixing and decay, as illustrated in Figure 1.12, leads to an overall phase difference of $-\gamma + 2\delta \gamma$.

Since $D_s K$ is not a CP-eigenstate the overall phase difference is not simply equal to $\eta$, because a possible non-CP violating strong phase difference must be considered. Then $\eta$ and $\bar{\eta}$ can be written

$$\eta = |\eta| e^{i(\Delta CP + \Delta_{qcd})}$$

$$\bar{\eta} = |\bar{\eta}| e^{i(-\Delta CP + \Delta_{qcd})}$$

(1.68)

where $\Delta_{qcd}$ is the strong phase difference between the amplitudes $B \rightarrow F$ and
\( \overline{B} \to F \), and \( \Delta_{CP} \) is the CKM phase difference, \(-\gamma + 2\delta \gamma \). Here, \( \eta \) is related to decays to the final state \( D_s^+ K^- \).

To extract the value of the CKM phase difference and the strong phase difference it is necessary to measure both time dependent asymmetries, \( A_\rho \) and \( \overline{A}_\rho \), as defined in Equations 1.66 and 1.67. The final precision on the measurement of \( \gamma \) depends on the value of the \( B_s \) mixing angle, \( 2\delta \gamma \), and the strong phase as well as the \( B_s \) oscillation parameter, but is expected to be of the order of \( 10^\circ \) per year of LHCb data-taking [Reo03].

### 1.6 Tagging Neutral \( B \) Mesons

In order to carry out time-dependent asymmetry measurements the flavour of the neutral \( B \) meson at the time of production must be known. This tagging of the selected \( B^0 \) in the decay of interest is achieved by estimating its flavour at production from the decay products of the other \( B \) hadron in the event. Since the mesons originate from \( b \bar{b} \) pair production, the flavour of the other \( B \) hadron indicates the flavour of the reconstructed \( B^0 \). The flavour can, for example, be determined if the other \( B \) meson decayed semi-leptonically, from the charge of the lepton. Alternatively, the tagging of the reconstructed \( B^0 \) can be carried out using its own decay products.

For each algorithm used to tag the reconstructed \( B \) there is a finite probability of an incorrect experimental identification, called the mistag fraction. Additionally, if the tag is based on the flavour of the other \( B \) hadron and it is a neutral \( B \) meson which may oscillate before decaying, this will contribute to the mistag fraction.

The mistag fraction, \( \omega \), affects the experimental measurement of the CP vio-
lating asymmetries, such that the measured asymmetry $A_{\mathcal{F}^\text{meas}}$ is related to the true asymmetry $A_{\mathcal{F}^\text{true}}$ by

$$A_{\mathcal{F}^\text{meas}} = (1 - 2\omega)A_{\mathcal{F}^\text{true}}. \quad (1.69)$$

Therefore the mistag fraction increases the statistical error on the experimental measurement of the CP violating asymmetries by a factor $\frac{1}{1-2\omega}$. This does not take into account other resolution effects such as background or the finite proper time resolution of the detector. Consequently, an accurate measurement of the mistag fraction is vital to control the systematic error in the measurement of the true asymmetry.

In this thesis a study was carried out to investigate how to measure the tagging performance for several $B_d^0$ and $B_s^0$ decays, described in Chapter 5, where Equation 1.69 is applied in Section 5.1.3.
Chapter 2

The LHCb Detector

Introduction

The LHCb experiment has been designed to study CP violation and other rare phenomena in the decays of $B$ mesons at the Large Hadron Collider, the $LHC$, currently under construction at CERN [RIC98]. Proton-proton collisions at a centre of mass energy of 14 TeV will provide large statistical samples of $B$ hadrons; the expected $b\bar{b}$ cross-section of $\sim 500$ $\mu$b gives a production rate of $\sim 10^{12}$ $b\bar{b}$ per year at a luminosity of order $10^{32}$ cm$^2$s$^{-1}$. All the $b$ species will be produced in a ratio of $B_u : B_d : B_s : B_c : \Lambda_b \approx 4 : 4 : 1 : 0.1 : 1$ [Bar05].

At LHC energies the production of the $b$ and $\bar{b}$ quarks is highly correlated, so that the two hadrons are produced predominantly in the same forward or backward hemisphere. For this reason the LHCb detector is designed as a single arm spectrometer. The aim is to exclusively reconstruct the $B$ meson decay of interest and provide reliable tagging of the $b$ flavour, indicating whether the decay originates from a $b$ or a $\bar{b}$ quark. To carry out precise measurements of $B$ hadrons, the experiment requires excellent mass and decay time resolution,
Figure 2.1: A side view of the LHCb detector, in the non-bending plane.

particle identification, and a robust and efficient trigger, both for final states including leptons and those which are purely hadronic. In this chapter a brief overview is given of the main features of the detector. A large portion of the work in this thesis is related to the performance of the RICH detectors for particle identification, and so these are described in greater detail.

2.1 Overview of the LHCb Detector

A schematic of the LHCb detector is shown in Figure 2.1. It has a forward angular acceptance of 10-300 mrad in the bending plane of the magnet, and 250 mrad in the non-bending plane. A right-handed coordinate system is defined with the origin at the nominal interaction point, the z-direction along the beam axis, x towards the centre of the accelerator ring and y vertically upwards.

The LHCb detector operates at a bunch crossing frequency of 40 MHz. At
the nominal LHCb luminosity of $2 \times 10^{32}$ cm$^2$s$^{-1}$, the frequency of bunch crossings containing interactions visible to the LHCb detector is $\sim 10$ MHz. Taking into account the $b\bar{b}$ cross-section, the detector acceptance, the fraction of $b$ quarks which undergo hadronisation to $B$ mesons, and the typical inclusive branching ratios for the decay channels under study, of order $10^{-3}$ to $10^{-2}$, events of interest for LHCb physics analyses occur at a frequency $O(10 \text{ Hz})$.

To reduce the 10 MHz visible interaction rate, a multi-stage trigger is implemented. At Level-0, the events are filtered in favour of $B$ hadron decays by requiring a particle with high transverse momentum. This trigger operates at an input rate of the 40 MHz bunch crossing frequency and an output rate of 1 MHz. The second trigger level, Level-1, is designed to further suppresses the rate by a factor of 25. The final stage of triggering, the higher level trigger or $HLT$, reduces the rate to $\sim 2$ kHz. The trigger is described in detail in Section 2.3.

An important consideration in the design of any detector is the material budget. In principle all parts of the detector, apart from the calorimeters and muon detector, should introduce a minimum amount of material that might affect the motion of the particles. The interaction of particles with matter causes energy loss and angular scattering. These quantities are characterised by the radiation length, $X_0$, and the nuclear interaction length, $\lambda_I$. The radiation length quantifies the energy loss due to electromagnetic interactions, defined as the average length of material over which the energy of the electron is reduced to $1/e$ of its original value. High-energy electrons predominantly lose energy via bremsstrahlung, and high-energy photons via pair production. In addition to scattering, the interaction of particles with matter may cause showering before the particle reaches the calorimeters. The interaction length quantifies the interactions of hadronic particles with nuclei. It is defined similarly to $X_0$, but taking into account elastic
or inelastic interactions with nuclei due to the strong force. Hence, as $X_0$ and $\lambda_I$ increase the occupancy of the detector increases, resulting in a decrease in the number of reconstructed $B$ mesons.

A comprehensive description of the LHCb experiment can be found in Refs. [RIC98], [Reo03]. Below, a brief overview of the different sub-detectors is given, moving downstream from the interaction point with reference to Figure 2.1.

**Beam Pipe**

The beam pipe maintains the vacuum environment required by the beam. It is shaped so as to minimise the interaction length presented to particles produced in the interaction region. Each of the two proton beams nominally consists of bunches of $1.15 \times 10^{11}$ particles, with a bunch crossing rate of 25 ns. The LHC design luminosity of $10^{34}$ cm$^{-2}$s$^{-1}$ leads to around 20 pp interactions per bunch crossing. To simplify the measurement of the primary and secondary $B$ decay vertices and the tagging of neutral $B$ mesons, the beams are defocused in the LHCb collision region to optimise the luminosity for single interactions, hence reducing the luminosity by a factor of 100. Additionally, the LHCb trigger is designed to veto multiple-interaction events.

A 1.8 m section of the beam pipe around the interaction point has a large diameter of approximately 120 cm, to accommodate the vertex detector which is placed inside. This is followed by two conical sections. The first is $\sim 1.5$ m long with a 25 mrad opening angle, made of 1 mm thick aluminium-beryllium alloy. The second is $\sim 16$ m long with a 10 mrad opening angle. The second section is made from aluminium and runs the full length of the detector in three mechanical
sections of increasing thickness.

The Vertex Locator

Vertex reconstruction is a fundamental requirement for the LHCb experiment. The displacement of the $b$ decay vertices from the primary pp interaction vertex, of order 1 cm, is a distinctive feature of $B$ hadron decays that is exploited in the Level-1 trigger. The VErtex LOcator, $VELO$, must provide precise measurements of track coordinates close to the interaction region to allow reconstruction of production and decay vertices of $b$ and $c$ hadrons of interest, to provide an accurate measurement of their decay lifetimes and to measure the impact parameter, $IP$, of each particle, as shown in Figure 2.2. The VELO is designed to achieve a proper time resolution of 40 fs for lifetime measurements and, for the highest transverse momentum tracks, an impact parameter resolution of $20\mu m$ [VEL01].

The VELO consists of 21 silicon stations positioned perpendicular to the beam between $z = -18$ cm and $z = 80$ cm. Each station is composed of two 220 $\mu m$-thick silicon sensors mounted back-to-back, one with circular strips to measure $r$, and
one with radial strips to measure $\phi$. Two stations upstream of the interaction point, with only $r$-measuring strips, are dedicated to the pile-up veto counter and are read out within 25 ns. This ensures that a rejection of multiple interactions is available for the Level-0 trigger decision. The full readout of the VELO is completed within 1 $\mu$s, so the information is available to the Level-1 trigger.

**Ring Imaging Cherenkov Detectors**

A full description of both the RICH detectors can be found in Section 2.2.

**Magnet**

The magnet provides an integrated magnetic field of 4 Tm, which ensures a better than 0.5% precision on the momentum measurement up to 200 GeV/c. It has a dipole structure, with bending of charged tracks in the horizontal plane. The magnet, with an aperture size $\pm 300$ mrad in $x$ and $\pm 250$ mrad in $y$, is situated immediately downstream of the first RICH detector, RICH-1. The position of the magnet is chosen to minimise track curvature in the VELO, where the validity of a fast straight-track fitting algorithm is important for the trigger performance. However, the magnet must also ensure sufficient field integral upstream of the tracking detectors to provide the required momentum resolution for the track trigger. The field integral is boosted in the region immediately upstream of the magnet by careful design of the iron shielding of the RICH-1 detector. The normal-conducting magnet design permits regular field inversions to combat systematic errors due to possible left-right asymmetries in the detector [Mag00].
Tracking System

The tracking system is composed of several sub-detectors: the VELO, described above, the Trigger Tracker, \( TT \), the Inner Tracker, \( IT \), and the Outer Tracker, \( OT \). The furthest upstream, the TT, has two main purposes. Firstly, it is used by the Level-1 trigger to assign transverse momentum information to tracks with large impact parameters. Secondly, it is used in offline analysis to reconstruct the trajectories of long-lived neutral particles which decay outside of the volume of the VELO and it is vital for the measurement of low-momentum particles that are swept out of the acceptance by the magnetic field before reaching the downstream tracking detectors. Both the TT and the IT are silicon microstrip detectors, having a spatial resolution of \( \sim 70 \, \mu \text{m} \).

Three tracking stations, \( T1, T2 \) and \( T3 \), are located between the magnet and the RICH-2 detector. Each station consists of an IT and an OT section. The IT covers the region closest to the beam pipe where the particle flux can reach up to \( 5 \times 10^5 \, \text{cm}^{-2}\text{s}^{-1} \). The OT is based on straw drift chamber technology and extends in \( x \) and \( y \) to cover the LHCb angular acceptance. Each tube is 5 mm in diameter and filled with a mixture of argon, \( \text{CO}_2 \) and \( \text{CF}_4 \). The gas mixture can be adjusted to fine-tune the maximal drift time, which is 50 ns, i.e. two bunch crossings. The drift time is kept as short as possible to minimise the time overlap between drift signals from different bunch crossings. The boundary between IT and OT in each tracking station is designed such that the occupancy in the OT is kept below 10%, with a particle flux less than \( \sim 1 \times 10^5 \, \text{cm}^{-2}\text{s}^{-1} \) [Out01] [Inn02].

The momentum resolution of the LHCb tracking system, measured on simulated \( B_s \rightarrow D_s K \) decays, is \( \delta p/p = 0.37\% \) [RIC98]. The track finding efficiency is 94\% for tracks with hits in all tracking stations. The false or ghost track rate
is 9% for all tracks, improving to 3% for tracks with transverse momenta greater than 0.5 GeV/c, which represents the majority of tracks from $B$ decays [Reb03]. Precise measurements must be made of the directions of the tracks as they pass through the two RICH detectors, as this is important for the particle identification algorithms. The error on the directions of the tracks is much better than the Cherenkov angle resolution achieved in the RICH detectors, described in Section 2.2.

**Calorimeter System**

The calorimeter system identifies hadrons, electrons and photons, and measures their position and energy. Situated downstream of RICH-2, the system consists of the scintillator pad detector, $SPD$, the pre-shower detector, $PS$, plus the electromagnetic, $ECAL$, and hadronic, $HCAL$, calorimeters. The particle identification capability of the calorimeter system is important for the Level-0 trigger decision, which selects high transverse energy electrons, photons and hadrons. As a consequence, the data are read out and processed within the 25 ns bunch crossing time.

Data from the SPD, PS, ECAL and HCAL are used in coincidence to perform particle identification and energy measurements. The SPD and PS are designed, respectively, to validate the charged and neutral components of the electromagnetic showers. The SPD is a 15 mm thick layer of scintillator tiles. This detector responds to charged particles and is therefore used to provide additional rejection between electrons and photons, which cannot be achieved once showering has begun in the calorimeter. The next downstream component of the calorimeter is the PS. This consists of a 12 mm thick layer of lead, equivalent to 2.5 $X_0$, followed
Figure 2.3: The structure of the LHCb HCAL and ECAL. The different tile arrangements, with respect to the beam axis, are designed to match the different profiles of electromagnetic and hadronic showers. The wavelength-shifting fibres for readout are also indicated in each case.

...by another layer of scintillator tiles. The PS is designed to distinguish between electrons and charged pions using the feature of their different interaction lengths.

Both the ECAL and HCAL are sampling calorimeters, with an alternating structure of scintillator tiles and high-$X_0$ material to encourage showering. Figure 2.3 shows the structure of the ECAL and HCAL.

The ECAL has a sandwich of 2 mm thick scintillator tiles and 4 mm thick lead tiles, with the scintillator/lead tile surface aligned perpendicular to the $z$-axis. The ECAL presents a total length equivalent to 25 $X_0$. The ECAL, and the SPD and PS, are read out using the *Shashlik* design, so that the photons are collected from the scintillator tiles via wavelength-shifting fibres embedded in each tile. The fibres are then read out to multi-anode photo-multiplier tubes.

The hadronic showers in the HCAL have a larger lateral profile than electromagnetic showers, and so the HCAL consists of scintillator tiles and iron posi-
tioned with the tile surface running parallel to the beam. The scintillator tiles have an average thickness of 4 mm sandwiched by iron tiles 16 mm thick, and 200 mm of this structure in the z direction is followed by 200 mm of iron; this structure recurs to give a total length of 5.6 \( \lambda_f \). A wavelength-shifting fibre in the HCAL is connected to a single edge of the tile, and is read out via a single-anode photo-multiplier tube.

The transverse dimension of the tiles in the calorimeter system is structured to match the hit density, which increases by two orders of magnitude for the tiles closest to the beam line in the \( x-y \) plane. This segmentation, and the detector acceptance, is matched for the SPD, PS, ECAL and HCAL. This facilitates the combination of data in the trigger decision [CAL00]. In the ECAL, the tiles are 121.2 mm square in the outer section and 40.4 mm square in the innermost section.

The ECAL energy resolution, for \( E \) in GeV, is given by

\[
\frac{\sigma_E}{E} = \frac{10\%}{\sqrt{E}} \oplus 1.5\%, \tag{2.1}
\]

where \( \oplus \) indicates addition in quadrature. The HCAL energy resolution is given by

\[
\frac{\sigma_E}{E} = \frac{80\%}{\sqrt{E}} \oplus 10\%. \tag{2.2}
\]

**Muon Detector**

The muon detector provides muon identification for the offline analysis and for the Level-0 trigger, which searches for muons with high transverse momentum. Efficient muon detection is vital to the physics goals of LHCb, since muons are
present in the final state of many decays which are important for CP asymmetry measurements, and also occur in the semi-leptonic $B$ decays exploited by the tagging algorithms. Four muon stations, M2 to M5, are interspersed with 80 cm plates of iron and placed downstream of the calorimeters, while the M1 tracking station is immediately in front of the PS. The total absorber length, including the calorimeter system, is approximately 20 $\lambda_I$ and the muons which reach station M5 will have an energy of at least 6 GeV.

Each station is divided into four sections, with a granularity decreasing with increasing distance from the beam axis to keep the occupancy approximately constant over the detector. The granularity is higher in the horizontal bending plane to increase the accuracy of the transverse momentum measurement. The stations M2 to M5 are constructed as arrays of multi-wire proportional chambers, MWPC; each station is made up of four layers of MWPC. However, M1 consists of only two layers of MWPC, to reduce the amount of material the particles traverse before the calorimeter system. In addition, for the innermost $\sim 1$ m$^2$ of M1 closest to the beam, the MWPCs are replaced by triple Gas-Electron-Multiplier detectors, GEM. These were chosen for their improved ageing properties, given the high rate in this portion of the detector.

For inclusive $b \to \mu X$ decays inside the acceptance, the efficiency for the muon trigger is $\sim 46\%$, based on a five-fold coincidence of the stations. In the offline analysis, the muon identification efficiency is greater than 90\% for muons with momenta in excess of 3 GeV/c, and the probability of a pion mis-identified as a muon is less than 1.5\% [Muo01].
2.2 Cherenkov Radiation and Particle Identification

When a charged particle travels through a dielectric medium at a velocity greater than the local speed of light, single photons in a cone of Cherenkov light are emitted. The effect was observed by P. Cherenkov in 1934 [Che34] [Che86], and a model describing the properties of the radiation was developed by I. Frank and I. Tamm [FT37]. The three shared the 1958 Nobel prize for their work. The opening angle $\theta_C$ of the cone is

$$\cos \theta_C = \frac{1}{\beta n(\omega)}$$

(2.3)

where $\beta = v/c$ is the velocity of the charged particle as a fraction of the speed of light in vacuum and $n$ is the refractive index of the medium, which in general is a function of the frequency, $\omega$, of the emitted Cherenkov photon. The distribution of the Cherenkov photons is uniform in the azimuthal angle $\phi$.

For a given $n$, the Cherenkov angle $\theta_C$ saturates for $\beta = 1$, such that

$$\cos \theta_{C_{\text{max}}} = \frac{1}{n}.$$  

(2.4)

A useful approximation when $n - 1 \ll 1$ is

$$\theta_{C_{\text{max}}} \approx \sqrt{2\left(1 - \frac{1}{n}\right)}.$$  

(2.5)

The distribution of the number of photons, $N_\gamma$, emitted per unit length of the radiator medium, $l$, per unit of the photon energy, $E = h\omega$, is given by the
relation

\[ \frac{dN_{\gamma}}{dl \, dE} = \frac{\alpha}{\hbar c} Z^2 \left( 1 - \frac{1}{(\beta n(E))^2} \right) \]

(2.6)

= \frac{\alpha}{\hbar c} Z^2 \sin^2 \theta_C(E).

Here \( Z \) is the charge of the particle in units of \( e \) and \( \alpha \) is the fine structure constant. Then, substituting the values of the constants in units of eV and centimetres, and integrating over the length of the particle track in the medium, \( L \), gives for a particle with \( Z = 1 \)

\[ \frac{dN_{\gamma}}{dE} = 370 \sin^2 \theta_C(E) L. \]

(2.7)

### 2.2.1 RICH Detectors

Two Ring Imaging Cherenkov Detectors, *RICH-1* and *RICH-2*, provide particle identification for charged tracks over a momentum range from \( \sim 1 \text{-} 100 \text{ GeV/c} \). Many \( B \) decay channels important for CP violation studies have identical topologies, with the final states distinguished by the presence of a kaon or pion. Hence, the separation of pions and kaons is crucial for the CP asymmetry studies planned for LHCb. Positive kaon identification is also important for the flavour tagging of neutral \( B \) mesons, as discussed in Chapter 5.

The importance of the RICH detectors in the measurement of the decay channel \( B^0_d \rightarrow \pi^+\pi^- \) is illustrated in Figure 2.4. The simulated invariant mass distribution for the decay is shown, along with the invariant mass distributions for the topologically identical decays, \( B^0_d \rightarrow \pi K, B^0_s \rightarrow \pi K \) and \( B^0_s \rightarrow KK \). On the left, the mass distributions are shown without any particle identification information available to the selection algorithm. On the right, the same events are shown,
Figure 2.4: The mass spectrum of $B_d^0 \rightarrow \pi^+ \pi^-$ candidates, before (left) and after (right) the RICH detectors have been used to identify both tracks as pions or lighter particles.

now using the RICH detectors to identify both tracks as a pion or lighter particle. The signal is no longer obscured by the background [RIC00].

The LHCb RICH detectors measure $\theta_C$ by detecting the Cherenkov light with an array of photon detectors. The photons emitted along the particle track in the radiator are reflected from a spherical mirror and focused onto a single characteristic ring at the focal plane of the mirror; hence the name *Ring Imaging* detectors. Figure 2.5 illustrates the principle. The spherical mirror is tilted to allow the array of photon detectors to be placed outside the detector acceptance, reducing the material budget. The position of the centre of the ring corresponds to the projection of the track reflected in the mirror, and the radius is dependent on the velocity of the particle. By matching rings to charged particle tracks, thus measuring their velocity from the radius of the ring and their momentum from the track curvature in the magnetic field of the magnet, the mass can be determined. This allows the particle to be identified.
The two RICH detectors are constructed with three different radiators, to cover the necessary range from ~1-100 GeV/c. Figure 2.6 indicates the strong correlation between the momentum and polar angle of the decay products in $B_d^0 \rightarrow \pi^+\pi^-$ events, which motivates the design of RICH-1 and RICH-2. RICH-1 is located upstream of the magnet and covers the full angular acceptance of the spectrometer. A schematic of the detector is shown in Figure 2.7. The first radiator is 5 cm of aerogel, with a refractive index $n=1.03$, designed to provide positive kaon identification above 2 GeV/c and pion-kaon separation up to about 10 GeV/c. This is followed by a C$_4$F$_{10}$ gas radiator with refractive index $n=1.0014$, extending 85 cm in $z$, providing pion-kaon separation up to about 60 GeV/c.

The RICH-1 detector is within the fringe field of the magnet. A stray field between the VELO and the TT tracking detector is necessary to provide momentum information for the Level-1 trigger, as described in Section 2.3. As a consequence, the magnetic field in the region of RICH-1 is ~60 mT. To reduce
Figure 2.6: Polar angle $\theta$ versus momentum for tracks from simulated $B_d^0 \rightarrow \pi^+\pi^-$ decays, with the coverage of the RICH-1 and RICH-2 detectors indicated.

this to a level compatible with the operation of the photon detectors, RICH-1 has a magnetic shielding box of high magnetic permeability iron. The maximum measured magnetic flux density inside the shielding is less than 2.5 mT [Pat05]. The shielding box was also carefully designed to help focus the magnet flux into the region most critical for the Level-1 momentum measurement [A⁺02a].

The RICH-1 spherical mirror system focuses the photons onto the photon detector plane. The mirrors have a radius of curvature of 2.4 m. The mirrors are of carbon fibre structure, coated with aluminium and a reflective layer of SiO$_2$ and HfO$_2$. To accommodate the shielding outside the acceptance, a second, flat mirror reflects the Cherenkov photons onto the photon detectors, located above and below the beam pipe.

The angular acceptance of RICH-2 is 120 mrad in the horizontal bending plane and 100 mrad in the vertical plane. It contains a CF$_4$ gas radiator, n=1.0005,
2.2 Cherenkov Radiation and Particle Identification

![Diagram of the RICH-1 detector](image)

Figure 2.7: The RICH-1 detector.
of 2 m length, providing pion-kaon separation from \( \sim 20-100 \) GeV\( /c \). The optical design of RICH-2, shown in Figure 2.8, is similar to RICH-1. There is a system of spherical mirrors, radius of curvature 8.6 m, with flat mirrors to reflect the Cherenkov photons onto the detector plane. Here the two photon detector planes are placed to the left and right of the beam pipe. Both the flat and the spherical mirror systems are made of aluminium coated glass. A magnetic shielding box encloses the photon detectors, reducing the peak magnetic flux density to less than 1.0 mT [B+05].

The total material budget for RICH-2, directly upstream from the calorimeter system and muon detector, is less than 0.124 of \( X_0 \), of which 0.02 is due to the CF4 radiator. For RICH-1, closer to the interaction point, the total material
budget is less than 0.08 of $X_0$, dominated by the 0.059 of $X_0$ due to the aerogel and gas radiators.

### 2.2.2 Pattern Recognition

The position of the photon hit, along with the corresponding charged particle track information, can be used to reconstruct the Cherenkov angle for each individual photon. The practical implementation of this method, essentially the inverse of ray-tracing, is described as part of the Cherenkov angle resolution studies in Chapter 4. The resolution on the reconstructed Cherenkov angle has the following contributions.

- **Chromatic error**: the refractive index of the radiator, and hence the Cherenkov angle, depends on the energy of the Cherenkov photon.
- **Emission point error**: the tilting of the spherical focusing mirror means that the position of the photon hit has a slight dependence on its emission point along the particle track.
- **Pixel error**: caused by the finite granularity of the photon detectors, described below.
- **Tracking**: the resolution on the measured track direction directly affects the knowledge of the centre of the reconstructed ring and hence the error on the reconstructed Cherenkov angle.

For a triggered signal event there are typically 30 tracks in the RICH detectors which produce Cherenkov rings on the photon detector plane. Because of the tilting of the spherical mirrors the rings are not perfect circles but are elliptical in shape. The Cherenkov angle resolution for the complete ring depends on the number of detected photon hits from the track. Assuming the errors on the Cherenkov angle reconstruction for each individual photon detected, listed above,
are independent, the resolution of the whole ring is given by

$$\sigma_{ring} \propto \frac{1}{\sqrt{N_{\text{photons detected}}}}. \quad (2.8)$$

To associate the rings with the correct corresponding track and therefore assign a particle type to each track, a global likelihood fit is performed. This method uses information from all three radiators simultaneously [RIC00].

The intrinsic RICH performance can hence be quantified by the resolution on the reconstructed Cherenkov angle for single photon hits, plus the number of photon hits detected for a track with $\beta \approx 1$. The photons are detected by conversion into photoelectrons at a photocathode, followed by the collection and amplification of the electrical signal generated by the photoelectrons. Details of the photon detector design are given in Section 2.2.3.

The number of detected photoelectrons is determined by several parameters [YS94]:

- $L$ - the length of the radiator.
- $T(E)$ - the probability that a photon reaches the photon detector plane, which includes the optical transmission losses in the radiator gas and other elements of the detector optics, as well as the reflectivity of the flat and spherical mirrors, as a function of energy.
- $\epsilon_A$ - the geometric efficiency, the probability that a photon reaching the detector plane hits the active area of a photon detector.
- $Q(E)$ - the quantum efficiency of the photocathode. This is the probability that a photon is converted into a photoelectron and is a function of the photon energy, $E$. 
Table 2.1: Summary of the characteristics of the three radiators used in the LHCb RICH detectors, including \( N_{\text{pe}} \), the mean number of detected photoelectrons in the ring.

- \( \eta_D \) - the single photoelectron detection efficiency, where the photon has been converted into a photoelectron at the photocathode.

The number of detected photoelectrons is then given by the expression

\[
N_{\text{pe}} = \frac{\alpha}{\hbar c} L Z^2 \epsilon_A \eta_D \int Q(E) T(E) \sin^2 \theta_C(E) dE. \tag{2.9}
\]

Table 2.1 summarises the characteristics of the three radiators including the number of detected photoelectrons, as determined from simulation. The occupancy of the photon detectors for triggered and accepted signal events varies across the photon detector plane. It reaches 8% in the region of RICH-1 illuminated by photons from low-angle tracks, and in the rest of RICH-1 and all of RICH-2 it is typically below 1%. The contributions to the resolution on the reconstructed Cherenkov angle are listed in Table 2.2, for single photons in the three radiators.

Figure 2.9 shows an example of the pattern of detected photoelectrons in RICH-1 and RICH-2 for a simulated \( B_d^0 \rightarrow \pi^+ \pi^- \) decay, with the result of the pattern recognition algorithm superimposed. A likelihood function is calculated for the whole event assuming that all the hits were produced by the reconstructed
Table 2.2: The contributions to the resolution of the reconstructed Cherenkov angle, per photoelectron, for the three radiators of the two RICH detectors, in mrad.

<table>
<thead>
<tr>
<th></th>
<th>RICH-1</th>
<th>RICH-2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Aerogel</td>
<td>C_4F_{10}</td>
</tr>
<tr>
<td>\sigma_{\text{emission}}</td>
<td>0.29</td>
<td>0.69</td>
</tr>
<tr>
<td>\sigma_{\text{chromatic}}</td>
<td>1.61</td>
<td>0.81</td>
</tr>
<tr>
<td>\sigma_{\text{pixel}}</td>
<td>0.62</td>
<td>0.62</td>
</tr>
<tr>
<td>\sigma_{\text{track}}</td>
<td>0.52</td>
<td>0.40</td>
</tr>
<tr>
<td>\sigma_{\text{total}}</td>
<td>1.89</td>
<td>1.27</td>
</tr>
</tbody>
</table>

tracks, for a given choice of particle type for each track. The set of particle types that maximise the likelihood is found. The CPU time required to carry the RICH reconstruction and pattern recognition algorithms is dependent on the number of hit pixels in the event, but has a mean of \( \sim 40 \) s.

Figure 2.10 shows the number of sigma separation between pions and kaons, \( N_\sigma (\pi - K) \), for true simulated pions from \( B_d^0 \rightarrow \pi^+ \pi^- \) decays, as a function of momentum. The precise choice of the log likelihood cut value affects the balance between the purity and efficiency of the sample, and can be adjusted to suit the needs of a particular physics analysis.

2.2.3 Photon Detector and Readout Electronics

The photon detector chosen for the RICH detectors is the pixel Hybrid Photon Detector, known as the HPD [Gys00]. This technology was developed in close collaboration with industry to satisfy the specific requirements of the LHCb RICH detectors. Single photons from Cherenkov radiation must be detected with high efficiency over a total area of 2.6 m\(^2\), with a high active-to-total area ratio (70\%) and with a granularity of 2.5 mm x 2.5 mm. This pixel size gives the contribution to the uncertainty on the reconstructed Cherenkov angle noted in Table 2.2, which
Figure 2.9: Event display for a simulated $B^0_d \rightarrow \pi^+\pi^-$ decay on the photon detector plane, showing the detected photoelectrons for RICH-1 (left) and RICH-2 (right), including background. The Cherenkov rings found by the pattern recognition algorithm are superimposed. The rings without filled hits are those with non-associated tracks.

Figure 2.10: Significance of the pion-kaon separation, $N_\sigma$, for true pions from simulated $B^0_d \rightarrow \pi^+\pi^-$ decays, as a function of momentum. The solid line indicates the average pion-kaon separation significance.
by design is comparable to the other sources of uncertainty. A smaller pixel size
would not improve the overall resolution, and would increase the complexity and
cost of the detector. The time resolution of the HPD must be better than the
LHC bunch crossing rate, 25 ns. In addition, the photon detectors will be subject
to a maximum ionising radiation dose of 3 krad per year and to the fringe field
of the dipole magnet, ∼2.5 mT inside the RICH-1 shielding box.

The HPD combines vacuum photocathode and solid-state technology in a
single device; a vacuum tube with a multi-alkali photocathode, high voltage cross-
focused electron optics and an anode consisting of a silicon pixel detector bump-
bonded to a CMOS readout chip. The anode assembly is fully encapsulated in
the device. A detailed description of the HPD design can be found in Chapter 3.
Here, a brief overview of the HPD and the readout electronics architecture will
be given.

Figure 2.11 is a photograph of an HPD, together with a schematic diagram.
The photon hits the photocathode, deposited on the internal surface of a trans-
parent quartz entrance window, and creates a photoelectron with a probability
determined by the quantum efficiency of the photocathode. This photoelectron
is accelerated by an electrostatic field with a 20 kV potential gradient onto the
pixelated silicon sensor. The silicon sensor is bump-bonded to the pixel readout
chip. The single photoelectron detection efficiency, $\eta_p$, after the photon con-
version, is ∼90%.

The pixel chip reads out the hits from the silicon sensor within the 25 ns
bunch crossing time, then digitises and buffers the data until the Level-0 trigger
decision is made. When an event is accepted the data from each chip are read
out at 40 MHz in less than 900 ns through 32 parallel lines, keeping dead time
losses in the data acquisition below 1%. The design and testing of this stage of
the readout electronics forms a substantial part of the work covered in this thesis, fully described in Chapter 3.

On receipt of a Level-0 trigger accept decision the data from each of the 484 HPDs in the RICH detectors are read out to the Level-1 electronics, via an on-detector Level-0 interface board and optical fibre links. The Level-1 electronics receives and buffers the data before receiving the Level-1 trigger decision. On a Level-1 accept, the data are passed to the DAQ system [Chr00]. Synchronisation of the electronics is achieved by the Timing Trigger and Control system, TTC, through an ASIC chip, the TTCrx, which distributes several signals including the 40 MHz global clock. The Level-0 and Level-1 trigger also accept a bunch count ID [Tay02]. A diagram summarising the main components of the Level-0 and Level-1 readout electronics is shown in Figure 2.12.

A single Level-0 board connects to two HPDs. The board distributes the clock
Figure 2.12: A schematic of the LHCb RICH Level-0 and Level-1 readout architecture.
and trigger signals to the pixel readout chip, further multiplexes the data, and then drives the data to the Level-1 electronics, located off-detector in the counting room [Wyl00]. The Level-0 board also provides the interface to the LHCb Experimental Control System, ECS [Onl01], which determines the configuration of the HPDs and the voltages and biases required for the HPD operation. The operating voltages and currents to the HPDs are controlled by the Analogue PILOT chip [Klu00]. A quartz crystal-controlled phase-locked loop chip, QPLL, is included to reduce the peak-to-peak jitter on the clock delivered by the TTCrx from ~800 ps to ~100 ps, as required by the gigabit-per-second serialisers [QPL].

Control of the Level-0 electronics is achieved via a dedicated Field Programmable Gate Array, FPGA chip, the Pixel Interface or PINT. An FPGA is made up of an array of Configurable Logic Blocks (CLBs), connected by programmable interconnections and surrounded by programmable I/O blocks [Bro92]. The ACTEL AX anti-fuse FPGA used to implement the PINT algorithms has been shown to withstand a radiation dose greater than 20 krad, corresponding to 10 years of LHCb operation, without failure. The PINT adds the bunch count and event ID as a header to the data, along with error and control flags. Two trailer words, including parity and check sum information, are also added. For each HPD, per event, 35 32-bits words are serialised and driven into the multimode optical fibres using Gigabit Optical Link, GOL, technology and Vertical Cavity Emitting Lasers, VCSEL [GOL] [S+01].

The Level-1 electronics receives the multiplexed data from the optical links and buffers them for the duration of the Level-1 latency, up to 58 ms. With the 1 MHz average Level-0 accept rate, this implies a buffer depth of at least 58254 events. The Level-1 electronics then discards events that fail the Level-1 trigger and passes accepted data onto the next stage of the data acquisition network.
2.2.4 HPD Implementation

In both RICH detectors the HPDs are close-packed in an hexagonal array on the photon detector planes, as illustrated in Figure 2.13. In RICH-1, the HPDs are organised in columns of 14 HPDs, with 16 per column in RICH-2. Each column contains the Level-0 interface boards, plus the high and low voltage distribution boards. To provide additional magnetic shielding every HPD is surrounded by a 0.9 mm thick μ-metal cylinder, reducing the 2.5 mT field in RICH-1 below \( \sim 1 \) mT. Studies have shown that distortions in the electron optics caused by magnetic fields at this level can be corrected [Rin05]. In total, there are 2 x 98 HPDs in RICH-1 and 2 x 144 in RICH-2, covering the total photon detector plane area of 2.6 m\(^2\). Since the HPD has an active diameter of 75 mm, with 87 mm between the centres of adjacent HPDs and a close-packing fraction of 97\%, the geometric efficiency, \( \epsilon_A \), is 67\%.

2.3 The Trigger

Figure 2.14 gives an overview of the Level-0 and Level-1 trigger architecture, indicating the sub-detectors which contribute to the each stage of the decision. The trigger exploits two particular features of \( B \) decays, differentiating them from other inelastic proton-proton interactions. Firstly events with \( B \) hadrons have decay products with high transverse momentum due to the relatively large \( B \) mass. Secondly \( B \) hadron decays have a displaced secondary vertex due to the long \( B \) lifetime [Tri03].

The first level of triggering, Level-0, uses the pile-up veto counter of the VELO to reject events with multiple proton-proton interactions. Furthermore, at Level-0, the highest transverse energy photon, electron and hadron clusters
Figure 2.13: An illustration of the hexagonally packed HPD detector plane, made up of columns of HPDs. Each column also carries the Level-0 interface board, and the LV and HV distribution boards.
Figure 2.14: Schematic of the Level-0 and Level-1 trigger architecture.
<table>
<thead>
<tr>
<th>particle</th>
<th>$E_t$ thresholds (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron</td>
<td>2.8</td>
</tr>
<tr>
<td>photon</td>
<td>2.6</td>
</tr>
<tr>
<td>hadron</td>
<td>3.6</td>
</tr>
<tr>
<td>muon</td>
<td>1.1</td>
</tr>
<tr>
<td>local $\pi^0$</td>
<td>4.5</td>
</tr>
<tr>
<td>global $\pi^0$</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Table 2.3: Summary of transverse energy thresholds for the Level-0 trigger decision.

in the calorimeter are reconstructed, as are the two muons with the highest transverse momentum in the muon chambers. Table 2.3 summarises the trigger thresholds that are accepted by the Level-0 trigger. An event is thus accepted by the Level-0 trigger if it has one or more particles which exceed these thresholds, and is not flagged as a multiple interaction. However, if the sum of the transverse momentum of the two muons exceeds 1.3 GeV, the event is accepted regardless of the outcome of the pile-up veto counter. The distinction between the local and global neutral pion threshold is a feature of the clustering of photon pairs from energetic $\pi^0$ decays, related to the calorimeter readout architecture [DP03].

The Level-0 trigger processing is implemented by dedicated electronics hardware mounted on the VELO, calorimeter system and muon detector. The hardware operates at the bunch crossing rate of 40 MHz and has an output of 1 MHz, with a fixed latency of 4 $\mu$s. The latency is the maximum time elapsed between each pp interaction and the arrival of the Level-0 trigger decision at the front-end electronics. The trigger decision is made by the Level-0 Decision Unit, which combines each detector signature into a single trigger decision per candidate. The Level-0 Decision Unit is fully programmable, designed so that the trigger can be readjusted depending on the experimental running conditions.

At the next stage of triggering, the Level-1 trigger is designed to select events
with high transverse momentum tracks from a displaced secondary vertex. The algorithm is split into two parts. Firstly, a cut is made on an impact parameter variable, calculated from the properties of the two tracks with the highest transverse momentum. This generic trigger algorithm is sensitive to all $B$ hadron decays. A second set of Level-1 trigger algorithms are designed to identify the signatures of specific $B$ decays, and if found, relax the generic trigger requirement.

These signatures are:

- The photon with the highest transverse energy, if greater than 3 GeV. This increases the trigger sensitivity for channels such as $B \rightarrow K^{*}\gamma$.
- The highest transverse energy electron, if greater than 3 GeV, to enhance the selection of $J/\psi \rightarrow e^+e^-$ decays.
- The highest invariant mass of a muon pair. If the invariant mass of the muon pair is within $\pm 500$ MeV of the $J/\psi$ or the $B$ mass, or greater than the $B$ mass, the event passes the Level-1 trigger regardless of the outcome of the generic algorithm. This enhances the selection of decay channels such as $J/\psi \rightarrow \mu^+\mu^-$ and $B \rightarrow \mu^+\mu^-X$ decays.

The Level-1 trigger operates at the Level-0 accept rate of 1 MHz and has a maximum output rate of 40 kHz, with a variable latency of up to 58 ms. Data from the calorimeters, the muon detectors, plus tracking information from the VELO and the TT tracking detector are input to the algorithm, as well data from the Level-0 Decision Unit. The transverse momentum measurement is made for each selected track by matching tracks in the VELO and the TT, and using the fringe field in front of the magnet, achieving a transverse momentum resolution of $\sim 20\%$ at 1 GeV/c and $\sim 40\%$ at 5 GeV/c. To achieve this, the magnet provides an integrated bending power of 0.15 Tm between the interaction point and the TT.
Both the Level-1 trigger and the HLT are implemented in software, running on a farm of ~1800 CPUs in the detector counting room. The HLT will have access to all the data from the LHCb detector. The generic HLT is designed to enhance the $b$ content and identify events with $\mu$ and $J/\psi$, while the specific HLT looks for particular $B$ meson decays. First, the generic HLT reduces the rate down to 10-15 kHz by re-running the Level-1 cuts with improved matching between the primary vertex, Velo and TT tracks, as well as further track matching to the full tracking system, including the stations T1-T3. As a result, the momentum resolution is better than $\sigma_p/p = 0.6\%$. Secondly the specific HLT is run, with individual cuts defined for each decay channel.

The HLT is designed to have an average output rate to disk storage of 2 kHz, consisting of four separate streams:

- 200 Hz of exclusively reconstructed $B$ decays for the core LHCb physics studies, including control channels for flavour tagging.
- 600 Hz of high mass di-muons, used to study the lifetime of unbiased $b \rightarrow J/\psi X$ events and hence measure the decay time resolution.
- 900 Hz of inclusive $b$ decays, with one high transverse momentum and high IP muon. This stream is used for systematic studies of the trigger efficiency and for data mining studies of $B$ decays.
- 300 Hz of inclusive decays to $D^*$, to calibrate the particle identification and also to study CP violation in charm decays.

A summary of the three different trigger levels is given in Table 2.4.
Table 2.4: Summary of the characteristics of the three LHCb trigger levels.

<table>
<thead>
<tr>
<th>Sub-detectors</th>
<th>Level-0</th>
<th>Level-1</th>
<th>HLT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input rate</td>
<td>40 MHz</td>
<td>1 MHz</td>
<td>40 kHz</td>
</tr>
<tr>
<td>Output rate</td>
<td>1 MHz</td>
<td>40 kHz</td>
<td>2 kHz</td>
</tr>
<tr>
<td>Sub-detectors</td>
<td>VELO pile-up counter calorimeter system muon detector</td>
<td>VELO calorimeter system muon detector TT tracking detector</td>
<td>All</td>
</tr>
</tbody>
</table>

Figure 2.15: The logical structure of the LHCb software, from the simulation of Monte Carlo data to analysis of the data, built within the GAUDI framework.

## 2.4 Software Framework

The LHCb software is built within the C++ GAUDI framework [B+01]. It provides administrative tools such as data persistency and histogramming, as well as allowing run-time configuration via custom-format options files. The software can be divided into two parts: the simulation system, and the system for analysing the data. When data-taking begins, the analysis software optimised on the Monte Carlo data will then be applied to the real data. The logical framework is shown in Figure 2.15 and described below [Com05].
Simulation

Gauss

The Monte Carlo events are generated by the simulation application, Gauss, combining event generation and the simulation of the passage of particles through the detector. The first phase is the generation of pp collisions and the subsequent decay of \( B \) hadrons. This is carried out using a combination of Pythia and a specialised \( b \)-decay package, EvtGen [SLMS03] [Lan01]. The second phase, the simulation of the physics processes that the particles undergo as they travel through the material of the LHCb detector, is based on the GEANT4 toolkit [A+03b].

Boole

A separate application, Boole, simulates the response of each sub-detector to the output from Gauss, producing raw \textit{digi} data files in exactly the same format as the experimental electronics and data acquisition systems. The effect of adjacent bunch crossings in sensitive detectors is included, as is information on the resolution and imperfections of each detector, measured in test beam data.

Analysis

Brunel

The essential aim of the analysis software is to recover the fundamental physics of the \( B \) decays. In the case of simulated data this means effectively inverting the simulation process. Brunel is the first tool in this procedure, analysing the raw \textit{digi} data files to reconstruct physics objects, like calorimeter clusters, tracks or rings in the RICH detectors. The output from Brunel is in the form of a \textit{DST} file containing all the reconstructed objects, including the combined particle
identification information from the RICH detectors, calorimeter system and muon detector, as well as the tracks, calorimeter clusters etc. The Brunel application is designed to treat raw digi files identically whether they originate from the simulation or real data. It also interfaces to the detector description and the conditions database, describing the running conditions of the experiment, such as the alignment and calibration.

*DaVinci*

The DST files are input to the LHCb offline analysis application, *DaVinci*, which creates high level objects like particles and vertices, and ultimately performs the event selection. Various DaVinci tools are provided, for example to combine particle objects into decay chains, identify signal events, and perform the flavour-tagging algorithms. When run on simulated data, these tools are used for the evaluation of the physics performance of the code, since they allow access to the Monte Carlo truth information recorded at each stage of the simulation. The output from DaVinci can take several forms. For example, it can be output to the LHCb event visualisation system, *Panoramix*, output to reduced DST files containing only events which satisfy a certain selection criteria, or output to ntuples suitable for analysis using the ROOT framework [BR97].
Chapter 3

Pixel Hybrid Photon Detectors

Introduction

To make precision measurements of CP violation in the B meson system charged particle identification is essential, in particular the ability to separate pions and kaons in the final state. To achieve this the LHCb detector includes two Ring Imaging Cherenkov (RICH) detectors [RIC00]. The design of the RICH detectors is described in Chapter 2. Details of the HPD design and fabrication are given in this chapter.

The suitability of HPDs had to be proven by a thorough investigation of their performance. This chapter describes one part of this investigation, the study of the HPD anodes. The development of a rigorous testing programme to ensure quality-control at each stage of the manufacturing process is also discussed. Accelerated ageing tests were performed to prove the suitability of the HPD anode for ten years of operation in the LHCb environment.
3.1 HPD Design

The HPD has a 7 mm thick quartz optical input window with a diameter of 83 mm, 75 mm of which is active. Figure 3.1 shows a schematic of the HPD. A photon incident on the quartz window releases a photoelectron at the S20 multi-alkali photocathode which is deposited on the inside of the window, within the vacuum tube [Tow03]. The sensitive wavelength range is typically 200 nm to 600 nm, and quantum efficiencies for this photocathode reach 25% at ~270 nm wavelength.

The photoelectron is accelerated by an applied high voltage of ~20 kV onto the anode assembly. The anode consists of a 300 μm thick pixel detector array with 32 x 256 p-n junctions, each forming a reverse-biased diode, bump-bonded to an LHCBIPIX1 binary readout chip [W+99]. Due to the cross-focused electron optics there is a 5 times demagnification from the cathode to the anode. The
spatial resolution of the electron optics is indicated by the width of the *point spread function*. The point spread function describes the radial distribution on the anode of photoelectrons generated by a point source on the cathode. In prototype tests, the RMS of the point spread function was measured to be between 33 μm and 45 μm [Rad].

The photoelectrons hit the back of the silicon sensor and create electron-hole pairs in the substrate. The incident photoelectron excites electrons from the valence band, so they are transferred to the conduction band, leaving behind holes in the valence band. An ionisation energy of 3.6 eV is needed to form each electron-hole pair; a single photoelectron releases ~5000 electron-hole pairs at 20 kV. The silicon is reverse-biased to generate a high electric field, to ensure the electrons are collected at the anode, and the holes at the cathode, before recombination can occur.

Figure 3.2 shows a schematic of the silicon sensor, manufactured by Canberra, Belgium [Can]. To minimise energy losses in the insensitive n⁺ surface layer on the rear side, which makes the ohmic contact to the bulk, the n⁺ implant is thinner than in standard processes; the peak of the implant distribution is 150 nm below the silicon surface. Additionally the sensor features an aluminium contact frame around the edge rather than a fully aluminised backplane, again minimising the material traversed by the incident photoelectron and hence the energy loss.

Each pixel has dimensions 62.5 μm x 500 μm and is connected by a solder bump bond to an input of the LHCbPIX1 readout chip with matching dimensions. The full pixel matrix consists of 256 rows x 32 columns. The binary readout chip is a custom designed ASIC manufactured by IBM in 0.25 μm CMOS technology, using layout techniques adapted for tolerance to ionising radiation and
Figure 3.2: Diagram of the silicon sensor.

single event upset [S⁺00]. The bump-bonds are formed by microscopic balls of solder, ~20 μm in diameter, which are deposited on the substrates of the silicon sensor and the readout chip. Initially a layer of nickel is electroplated onto both surfaces, referred to as the under-bump metallization, followed by the solder. The connection between the silicon sensor and the readout chip is made by a high temperature reflow of the bump-bonds.

In the LHCPIX1 chip a logical OR is made of the hit information from eight adjacent pixels to form a super-pixel 500 μm x 500 μm, matching the dimensions required by the RICH detectors. Readout of the OR-ed super-pixel is called LHCb mode, as opposed to the ALICE mode where data from each of the 8192 pixels are read out individually. Much of the testing of the LHCPIX1 chip and the HPD itself is carried out in ALICE mode to maximise the information available. There are many advantages to this approach. In LHCb mode, the
3.1 HPD Design

![Diagram of a single pixel readout channel](image)

Figure 3.3: Schematic of a single pixel readout channel.

The capacitive load on the analogue front end of the pixels is still that of the small intrinsic pixel size and so the low noise characteristics are maintained. The time-averaged hit occupancy across the RICH detectors reaches a maximum of 4% in some regions, however since every LHCb logical pixel is made up of these eight 62.5 μm x 500 μm pixels, the hit occupancy seen by each individual front-end electronics channel is significantly lower.

The typical signal of ~5000 electrons for each photoelectron can be reduced to below 2500 electrons by charge sharing between channels. Hence the chip must be sensitive to a signal size of 2000 electrons. This defines the minimum threshold, together with a pixel to pixel RMS spread of less than 250 electrons. The threshold to noise ratio has been designed to be better than 6.

The pixel is split into an analogue and digital part, shown schematically in Figure 3.3. The design of the chip is described in full in Ref. [W+99]. The analogue front-end has a differential pre-amplifier followed by a differential shaper stage with a 25 ns peaking time. There is a 16 fF injection capacitance in series with the input of the preamplifier to allow a test-pulse input to be sent to each pixel, with one test-pulser per 32-channel column.

A discriminator compares the output of the shaper to a global threshold fixed
for the whole chip. Each pixel also has a 3-bit threshold adjust to allow fine
adjustment of the thresholds on a by-pixel basis. The output of the discriminator
is fed into the digital part of the pixel cell. The digital circuitry stores hits until
a trigger is received, buffers triggered events and reads out the data synchronous
with the clock to the next stage of the data acquisition system (DAQ). The
discriminator output is first synchronised to the 25 ns clock and then enters the
digital delay units, designed to store the hit for the duration of the trigger latency.

A logical fast-OR signal is taken from all the discriminator outputs of each
pixel. An important practical use of the logical fast-OR will be described later in
the chapter. A single mask bit can be set for each pixel to disconnect the output
of the discriminator from the downstream circuitry, used to inhibit readout of
noisy channels.

There are two digital delay units per pixel cell, as represented in Figure 3.4.
The delay can be adjusted to exactly match the trigger latency, as required by the
experiment. When a trigger accept occurs, the delayed hit is loaded into the next
available cell of a four event FIFO (First In First Out) buffer, the multi-event
buffer and de-randomiser. On receiving a signal from the next stage of the DAQ,
the contents of the FIFO cells are loaded into a flip-flop. The flip-flops of a single
column form a shift register, allowing the data to be shifted out synchronously
with the system clock. All the digital logic in the pixel cell switches under current
control. This means that external bias voltages limit the switching speed for each
digital block, therefore minimising the noise injected into the analogue circuitry.

In ALICE mode, each pixel cell is read out as an individual channel. Using
the two delay units each pixel cell can simultaneously store two hits during the
trigger latency time. The readout of the pixel cells in ALICE mode is shown in
Figure 3.4. An external control signal is used to choose either ALICE or LHCb
Figure 3.4: Readout of pixel cells in ALICE mode.
operational mode. For LHCb mode the clock-synchronised discriminator outputs of eight vertical pixels are OR-ed together, making up the super-pixel. This configuration is illustrated in Figure 3.5. The sixteen delay units of these eight pixel cells are configured in an array, hence a maximum of sixteen hits can be stored during the trigger latency. Four of the FIFOs are connected to form a sixteen event FIFO, meeting the LHCb requirement for a Level-0 de-randomiser depth of sixteen. This FIFO output is loaded into the flip-flop of the top pixel in the group and the other seven are bypassed. Using this scheme, data from the
32 x 32 super-pixels can be read out in less than 900 ns on 32 parallel lines at 40 MHz, keeping dead time losses in the DAQ below 1%.

Figure 3.6 is a schematic floorplan of the chip. The sensitive area, 16 mm x 16 mm, is divided into the 32 rows and 32 columns of the 500 μm x 500 μm super-pixels which are read out in LHCb mode. In ALICE mode, where the eight separate channels of each super-pixel are read out individually, the pixel matrix has 32 columns and 256 rows. The rest of the chip consists of peripheral control logic, the biasing circuitry, the input/output wire-bond pads, and a JTAG serial interface. JTAG is a standard for providing external serial access to integrated circuits. The acronym stands for the ‘Joint Test Action Group’, which defined the standard [IEE]. This JTAG interface is used to send all the command and
configuration data to the readout chip. There are 42 8-bit digital-to-analogue-converters (DACs) on the readout chip, to provide bias voltages and currents to the analogue and digital circuits of the pixel cell, and all can be configured via JTAG. The test-pulse, threshold adjust and mask bits of each pixel cell are all set through the JTAG interface.

The silicon sensor bump-bonded to the readout chip, referred to as the HPD sensor assembly, is mounted onto a custom-designed ceramic carrier supplied by Kyocera [Kyo]. Connections are made with gold wire bonds to the wire-bond pads at the top and bottom of the readout chip, as indicated in Figure 3.6. The assembly is glued into the ceramic carrier, forming the HPD anode, using a high curing temperature (400°C) silver glass glue specifically chosen for its low out-gassing characteristics. This is because the last stage of HPD manufacture involves a three hour bake-out cycle where the temperature reaches 300°C. Since the HPD anode is encapsulated within the HPD vacuum tube, it must be compatible with all stages of the manufacturing process.

The chosen HPD tube geometry is robust against external electrical field deviations. However it is susceptible to magnetic field perturbations, due to the large tube diameter, and hence needs an externally mounted μ-metal shield to perform correctly in the LHCb environment, as described in Chapter 2.

The total number of HPDs required by LHCb is 550. This allows for 196 HPDs in RICH 1, 288 HPDs in RICH 2, plus 66 (15%) additional HPDs for pre-series testing and spares.
### 3.2 Overview of HPD Manufacture and Testing

The cost of each stage of the HPD manufacture and the tight time schedule of production meant it was essential that no effort was wasted on substandard components. A strict quality-control programme was designed and implemented to ensure the final HPDs meet all the requirements of the experiment.

The LHCbPix1 readout chips were manufactured in 200 mm CMOS wafers of 71 chips. Each chip on a wafer underwent extensive electrical testing on a probe station. This electrical testing of the readout chip occurred at each stage of the process and is described in Section 3.2.1. A map of the wafer was created to indicate which chips had passed the tests. Once tested, the readout chip wafers, silicon sensor wafers and the wafer maps were shipped to the bump-bonding contractor, VTT [VTT]. Readout chips which passed the tests were diced and then bump-bonded to the silicon sensor chips to form the sensor assemblies. The assemblies were returned to CERN where the electrical testing on the probe station was repeated, to check there had been no degradation in the readout chip performance during bump-bonding. In addition, a radioactive source was used to test the quality of the bump bonds. In Section 3.3 this testing and the bump-bonding process are discussed in detail.

The assemblies which passed the quality-control tests were glued into the ceramic carriers, and the input and output pads of the readout chips were wire bonded to the ceramic carrier. This was carried out by HCM [HCM]. These completed HPD anodes underwent a series of quality-control tests, including the standard set of electrical tests, before being sent to DEP [DEP] for the final stage of manufacturing the HPD. A diagram of the HPD manufacturing process is shown in Figure 3.7.
At DEP the HPD anodes were encapsulated into the vacuum tubes. A high temperature bake-out is needed to remove contaminants before depositing the photocathode on the inside of the quartz window and sealing the tube. It is vital that the HPD anodes, and in particular the bump-bond connections, are robust enough to withstand this high temperature cycle. For this reason an oven laboratory process to simulate the vacuum bake-out cycle was an important feature of the accelerated ageing tests described in Section 3.3.3.

### 3.2.1 Electrical Testing Procedure for the HPD Readout Chip

A standard electrical testing procedure was designed, to test the readout chip at each stage of the manufacturing process as part of the quality-control programme. To carry out testing of chips on the wafer, control signals and biases must be provided to the readout chips, and data from the chips must be read out. The laboratory test system is illustrated in Figure 3.8. The system is controlled by a PC running LabVIEW software. It uses the JTAG protocol to set the values of the 42 DACs on the readout chip. A mother-board supplies the bias voltages to the readout chip and the DACs controlling these values are also set by JTAG. The mother-board is connected to a probe-card that sits directly above each chip.
Figure 3.8: Diagram of the system for carrying out the electrical testing procedure on a wafer of readout chips.
in turn on the wafer. Needles on the probe-card make an electrical connection to the input/output pads of each readout chip. The probe station allows the controlled precision movements of the wafer and probe-card which are needed to place the needles on the 100 μm x 100 μm pads of the readout chip without damage.

Referring to Figure 3.8, a transmitter board contains an FPGA (Field-Programmable Gate Array, a programmable logic chip) to generate the control signals for the readout chip, plus a crystal oscillator to provide the 40.08 MHz clock. Data arriving from the readout chip via the mother-board are buffered on the transmitter board, which then serialises the data and transmits them on a twisted pair data link to the receiver. The S-LINK protocol used to transmit the data was developed at CERN [S-L]. A FLIC (Flexible Input/Output Card), also developed at CERN [G+02], is connected to the S-LINK receiver. The FLIC stores the data in its memory before uploading to the PC. It sends a software-controlled trigger to the transmitter board to control the shifting out of data from the readout chip.

Readout chips are classified as Class 1, and therefore suitable to go on to the next stage of production, Class 2 or Class 3. Class 2 and 3 readout chips are rejected. This classification is explained in detail below. The typical production yield for Class 1 readout chips was ~60%. At this first stage of production, of order 1000 Class 1 chips were required to produce 800 HPD assemblies, allowing a suitable safety margin.

The electrical testing is carried out in several stages. First the analogue and digital currents of the chip are measured. The digital current must be below 700 mA and the analogue current must be below 600 mA. If this is not the case the chip is classified as Class 3 and no further testing is necessary. For chips accepted
at the first stage, a simple JTAG test is then performed. A code is sent to the readout chip and read back, and any inconsistency reveals a JTAG failure. If the interface to the DACs fails, this means one or more of the DACs cannot be set to the correct value and the chip is listed as Class 3. The JTAG interface is also used to control the configuration of the readout chip, setting the mask, test and threshold adjust bits for each pixel. If there is a failure of the JTAG in setting these bits the readout chip is not immediately classified as failed, as it may be due to a stuck bit which affects only one pixel. Ultimately a Class 1 chip must have more than than 99% operational pixels.

Secondly, the pixel threshold circuitry is investigated, in particular the threshold sensitivity and uniformity across the pixels. All the testing is carried out in ALICE mode, so the 8192 pixels are read out individually rather than being combined into 1024 super-pixels. A signal is simulated by directly injecting charge at the input of the analogue readout circuitry using the test-pulse. The amplitude of the test-pulse is controlled externally; it is determined by the difference between two voltages and is calibrated such that 1 mV is equivalent to the injection of 100 electrons. The readout of hits resulting from the test-pulse is triggered by the LabVIEW software.

In the first stage of threshold testing the minimum setting of the global threshold is found. The value of the global threshold is controlled by a single DAC labelled \( \text{pre}_VTH \). It is initially set to the lowest possible value, just above the intrinsic noise level of the chip. This is found by setting the amplitude of the test-pulse to zero, and first setting the threshold to a very low value. Any hits read out from the chip are due to noise. The threshold is slowly increased until the point at which the noise rate is negligible, and this point is the minimum global threshold.
<table>
<thead>
<tr>
<th>DAC name</th>
<th>function</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre_VTH</td>
<td>sets the imbalance of the discriminator stage to define the pixel charge threshold</td>
</tr>
<tr>
<td>Eu_VBIAS</td>
<td>bias for timestamp latch in delay units</td>
</tr>
<tr>
<td>Ken_VBNS</td>
<td>controls current-starving on shift register</td>
</tr>
<tr>
<td>Pre_VI2</td>
<td>second shaper stage current</td>
</tr>
<tr>
<td>Pre_VREF1</td>
<td>reference input for differential preamplifier</td>
</tr>
</tbody>
</table>

Table 3.1: List of the DACs tested in the standard electrical testing procedure.

Data are collected for the study of the threshold behaviour of the pixels with the pre_VTH DAC set at the minimum global threshold. For each pixel in the column of 32, a test-pulse is injected, followed by readout of the pixel data. This is repeated 40 times. If the test-pulse is above threshold in an individual pixel, a hit is registered in that pixel. This process is repeated for all 256 rows of the readout chip. Next, the size of the test-pulse is reduced and the measurement is repeated for all the rows and columns. By injecting a varying amount of a known charge in this way, the threshold of each pixel is calibrated and the noise can be measured. Hence data from these test-pulse scans are used to understand the threshold and noise distribution across the chip. This is discussed in detail in Section 3.2.2 below.

The final step in the electrical testing procedure is to test five of the most important DACs by varying the configuration sent to each one by JTAG and recording the output voltage or current. Table 3.1 lists the DACs tested and their function.

In summary, the criteria that define which readout chips are Class 1 and which are Class 2 and or Class 3, and therefore rejected, are:

Class 1 - pass:

- less than 1% non-operational pixels and
3.2 Overview of HPD Manufacture and Testing

- digital current < 700 mA and
- analogue current < 600 mA and
- DAC scans pass.

Class 2 - reject:
- more than 1% non-operational pixels and
- digital current < 700 mA and
- analogue current < 600 mA and
- DAC scans pass.

Class 3 - reject:
- no digital output OR
- digital current > 700 mA OR
- analogue current > 600 mA OR
- JTAG DAC test failed OR
- no minimum global threshold can be found OR
- DAC scans fail.

Careful design of the LabVIEW software allowed much of the testing procedure to be automated. For example, the series of threshold scans at varying test-pulse values could be performed with a single command. The results of each test were saved in a suitable format to a location set by the software. Once an entire wafer had been tested, this simplified the process of backing up all the data to a database.

This electrical testing procedure was repeated for the HPD assemblies, anodes and completed HPDs where the same criteria were applied; only those with Class 1 readout chips pass the quality-control requirements. All the test results for a completed HPD are stored in the database.
3.2.2 Analysis of Test-Pulse Scans

A test-pulse scan is performed to study the threshold and noise distribution across the readout chip. As described above, the response of each pixel to a particular amplitude of the test-pulse is recorded for a scan over the test-pulse amplitude, while the value of the global threshold is kept constant at the minimum threshold. The efficiency of each pixel is defined as the number of hits recorded divided by 40, the number of test-pulses injected, and is plotted against the amplitude of the test-pulse. The pixel threshold is then defined as the value in millivolts where the efficiency falls to 50%. In the absence of noise hits this curve, called the s-curve of the pixel, would be a step function, with an efficiency of zero for a test-pulse amplitude below the pixel threshold and 100% above. However in reality noise in the pixel causes a rounding of the s-curve from a perfect step function.

The noise distribution is the derivative of the s-curve [Pul95]. In the case of no noise, the signal is a delta function, which is the derivative of a step function. Given in addition a Gaussian noise distribution, the appropriate form for the s-curve is the error function, Equation 3.1.

\[
erf(z) = \frac{2}{\sqrt{\pi}} \int_{z}^{\infty} \exp(-t^2) \, dt.\tag{3.1}\]

Here

\[
z = \frac{|x - \mu|}{\sigma}\tag{3.2}\]

and, since the derivative of an error function is a Gaussian:

\[
N(x, \mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x - \mu)^2}{2\sigma^2}},\tag{3.3}\]

where \(\sigma\) is the noise, \(\mu\) is the 50% point of the test-pulse scan and \(x\) is the test-
Figure 3.9: The error function, \( erf(z) \), where \( z = \frac{|x-\mu|}{\sigma} \) with \( \mu = 0 \) and \( \sigma = 1 \).

pulse voltage. A plot of the function \( erf(z) \) is shown in Figure 3.9, for \( \mu = 0 \) and \( \sigma = 1 \).

For each pixel, a measurement of the RMS value of the noise is derived from the s-curve, using the fact that the difference between the test-pulse magnitude at 98% and 2% efficiency is \( \pm 2\sigma \) of the noise distribution. Taking the 50% point as the peak of a Gaussian, the values for the efficiency recorded above the 50% point are simply inverted about the line of 50%, and then the variance of the distribution is calculated. This method allows a fast estimate of the noise in the pixel, where an accurate fit to the error function is not possible with the available data. An illustration of this method is given in Figure 3.10 for a typical pixel, using the results of such a test-pulse scan, in black. Indicated in red is the inversion of the data points above the 50% point, from which an estimate of the \( \sigma \) of the noise distribution is calculated.

An s-curve is recorded for every pixel of the readout chip in turn and a mea-
Figure 3.10: Test-pulse scan results for a typical pixel, black crosses. The global threshold was fixed at the minimum, and the efficiency measured for a scan over the test-pulse values. The squares in red indicate the inversion of the data points above the 50% point, from which an estimate of the $\sigma$ of the noise distribution is calculated.
Figure 3.11: Pixel threshold (top) and noise distribution (bottom) for all readout channels for a typical chip on a wafer.

Measurement is made of the pixel threshold and RMS noise of each. It is then possible to plot the distribution of the pixel threshold and noise for all 8192 pixels. An example of the two measured distributions is given in Figure 3.11, for all 8192 channels of a typical bare readout chip on a wafer. These results are used to determine if the readout chip meets specifications; i.e., the minimum threshold of every pixel must be sensitive to a signal of 2000 electrons, and the RMS noise of each pixel must be less than 250 electrons. Since these tests are repeated at each manufacturing stage as part of the quality-control programme, the results stored in the database for the readout chip as the HPD is constructed can be compared. It can be seen from Figure 3.11 that the chip in question, prior to bump bonding, far exceeds the specifications.
3.3 Testing of HPD Anodes

A wafer of readout chips is diced and those chips which have passed the electrical testing procedure as Class 1 are bump-bonded to silicon sensor chips. The silicon sensors are produced as 125 mm wafers and all the sensor chips are assumed to be good. The bump-bonding process is carried out in several steps.

Firstly field metal layers are deposited on the surface of the wafers to act as an electrode for the subsequent electrolytic deposition. Then thick photoresist is patterned onto the wafer around the openings in the passivation where the bumps will be grown. Next the nickel under-bump metallization and solder are electroplated on both substrates. Finally, the photoresist and the field metal layers between the bumps are removed, and the bumps are reflowed on both sides. The wafers are diced and the Class 1 readout chips are bump-bonded to sensor chips, producing the sensor assemblies.

These assemblies are shipped to CERN and the next stage of quality-control testing is carried out. The electrical testing procedure for the readout chip is repeated and additionally the integrity of the bump bonds is tested using a radioactive source. More than 99% of the pixels must be operational or the assembly is failed. Details of this test are given in Section 3.3.1.

A further complication is introduced by subsequent steps in the process of manufacturing the HPD, specifically the high temperature vacuum bake-out after the anode is encapsulated in the HPD tube. During the bake-out cycle the temperature reaches \( \sim 300^\circ\text{C} \). The standard VTT bump-bonding procedure uses eutectic solder, 63% Sn and 37% Pb, with a melting point of 183\(^\circ\)C. On the first batch of HPDs produced many of these bump bonds melted and the under-bump metal dissolved into the molten solder, severely reducing the number of
operational pixels. To solve this problem the solder used in the bump-bonding was changed. Instead of a single layer of eutectic solder, a thin layer of eutectic was followed by a thick layer of pure lead solder deposited on the readout chip wafer. The solder on the sensor wafer was unchanged. This resulted in a final composition of 10% Sn and 90% Pb, with a melting point above $\sim 300^\circ$C [Cam04].

A pre-production batch of HPD assemblies was produced with these high-lead content bump bonds. The assemblies were tested before being packaged in the ceramic carriers and were then shipped back to CERN, where the quality-control tests were repeated. As well as the standard quality-control tests, these pre-production HPD anodes were subjected to a rigorous accelerated ageing programme, including a series of simulated bake-out cycles. The conclusions drawn from this programme are discussed in Section 3.3.3.

### 3.3.1 Bump Bond Integrity Tests

A radioactive source is used to check the quality of the bump bonds, first for the sensor assembly and then again when the assembly is glued into the ceramic carrier to form the HPD anode. Pixel hits caused by energy deposited in the reverse-biased silicon by the radioactive particles are read out from the chip to a PC controlled by LabVIEW software. The data are used to make a map of the 8192 pixels, indicating the hit multiplicity in each one over a period of time.

The test setup is similar to that used in the standard electrical testing procedure, with several important differences. Contrary to the test-pulse, the pixel hits due to energy deposited by the radioactive particles are random in time. It would have been possible to run the test with a random trigger, however a more efficient method was developed by triggering on the fast-OR signal provided by the front-end electronics. As seen in Figure 3.3, this signal is constructed from
a logical OR of the discriminator outputs of every pixel and therefore is asserted each time any single pixel records a hit above threshold.

There is a problem with the fast-OR signal that had to be considered in the trigger logic design. Spurious fast-OR signals are often generated by noise during the readout process, when data in the FIFOs are parallel-loaded into the output shift registers and shifted out. A noise veto is therefore applied for 20 $\mu$s after each trigger to block these false signals. A second veto is required to ensure the finite memory of the FLIC card does not overflow during readout of the data, referred to here as the FLIC veto.

Figure 3.12 describes the trigger logic. The fast-OR signal from the readout chip is passed to a NIM crate where the noise veto is asserted, so that for 20 $\mu$s subsequent fast-OR signals are ignored. An external trigger is then fed back to the chip to trigger readout of the data, unless the FLIC veto is asserted. In this event, the Flexible Input/Output Card connected to the PC cannot store any more events and there is a pause in the data acquisition while the data are transferred to the PC.

The FLIC veto is controlled by the PC via a National Instruments PCI-6036E DAQ card and the maximum number of events to store in the FLIC memory is set by the software. The PCI-6036E receives the external triggers and counts them until this limit is reached, using a built in counter that is capable of handling a maximum trigger rate of 20 MHz. When the number of triggers reaches the limit the FLIC veto signal is sent to the NIM crate that applies the veto logic.

Using this test system and irradiating the HPD assemblies or anodes with a Sr-90 source, 80,000 fast-OR triggers could be collected in approximately five minutes, with each pixel sampling $\sim$40 hits. Operating with an uncorrelated random trigger required around half an hour to record a million random triggers,
Figure 3.12: Diagram of the data acquisition system with the fast-OR trigger, used to test the integrity of the bump bonds. The radioactive source is positioned directly above the sensor assembly or anode.
Figure 3.13: Radioactive source (Sr-90) measurement of a typical HPD anode, showing the 32 by 256 pixel array. White represents no hits. The average occupancy is 49 hits.

and each pixel sampled ~10 hits. Therefore the fast-OR system was a great improvement since it allowed the tests required for the quality-control programme to be carried out in a much shorter time period.

For the source test, the global pixel threshold is set to the minimum threshold, which is determined as described above for the standard electrical testing procedure. Results for a typical HPD anode are shown in Figure 3.13. White areas represent non-operational pixels. This anode has 8189 working pixels, far exceeding the 99% requirement. A summary of the bump-bond integrity tests for the packaged HPD anodes in the pre-production batch is given in Table 3.2. The ID is a code that uniquely identifies each anode, allowing access to the data recorded in the database at each stage of manufacture. All are found to meet the requirement of at least 99% operational pixels. The sources used are summarised in Table 3.3.

As discussed in Section 3.1, the average energy required to create an electron-hole pair in silicon is 3.6 eV at room temperature, and this is independent of the
### 3.3 Testing of HPD Anodes

<table>
<thead>
<tr>
<th>Anode ID</th>
<th>operational pixels (%)</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>2.7</td>
<td>99.8</td>
</tr>
<tr>
<td>3.1</td>
<td>99.6</td>
</tr>
<tr>
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<td>99.8</td>
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<tr>
<td>4.1</td>
<td>99.9</td>
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<tr>
<td>4.2</td>
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</tr>
<tr>
<td>4.8</td>
<td>99.9</td>
</tr>
</tbody>
</table>

Table 3.2: Results of bump bond integrity tests for the pre-production batch of HPD anodes.

<table>
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<tr>
<th>Sr-90</th>
<th>type</th>
<th>average energy (keV)</th>
<th>max. energy (keV)</th>
</tr>
</thead>
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<tr>
<td>Beta</td>
<td>196</td>
<td>546</td>
<td></td>
</tr>
<tr>
<td>Beta (Y)</td>
<td>935</td>
<td>2284</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3: Details of radioactive sources used to test HPD anodes and assemblies.
type of ionising radiation. The Sr-90 source used to carry out the quality-control testing of the bump bonds produces of the order of 100,000 electron-hole pairs. Hence, in a single triggered event, often more than one pixel records a hit above threshold. This allows data on the number of working bump bonds to be collected in a shorter amount of time.

Several HPD anodes from the pre-production batch were also tested with a Cd-109 source. The lower energy gamma radiation from this source creates a smaller number of electron-hole pairs in the silicon sensor, and is thus a closer approximation to the minimum threshold requirement for the HPD, which is 2000 electrons. Tests with this source revealed an unexpected limitation of the fast-OR signal. The output from the discriminators is collected at the base of each column and the discriminator current should be proportional to the total number of hit pixels in the column. However in most columns it was found that two or more pixels had to be hit to produce a large enough signal for the global fast-OR to be asserted at the output of the readout chip. As result, the signal is not a true OR, since the sensitivity varies strongly from column to column and may require two or more pixels in each column to be hit simultaneously. In this test setup this will in principle degrade the readout sensitivity of the chip, since the fast-OR is used to trigger the readout.

Figure 3.14 illustrates this effect. It shows the 32 by 256 pixel array, shaded to indicate the hit multiplicity for this typical HPD anode, after exposure to the Sr-90 and the Cd-109 sources. In each case the data acquisition was run until 80,000 fast-OR triggers were collected. For the Sr-90 source the hit multiplicity is fairly uniform across the pixels. There is a region of higher multiplicity in the centre of the anode due to the finite size of the source, which is circular and has a diameter less than the 16 mm x 16 mm sensitive area of the anode. In contrast
the data from the Cd-109 source has a large variation in hit multiplicity from column to column. Since both plots are for the same anode this effect cannot be due to missing bump bonds or failures in the pixel readout electronics. It is caused by the inadequate fast-OR signal.

To conclude, this problem with the fast-OR signal means it is more efficient to use the Sr-90 source to test the integrity of the bump bonds. The high energy beta particles create a large number of electron-hole pairs and ensure a cluster of adjacent pixels is hit in each event, therefore reliably producing a fast-OR signal to trigger the readout. The lower energy gamma radiation from the Cd-109 source is less likely to produce a hit in more than one pixel.

### 3.3.2 Threshold and Noise Performance

Using data from the electrical testing procedure the behaviour of every readout chip can be studied at each stage of manufacture; from bare chip on the wafer, to bump-bonded assembly, to packaged anode. Empirically, the performance of
the readout chip does not change as a result of the ceramic carrier packaging. A
difference in performance is clear, however, when comparing the bare chip on the
wafer with the bump-bonded assembly. As a result of the silicon sensor which
is bump-bonded to the chip, the capacitive load on the front-end electronics
increases and therefore so does the noise. A higher minimum threshold is also
measured.

Figure 3.15 shows the distribution of thresholds measured for each pixel of a
typical anode from the pre-production batch, compared to the pixel thresholds
measured for the same readout chip at the wafer-probing stage, before bump-
bonding to the sensor chip. It is clear that the mean pixel threshold has shifted
to a higher value after the bump-bonding step, although it remains below the
2000 e\textsuperscript{-} requirement. The pixel noise distribution is shown in Figure 3.16, again
before and after the bump-bonding to the sensor wafer. The mean of the noise
distribution has also shifted to a higher value, but the readout chip is still well
within the specifications.

The result of a Gaussian fit to the pixel threshold distributions, before and
after bump-bonding, is given in Table 3.4, along with the results for the entire pre-production batch of HPD anodes. Table 3.5 gives the results of a Gaussian fit to the noise distributions, before and after bump-bonding. The results for both the pixel threshold and noise show an increase in the mean value after the readout chip is bump-bonded to the sensor. The average shift in the mean threshold is $275 \pm 30e^-$, and the average shift in the mean noise is $32 \pm 5e^-$. The pixel threshold and noise thus remain well within the requirements of the experiment for all the pre-production anodes.

Following this study, and the satisfactory results of both the bump-bonding integrity tests and accelerated ageing tests, described in Section 3.3.3 below, the decision was made to start the full production of HPD anodes. The testing of the chips on the wafer has been completed, and 1300 chips were found to be Class 1 standard, from a total production of 3408 chips. To date, 234 production anodes have passed the quality-control tests. Figure 3.17 shows the mean of the pixel threshold distribution for each chip and anode tested, from Gaussian fits to the distributions, and similarly Figure 3.18 shows the mean of the noise distribution.
<table>
<thead>
<tr>
<th>Anode ID</th>
<th>Bare Chip Results</th>
<th>Anode Results</th>
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</thead>
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<td></td>
<td>Pixel thresholds (e⁻)</td>
<td>Pixel thresholds (e⁻)</td>
</tr>
<tr>
<td>Anode ID</td>
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<tr>
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<td>92</td>
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Table 3.4: Comparison of pixel threshold distribution, for readout chips before and after bump-bonding to sensor chips, for anodes in the pre-production batch.
### 3.3 Testing of HPD Anodes

<table>
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<tr>
<th>Anode ID</th>
<th>Noise (e⁻) mean</th>
<th>Noise (e⁻) sigma</th>
<th>Anode Results Noise (e⁻) mean</th>
<th>Anode Results Noise (e⁻) sigma</th>
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<td>4.8</td>
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<td>143</td>
<td>16</td>
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Table 3.5: Comparison of pixel noise distribution, for readout chips before and after bump-bonding to sensor chips, for anodes in the pre-production batch.
Figure 3.17: The mean value of the pixel threshold distribution for each chip, for the bare chips and the packaged anodes of the HPD production run. Data are included for the total production run of 1300 Class 1 bare chips, and from testing of the 234 anodes produced to date.

for each.

3.3.3 Accelerated Ageing of HPD Anodes

The ability of the HPDs to perform as required throughout the lifetime of the LHCb experiment had to be proven. Twelve HPD anodes from the pre-production batch with high-lead bump bonds were therefore selected to undergo accelerated ageing tests. The anodes tested were taken from those which had passed the standard quality-control tests at every stage of manufacture, including the standard electrical testing procedure and the radioactive source test of the bump bond integrity.

An accelerated ageing programme was devised to prove the performance of the anodes would not degrade over the ten year lifetime of the experiment. It was particularly important to demonstrate the bump bonds were not damaged
Figure 3.18: The mean value of the pixel noise distribution for each chip, for the bare chips and the packaged anodes of the HPD production run. Data are included for the total production run of 1300 Class 1 bare chips, and from testing of the 234 anodes produced to date.

by the high temperature vacuum bake-out required at the last stage of HPD manufacture. For this reason all twelve anodes underwent a simulated bake-out cycle in a laboratory oven, in vacuum, at the start of the ageing programme, to replicate the effect of the HPD manufacture.

After the bake-out step, several anodes were set aside as references and the rest were split into two groups. Half underwent a series of heat cycles designed to simulate five years of normal LHCb operation and half did not. This heat cycle consisted of 72 cycles from $5^\circ C$ to $80^\circ C$ over approximately five days, in a laboratory oven in air. The next step was a second simulated vacuum bake-out cycle, in a laboratory oven, in vacuum. The contract with the HPD manufacturers allows some flexibility in the number of bake-out cycles, typically one or two but in rare cases up to three. A subset of the anodes underwent a second series of heat cycles to give an equivalent accelerated lifetime of ten years normal LHCb operation. Figure 3.19 is a flow diagram summarising the accelerated ageing
Figure 3.19: Flow diagram illustrating the programme of accelerated ageing tests. The numbers are unique identifiers for the 12 anodes under investigation.

programme carried out on these pre-production anodes.

After every step of the ageing programme the standard electrical testing procedure was performed. No degradation in the readout chip performance was observed for any of the anodes. The integrity of the bump bonds was also checked using a Sr-90 source as previously described in Section 3.3.1. Table 3.6 summarises the results of these tests, listing the number of operational pixels measured at each stage.
### Table 3.6: Results of bump bond integrity tests throughout the accelerated ageing programme. The broken wire bonds on anode 4.8 were caused by poor handling of the anode and were not related to the programme. NA stands for *not applicable.*

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<th>Anode ID</th>
<th>before bake-out</th>
<th>after bake-out</th>
<th>after heat cycle</th>
<th>after second bake-out</th>
<th>after second heat cycle</th>
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<td>100.0</td>
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<td>NA</td>
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</tr>
<tr>
<td>4.8</td>
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<td>99.9</td>
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<tr>
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<td>100.0</td>
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<tr>
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<td>NA</td>
<td>100.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

The conclusion that can be drawn from these results is that absolutely no degradation in the performance of the HPD anodes due to heat cycling is observed for a lifetime equivalent to ten years normal operation. This is true for those anodes which have undergone two high temperature vacuum bake-out cycles, the typical maximum in the HPD manufacturing process. This was a crucial result to validate the HPD for use in the RICH detectors of LHCb, particularly to guarantee the robustness of the high-lead solder bump bonds.

### 3.4 Conclusions

A novel photon detector technology, the HPD, was developed for use in the RICH detectors of LHCb. A rigorous quality-control programme, applied at each stage of the HPD manufacturing process, was devised so that the final HPDs meet
the requirements of the experiment. This included a standard electrical testing procedure for the readout chip, to check the threshold and noise of every pixel were within specifications. A study of a pre-production batch of HPD anodes indicated the mean threshold and noise for each readout chip increased after bump-bonding to the silicon sensor, but still remained well within the specifications. Results from the production run confirmed this result.

An efficient test of the integrity of the bump-bonds was designed, using the fast-OR signal to trigger the readout of hits caused by radioactive particles, allowing the bump-bonds to be checked as part of the quality-control programme. Additionally, an accelerated ageing study was carried out for a set of pre-production HPD anodes, proving that no degradation in the performance of the HPD anodes will occur during ten years of LHCb operation. As a result of this work full production of the HPD anodes was able to proceed. The quality-control programme will play a crucial role in ensuring that the final photon detectors produced will be suitable for installation in the RICH detectors of LHCb.
Chapter 4

HPD Testbeam Results

Introduction

A demonstrator Ring Imaging Cherenkov detector was constructed and used to collect data from a testbeam at CERN in October 2003. This RICH detector consisted of an aerogel radiator, a spherical mirror with a radius of curvature of 949 mm, and three prototype HPDs, similar in optical configuration and design to the LHCb RICH-1 detector. The purpose of the testbeam programme was to test the performance of the HPDs and also to investigate the Cherenkov angle resolution and photon yield that could be achieved with the aerogel radiator.

4.1 Testbeam Setup

A schematic of the RICH detector is shown in Figure 4.1. The three HPDs, the spherical mirror and the aerogel radiator were located inside a light-tight and gastight vessel, which was flushed with nitrogen to protect the hygroscopic aerogel. The vessel was sited in the T9 testbeam facility at the CERN PS accelerator.
Figure 4.1: A schematic of the RICH detector used in the testbeam. The photon detector plane is indicated by the section A-B.

From the 24 GeV/c proton primary beam a secondary beam was selected with a momentum of 10 GeV/c. Data were collected from runs with either a pure $\pi^-$ beam or with a mixture of $\pi^+$ and protons. A photograph of the RICH detector is shown in Figure 4.2.

The passage of the beam through the vessel is indicated in Figure 4.1. The particles produced Cherenkov photons in the aerogel and exited the vessel after passing through the mirror. The Cherenkov photons were reflected from the spherical mirror and focused onto the photon detector plane. The HPDs were placed on the plane at the vertices of an equilateral triangle, as indicated in Figure 4.3.

A set of scintillators placed in the beam line provided the coincidence signal used to trigger the data readout. The beam divergence was limited to a maximum of $\sim 1.5$ mrad by two $1 \text{ cm}^2$ scintillators located $\sim 8$ m apart. Three pixellated silicon detector planes were also situated in the beam line to act as a tracking detector, the *silicon telescope*. Figure 4.4 shows a schematic of the testbeam
Figure 4.2: A photograph of the RICH detector used in the testbeam.

Figure 4.3: View of the photon detector plan showing the location of the three HPDs.
4.1.1 Photon Detectors

The design of the HPD is discussed in detail in Chapter 3. The probability of a photon producing a photoelectron at the cathode is given by the quantum efficiency curve, shown in Figure 4.5, for the three HPDs used in the testbeam. These curves are for photons which impact at a right angle to the quartz entrance window and were factory-measured when the HPDs were produced. Photon absorption in the quartz window causes the sharp cut off at $\sim 200$ nm. These quantum efficiency curves were used to evaluate the expected photoelectron yield, as part of the detailed Monte Carlo simulation described in Section 4.1.4 below.

The double peak structure is due to a subtlety of the photoelectron emission via the photoelectric effect. An incoming photon can excite an electron from the conduction band to the valence band. This primary excited electron has an energy inside the photocathode of $E_\gamma$, the energy of the incoming photon, minus the threshold bandgap energy, $E_{gap}$, of the photocathode. The excited electron will lose part of this energy, mainly via interactions with the photocathode lattice.

The electron may escape to the vacuum with a probability giving rise to the quantum efficiency, after having lost a further energy $E_A$, the electron affinity, which is the potential barrier of the vacuum. For this photocathode $E_{gap} = 1.38$ eV. Now, if the incoming photon has an energy at least twice $E_{gap}$, i.e. a wavelength smaller than 450 nm, the primary excited electron has an energy at least larger than $E_{gap}$ and may excite a secondary electron, via a scattering process, from the conduction band to the valence band. In the process the primary electron will lose an energy at least equal to $E_{gap}$, and therefore will have less chance to escape to the vacuum. This effect causes the dip below $\sim 450$ nm in Figure 4.5. The size
Figure 4.4: Schematic overview of the testbeam setup.
of the dip is related to the photocathode thickness, since a thicker photocathode would increase the probability for the primary and secondary electron to interact with the photocathode material.

A photoelectron produced at the cathode is accelerated to the anode by the cross-focused electron optics. The bias for the three electrodes of the electron optics is obtained from the cathode voltage supply by an external resistive voltage divider. A single voltage divider set the three bias values for all three HPDs in the testbeam. The nominal cathode operating voltage is 20 kV, however due to problems with the high voltage insulation, an operating voltage of 18 kV was used. The cross-focused electron optics cause a 5 times demagnification from the cathode to the anode. The demagnification law is the algebraic relationship between the radial distance of the photoelectron hit on the anode and the radial distance of the cathode emission point.

The bias applied to the three electrodes of the electron optics determines the form of the demagnification law. Unfortunately the testbeam setup did not
Figure 4.6: The demagnification law assumed for the HPDs, showing the points from the simulation and the linear fit used in the analysis of the testbeam data.

include accurate monitoring of the HV values. This problem is solved in the baseline design of the high voltage supply for the final RICH detectors. For the analysis of data from this testbeam, the demagnification law was obtained from numerical simulations of the electron trajectories, tuned to match the features seen in the data. Figure 4.6 is a plot of the simple linear relationship which defines the demagnification law, with a gradient of 0.2 and zero offset.

Two of the HPDs used in this testbeam, HPD0 and HPD1, were final pre-production prototypes developed for the LHCb RICH detectors, with the LHCbPIXI chip described in Chapter 3. The pixel size at the silicon sensor is 500 μm x 62.5 μm, in an array of 32 x 256 pixels, giving a effective pixel size at the quartz entrance window of 0.31 mm x 2.5 mm. The third HPD, HPD2, was an older prototype developed jointly by the ALICE and LHCb collaborations. It had a smaller pixel size, 50 μm x 400 μm.

The charge signal from the ∼5000 electron-hole pairs released by the photoelectron is integrated, amplified and then discriminated to indicate the pixel has
been hit. The detection efficiency, defined as the probability of registering a pixel
hit given a photoelectron has been converted at the photocathode, is dependent
on both the photoelectron energy and on the discriminator threshold. The pho-
toelectron energy is controlled by the high voltage applied. The testbeam setup
allowed the high voltage bias to be varied over a range from 14 kV to 20 kV
and, as stated above, 18 kV was chosen as the nominal value. The discriminator
threshold was kept constant for all three HPDs throughout the testbeam run-
ning period. A measurement of the single photoelectron detection efficiency, $\eta_p$, was made using a dedicated setup in the laboratory, with the same discrimina-
tor threshold and high voltage settings, and was found to be 82% [Mor04]. It
is reduced from 100% because of inefficiencies in charge generation and collection
caused by back-scattering of the photoelectrons from the silicon surface, and due
to charge sharing between adjacent pixels.

Readout of the HPD binary data was performed using existing laboratory
test hardware and software, extended for the purposes of the testbeam. Data
acquisition and control software were implemented in LabVIEW. This included a
JTAG system to simultaneously control the three readout chips and the voltage
bias settings. Three PILOT VME boards were used to read out the HPDs during
each beam-spill, at a clock speed of 10 MHz [Klu00]. The data were buffered on
each PILOT board for the duration of the beam-spill, then sent to the PC, as
indicated in Figure 4.4.

To use the laboratory test system in the asynchronous testbeam setup a syn-
chronised readout trigger had to be provided. The trigger signal was generated by
the coincidence of signals from the scintillators, then sent to the PILOT boards
to start the readout sequence for the chips. A veto was enforced by the PC to
ensure no further triggers were sent until the data had been stored in the memory
of the PILOT boards. At the end of each beam-spill period, the data were transferred from the PILOT board to the PC for storage. The readout rate during the beam-spill, of a few kHz, was limited by the readout from the silicon telescope tracking detector, which was also synchronised to the trigger.

Analysis of data from the silicon telescope is discussed in Section 4.1.3. For each triggered event, the data stored from the HPDs were in the form of a binary map of the hits in the 32 x 256 pixels, since the chips were read out in ALICE rather than LHCb mode. In LHCb mode, groups of eight pixels are OR-ed together to from an array of 32 x 32 super-pixels, as explained in Chapter 3. Software routines were developed to convert the raw binary data to the ROOT file format, for analysis within the ROOT framework [BR97].

### 4.1.2 Aerogel Radiator

The first RICH detector downstream of the interaction point in LHCb, RICH 1, will have an aerogel radiator with a refractive index, $n$, of 1.03. Aerogel is an extremely low density material, composed of silicon dioxide. It is important that the Cherenkov photons produced in the radiator are not scattered as they pass through the radiator material, since any angular dispersion introduced will reduce the precision on the final measured Cherenkov angle. The dominant cause of scattering in the aerogel is Rayleigh scattering. One of the aims of this testbeam programme was to understand the Cherenkov angle precision and photon yield that could be achieved with this aerogel radiator.

The light transmission $T$ at a wavelength $\lambda$ through a sample of aerogel of thickness $L$ is given by Equation 4.1 [BGK+99],

$$ T = A e^{-\frac{\alpha L}{\lambda^4}} $$

(4.1)
Table 4.1: Measurements made prior to the testbeam of the optical properties of the four aerogel tiles.

where $A$ denotes the transmission in the high wavelength region and $C$ the clarity coefficient. Four aerogel tiles were used in this testbeam study. Each had approximate dimensions 10 x 10 x 4 cm³, mounted in a 2 x 2 array to give a total aerogel volume of 20 x 20 x 4 cm³. The tiles were produced by the Boreskov Institute of Catalysis [K+96]. Measurements were made of the optical properties of these aerogel samples, the results are given in Table 4.1.

The refractive index of the aerogel radiator, and hence the Cherenkov angle, is a function of the photon wavelength. The Lorentz-Lorentz equation, Equation 4.2, was used to parameterise the refractive index $n$ as a function of the energy, $E$, of the photon [YS94].

$$\frac{n(E)^2 - 1}{n(E)^2 + 1} = cf(E), \quad (4.2)$$

where

$$c = \frac{4\pi a \rho N_A}{3M} = (0.3738 \text{ cm}^3) \frac{\rho}{M}, \quad (4.3)$$

and

$$f(E) = \frac{F_1}{G_1^2 - E^2} + \frac{F_2}{G_2^2 - E^2}, \quad (4.4)$$

is the Sellmeier fit to the molar refractivity. Here $a$ is the Bohr radius, $\rho$ is
the density of the aerogel, \( N_A \) is Avogadro’s number and \( M \) is the molecular weight. The Sellmeier coefficients \( F \) and \( G \) were extrapolated from measurements made on fused quartz, scaled by the relative density. The resulting curve is then corrected to match the refractive index of aerogel measured at a wavelength of 543.5 nm, as indicated by the black curve in Figure 4.7.

The blue curve of Figure 4.7 results from an alternative calculation of the dispersion curve. Equation 4.5 is the Clausius-Mossotti equation for a binary mixture of air and silica particles, assuming a refractive index of 1 for air [Jac75].
Here $\rho$ is the density of aerogel and $\rho_s$ is the density of solid silica. The refractive index of solid silica, $n_s$, is taken from experimental measurements performed on UV grade fused silica.

$$n - 1 = \frac{3 \rho \frac{n_s^2}{\rho_s} - 1}{2 \rho_s n_s^2 + 1}$$  \hspace{1cm} (4.5)

The density measured for the aerogel does not take into account residues such as water or ethanol. To correct for this, the value of $\rho$ was tuned so that the result of Equation 4.5 at a wavelength of 543.5 nm matched the value measured for the aerogel tiles used in the testbeam. The resulting dispersion curve is plotted in Figure 4.7.

During testbeam running, some data were collected with a 100 $\mu$m thick filter of D263 glass mounted on the photon exit surface of the aerogel tiles. This filter absorbed photons with a wavelength below $\sim$300 nm. These high energy photons are more likely to be Rayleigh scattered, and thus contribute to the uncertainty on the Cherenkov angle. Additionally, from Figure 4.7, it is clear that photons in this energy range show the strongest dependence of refractive index with wavelength. Comparisons between data collected with and without the filter in place are part of the Cherenkov angle resolution studies presented in Section 4.2 below.

### 4.1.3 Silicon Telescope Tracking Detector

The silicon telescope was used in the testbeam to provide an event by event track direction for the Cherenkov angle reconstruction, and to measure the beam divergence. The silicon telescope consisted of three planes of silicon pad detectors. Each plane had 22 x 22 pixels and each pixel was 1.3mm x 1.3mm in area. Two planes were placed upstream of the vessel containing the aerogel and detector system, the third was placed downstream. The first and third plane were separated
by 5 m. From upstream to downstream, they were numbered Plane 1, Plane 2, Plane 3, as shown schematically in Figure 4.8.

For each hit pixel in the silicon telescope the amplitude was recorded and converted to a digital signal using an ADC. The distribution of this value has a peak corresponding to the mean energy loss of a minimum ionising particle travelling through the silicon, with a tail towards higher values. Figure 4.9 is a plot of the ADC hits in the three telescope planes for a representative run. There is a saturation effect towards higher ADC counts, particularly in Plane 1. The
Table 4.2: Results of cluster-finding algorithm for the silicon telescope.

<table>
<thead>
<tr>
<th></th>
<th>single</th>
<th>double</th>
<th>triple</th>
<th>four</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plane 1</td>
<td>97.4%</td>
<td>2.5%</td>
<td>0.04%</td>
<td>0.009%</td>
</tr>
<tr>
<td>Plane 2</td>
<td>97.5%</td>
<td>2.5%</td>
<td>0.04%</td>
<td>0.004%</td>
</tr>
<tr>
<td>Plane 3</td>
<td>97.2%</td>
<td>2.6%</td>
<td>0.03%</td>
<td>0.003%</td>
</tr>
</tbody>
</table>

noise pedestal can be clearly seen towards low ADC values.

To remove noisy pixels, a cut on the ADC count is applied. However this method is complicated by the possibility of charge sharing between pixels. The energy from an ionising particle may be shared between two or more pixels, giving a lower ADC count in each that could be misinterpreted as noise. For this reason a cluster-finding algorithm was applied to the data before performing a hard pedestal cut. When a hit cluster was found, a centre of gravity calculation was carried out to produce a single hit coordinate. Once the noise had been removed a further cut was made on the data to accept only events where a single hit was recorded in each plane of the silicon telescope.

To summarize the cuts applied:

- First, every hit must have an ADC value >125 AND <770.
- Then events which have any clusters with size>4 are rejected.
- If the total ADC of the cluster is not >300, that cluster is discarded.
- Only events where each of the three silicon telescope planes has exactly one cluster pass the final cut.

Consistently, for all runs, 65% of the events pass the cuts. The cluster results for each plane in the representative run are detailed in Table 4.2.

The hit density in the three silicon telescope planes is shown in Figure 4.10, after the cuts described in the previous section. The limits of the beam profile are set by the smaller of the two scintillators that triggered the readout. The next
Figure 4.10: The hit density in the three planes of the silicon telescope after applying the cuts described in the text.
step was to convert the pixel hit within the silicon telescope plane to the global coordinate system, defined by the nominal beam direction (0,0,1), with the origin in the middle of the front plane of the vessel, at the nominal beam entry point. The plots in Figure 4.10 were used to locate the central spot where the majority of particles hit each plane. This indicated the nominal beam direction, the $z$-axis in the global coordinate system. For Plane 3 in particular the limits on the beam profile are apparent, caused by the placement of the scintillators used to trigger the readout. This may introduce a bias in the particle direction, as only particles which pass through the region defined by the coincidence of the scintillators are able to cause a trigger.

The hits in Plane 2 were cut on to define the central region. Information on the accuracy of the silicon telescope alignment was obtained from a straight-line fit through the hits in this central region of the three planes. For every event, the hits in each plane were fitted using least squares fits in the $xz$ and $yz$ planes independently. Plots were made of the residue for all three planes, defined as

$$\text{residue} = \text{fitted coordinate} - \text{actual hit coordinate}. \quad (4.6)$$

The alignment of the planes was adjusted to minimise the value of the residue around zero. The mean and rms of the residue after this alignment is detailed in Table 4.3. The rms of the residue is less than the pixel size ($1.3 \text{ mm}^2 \times 1.3 \text{ mm}^2$)}
4.1 Testbeam Setup

Figure 4.11: The residue (defined in Equation 4.6) for the three silicon telescope planes, after the alignment procedure. The left hand plot shows the residue for the $x$ coordinate, the right hand plot the same for the $y$ coordinate. The effect of the pixelisation is visible in the right hand plot.

and therefore the alignment is assumed to be correct within the size of a pixel. Figure 4.11 indicates the residues in $x$ and $y$ after the alignment procedure.

Figure 4.12 shows the distribution of the angle of the track fitted through the silicon telescope hits for each event. From this, the beam divergence can be obtained from a Gaussian fit. For the pure $\pi^-$ beam runs, the divergence was found to be 1.6 mrad in $x$ and 0.7 mrad in $y$. The contribution to the beam divergence from the silicon telescope pixelisation is $\sim 0.05$ mrad. The bias introduced to the particle direction in the $yz$ plane is due to the limits on the beam profile seen in plane 3, caused by a small misalignment of the scintillators used to trigger the readout.
Figure 4.12: Angle in milliradians of the track fitted through the silicon telescope hits, in the $xz$ (top) and $yz$ (bottom) plane. A Gaussian fit to this distribution gives the beam divergence. Indicated in red are the subset of events where the track passed through the central region of plane 2.
4.1.4 Monte Carlo Simulation

To simulate the results of the aerogel data a software program was developed using the GEANT4 toolkit [A+03b]. The detailed geometry of the testbeam setup was described and the passage of a 10 GeV/c beam through the RICH detector was simulated, both for a pure $\pi^-$ beam and for a mixture of $\pi^+$ and protons. Most testbeam data were collected in runs with a pure $\pi^-$ beam, and the results presented in this Chapter refer to a pure $\pi^-$ beam unless otherwise indicated.

As the charged particles pass through the aerogel radiator Cherenkov photons are produced with an angle determined by the standard relationship, Equation 4.7,

$$\cos \theta_C = \frac{1}{\beta n(\omega)}.$$  \hspace{1cm} (4.7)

Here $\theta_C$ is the Cherenkov angle, $\beta$ is the velocity of the charged particle as a fraction of the speed of light in vacuum and $n(\omega)$ is the refractive index of the radiator, which is a function of the energy of the photon. The azimuthal angle, $\phi_C$, is uniformly distributed. Each photon is assigned an energy between the limits 1.5 eV to 7.3 eV according to the distribution

$$\frac{dN_\gamma}{dE} = 370 \sin^2 \theta_C(E)L$$  \hspace{1cm} (4.8)

where $\theta_C$ is the Cherenkov angle and $L$ is the radiator length in millimetres, discussed in Chapter 2, Equation 2.7.

The nominal dispersion curve used in the simulation was taken from the Lorenz-Lorentz equation calculation, Equation 4.2, described above and shown in Figure 4.7. A second set of simulated results was produced for comparison
Figure 4.13: Reflectivity of the mirror as a function of the photon wavelength in nm.

studies using the corrected Clausius-Mossotti dispersion curve, Equation 4.5.

The transmission of the photons through the aerogel was described by Equation 4.1, above. Optical boundaries in the testbeam setup, such as the interface between the aerogel and the filter, were modelled using the GEANT4 toolkit. Photons incident on these boundaries were refracted and reflected according to the Fresnel equations. Reflection of the photons from the spherical mirror as a function of wavelength was governed by the mirror reflectivity, which was measured experimentally with a systematic error of 2%, shown in Figure 4.13.

The conversion of photons to photoelectrons at the cathode of each HPD was simulated using the measured quantum efficiency curves in Figure 4.5, which had a 6% uncertainty on the measurements. Subsequently the demagnification law was used to translate the photon conversion point on the photocathode to a hit on the sensor assembly. A detection efficiency of 82% was also included in the simulation.
4.2 Cherenkov Angle Resolution Studies

A program was written to analyse the data collected, to measure the Cherenkov angle of the particles in the beam and determine the resolution that could be achieved. By also analysing data generated by the GEANT4 simulation of the testbeam setup, the different contributions to the uncertainty of the measured Cherenkov angle could be investigated.

In the testbeam setup the silicon telescope gives an event-by-event track direction for the particle. The emission point for the Cherenkov photon is assumed to be the point along this track that lies at the midpoint of the aerogel radiator, as the precise emission point is not known. Given the known detection point of the photon on the photocathode, combined with the emission point and the centre of curvature of the spherical mirror, a quartic equation can be defined to solve for the angle between the emission and reflection points about the centre of curvature, \( \alpha \).

The method is illustrated in Figure 4.14. The definitions are

- \( R \equiv \) radius of curvature of the mirror.
- \( e \equiv \) distance from emission point to centre of curvature.
- \( P \equiv \) distance from detection point to centre of curvature.
- \( P_\parallel \equiv \) distance from detection point to centre of curvature, parallel to \( e \).
- \( P_\perp \equiv \) distance from detection point to centre of curvature, perpendicular to \( e \). 1
- \( \alpha \equiv \) angle between the emission and reflection points, about the centre of curvature, in the plane defined by these three points.

The quartic equation which must be solved to find \( x = \sin \alpha \) is

\[
x^4 + ax^3 + bx^2 + cx = d = 0 \tag{4.9}
\]
Figure 4.14: Diagram illustrating the method used to find the reflection point of the Cherenkov photon. Details are given in the text.

where

\[
\begin{align*}
a &= \frac{-4e^2 P_{\perp} R}{(2eP)^2} \\
b &= \frac{P_{\perp}^2 R^2 + (P_{\parallel} + e)^2 R^2 - (2P_e)^2}{(2P_e)^2} \\
c &= \frac{2P_{\perp} e(e - P_{\parallel}) R}{(2P_e)^2} \\
d &= \frac{P_{\perp}^2 (e^2 - R^2)}{(2P_e)^2}.
\end{align*}
\]

This quartic equation was solved in the analysis programme using a routine provided by the CERN software library [Kol94]. Two real solutions are returned for \( \sin \alpha \), one in the forward and one in the backward direction. The backward solution is discarded since it is unphysical, as the mirror is not a complete sphere. From \( \alpha \), the reflection point on the mirror can be found. Finally, the Cherenkov angle can be calculated from the reflection point, emission point and direction of the particle. Further details of this method can be found in Ref. [R. 98].

For the \( \pi^- \) data with the filter in place, Figure 4.15 shows the hits on the plane
Figure 4.15: The hits on the plane containing the three HPDs, integrated over 20,000 events, with the filter included.

containing the three HPDs, as illustrated schematically in Figure 4.3, integrated over 20,000 events.

For the same 20,000 events the reconstructed Cherenkov angle is plotted in Figure 4.16, for all three HPDs individually. The Cherenkov angle is reconstructed in the analysis program for every detected photon. A single Gaussian has been fitted to each distribution. A large contribution to the width is due to the chromatic error introduced by the variation of the aerogel refractive index with wavelength, which gives a non-Gaussian tail at higher Cherenkov angles. This is investigated in more detail in Section 4.2.1. The filter is intended to reduce this effect by cutting out shorter wavelength photons. For comparison, Figure 4.17 is the reconstructed Cherenkov angle for a run with no filter.

Figure 4.18 shows the reconstructed Cherenkov angle for the Monte Carlo,
Figure 4.16: Distributions of the reconstructed $\theta_C$ for the three HPDs, showing single Gaussian fits, for 20,000 events. These data are for a run with the filter.
Figure 4.17: Distributions of the reconstructed $\theta_C$ for the three HPDs, for 20,000 events. These data are for a run without the filter.
Figure 4.18: Monte Carlo distributions of the reconstructed $\theta_C$ for the three HPDs, simulated for 20,000 events with the filter. Single Gaussian fits are shown.

with filter, for 20,000 events. Single Gaussians, as shown, are poor fits to these distributions. This is caused by the non-Gaussian tails due to the chromatic error.

The effect of refraction at the HPD quartz window is not taken into account in the reconstruction of the photon direction from the detection point in data. Truth information from the Monte Carlo simulation shows that, as a result of this effect, a positive shift of 1 mrad is observed in HPD2. No shift is seen for HPD0 or HPD1. This is discussed in Section 4.2.1 below.

The results of a single Gaussian fit to data and Monte Carlo, with the necessary refraction correction (-1 mrad) applied to the mean of HPD2, are listed in Table 4.4. It can be seen that the resolution in data is $\sim 30\%$ worse than in Monte Carlo. One possible reason for this could be misalignments in the geometry de-
<table>
<thead>
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<td>HPD0</td>
<td>238.2±0.01</td>
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</tr>
<tr>
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<td>238.2±0.02</td>
<td>3.2</td>
<td>HPD 1</td>
<td>239.3±0.01</td>
<td>2.4</td>
</tr>
<tr>
<td>HPD 2</td>
<td>240.5±0.02</td>
<td>3.5</td>
<td>HPD 2</td>
<td>239.7±0.01</td>
<td>2.4</td>
</tr>
</tbody>
</table>

Table 4.4: Single Gaussian fits (in mrad) to reconstructed Cherenkov angle distributions, for data and Monte Carlo simulated data.

fined in the software, compared to the actual geometry of the testbeam. The photoelectron yield achieved is investigated in Section 4.3, with the calculated yield per HPD listed in Table 4.7.

To check the alignment a photon counting method was used. For a perfectly aligned system, the angular resolution of the full Cherenkov ring is expected to be inversely proportional to the square root of the number of photon hits in the ring. Any residual misalignment will cause a deviation from this square root dependence, as given by Equation 4.14. Here \( \sigma \) is the width of the Gaussian distribution of reconstructed Cherenkov angles, \( \sigma_{dep} \) is the simple square root dependence and \( \sigma_{mis} \) the contribution of any misalignment, which does not scale with the number of photon hits. The number of photon hits in the ring is \( N \).

\[
\sigma^2 = \frac{\sigma_{dep}^2}{N} + \sigma_{mis}^2 \tag{4.14}
\]

The resolution dependence was investigated by calculating the mean Cherenkov angle in data for each event. First, the data were separated into subsamples depending on the number of photon hits in the ring; events with up to five hits were studied. Then, for each subsample, the Cherenkov angle for the individual photon hits was reconstructed. The mean Cherenkov angle for the event was taken as the sum of the individual Cherenkov angles divided by the number of photon
Figure 4.19: The Cherenkov angle resolution as a function of the number of
photoelectrons in the ring is given by the black square points, fitted with the
black dashed line. The red circles show the expected dependence for a perfectly
aligned system, normalised to the single hit result, and fitted with the red dotted
line.

hits. To reject background, a cut was applied to include only those events where
the individual Cherenkov angle fell within ±3σ of the mean defined in Table 4.4.

The distribution of the Cherenkov angle in each subsample was fitted with a
single Gaussian and the sigma of each distribution was plotted versus the number
of hits, the black points on Figure 4.19. The red points indicate the expected
sigma dependence on the number of photon hits for a perfectly aligned system,
normalised to the single hit result. The dashed black line is a fit of Equation 4.14
to the data. The result of the fit gives σ_{dep}=3.2 mrad, and σ_{mis}=0.7 mrad. This
indicates there is a contribution to the width of the Cherenkov angle distribution
from misalignment of approximately 0.7 mrad.

Another possible cause of degraded resolution in the data is uncertainty in the
demagnification law. A check was therefore performed to understand the effect of any distortions or inaccuracies in the demagnification law used. The resolution in data was determined only for photon hits on a strip in the centre of each HPD, 100 mrad wide in $\phi_C$ from the centre of the image of the photocathode on the anode, indicated in Figure 4.20. Hits in this central part of the anode are least affected by the radial demagnification law. Therefore, an improvement in the resolution for these events, compared to the full set of events, indicates a contribution to the Cherenkov angle resolution from the demagnification. An improvement in the resolution of $\sim 0.3$ mrad for each HPD was observed for this data sample. Taking the $\sigma=3.2$ mrad measured for the full data set, the difference in quadrature of the two resolutions indicate a contribution of $\sim 1.3$ mrad from the demagnification.

A non-uniformity in the refractive index of the aerogel can also contribute to the angular spread in the reconstructed Cherenkov angle. For the aerogel used in the testbeam later measurements revealed a spread of 1.0%, as defined in Equation 4.15 [ABC04].

$$\frac{\sigma(n-1)}{(n-1)} = 1.0\%$$

This was not included in the Monte Carlo simulation. A special set of Monte Carlo was generated to understand this effect, and it was found that the effect contributes an increase of 1.1 mrad to the Cherenkov angle resolution.

In summary, the discrepancy between the resolution measured in data and Monte Carlo can be attributed to several uncertainties not accounted for in the Monte Carlo simulation. These are listed in Table 4.5. Taking the resolution measured from the Monte Carlo simulation as the intrinsic resolution, $\sigma_{\text{intrinsic}}$, 
Figure 4.20: Cherenkov angle $\theta_c$ versus $\phi_C$ along the ring, for the three HPDs. In red, the region selected for the check of the demagnification law used in the simulation and reconstruction.

<table>
<thead>
<tr>
<th>contribution</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>misalignment ($\sigma_{\text{mis}}$)</td>
<td>0.7</td>
</tr>
<tr>
<td>demagnification ($\sigma_{\text{demag}}$)</td>
<td>1.3</td>
</tr>
<tr>
<td>non-uniformity of aerogel ($\sigma_{\text{non-uniform}}$)</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Table 4.5: Contributions to the Cherenkov angle resolution in data, in mrad.
4.2 Cherenkov Angle Resolution Studies

The measured resolution is then the sum of these,

$$\sigma_{\text{measured}}^2 = \sigma_{\text{intrinsic}}^2 + \sigma_{\text{mis}}^2 + \sigma_{\text{demag}}^2 + \sigma_{\text{non-uniform}}^2. \quad (4.16)$$

This gives $\sigma_{\text{measured}}=3$ mrad for the Monte Carlo, to be compared with the $\sigma_{\text{measured}}$ in data of $\sim3.2$ mrad (Table 4.4). Thus, the sum of these additional effects not included in the simulation; misalignments, uncertainties in the demagnification law and non-uniformity in the refractive index of the aerogel, can explain the $\sim30\%$ difference between the resolution in data and in Monte Carlo.

4.2.1 Contributions to Intrinsic Resolution

Chromatic Error

Chromatic dispersion in the aerogel means the Cherenkov angle depends on the photon energy. The dispersion curve for the aerogel is derived from the Sellmeir curve for quartz as described above. Since the photon energy is unknown, this clearly affects the resolution that can be achieved. The Monte Carlo simulation was used to study this effect. By reconstructing the Cherenkov angle using truth information, the component of the Cherenkov angle resolution due to chromatic error can be isolated.

Figure 4.21 shows the distribution of the reconstructed Cherenkov angle for the three HPDs, using Monte Carlo truth information for the photon emission, reflection and detection points, assuming a pure 10 GeV/$c$ $\pi^-$ beam and the filter in place. A single Gaussian fit is included in red. The width of these distributions is therefore dominated by the chromatic error, and as such is not well fitted by a Gaussian but has a tail towards higher Cherenkov angles. From the fits, the contribution of the chromatic dispersion error to the Cherenkov angle resolution
Figure 4.21: The Monte Carlo reconstructed $\theta_C$ distribution for the three HPDs, for 20,000 events, using truth information on the detection, reflection and emission points of the photon. Data are generated for runs with a 10 GeV/c $\pi^-$ beam, with the filter included.

is estimated to be $2.3\pm0.01$ mrad.

This study was repeated for the Monte Carlo generated with the alternative, Clausius-Mossotti, formulation for the dispersion curve. In this case the contribution to the Cherenkov angle resolution is $\sim 1.7\pm0.01$ mrad. The Monte Carlo generated with the nominal, Sellmeier, curve is a better match to the distribution seen in the testbeam data. However it is interesting that a minor difference in the dispersion curve leads to a large difference in the chromatic error. For this reason it is planned to accurately measure the refractive index of the aerogel tiles at a number of different wavelengths, before installing them in the RICH-1 detector.

Monte Carlo data were also generated without the filter in place. The chromatic error contribution to the Cherenkov angle resolution in the Monte Carlo
Figure 4.22: Reconstructed Cherenkov angle in data (left) and Monte Carlo (right) in HPD1, with and without the filter. The distributions are normalised to the height of the peak in each case.

increases to 2.7±0.02 mrad. Figure 4.22 compares the reconstructed Cherenkov angle distribution with and without the filter in place, in data and Monte Carlo. A Gaussian fit to the no-filter data indicates the Cherenkov angle resolution increases from 3.2 mrad to ∼4.3 mrad, although the fit is poor due to the large tail. The effect of the filter on the number of photons detected is discussed in Section 4.3 below.

**Quartz Window Refraction**

Photons that hit the outside of the quartz window of the HPD are refracted before reaching the photocathode and producing a photoelectron. This effect is included in the simulation which generates the Monte Carlo. However, it is not included in the reconstruction software which calculates the Cherenkov angle from the recorded hits. This is because accurate information on the angle of incidence of the photon is not available. The reconstruction software was modified to remove the refraction effect from the Monte Carlo to allow direct comparison with the data, by taking the detection point as the photon hit on the outside surface of the quartz window rather than the photoelectron origin. The principle is indicated
Figure 4.23: Diagram explaining the two different detection points which can be used when reconstructing the Cherenkov angle, in Monte Carlo. These are the hit on the outside surface of the quartz window or the point of photoelectron conversion.

in Figure 4.23. The coordinates of the hit on the quartz window were determined from Monte Carlo truth information.

The reconstructed Cherenkov angle for each photoelectron was compared in Monte Carlo in the two instances, firstly where the detection point was taken as the photoelectron origin and secondly where it was taken as the hit on the outside surface of the quartz window. All the other parameters were taken from the truth information and were identical in the two instances. Figure 4.24 shows the difference in the reconstructed Cherenkov angle for these two different Monte Carlo truth variables. The distribution for HPD2 it is centred at ~1 mrad, showing that an appreciable change in the photon impact point is caused by refraction at the quartz window. This is because photons incident on HPD2 have a higher average angle of photon incidence, due to the positioning of the HPDs within the vessel. The distribution for HPD0 is centred at 0.5 mrad, indicating there is a small contribution from this refraction effect. A single Gaussian fit to the width of the distribution for all three HPDs, as shown, indicates this effect contributes approximately 0.15±0.001 mrad to the resolution.
Figure 4.24: For the three HPDs, the difference in Monte Carlo reconstructed $\theta_C$ from using either the photoelectron origin or the hit on the quartz window.
Emission Point Error

The tilt of the focusing mirror causes the photon hit position on the HPD plane to depend on its emission point along the particle track. Since the reconstruction software has to assume the emission point is the midpoint of the aerogel radiator, an error is introduced to the reconstructed Cherenkov angle.

The contribution of this effect was studied using the Monte Carlo simulation. For each detected photon, the Cherenkov angle reconstructed by taking the emission point at the centre of the aerogel was subtracted from the Cherenkov angle obtained using the true emission point. All the other parameters, including the detection point and the particle direction, were taken from the truth information. Figure 4.25 shows this distribution for the three HPDs. The RMS width, giving the contribution of the emission point error to the resolution, was calculated to be 0.7±0.003 mrad.

Pixel Error

The finite HPD pixel size also contributes to the reconstructed Cherenkov angle resolution. This was studied in the simulation by comparing the Cherenkov angle reconstructed using the true photoelectron detection point on the silicon sensor with the central coordinate of the hit pixel, where all the other parameters were the Monte Carlo truth values. The effect on the Cherenkov angle resolution was found to be ~0.5±0.002 mrad, with a small variation between the HPDs due to the different orientation of the Cherenkov ring with respect to the rectangular pixels. This also includes the error caused by the HPD point spread function, as described in Chapter 3, Section 3.1.
Figure 4.25: For the three HPDs, the difference in the Monte Carlo reconstructed \( \theta_C \) from using the true emission point and assuming the emission point is the centre of the aerogel tile.
(mrad) with filter no filter
\hline 
chromatic & 2.3±0.01 & 2.7±0.02 \\
emission point & 0.7±0.003 & 0.7±0.003 \\
pixel & 0.5±0.002 & 0.5±0.002 \\
quartz window refraction & 0.15±0.001 & 0.2±0.001 \\
Sum & 2.5±0.01 & 2.8±0.02 \\
\hline

Table 4.6: Summary of contributions to Cherenkov angle resolution (in mrad), with and without the filter. The values are the averages of the values for the three HPDs.

**Particle Direction**

The contribution to the resolution from uncertainty in the determination of the beam direction is negligible, since the event-by-event beam direction was measured using the silicon telescope. The contribution from the pixelisation of the silicon detector is ~0.05 mrad. Fully neglecting the beam divergence, i.e. not using the silicon telescope information, would cause an increase in the Cherenkov angle resolution of 1.4±0.02 mrad in HPD0 and HPD1, and 0.7±0.01 mrad in HPD2.

**Summary**

A summary of the different contributions to the angular resolution in the Monte Carlo simulation is listed in Table 4.6, for Monte Carlo simulated with and without the filter. In this testbeam setup the the resolution is dominated by the chromatic error.

**4.2.2 Proton-Pion Separation**

Data were collected from runs where the beam was a mixture of π⁺ and protons, at a momentum of 10 GeV/c, without the filter in place. The Cherenkov angle
Figure 4.26: Reconstructed Cherenkov angle for data from the $\pi^+$/p beam run, for all three HPDs, without the filter. A double Gaussian fit is shown.

was reconstructed for each photon using the standard reconstruction software. Figure 4.26 shows the reconstructed Cherenkov angle for each photon hit, for all three HPDs. No correction was made for the effect of refraction at the quartz window for HPD2. Fitting a double Gaussian gives $\theta_\pi=238.0\pm0.06$ mrad, and $\theta_p=221.4\pm0.04$ mrad. The single photon resolution is found to be about 5 mrad.

4.3 Photoelectron Yield

The number of photons generated in the aerogel radiator is an important measure of the performance of the RICH detector. The resolution on the Cherenkov angle depends strongly on the number of photoelectrons detected. An investigation of the photoelectron yield in this testbeam was reported in Ref. [R+06]. Here the
Figure 4.27: Number of photoelectrons per event in data for the three HPDs, with filter. The solid curves indicate Poisson fits.

results of an independent study of the photoelectron yield are presented.

The photoelectron yield is defined as the average number of photoelectrons detected per event. A signal region was defined, for both the testbeam and the Monte Carlo, as $\pm 3\sigma$ around the mean reconstructed Cherenkov angle indicated in Table 4.4. Then, for hits in this signal region, a Poisson fit was performed to the distribution of the number of photoelectrons detected in each event, giving the mean photoelectron yield for each HPD. This is shown in Figure 4.27 for data from a testbeam run with the filter.

In the data the effect of charge sharing must be considered by clustering hits in adjacent pixels. Each cluster is then assumed to originate from one photoelectron. However, this means that events where two photoelectrons hit neighbouring pixels will be treated as a single cluster. This results in an underestimation of the
number of photoelectrons. The Monte Carlo simulation was used to understand the probability of two photoelectrons hitting neighbouring pixels. Charge sharing is not modelled in the simulation, and hence two pixel clusters can only result from two genuine separate hits. From the simulation, the clustering results in an underestimation of the total photoelectron yield in the signal region of $\sim 0.01$.

Several methods were used to estimate the background contribution to the photoelectron yield. For a special testbeam run, data were collected during the beam-spill with a random trigger. A black screen was placed over the exit side of the aerogel, to block Cherenkov photons. For this background run, which includes the contribution of beam-related photons, the number of photoelectrons within the $\pm 3\sigma$ signal band was found to be approximately 0.01 per event. Another background run was performed outside the beam-spill, to measure the dark count rate. Dark counts are mainly caused by thermionic electron emission from the photocathode. The rate was found to be negligible.

Additionally, an estimation of the background contribution was made from fits to sideband regions of the Cherenkov angle distribution, well separated from the $\pm 3\sigma$ signal region. Hits in this region are predominantly caused by scattered Cherenkov photons. The fits were used to estimate the photoelectron yield due to these scattered Cherenkov photons in the $\pm 3\sigma$ signal region, for each HPD. The sideband regions of the Cherenkov angle distribution for the data and the Monte Carlo simulation, normalised to 20,000 events, are shown in Figure 4.28, with an appropriate function fitted to each.

The shape of the background due to scattered Cherenkov photons in the testbeam data and in the Monte Carlo is different. The Monte Carlo has a fairly flat distribution, whereas the data can be fitted with a Gaussian. The main contributions to the background included in the Monte Carlo simulation
Figure 4.28: Sideband region of the Cherenkov angle distributions, showing background effects, for 20,000 Monte Carlo (left) and data (right) events. A straight line function has been fitted to the Monte Carlo, and a Gaussian has been fitted to the data. Possible reasons for the difference in the shapes of these distributions are discussed in the text.

are Rayleigh scattering of the photons produced in the aerogel, plus reflections from the kovar ring of the HPD and the vessel walls. Rayleigh scattering is the dominant contribution, causing the flat distribution in the angle of the scattered photons.

There are additional contributions to the background in the data, observed in laboratory tests of the HPDs but not simulated in the Monte Carlo, which cause the background distribution for scattered photons to be Gaussian rather than flat. These include deviations of the photon trajectories due to internal reflections in the quartz window, photoelectron reflections inside the HPD from the electrodes and from the anode silicon surface, and hits of back-scattered photoelectrons in neighbouring pixels [KARG+05]. The aerogel refractive index inhomogeneity, not included in the simulation, is also expected to have an effect on the background photon yield.

A measurement was made of the photoelectron yield in the data using the results of the Poisson fits to the distributions of the number of photoelectrons in each HPD, and subtracting the background contribution. The estimate of the
background contribution was made in several stages. First, the results of the two special background runs were considered. The run with a black screen placed over the exit side of the aerogel indicated an overestimation of the photoelectron yield by 0.01. Also, the underestimation of the photoelectron yield by 0.01, due to clustering of genuine adjacent pixel hits, was taken into account. Secondly, the background due to scattered photons, estimated from the sideband fits, was subtracted.

Table 4.7 gives the measured photoelectron yield in the data. The total in ±3σ is the total photoelectron yield measured from the Poisson fits, corrected for the contribution measured in the background runs and the underestimation due to clustering. The background contribution from scattered Cherenkov photons was subtracted separately. This was done for each HPD, using the fit to the sideband regions to estimate the contribution in the ±3σ signal region. This contribution is indicated in the table as the fitted background in ±3σ. The final, corrected, number for the photoelectron yield is also given.

The same procedure was carried out for Monte Carlo, Table 4.8. Here, the total in ±3σ is the total photoelectron yield measured from the Poisson fits, without correction. The background contribution from scattered Cherenkov photons in the signal region, fitted background in ±3σ, is estimated from the fits to sideband regions of the Cherenkov angle distribution in Monte Carlo and is subtracted from the total to give the final, corrected number. The ratio of the photoelectron
Table 4.8: Photoelectron yield for Monte Carlo, with filter. The ratio of the background corrected yield in data and Monte Carlo is given.

<table>
<thead>
<tr>
<th></th>
<th>HPD0</th>
<th>HPD1</th>
<th>HPD2</th>
</tr>
</thead>
<tbody>
<tr>
<td>total in ±3σ</td>
<td>1.65±0.1</td>
<td>1.47±0.1</td>
<td>1.59±0.1</td>
</tr>
<tr>
<td>fitted background in ±3σ</td>
<td>0.003±0.001</td>
<td>0.003±0.001</td>
<td>0.003±0.001</td>
</tr>
<tr>
<td>corrected photoelectron yield</td>
<td>1.65±0.1</td>
<td>1.47±0.1</td>
<td>1.47±0.1</td>
</tr>
<tr>
<td>(\frac{\text{data}}{\text{MC}})</td>
<td>1.01±0.06</td>
<td>0.96±0.06</td>
<td>0.76±0.05</td>
</tr>
</tbody>
</table>

Table 4.9: Photoelectron yield for data, without filter.

yield in data and Monte Carlo is indicated.

The study was repeated for testbeam data and Monte Carlo from a run without the filter in place. For the data, the results are given in Table 4.9, and for the Monte Carlo, in Table 4.10.

The quoted errors on the photoelectron yield in data are statistical. The systematic error on the Monte Carlo yield is calculated from the 6% uncertainty in the quantum efficiency measurements and the 2% uncertainty on the mirror transmissivity.

<table>
<thead>
<tr>
<th></th>
<th>HPD0</th>
<th>HPD1</th>
<th>HPD2</th>
</tr>
</thead>
<tbody>
<tr>
<td>total in ±3σ</td>
<td>2.27±0.01</td>
<td>2.0±0.01</td>
<td>1.75±0.01</td>
</tr>
<tr>
<td>fitted background in ±3σ</td>
<td>0.06±0.02</td>
<td>0.06±0.02</td>
<td>0.08±0.02</td>
</tr>
<tr>
<td>corrected yield</td>
<td>2.21±0.03</td>
<td>1.94±0.03</td>
<td>1.67±0.03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(\frac{\text{data}}{\text{MC}})</th>
<th>(\frac{\text{data}}{\text{MC}})</th>
<th>(\frac{\text{data}}{\text{MC}})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.96±0.06</td>
<td>0.92±0.06</td>
<td>0.75±0.05</td>
</tr>
</tbody>
</table>

Table 4.10: Photoelectron yield for Monte Carlo, without filter. The ratio of the background corrected yield in data and Monte Carlo is given.
Table 4.11: Photoelectron yield for data and Monte Carlo, with filter, estimated for the full Cherenkov ring, assuming each HPD has 100% coverage.

Within the errors, the photoelectron yields in the data are compatible with the Monte Carlo expectations for HPD0 and HPD1. There is an overestimation of the photon yield in the Monte Carlo simulation for HPD2. This is not fully understood, but an intrinsic timing efficiency is the probable cause. These results show that the filter reduces the photoelectron yield by approximately 25%.

An estimate of the acceptance of each HPD in $\phi_C$, with respect to the full ring, was made using the Monte Carlo simulation. This was then used to estimate the expected photoelectron yield for each HPD for the complete Cherenkov ring, to correct for the incomplete coverage of the HPDs. The results are shown in Table 4.11.

4.4 Resolution and Yield as a Function of High Voltage

Data were collected for several runs with different settings of the high voltage supplied to the HPDs, ranging from 14 kV to 20 kV. The filter was not used in these runs. Results for the Cherenkov angle resolution as a function of high voltage are shown in Table 4.12. The values listed are for a single Gaussian fit. This confirms the stable response of the HPD devices with respect to the voltage supply. The photoelectron yield was also studied, following the method
Table 4.12: Single photon resolution for the three HPDs as a function of applied voltage. All results are for runs without filter.

<table>
<thead>
<tr>
<th>HV (mrad)</th>
<th>20kV</th>
<th>18kV</th>
<th>16kV</th>
<th>14kV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>sigma</td>
<td>mean</td>
<td>sigma</td>
</tr>
<tr>
<td>HPD 0</td>
<td>239.2</td>
<td>4.0</td>
<td>241.9</td>
<td>3.9</td>
</tr>
<tr>
<td>HPD 1</td>
<td>239.0</td>
<td>4.1</td>
<td>239.3</td>
<td>4.1</td>
</tr>
<tr>
<td>HPD 2</td>
<td>240.8</td>
<td>5.0</td>
<td>243.0</td>
<td>4.8</td>
</tr>
</tbody>
</table>

Figure 4.29: Photoelectron yield as a function of the HPD high voltage.

described in the previous section. Figure 4.29 indicates that the yield increases with voltage, due to the increased probability of detecting charge-shared clusters.

4.5 Conclusions

A successful testbeam programme was carried out with a demonstrator RICH detector, with an aerogel radiator and three HPDs. The Cherenkov angle reso-
olution was studied in detail. Insight was gained into the effect of a filter, used in combination with the aerogel, and measurements were made of the contributions to the Cherenkov angle reconstruction from chromatic dispersion, emission point error and pixelisation. The resolution in data was degraded with respect to the expected resolution from the Monte Carlo simulation. This 30% difference was due to geometrical misalignments of the HPDs, uncertainties in the electron optics corrections and the parameterisation of the aerogel dispersion curve. The results confirm that accurate measurements of the geometrical alignment of the photon detectors and the HPD electron optics corrections are crucial. In addition, the aerogel dispersion curve must be parameterised precisely, to allow the Monte Carlo simulation to reproduce the data as accurately as possible.
Chapter 5

Determination of Flavour
Tagging Performance From Data

Introduction

To study asymmetries in the decays of neutral $B$ mesons, the flavour of the $B^0$ at production, defined here as either a $B^0$ or $B^0$, must be known. Several methods of determining the initial flavour of the reconstructed $B^0$ meson, called tagging the $B$, will be used in LHCb. The statistical error on the asymmetry measurement is affected by the number of wrongly tagged events, as was discussed in Chapter 1, so the number of wrongly tagged events must be well known. For clarity, the reconstructed $B^0$ meson in the channel of interest will be referred to as the selected $B$ throughout, and the other $B$ hadron in the event as the other $B$.

To measure the number of wrongly tagged events, the tagging algorithms are calibrated on a channel where the charge of one of the final state particles unambiguously determines the flavour of the selected $B$. The result of the tagging algorithms for this control channel measures how often the flavour is wrongly
tagged. Then, the challenge is to understand how valid this measurement is for the CP channels, where the CP violation measurements are made. Differences between the triggering and selection of the CP channels with respect to the control channel can affect the performance of the tagging, and hence introduce systematic errors to the measurement.

In this chapter, a simulation-based investigation of the tagging of several $B_d^0$ and $B_s^0$ channels is presented, to study the systematic effects of both the trigger and the selection. A strategy has been developed to understand and correct for differences in the tagging performance between control and CP signal channels, using only information that will be available in real data.

5.1 Flavour Tagging

There are two main approaches to the problem of tagging. The flavour of the selected $B$ can be determined either by studying the decay of this particle, or from the decay of the other $B$ hadron produced in the event. For historical reasons, since at LEP the two $B$ mesons were produced in separate hemispheres, these approaches are known as same side and opposite side tagging, respectively.

For both event selection and flavour tagging, particle identification is extremely important. Particle identification is provided by the two RICH detectors, and by the calorimeters and the muon chambers, as described in Chapter 2. Below, the tagging algorithms are described in detail. Each tagging algorithm uses different cuts to select the tagging particles. Work is ongoing within the LHCb collaboration to improve the flavour-tagging algorithms, and those used in this study represent a snapshot of an evolving analysis package [CDM03]. In the descriptions below, the exact values of the standard cuts used to achieve the
results described in this thesis are listed for reference.

5.1.1 Opposite Side Tagging Algorithms

The flavour of the selected $B$ meson can be deduced from the flavour of the other $B$ hadron produced in the event, since if the selected $B$ contains a $b$ quark at production, the other $B$ hadron must contain a $\bar{b}$ quark. Various tagging algorithms are applied to find a suitable particle from the decay of the other $B$ hadron, determine the charge of the $b$ quark, and therefore infer the flavour of the hadron at production. The possibility that the other $B$ hadron may have been a neutral meson and have oscillated before decay introduces an inherent uncertainty to this opposite side tagging approach.

Three independent algorithms are used in LHCb to determine the flavour of the other $B$ hadron, based on three different measurements:

- The charge of the lepton from the prompt semileptonic $B$ decay, eg $b \rightarrow Xl^-\nu$.
- The charge of the final kaon from the $b \rightarrow c \rightarrow s$ decay chain.
- The vertex charge, defined as the sum of the charges of the particles from the inclusive secondary vertex reconstructed from the $b$-decay products.

This tag is only useful when the other $B$ hadron is charged.

To select candidates for opposite side leptons from semileptonic $B$ decay the cuts are designed to reduce the number of $b \rightarrow c \rightarrow l$ decays, which would give an incorrect tag. The lepton momentum and transverse momentum must be larger than the following cut values:

- Lepton momentum $> 5$ GeV/c.
- Lepton transverse momentum $> 1.2$ GeV/c.

If more than one electron or muon passes the cuts, the one with the highest
transverse momentum is chosen as the tagging particle.

For the opposite side kaon tag, the kaon identification is reliant on information from the RICH detectors. A suitable tagging kaon is selected by cutting on the kaon momentum to preferentially select kaons from the decay chain of the $b$ over kaons produced in the fragmentation process. Additionally, the kaon must have a large impact parameter significance with respect to the primary vertex, to reject kaons originating from the primary vertex. A similar cut was considered for the lepton tagging candidates but was not adopted because the effective efficiency was found to decrease as a function of the cut on the impact parameter significance [Jac]. The specific cut values used for the opposite side kaon tag were:

- Kaon momentum $> 3$ GeV/c.
- Kaon transverse momentum $> 0.4$ GeV/c.
- Impact parameter significance with respect to the primary vertex,

$$IP/\sigma_{IP} > 3.7,$$ where $IP$ is the impact parameter and $\sigma_{IP}$ is its measurement error.

The vertex charge tag works by performing an inclusive reconstruction of the decay vertex of the other $B$ hadron. First, two tracks are taken as seeds. These tracks must satisfy kinematic and impact parameter cuts designed to enhance the probability they originate from the $B$ hadron decay. Additionally, pairs of tracks which are compatible with the decay of a $K_s^0$ are not considered. Then, other tracks are added if they satisfy kinematic cuts on the distance from the primary vertex, the $\chi^2$ of the secondary vertex, and the impact parameter. At each step, when another track is added, the impact parameter of each selected track in turn is calculated with respect to the vertex formed by all the other selected tracks. The track giving the largest impact parameter significance is then rejected if the
impact parameter significance is larger than 3. Finally, the vertex charge is taken as the sum of the charges of all tracks associated to the vertex.

### 5.1.2 Same-Side Tagging Algorithms

The same-side tagging algorithms are designed to determine the flavour of the selected $B$ hadron from flavour correlations in the fragmentation decay chain. When a $B_s^0$ meson, made up of a $\bar{b}s$ antiquark-quark combination, is produced in the fragmentation of a $B$ quark, there is a $\bar{s}$ quark available to form a $K$ meson. The probability for this kaon to be charged or neutral is approximately equal. Figure 5.1 gives an example of this fragmentation process for a $B_s^0$ producing a $K^+$ meson. Cuts are applied to select the charged kaons, which are correlated in phase space with the selected $B_s^0$ and emerge from the primary vertex.

A cut is imposed on the kaon impact parameter significance with respect to the primary vertex, and minimum momentum and transverse momentum cuts are also applied. Additional cuts are designed to select kaons correlated in phase space with the selected $B_s^0$. The specific cuts used here were:

- Kaon momentum $> 4$ GeV/c.
• Kaon transverse momentum > 0.4 GeV/c.
• Impact parameter significance with respect to the primary vertex,
  \( IP/\sigma_{IP} < 2.5. \)
• Difference in the reconstructed azimuthal angle of the \( B_s^0 \) and \( K < 1.1 \) radians.
• Difference in the pseudo-rapidity of the kaon and the reconstructed \( B_s^0 < 1. \)
• Mass difference between the \( B_s^0 K \) combination and the reconstructed \( B_s^0 < 1.5 \) GeV.

If more than one kaon is selected, the flavour tag is taken from the charge of the kaon with the highest transverse momentum.

A similar procedure can be followed when the selected \( B \) is a \( B_d^0 \) meson, a \( \bar{t}d \) antiquark-quark combination. Then a \( \bar{d} \) quark is available to form a pion in the hadronisation process by combining with a \( u \) quark from the sea to give a \( \pi^+ \). This nearest pion in the fragmentation chain can also originate from the decay of an excited \( B \) meson state, \( B^{**} \). As in the \( B_s^0 \) same-side kaon case, cuts are applied to select these particles and, if more than one pion is selected, the flavour tag is taken from the pion with the highest transverse momentum. The cuts used were:

• Pion momentum > 2 GeV/c.
• Pion transverse momentum > 0.2 GeV/c.
• Impact parameter significance with respect to the primary vertex,
  \( IP/\sigma_{IP} < 3.0. \)
• Mass difference between the \( B_d^0 \pi \) combination and the reconstructed \( B_d^0 < 1.5 \) GeV.
5.1.3 Flavour Tagging Performance

The results from each of the separate tagging algorithms described above are combined to produce a final tag decision, giving the best estimate of the flavour of the selected $B$ hadron. Studies are underway to determine the most effective method of combining the separate tag decisions, including the possibility of a neural net decision process. In the results discussed here, the procedure described below is used when more than one tag is available.

In the case when more than one tag hypothesis exists, the vertex charge tag is ignored. If both the muon and electron tags are present the tag with the highest transverse momentum is preferred, since this is most likely to originate from a $b \rightarrow l$ decay, and the other tag decision is discarded. If, after these two steps have been taken, only two tags are available and they are in disagreement then no tagging decision is made. If however the two tags agree, this is taken as the decision. Finally, in the case where three tags are available, the majority tag is taken as the final tag decision. This is the baseline decision procedure, used for example to produce the physics performance numbers given in Ref. [Reo03].

The tagging efficiency, $\epsilon_{\text{tag}}$, is the probability that the tagging algorithm gives a result:

$$
\epsilon_{\text{tag}} = \frac{R + W}{R + W + U}.
$$

(5.1)

Here, $W$ is the number of wrongly tagged events, $R$ is the number of correctly tagged events and $U$ is the number of untagged events. The mistag fraction, $\omega$, is the fraction of tagged events which are wrongly tagged:

$$
\omega = \frac{W}{R + W}.
$$

(5.2)
As detailed in Chapter 1 and Section 5.9, the CP asymmetry measured in a particular channel is directly related to the mistag fraction:

\[ A_{\text{F,meas}} = (1 - 2\omega)A_{\text{F,true}}. \]  

(5.3)

The statistical uncertainty on the CP asymmetry is directly related to the effective tagging efficiency, \( \epsilon_{eff} \),

\[ \sigma_{A_{\text{F,meas}}}^2 \propto \frac{1}{\epsilon_{eff}} \]  

(5.4)

where \( N \) is the number of triggered and selected events. The effective tagging efficiency is:

\[ \epsilon_{eff} = \epsilon_{tag}(1 - 2\omega)^2. \]  

(5.5)

It is straightforward to obtain the tagging efficiency in data, but measuring the mistag fraction is more difficult. For decays to flavour-specific final states, the mistag fraction can be measured in data through the amplitude of \( B^0_d \) or \( B^0_s \) oscillations, measured in a control channel. This process is described in detail in Section 5.8. The aim of this study is to understand how the mistag fraction measured in such a control channel can be used to estimate the mistag fraction in the CP channels of interest.

### 5.2 Systematic Effects of the Trigger and Selection

Measurements of the mistag fraction made in control channels will be used to estimate the mistag fraction in the CP channels. However, the tagging performance in each channel is dependent on the particular kinematical properties of the event. This is because the four-momentum and proper lifetimes of the selected \( B \) and the other \( B \) produced in the event, as well as the kinematics and
multiplicity of the underlying event, determine the probability that the selected \( B \) meson is triggered, fully reconstructed, selected and tagged.

The principle of using control channels to estimate the tagging performance in CP channels is that, since the opposite side tagging algorithms use particles from the decay of the other \( B \) in the event, the specific decay channel of the selected \( B \) does not affect the mistag fraction measured for these opposite side tags. This simple assumption is invalid for two reasons. Firstly, the LHCb Level-0 and Level-1 triggers are designed to find events based on the generic characteristics of \( B \) decays, and so it is possible for a selected \( B \) decay to have been triggered by particles from the decay of the other \( B \) in the event. Secondly, even for an event triggered solely on the selected \( B \) decay, correlations between the selected \( B \) and the other \( B \) affect the phase space of the other \( B \), and hence bias the opposite side tagging performance [DTB03].

If the four-momenta and proper lifetimes of all the decay products of both \( B \) hadrons were known, it would be possible to determine the mistag fraction from the control channel for every point in phase space, and thus understand the effect on the mistag fraction of the CP channel. However, in general, the other \( B \) in the event is not fully reconstructed and its four-momentum and proper lifetime remain unknown. Instead, any correction between the phase space of the control and CP channels can be made using only the properties of the selected \( B \) in the event, which is reconstructed in the data. Before such a correction is possible, the events must be sorted into those which were triggered by the selected \( B \) decay and those which were triggered by the other \( B \) decay.

The LHCb trigger was described in Chapter 2. The trigger exploits two particular features of \( B \) decays compared to other inelastic proton-proton interactions; events with \( B \) hadrons have particles with high transverse momentum due to the
relatively large $B$ mass and generally they have a displaced secondary vertex due to the long $B$ lifetime. Consequently, the transverse momentum of the selected $B$ was the first property to be investigated when considering how to correct for any systematic bias of the trigger.

In this thesis the data studied were Monte Carlo data generated by the full simulation of the LHCb detector for a variety of control and CP decay channels. The aim of the study was to investigate the systematic effects of the trigger and the selection on the tagging performance in each channel, and develop a strategy to allow an unbiased estimate of the mistag fraction in a number of CP channels from the control channels. Three $B^0_d$ and three $B^0_s$ channels were studied, involving a range of decay topologies. For the $B^0_d$ system, the self-tagging control channel $B^0_d \rightarrow J/\psi(\mu\mu)K^*$ was compared to the CP channels $B^0_d \rightarrow J/\psi(\mu\mu)K^0_s$ and $B^0_d \rightarrow \pi^+\pi^-$. The $B^0_s$ control channel $B^0_s \rightarrow D^-\pi^+$ compared to the CP channels $B^0_s \rightarrow D^\pm K^\pm$ and $B^0_s \rightarrow J/\psi(\mu\mu)\phi$ was also considered. Hence in each case, one CP channel with a similar and one with a dissimilar topology to the control channel was chosen. The intention was to investigate a range of systematic biases between the control and CP channels.

The event samples considered in this study consisted of events which satisfied the LHCb trigger and passed the offline selection criteria specific to each decay channel. Details of the offline selection cuts for each $B^0_d$ and $B^0_s$ decay are given in Ref. [Reo03].

5.2.1 Classification of Triggered Events

To understand the systematic effect of the trigger on the tagging performance the events must be separated into several sets, depending on how the event was triggered. It is necessary to split the events into two mutually exclusive subsam-
5.2 Systematic Effects of the Trigger and Selection

...
ger decision, and the events triggered only by the selected $B$. Hence all events
Triggered Independently of the Selected $B$ are labelled as TIS for the purposes
of this study, and only those events exclusively Triggered On the Selected $B$
decay are labelled as TOS.

A third set of events exists, when the event does not trigger in either of
the two masking steps. In this case, neither the decay products of the selected
$B$ or the other $B$ in isolation are sufficient to give a positive trigger decision.
Instead, some combination of particles from both $B$ decays has triggered the
event. This third category of events, Triggered On Both $B$ decays, is labelled
as TOB. This classification is unlikely at Level-0, where the trigger decision is
based on finding individual particles with high transverse momentum. However,
the Level-1 trigger is designed to also search for a displaced secondary vertex
based on the properties of the two tracks with the highest transverse momentum.
If one of these tracks is from the decay of the selected $B$ and one from the decay
of the other $B$ hadron, the event can only be Triggered On Both. This situation
is illustrated in Figure 5.2.

To summarise:

- **TIS** - An event where particles from the decay of the other $B$ hadron
  contributed to the trigger decision.

- **TOS** - An event triggered exclusively by particles from the decay of the
  selected $B$.

- **TOB** - An event which triggers only when all the decay products of both
  $B$ hadrons are considered.

Each event will have two labels, describing the triggering at Level-0 and at
Level-1. The systematic effect of the trigger bias for TOB events is difficult
to disentangle and these events are discarded from this study. This leaves four
5.2 Systematic Effects of the Trigger and Selection

Figure 5.2: Diagram showing a typical event where the Level-1 trigger subsample is **TOB**, Triggered On Both. This event passes the Level-1 trigger requirement based on the impact parameter variable calculated from two tracks with high transverse momentum. However one of the tracks is from the decay of the selected $B^0$ meson and the other is from the decay of the other $B$ hadron in the event.

exclusive subsamples of the events, namely:

- Level-0 **TOS** and Level-1 **TOS**: TOSTOS or
- Level-0 **TOS** and Level-1 **TIS**: TOSTIS or
- Level-0 **TIS** and Level-1 **TOS**: TISTOS or
- Level-0 **TIS** and Level-1 **TIS**: TISTIS

The numbers of events in each of these categories, for the decay channels studied, is discussed below.

Since the buffer-tampering tool is designed to be applied to real data as well as simulated data, it does not require any Monte Carlo truth information. The trigger objects manipulated by the Level-0 buffer-tampering tool are reconstructed clusters in the calorimeters and muon chambers. At Level-1, the trigger objects also include reconstructed tracks. Unfortunately the simulated data used in this study did not contain sufficient reconstructed information, therefore the Level-1 buffer-tampering tool was temporarily modified to use Monte Carlo truth infor-
Table 5.1: Total numbers of events generated for the three $B_s^0$ channels, and the numbers of events selected and tagged.

<table>
<thead>
<tr>
<th>channel</th>
<th>generated events</th>
<th>selected &amp; tagged</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_s^0 \rightarrow D_s\pi$</td>
<td>3.3 M</td>
<td>34156</td>
</tr>
<tr>
<td>$B_s^0 \rightarrow D_sK$</td>
<td>4.6 M</td>
<td>35928</td>
</tr>
<tr>
<td>$B_s^0 \rightarrow J/\psi(\mu\mu)\phi$</td>
<td>1.6 M</td>
<td>101001</td>
</tr>
</tbody>
</table>

5.3 Tagging Performance for $B_s^0$ Channels

The three $B_s^0$ channels studied were $B_s^0 \rightarrow D_s^-\pi^+$, $B_s^0 \rightarrow D_s^+K^\pm$ and $B_s^0 \rightarrow J/\psi(\mu\mu)\phi$. For the control channel $B_s^0 \rightarrow D_s^-\pi^+$, a self-tagging mode, the flavour of the $B_s^0$ at decay is given by the charge of the bachelor pion. Table 5.1 lists the numbers of events generated for each of these channels, and the numbers of selected and tagged events which passed the offline selection cuts. These events were generated by GAUSS, the most complete current simulation of the LHCb detector, described in Chapter 2. Selection and analysis of the data were performed by the analysis application, DaVinci v12r11, requiring each event to pass the Level-0 and Level-1 triggers and the offline selection specific to each channel. Then the tagging algorithms and buffer-tampering tools were run on this reduced, or stripped, set of data. For reference, the flavour tagging algorithm used was version v5r5 and the buffer-tampering was version v1r1.

Table 5.2 lists the resulting numbers of tagged events in this analysis. The tagging efficiency and mistag fraction are indicated. Here, as throughout this study, the tag is the result of the tagging decision process described in Section 5.1.3 and the final efficiency and mistag fraction are calculated using Equations 5.1
<table>
<thead>
<tr>
<th>channel</th>
<th>selected &amp; tagged</th>
<th>$\epsilon_{tag}$ (%)</th>
<th>$\omega$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_s^0 \rightarrow D_s \pi$</td>
<td>34156</td>
<td>57.14±0.34</td>
<td>33.22±0.34</td>
</tr>
<tr>
<td>$B_s^0 \rightarrow D_s K$</td>
<td>35928</td>
<td>58.02±0.33</td>
<td>32.47±0.32</td>
</tr>
<tr>
<td>$B_s^0 \rightarrow J/\Psi(\mu\mu)\Phi$</td>
<td>101001</td>
<td>53.62±0.19</td>
<td>34.94±0.20</td>
</tr>
</tbody>
</table>

Table 5.2: Details of $B_s^0$ selected & tagged events, with tagging performance information.

and 5.2, based on the total number of right, wrong and untagged events as indicated by the Monte Carlo truth. Ultimately, in real data, the mistag fraction in the self-tagging control channel will be determined using the method detailed in Section 5.8.

From Table 5.2, the value of the mistag fraction for the $B_s^0 \rightarrow D_s \pi$ and $B_s^0 \rightarrow D_s K$ events agrees within the statistical error, and so an estimate of the mistag fraction for the $D_s K$ channel from the $D_s \pi$ control channel is possible. However, the mistag fraction for $B_s^0 \rightarrow J/\Psi(\mu\mu)\Phi$ is significantly different from that of $B_s^0 \rightarrow D_s \pi$. The assumption is that the difference in the mistag fractions can be attributed to the different phase space of the three channels. The $D_s \pi$ and $D_s K$ channels share an identical topology and very similar kinematics, and so are triggered and selected in approximately the same way. As a result, the systematic effect of the trigger and the offline selection on the phase space, and hence the tagging performance, is comparable for these channels. For the $J/\Psi(\mu\mu)\Phi$ decays, the trigger introduces a different systematic bias, as does the offline selection, resulting in a different tagging performance.

To test the above assumption, the trigger behaviour was investigated using the buffer-tampering tool. The percentages of events which fall into each exclusive trigger subsample are given in Table 5.3, for each of the three channels. The percentage of events which are TOB is recorded, for the Level-1 trigger, and
Table 5.3: The percentage breakdown of tagged $B_s^0$ events into the four exclusive trigger subsamples.

<table>
<thead>
<tr>
<th></th>
<th>$B_s^0 \to D_s\pi$</th>
<th>$B_s^0 \to D_sK$</th>
<th>$B_s^0 \to J/\Psi(\mu\mu)\Phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOSTOS</td>
<td>26.1±0.2</td>
<td>26.4±0.2</td>
<td>60.5±0.2</td>
</tr>
<tr>
<td>TOSTIS</td>
<td>4.0±0.1</td>
<td>4.1±0.1</td>
<td>8.3±0.1</td>
</tr>
<tr>
<td>TISTOS</td>
<td>38.5±0.3</td>
<td>38.7±0.3</td>
<td>19.4±0.1</td>
</tr>
<tr>
<td>TISTIS</td>
<td>16.1±0.2</td>
<td>15.3±0.2</td>
<td>6.9±0.1</td>
</tr>
<tr>
<td>L1TOB</td>
<td>15.2±0.2</td>
<td>15.4±0.2</td>
<td>4.8±0.1</td>
</tr>
<tr>
<td>L0TOB not L1TOB</td>
<td>0</td>
<td>0</td>
<td>0.2±0.01</td>
</tr>
</tbody>
</table>

Separately for events which are **TOB** for the Level-0 trigger but not the Level-1. These events are discarded from the sample. Each of the trigger subsamples in the table is an exclusive set, and the total for each channel is 100%.

For the $B_s^0 \to D_s\pi$ and $B_s^0 \to D_sK$ decays, the identical topologies and very similar kinematics result in similar trigger behaviour, as expected. The very different triggering mechanisms for $B_s^0 \to J/\Psi(\mu\mu)\Phi$ decays are highlighted. At Level-0, the majority of the $J/\Psi(\mu\mu)\Phi$ events are triggered by the muons from the $J/\Psi$ decay, whereas the $D_s\pi$ and $D_sK$ channels are dominated by events selected by the hadronic trigger. Since the efficiency of the muon trigger is much higher than the hadronic trigger in the $J/\Psi(\mu\mu)\Phi$ channel, $\sim 87\%$ as opposed to $\sim 40\%$ [Tri03], a much higher proportion of events in this channel are **TOS** at Level-0.

At Level-1, many $J/\Psi(\mu\mu)\Phi$ events again pass the trigger due to the muons from the $J/\Psi$ decay, since the Level-1 trigger accepts any event where the invariant mass of the muon pair is compatible with the $J/\Psi$ mass. In contrast, the $D_s\pi$ and $D_sK$ events are in general triggered by the impact parameter variable calculated from the two highest transverse momentum tracks. This results in the higher number of Level-1 **TOB** events.
Table 5.4: The mistag fractions, separately for each exclusive trigger subsample, for the three $B_s^0$ channels.

Using the information provided by the buffer-tampering tool, Table 5.4 gives the mistag fraction separately for each trigger subsample. The same information is presented graphically in Figure 5.3. The systematic effect of the Level-0 hadronic trigger on the $B_s^0 \rightarrow D_s \pi$ and $B_s^0 \rightarrow D_s K$ decays causes a clear difference in the mistag fraction between those decays and the $B_s^0 \rightarrow J/\Psi(\mu\mu)\Phi$ decays.

5.4 Phase Space Correction for Tagging Performance

As explained above, different mistag fractions are observed for the $B_s^0 \rightarrow D_s \pi$ control channel and the $B_s^0 \rightarrow J/\Psi(\mu\mu)\Phi$ CP channel due to the different systematic effects of the trigger. A strategy has been devised to correct for this systematic bias, to allow the control channel to be used to calculate the mistag fraction for the very different CP channel topology; a method that does not rely on Monte Carlo truth and can therefore be used in real data.

The method involves a weighting technique to correct for the phase space differences between the two channels, which are responsible for the trigger biases. Since the $B$ meson transverse momentum distribution is the relevant parameter for event acceptance in the trigger, this is the parameter on which the correction
Figure 5.3: A graphical representation of the mistag fractions, separately for each exclusive trigger subsample, for the three $B_s^0$ channels.

is based. The details of the method are described below.

Figure 5.4 shows the transverse momentum of the three selected $B_s^0$ channels for each trigger subsample, normalised to the same total number of events. For events triggered by the selected $B$ decay at Level-0, the hadronic trigger cuts into the lower end of the transverse momentum distribution for $D_s\pi$ and $D_sK$, much more than the muon trigger which dominates the $J/\Psi(\mu\mu)\Phi$ channel. This can be seen in the top two plots of Figure 5.4 which are TOS at Level-0; the left hand plot is TOS at Level-1 and the right hand plot is TIS at Level-1. Here, the selected $B$ transverse momentum distributions of the $B_s^0 \rightarrow D_s\pi$ and $B_s^0 \rightarrow D_sK$ decays are biased to a higher value. Clearly this will affect the tagging performance for the same-side tags and, because of the correlation between the selected $B$ and the other $B$, this also has an effect on the phase space of the other $B$ and thus affects the opposite side tagging performance. The transverse
momentum distributions of the other $B$ hadron for each channel, in each trigger subsample, are shown in Figure 5.5, again normalised to the same total number of events.

The correction for the systematic bias introduced by the trigger is achieved by matching the control-channel selected $B$ transverse momentum distribution to the transverse momentum distribution of the selected $B$ in the CP channel. A weighting method is used. Explicitly, the weighted mistag fraction of the $CP$ channel is given by

$$< \omega > = \frac{\sum_i N_i \omega_i}{\sum_i N_i},$$  \hspace{1cm} (5.6)$$

where:
Figure 5.5: The normalised transverse momentum distributions of the other $B$ in the event for the three $B_s$ channels, plotted separately for each trigger subsample.
Figure 5.6: The reconstructed transverse momentum minus the true transverse momentum of the selected $B_s^0$.

- $\omega_i$ is the bin-by-bin mistag fraction in the control channel, as a function of the selected $B$ transverse momentum.
- $N_i$ is the number of events in the $i^{th}$ bin of the selected $B$ transverse momentum distribution of the CP channel.

In this way, $N_i$ in the CP channel and $\omega_i$ in the control channel, both accessible in real data, are used to estimate the mistag fraction in the CP channel, $<\omega>$, of which a direct measurement is not available. Throughout this study, a bin width of 0.4 GeV is used, with 100 bins considered.

The transverse momentum distribution shown in Figure 5.4 which is used to perform the weighting is the true Monte Carlo transverse momentum. When this method is applied to real data the reconstructed momentum information will be used. To check that using the Monte Carlo truth momentum rather than the reconstructed momentum would not affect the results, the difference between the true and reconstructed transverse momentum of the selected $B$ is plotted for each event, shown in Figure 5.6. As expected, this plot is centred on zero, and the width is related to the finite momentum resolution of the reconstructed
transverse momentum. The distribution has an RMS width of 0.03 GeV. The excellent momentum resolution achieved by the LHCb detector ensures this width is smaller than the 0.4 GeV bin width used in the weighting method, and hence will not significantly affect the result.

Figure 5.7 shows the rebinned transverse momentum distribution of the selected $B$ for events which are TOS at Level-0 and TOS at Level-1, again normalised to the same total number of events, for the control channel and the CP channel. $N_i$ is obtained from the number of entries in each bin of the $J/\Psi(\mu\mu)\Phi$ distribution. Figure 5.8 shows the mistag fraction versus transverse momentum of the $B$ in the $B_s^0 \rightarrow D_s\pi$ control channel, which gives $\omega_i$. This is compared in the same figure with the mistag fraction for the CP channel $B_s^0 \rightarrow J/\Psi(\mu\mu)\Phi$.

It is possible to calculate the error on the weighted mistag fraction, Equation 5.6. The fractional uncertainty on the weighted mistag fraction is given by Equation 5.7.

$$\delta^2 < \omega > = \sum_i \left[ \left( \frac{\delta < \omega >}{\delta N_i} \right)^2 (\delta N_i)^2 + \left( \frac{\delta < \omega >}{\delta \omega_i} \right)^2 (\delta \omega_i)^2 \right]$$  \hspace{1cm} (5.7)

Here the error, $\delta N_i$, on the number of entries in each bin is

$$\left( \delta N_i \right)^2 = N_i$$  \hspace{1cm} (5.8)

and

$$\frac{\partial < \omega >}{\partial N_i} = \frac{\omega_i \sum_i N_i - \sum_i \omega_i N_i}{\left( \sum_i N_i \right)^2}$$  \hspace{1cm} (5.9)

$$\frac{\partial < \omega >}{\partial \omega_i} = \frac{N_i}{\sum_i N_i}.$$  \hspace{1cm} (5.10)
Figure 5.7: The transverse momentum of the selected $B$, for the CP channel $B_s^0 \rightarrow J/\Psi (\mu \mu) \Phi$ and the control channel $B_s^0 \rightarrow D_s \pi$.

Figure 5.8: The mistag fraction as a function of the transverse momentum of the selected $B$, for the CP channel $B_s^0 \rightarrow J/\Psi (\mu \mu) \Phi$ and the control channel $B_s^0 \rightarrow D_s \pi$. 
<table>
<thead>
<tr>
<th>$\omega$ (%)</th>
<th>$B_s^0 \rightarrow D_s \pi$</th>
<th>$B_s^0 \rightarrow J/\Psi(\mu\mu)\Phi$</th>
<th>$B_s^0 \rightarrow D_s \pi$ weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOSTOS</td>
<td>33.66±0.73</td>
<td>36.25±0.28</td>
<td>34.84±1.06</td>
</tr>
<tr>
<td>TOSTIS</td>
<td>29.47±1.48</td>
<td>32.48±0.62</td>
<td>31.54±2.01</td>
</tr>
<tr>
<td>TISTOS</td>
<td>34.82±0.56</td>
<td>34.32±0.45</td>
<td>34.62±0.56</td>
</tr>
<tr>
<td>TISTIS</td>
<td>30.38±0.72</td>
<td>30.89±0.64</td>
<td>30.13±0.73</td>
</tr>
</tbody>
</table>

Table 5.5: The mistag fractions for the decays indicated, plus the mistag fraction estimated for the $B_s^0 \rightarrow J/\Psi(\mu\mu)\Phi$ decay by weighting the $B_s^0 \rightarrow D_s \pi$ decay according to the transverse momentum distribution of the selected $B$.

Hence the result is:

$$
\delta^2 < \omega > = \sum_i \left[ \left( \frac{\omega_i}{\sum_i N_i} - \frac{< \omega >}{\sum_i N_i} \right)^2 N_i + \left( \frac{N_i}{\sum_i N_i} \right)^2 (\delta \omega_i)^2 \right].
$$

(5.11)

The fractional uncertainty for the weighted mistag fraction can therefore be written as:

$$
\delta^2 < \omega > = \frac{1}{(\sum_i N_i)^2} \sum_i \left[ (\omega_i - < \omega >)^2 N_i + N_i^2 (\delta \omega_i)^2 \right].
$$

(5.12)

5.5 Weighted Tagging Performance for $B_s^0$ Channels

Using the method described in the previous section, the transverse momentum distribution of the selected $B$ measured in the channel $B_s^0 \rightarrow J/\Psi(\mu\mu)\Phi$ is used to weight the mistag fraction measured in the self-tagging control channel $B_s^0 \rightarrow D_s \pi$. The error calculation was carried out according to Equation 5.12. This weighting was carried out separately for each trigger subsample. The results are given in Table 5.5 and graphically as the starred points in Figure 5.9.

For the events which are Level-0 TIS, the weighting does not change the
mistag fraction significantly, and the mistag fractions for the signal and control channels agree within the errors. The weighting has the largest effect for events which are TOS at Level-0, as expected. For these trigger subsamples, the weighted mistag fraction for the control channel now agrees, within the errors, with the CP channel mistag fraction. Whilst the conclusions are limited by the size of the statistical sample, these results indicate this method is a promising way of estimating the mistag fraction in the $B^0_s \to J/\Psi(\mu\mu)\Phi$ CP channel.

The weighting method was also applied to the $B^0_s \to D_s K$ CP channel where, since the decay topology and kinematics are very similar to the control channel, no significant difference in the mistag fraction is expected. The results are listed in Table 5.6, and shown in Figure 5.10. As expected, the weighting method does not change the mistag fraction estimated from the control channel, due to
<table>
<thead>
<tr>
<th>$\omega$ (%)</th>
<th>$B_s^0 \rightarrow D_s \pi$</th>
<th>$B_s^0 \rightarrow D_s K$</th>
<th>$B_s^0 \rightarrow D_s \pi$ weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOSTOS</td>
<td>33.66±0.73</td>
<td>32.49±0.69</td>
<td>33.76±0.74</td>
</tr>
<tr>
<td>TOSTIS</td>
<td>29.47±1.48</td>
<td>27.65±1.37</td>
<td>29.82±1.57</td>
</tr>
<tr>
<td>TISTOS</td>
<td>34.82±0.56</td>
<td>33.88±0.53</td>
<td>34.88±0.57</td>
</tr>
<tr>
<td>TISTIS</td>
<td>30.38±0.72</td>
<td>29.72±0.71</td>
<td>30.22±0.73</td>
</tr>
</tbody>
</table>

Table 5.6: The mistag fractions for the decays indicated, plus the mistag fraction estimated for the $B_s^0 \rightarrow D_s K$ decay by weighting the $B_s^0 \rightarrow D_s \pi$ decay according to the transverse momentum distribution of the selected $B$.

Figure 5.10: The mistag fractions for the decays indicated, plus the mistag fraction estimated for the $B_s^0 \rightarrow D_s K$ decay by weighting the $B_s^0 \rightarrow D_s \pi$ decay according to the transverse momentum distribution of the selected $B$. 


<table>
<thead>
<tr>
<th>channel</th>
<th>generated events</th>
<th>selected &amp; tagged</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0_d \rightarrow J/\Psi(\mu\mu)K^{*0}$</td>
<td>1.7 M</td>
<td>92640</td>
</tr>
<tr>
<td>$B^0_d \rightarrow J/\Psi(\mu\mu)K_s$</td>
<td>0.6 M</td>
<td>24606</td>
</tr>
<tr>
<td>$B^0_d \rightarrow \pi\pi$</td>
<td>1.9 M</td>
<td>53870</td>
</tr>
</tbody>
</table>

Table 5.7: Total numbers of events generated for the three $B^0_d$ channels, and the numbers of events selected and tagged.

<table>
<thead>
<tr>
<th>channel</th>
<th>selected &amp; tagged</th>
<th>$\epsilon_{tag}$ (%)</th>
<th>$\omega$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0_d \rightarrow J/\Psi(\mu\mu)K^{*0}$</td>
<td>92640</td>
<td>58.71±0.21</td>
<td>37.99±0.21</td>
</tr>
<tr>
<td>$B^0_d \rightarrow J/\Psi(\mu\mu)K_s$</td>
<td>24606</td>
<td>57.94±0.40</td>
<td>38.56±0.41</td>
</tr>
<tr>
<td>$B^0_d \rightarrow \pi\pi$</td>
<td>53870</td>
<td>56.35±0.27</td>
<td>37.66±0.28</td>
</tr>
</tbody>
</table>

Table 5.8: Details of $B^0_d$ selected & tagged events, with tagging performance information.

the very similar phase space of the $B^0_s \rightarrow D_s\pi$ and $B^0_s \rightarrow D_sK$ decays.

### 5.6 Tagging Performance for $B^0_d$ Channels

The tagging performance in three $B^0_d$ channels was also studied. The self-tagging control channel was $B^0_d \rightarrow J/\Psi(\mu\mu)K^{*0}$, and the CP channels were $B^0_d \rightarrow J/\Psi(\mu\mu)K_s$ and $B^0_d \rightarrow \pi\pi$. Table 5.7 gives the numbers of events generated for each channel.

Table 5.8 indicates the numbers of tagged events in each channel which passed the triggers and the offline selection. The tagging efficiency and the mistag fraction are also shown. The tag for each event is taken from the outcome of the tagging decision process, as for the $B^0_s$ analysis. The numbers of right and wrong tags were taken from Monte Carlo truth information. The percentages of events which fall into each trigger subsample are shown in Table 5.9.

Channels where a $J/\Psi$ decays to $\mu\mu$ are preferentially triggered by the decay of the selected $B$, by a muon with high transverse momentum at Level-0, and by
### Table 5.9: The percentage breakdown of tagged $B_d^{0}$ events into the four exclusive trigger subsamples.

<table>
<thead>
<tr>
<th>(percentages)</th>
<th>$B_d^{0} \rightarrow J/\Psi(\mu\mu)K^{*0}$</th>
<th>$B_d^{0} \rightarrow J/\Psi(\mu\mu)K_s$</th>
<th>$B_d^{0} \rightarrow \pi\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOSTOS</td>
<td>61.8±0.2</td>
<td>58.0±0.3</td>
<td>25.8±0.2</td>
</tr>
<tr>
<td>TOSTIS</td>
<td>7.7±0.1</td>
<td>8.1±0.2</td>
<td>4.4±0.1</td>
</tr>
<tr>
<td>TISTOS</td>
<td>20.2±0.1</td>
<td>20.2±0.3</td>
<td>22.4±0.2</td>
</tr>
<tr>
<td>TISTIS</td>
<td>6.5±0.1</td>
<td>6.9±0.2</td>
<td>11.9±0.1</td>
</tr>
<tr>
<td>L1TOB</td>
<td>3.6±0.1</td>
<td>6.5±0.2</td>
<td>35.6±0.2</td>
</tr>
<tr>
<td>L0TOB not L1TOB</td>
<td>0.2±0.02</td>
<td>0.2±0.03</td>
<td>0</td>
</tr>
</tbody>
</table>

The dimuon trigger at Level-1. For the $B_d^{0} \rightarrow \pi\pi$ decay, the two-body topology means the event is likely to trigger on a pion with high transverse momentum at Level-0, and on the impact parameter variable calculated from the two pion tracks at Level-1.

The transverse momentum distribution for each channel and trigger subsample is shown in Figure 5.11, normalised to the same total number of events. For the $B_d^{0} \rightarrow \pi\pi$ channel, events which are Level-1 TOS are more likely to have a lower $B$ transverse momentum than the $J/\Psi(\mu\mu)K^{*0}$ and $J/\Psi(\mu\mu)K_s$ decays. This is because the $B_d^{0} \rightarrow \pi\pi$ decay is two-body and is therefore less biased by the Level-1 trigger, as a pion with high transverse momentum can be produced even from the decay of a $B$ meson with low transverse momentum. In contrast, for the multi-body $B_d^{0} \rightarrow J/\Psi(\mu\mu)K^{*0}$ and $B_d^{0} \rightarrow J/\Psi(\mu\mu)K_s$ decays, the Level-1 trigger requirements bias the $B$ meson distribution towards higher values. The transverse momentum distributions of the other $B$ hadron for each channel in each trigger subsample are shown in Figure 5.12.

A significant number of $B_d^{0} \rightarrow \pi\pi$ decays are TOB at Level-1. This occurs when one track from the selected $B$ decay has a high transverse momentum, and passes the impact parameter cut. The event is then triggered by the addition
Figure 5.11: The normalised transverse momentum distributions for the three selected $B_d^0$ channels, plotted separately for each trigger subsample.
Figure 5.12: The normalised transverse momentum distributions of the other $B$ in the event, for the three $B_d^0$ channels, plotted separately for each trigger subsample.
<table>
<thead>
<tr>
<th>$\omega$ (%)</th>
<th>$B_d^0 \rightarrow J/\Psi(\mu\mu)K^{*0}$</th>
<th>$B_d^0 \rightarrow J/\Psi(\mu\mu)K_s$</th>
<th>$B_d^0 \rightarrow \pi\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>all events</td>
<td>37.99±0.21</td>
<td>38.56±0.41</td>
<td>37.66±0.28</td>
</tr>
<tr>
<td>TOSTOS</td>
<td>40.07±0.28</td>
<td>40.93±0.57</td>
<td>41.16±0.60</td>
</tr>
<tr>
<td>TOSTIS</td>
<td>34.09±0.65</td>
<td>33.80±1.23</td>
<td>32.14±1.12</td>
</tr>
<tr>
<td>TISTOS</td>
<td>36.79±0.46</td>
<td>37.13±0.90</td>
<td>38.31±0.61</td>
</tr>
<tr>
<td>TISTIS</td>
<td>31.71±0.68</td>
<td>32.65±1.29</td>
<td>32.18±0.67</td>
</tr>
</tbody>
</table>

Table 5.10: The mistag fractions, separately for each exclusive trigger subsample, for the three $B_d^0$ channels.

Figure 5.13: A graphical representation of the mistag fractions, separately for each exclusive trigger subsample, for the three $B_d^0$ channels.

of any track, not from the selected decay but from the other $B$ hadron or the underlying event, with a sufficiently high impact parameter. Both classes of events were excluded from the study, as for the $B_s$ channels, since the systematic bias resulting from such a combined trigger is difficult to disentangle.

The mistag fraction is listed separately for each trigger subsample in Table 5.10 and shown graphically in Figure 5.13. For three of the four trigger subsamples, within statistical errors, the mistag fraction for each channel is in
agreement. The disagreement in the Level-0 TIS and Level-1 TOS subsample is around $2\sigma$.

The results of Table 5.10 are to be compared with the results shown in Table 5.4 for the $B_s^0$ channels, where the different topology for the $B_s^0 \rightarrow J/\Psi(\mu\mu)\Phi$ decay compared to the $B_s^0 \rightarrow D_s\pi$ decay introduced different trigger biases, and hence affected the tagging performance. Here, the identical topology and similar kinematics of the $B_d^0 \rightarrow J/\Psi(\mu\mu)K^{*0}$ and $B_d^0 \rightarrow J/\Psi(\mu\mu)K_s$ decays is sufficient to explain the similarity in the mistag fractions. The $B_d^0 \rightarrow \pi\pi$ decays are less biased by the Level-1 trigger in particular, as shown in Figure 5.11. As a result, at the level of statistics available, the bias introduced by the differences in triggering does not significantly change the mistag fraction in the CP channel compared to the control channel.

### 5.7 Weighted Tagging Performance for $B_d^0$ Channels

The weighting method described in Section 5.4 was applied to the $B_d^0$ data, as a cross-check to investigate how the systematic correction would affect the tagging performance. The control channel $B_d^0 \rightarrow J/\Psi(\mu\mu)K^{*0}$ was weighted to match the transverse momentum distribution measured in the CP channels $B_d^0 \rightarrow \pi\pi$ and $B_d^0 \rightarrow J/\Psi(\mu\mu)K_s$, as defined in Equation 5.6. As for the $B_s$ channels, the Monte Carlo truth momentum was used.

The results of the weighting method for the $B_d^0 \rightarrow \pi\pi$ decay are shown in Table 5.11 and graphically in Figure 5.14. For all four trigger subsamples, no large shift in the mistag fraction is apparent as a result of the weighting process. Thus, for the subsample of events which are Level-0 TIS and Level-1 TOS, the
### Table 5.11: The mistag fractions for the decays indicated, plus the mistag fraction estimated for the $B^0_d \rightarrow \pi \pi$ decay by weighting the $B^0_d \rightarrow J/\Psi(\mu \mu)K^{*0}$ decay according to the transverse momentum distribution of the selected $B$. 

<table>
<thead>
<tr>
<th>$\omega$ (%)</th>
<th>$B^0_d \rightarrow J/\Psi(\mu \mu)K^{*0}$</th>
<th>$B^0_d \rightarrow \pi \pi$</th>
<th>$B^0_d \rightarrow J/\Psi(\mu \mu)K^{*0}$ weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOSTOS</td>
<td>40.07±0.28</td>
<td>41.16±0.60</td>
<td>40.84±0.35</td>
</tr>
<tr>
<td>TOSTIS</td>
<td>34.09±0.65</td>
<td>32.14±1.12</td>
<td>33.62±0.71</td>
</tr>
<tr>
<td>TISTOS</td>
<td>36.79±0.46</td>
<td>38.31±0.61</td>
<td>36.69±0.62</td>
</tr>
<tr>
<td>TISTIS</td>
<td>31.71±0.68</td>
<td>32.18±0.67</td>
<td>31.83±0.75</td>
</tr>
</tbody>
</table>

![Figure 5.14: The mistag fractions for the decays indicated, plus the mistag fraction estimated for the $B^0_d \rightarrow \pi \pi$ decay by weighting the $B^0_d \rightarrow J/\Psi(\mu \mu)K^{*0}$ decay according to the transverse momentum distribution of the selected $B$.](image-url)
weighted mistag fraction for the control channel remains around 2σ lower than the \( B_d^0 \to \pi \pi \) mistag fraction. For the \( B_d^0 \to \pi \pi \) CP channel, within the limited statistics, these results are inconclusive in indicating whether the the weighting methods improves the measurement of the mistag fraction. Further study awaits higher Monte Carlo statistics.

The same weighting method was applied to estimate the \( B_d^0 \to J/\Psi(\mu \mu)K_s \) mistag fraction, by weighting the \( B_d^0 \to J/\Psi(\mu \mu)K^{*0} \) events. The results are given in Table 5.12 and Figure 5.15. As expected, due to the similar transverse momentum distribution for \( B_d^0 \to J/\Psi(\mu \mu)K^{*0} \) and \( B_d^0 \to J/\Psi(\mu \mu)K_s \) decays, the weighting does not significantly alter the mistag fraction measured in the control channel.

### 5.8 Measuring the Mistag Fraction in a Control Channel

For the self-tagging control channel \( B_d^0 \to J/\Psi(\mu \mu)K^{*0} \), the flavour of the reconstructed and selected \( B_d^0 \) at decay is indicated by the charge of the kaon in the decay \( K^{*0} \to K^+ \pi^- \). In this section, results will presented showing how the mistag fraction in this channel can be measured from the amplitude of the \( B_d^0 \) decays.

<table>
<thead>
<tr>
<th>( \omega (%) )</th>
<th>( B_d^0 \to J/\Psi(\mu \mu)K^{*0} )</th>
<th>( B_d^0 \to J/\Psi(\mu \mu)K_s )</th>
<th>( B_d^0 \to J/\Psi(\mu \mu)K^{*0} ) weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOSTOS</td>
<td>40.07±0.28</td>
<td>40.93±0.57</td>
<td>40.53±0.31</td>
</tr>
<tr>
<td>TOSTIS</td>
<td>34.09±0.65</td>
<td>33.80±1.23</td>
<td>34.37±0.73</td>
</tr>
<tr>
<td>TISTOS</td>
<td>36.79±0.46</td>
<td>37.13±0.90</td>
<td>36.37±0.48</td>
</tr>
<tr>
<td>TISTIS</td>
<td>31.71±0.68</td>
<td>32.65±1.29</td>
<td>31.69±0.75</td>
</tr>
</tbody>
</table>

Table 5.12: The mistag fractions for the decays indicated, plus the mistag fraction estimated for the \( B_d^0 \to J/\Psi(\mu \mu)K_s \) decay by weighting the \( B_d^0 \to J/\Psi(\mu \mu)K^{*0} \) decay according to the transverse momentum distribution of the selected \( B \).
Figure 5.15: The mistag fractions for the decays indicated, plus the mistag fraction estimated for the $B^0_d \to J/\Psi(\mu\mu)K_s$ decay by weighting the $B^0_d \to J/\Psi(\mu\mu)K^{*0}$ decay according to the transverse momentum distribution of the selected $B$. 

oscillations, by comparing this self-tagged flavour with the flavour tag taken from the outcome of the tagging algorithms. Events tagged with the other side tagging algorithms and events tagged with the same side tag from the nearest pion in the fragmentation chain must be considered separately.

As described in Section 5.1, for events tagged with the other side tagging algorithms, the tag comes either from the charge of the lepton from the prompt semileptonic decay, the charge of the final kaon from the \( b \to c \to s \) decay chain, or the inclusive vertex charge. In each case the tagging particle, or inclusive reconstructed vertex, has a negative charge when the other \( B \) in the decay contains a \( b \) quark, i.e. when the flavour of the selected \( B_d^0 \) at production contains a \( \bar{b} \) quark. Then, the charge of the kaon from the decay of the selected \( B_d^0 \) in the self-tagging control channel is positive. Thus, the sign of the other side tag and the sign of the kaon from the decay of the \( K^{*0} \) will be different in the case where the tagging algorithms have correctly identified the flavour of the selected \( B \) at production, and identical when a mistag has occurred. However if the selected \( B_d^0 \) oscillates before decaying, an identical sign will also result.

The situation can be summarised as follows. Comparing the sign of the other side tag and the sign of the kaon from the decay of the \( K^{*0} \), if they are different, there are two possible explanations:

- The other side tag correctly identified the flavour of the other \( B \) hadron at decay, and hence the flavour of the selected \( B_d^0 \).
- The other side tag wrongly identified the flavour of the other \( B \) hadron at decay, but the selected \( B_d^0 \) had oscillated before decaying.

In contrast, if the sign of the other side tag and the sign of the kaon from the decay of the \( K^{*0} \) are identical, the like sign result, then again there are two possible explanations:
• The other side tag wrongly identified the flavour of the other $B$ hadron at decay, and hence the flavour of the $B_{d}^{0}$.

• The other side tag correctly identified the flavour of the other $B$ hadron at decay, but the selected $B_{d}^{0}$ had oscillated before decaying.

The mixing of neutral $B$ mesons was discussed in detail in Chapter 1. The time-dependent probability for an initial $B_{d}^{0}$ to remain a $B_{d}^{0}$ as a function of time can be written, neglecting direct CP violation, as

$$R_{B_{d}^{0} \rightarrow B_{d}^{0}} = K(t)(1 + \cos \Delta m t)e^{-\Gamma t}, \quad (5.13)$$

and the probability for a $B_{d}^{0}$ to oscillate to a $\overline{B}_{d}^{0}$ is

$$R_{B_{d}^{0} \rightarrow \overline{B}_{d}^{0}} = K(t)(1 - \cos \Delta m t)e^{-\Gamma t}, \quad (5.14)$$

where $K(t)$ indicates a time-dependent term identical for both, $t$ is the proper time, $\Delta m$ is the mass difference of the mass eigenstates and $\Gamma$ is the decay width.

Similarly, for an initial $\overline{B}_{d}^{0}$, the probability to remain a $\overline{B}_{d}^{0}$ as a function of time is

$$R_{\overline{B}_{d}^{0} \rightarrow \overline{B}_{d}^{0}} = K(t)(1 + \cos \Delta m t)e^{-\Gamma t}, \quad (5.15)$$

and the probability for a $\overline{B}_{d}^{0}$ to oscillate to a $B_{d}^{0}$ is

$$R_{\overline{B}_{d}^{0} \rightarrow B_{d}^{0}} = K(t)(1 - \cos \Delta m t)e^{-\Gamma t}. \quad (5.16)$$

Using these results, the probability that the sign of the other side tag and the sign of the kaon from the decay of the $K^{*0}$ are like or unlike can be constructed. This gives for the number of unlike sign events, $N_{\text{unlike}}(t)$:
\[ N_{\text{unlike}}(t) \]
\[ = [\text{right tag and no oscillation}] \ OR [\text{wrong tag and oscillation}] \]
\[ = \left[ (1 - \omega)R_{B_d^0 \to B_d^0} + (1 - \omega)R_{B_d^0 \to B_d^0} \right] + \left[ \omega R_{B_d^0 \to B_d^0} + \omega R_{B_d^0 \to B_d^0} \right] \]
\[ = 2(1 - \omega)K(t)(1 + \cos \Delta mt)e^{-\Gamma t} + 2\omega \frac{K(t)(1 - \cos \Delta mt)e^{-\Gamma t}}{1 - 2\omega}. \]

(5.17)

If the signs are the same, the appropriate expression is then:

\[ N_{\text{like}}(t) \]
\[ = [\text{wrong tag and no oscillation}] \ OR [\text{right tag and oscillation}] \]
\[ = \left[ \omega R_{B_d^0 \to B_d^0} + \omega R_{B_d^0 \to B_d^0} \right] + \left[ (1 - \omega)R_{B_d^0 \to B_d^0} + (1 - \omega)R_{B_d^0 \to B_d^0} \right] \]
\[ = 2\omega \frac{K(t)(1 + \cos \Delta mt)e^{-\Gamma t}}{1 - 2\omega} + 2(1 - \omega)K(t)(1 - \cos \Delta mt)e^{-\Gamma t}. \]

(5.18)

A ratio, Equation 5.19, is constructed to cancel out the terms common to the case where the signs agree, and where they disagree. In this way the mistag fraction, \( \omega \), can be measured directly from the amplitude of the \( B_d^0 \) oscillations.

\[ \frac{N_{\text{unlike}}(t) - N_{\text{like}}(t)}{N_{\text{unlike}}(t) + N_{\text{like}}(t)} = (1 - 2\omega)\cos \Delta mt. \]

(5.19)

From the \( B_d^0 \to J/\Psi(\mu\mu)K^{*0} \) events selected and tagged, a plot can be made of this ratio as a function of the proper time of the reconstructed and selected
$B_d^0$, as shown in Figure 5.16. Only the 73892 events tagged with the other side
tagging algorithms are included, 80% of the total number of tagged events. The
variable $\Delta m$ is fixed to the value $\Delta m = 500$ ns$^{-1}$, taken from the Particle Data
Group world average [E+04]. The mistag fraction is then measured by fitting
Equation 5.19, shown in blue in Figure 5.16, giving a value for the mistag fraction
$\omega = 36.5 \pm 0.2$. This agrees with the mistag fraction taken from the Monte Carlo
truth information, $\omega = 36.09 \pm 0.25$.

An exactly analogous method can be used to measure the mistag fraction for
the same side tagging algorithms, taking into account the change in sign of the
tagging particle. For the same side tag, the nearest pion in the fragmentation
chain has a positive charge when the selected $B_d^0$ at production contains a $\bar{b}$
quark. Hence, when comparing this particle with the charge of the kaon from the
decay of the self-tagging $B_d^0$, in this case also positive, the signs are in agreement
when the tagging algorithms have correctly identified the flavour of the selected
$B$ at production, or when the tag is incorrect, but the selected $B_d^0$ oscillated
before decaying, designated $N_{like}^s(t)$. Then Equation 5.17 and Equation 5.18
above become, for the same side tagging algorithm,

\[
N_{like}^s(t) \quad \text{for same side tag}
\]

\[
= [\text{right tag and no oscillation}] \ OR \ [\text{wrong tag and oscillation}]
\]

\[
N_{unlike}^s(t) \quad \text{for same side tag}
\]

\[
= [\text{wrong tag and no oscillation}] \ OR \ [\text{right tag and oscillation}].
\]

(5.20)
The combined ratio, for the other side tags and the same side tag, can be written

\[
\left[ N_{\text{unlike}}(t) + N_{\text{like}}^S(t) \right] - \left[ N_{\text{like}}(t) + N_{\text{unlike}}^S(t) \right] \\
\left[ N_{\text{unlike}}(t) + N_{\text{like}}^S(t) \right] + \left[ N_{\text{like}}(t) + N_{\text{unlike}}^S(t) \right] = (1 - 2\omega)\cos \Delta mt,
\]

where \( N(t) \) is the number of events tagged with the other side tagging algorithms, and \( N^S(t) \) is the number of events tagged with the same side tag. Plotting this ratio for the 92640 \( B_d^0 \to J/\Psi(\mu\mu)K^{*0} \) events selected and tagged in this thesis, and fitting Equation 5.21 indicates the overall mistag fraction is \( \omega = 38.3 \pm 0.1 \). This agrees with the mistag fraction calculated from the Monte Carlo truth, \( \omega = 37.99 \pm 0.21 \), quoted in Table 5.8. This illustrates the method to measure the mistag fraction for the tagging algorithms in real data using the self-tagging control channels. The measurement in real data will be affected by other systematic errors, for example due to the proper time resolution, background and acceptance.

### 5.9 Annual Signal Yield

It is possible to extrapolate from the statistics simulated in this study to the expected number of signal events after one year of data collection at LHCb. The effect on the errors introduced by the weighting method developed in this thesis can then be understood. In addition, the statistical error on the measurement of the CP violating asymmetry due to the tagging can be calculated from the error on the mistag fraction and the tagging efficiency, as discussed in Section 5.1.1.3. The annual yield of tagged signal events has been estimated by the LHCb collaboration [Reo03]. Table 5.13 lists these yields for the decays considered in this study, along with the number of simulated tagged events which were part of this
Figure 5.16: The ratio constructed by comparing the outcome of the other side tagging algorithms with the self-tagging given by the decay of the the $K^{*0}$, for the control channel $B^0_d \rightarrow J/\Psi(\mu \mu)K^{*0}$. The probability of the two tags agreeing or disagreeing is dependent on the the mistag fraction, and the probability that the $B^0_d$ oscillates before decaying. Details are given in the text.

analysis. The number of selected & tagged is significantly lower than the expected annual yield of tagged events for both the $B^0_s$ control channel, $B^0_s \rightarrow D_s \pi$, and the $B^0_d$ control channel, $B^0_d \rightarrow J/\Psi(\mu \mu)K^{*0}$.

The fractional error on the weighted mistag fraction was derived, given by Equation 5.12. To understand the effect on the fractional error of a different number of tagged events in the signal and control channels, the contributions to Equation 5.12 can be scaled appropriately. Hence the scaling is applied to $N_i$ and to $\delta \omega_i$; the statistical error on $N_i$ is simply $\sqrt{N_i}$, whereas the statistical error on $\delta \omega_i$ scales as $1/\sqrt{N_i^{control}}$, where $N_i^{control}$ is the number of control channel events in the ith bin.

This scaling process was applied to each weighting calculation described in this study, so that for each CP channel the scaled error on the weighted mistag fraction from the control channel was recalculated. This was then used to calculate the
<table>
<thead>
<tr>
<th>channel</th>
<th>annual yield of tagged events</th>
<th>selected &amp; tagged</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0_s \rightarrow D_s \pi$</td>
<td>80 k</td>
<td>34 k</td>
</tr>
<tr>
<td>$B^0_s \rightarrow D_s K$</td>
<td>5.4 k</td>
<td>36 k</td>
</tr>
<tr>
<td>$B^0_s \rightarrow J/\Psi(\mu\mu)\Phi$</td>
<td>100 k</td>
<td>101 k</td>
</tr>
<tr>
<td>$B^0_d \rightarrow J/\Psi(\mu\mu)K^{*0}$</td>
<td>670 k</td>
<td>74 k</td>
</tr>
<tr>
<td>$B^0_d \rightarrow J/\Psi(\mu\mu)K_s$</td>
<td>216 k</td>
<td>20 k</td>
</tr>
<tr>
<td>$B^0_d \rightarrow \pi\pi$</td>
<td>26 k</td>
<td>43 k</td>
</tr>
</tbody>
</table>

Table 5.13: The annual yield of events expected in each channel, compared to the number of simulated events available for this study.

<table>
<thead>
<tr>
<th>channel</th>
<th>combined relative error on the asymmetry $\mathcal{A}_{\text{meas}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0_s \rightarrow J/\Psi(\mu\mu)\Phi$</td>
<td>9%</td>
</tr>
<tr>
<td>$B^0_s \rightarrow D_s K$</td>
<td>6%</td>
</tr>
</tbody>
</table>

Table 5.14: The estimated relative error on the measured asymmetry, using the weighting method to estimate the mistag fraction, for selected $B^0_s$ channels after one year of data taking.

The combined relative error on the mistag fraction, from the contribution of each trigger subsample. The result is the combined relative error on the measured asymmetry using the weighting method, after one year of LHCb data taking.

The results for the $B^0_s$ channels are given in Table 5.14, where the mistag fraction is estimated by weighting the control channel $B^0_s \rightarrow D_s \pi$, and for the $B^0_d$ channels in Table 5.15, where the control channel is $B^0_d \rightarrow J/\Psi(\mu\mu)K^{*0}$.

<table>
<thead>
<tr>
<th>channel</th>
<th>combined relative error on the asymmetry $\mathcal{A}_{\text{meas}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0_d \rightarrow \pi\pi$</td>
<td>3%</td>
</tr>
<tr>
<td>$B^0_d \rightarrow J/\Psi(\mu\mu)K_s$</td>
<td>1%</td>
</tr>
</tbody>
</table>

Table 5.15: The estimated relative error on the measured asymmetry, using the weighting method to estimate the mistag fraction, for selected $B^0_d$ channels after one year of data taking.
5.10 Conclusions

The accuracy of the CP asymmetry measurements planned for LHCb will be affected by systematic errors, caused by the triggering, selection and tagging of events. In particular, for decays involving neutral $B$ mesons, the number of wrongly tagged events contributes directly to the error on the measurement of the CP asymmetry. It is crucial that this mistag fraction can be measured in real data.

A strategy has been developed to estimate the mistag fraction for several CP channels from a suitable self-tagging control channel. This was demonstrated for the CP channels $B_s^0 \rightarrow D_s K$ and $B_s^0 \rightarrow J/\Psi(\mu\mu)\Phi$ using the control channel $B_s^0 \rightarrow D_s \pi$. A similar study was carried out on the $B_d^0$ CP channels, $B_d^0 \rightarrow J/\Psi(\mu\mu)K_s$ and $B_d^0 \rightarrow \pi\pi$, using the control channel $B_d^0 \rightarrow J/\Psi(\mu\mu)K^{*0}$, but the results were limited by the Monte Carlo statistics available. The study presented in this thesis was performed using all the available Monte Carlo data for the channels considered, generated with the full LHCb simulation. The results are encouraging, suggesting this strategy for measuring the mistag fraction in real data should be pursued.

Throughout this study the baseline LHCb combined tagging algorithm was used to assess the tagging performance. This maximised the statistics available for the tests of the weighting method. In future studies, when more statistics are available, a more powerful approach would be to treat the result of each tagging algorithm as an independent subsample of events. In this case, it would be possible to individually choose control channels suitable for each tagging algorithm. For example, the control channel $B_d^0 \rightarrow K^+\pi^-$ could be used to estimate the mistag fraction for the other side tagging algorithms only in the CP channel $B_d^0 \rightarrow \pi\pi$. 
Chapter 6

Conclusions

The LHCb experiment will study CP violation in the decays of $B$ mesons at the LHC. The particle identification capability provided by two RICH detectors will be vital to the success of these studies. This thesis describes several contributions to the development of the RICH detectors, as well as a study of how best to determine the systematic errors on the CP asymmetry measurements in neutral $B$ meson decays.

The RICH photon detectors, the HPDs, are a novel technology developed in collaboration with industry. It was important to design and implement a quality-control programme for each stage of the HPD production, as reported in Chapter 3, to ensure the final HPDs meet the strict requirements of the experiment. The performance of the HPD readout chip was validated by an electrical testing procedure which included a study of the variation of threshold and noise across the chip, before and after the silicon pixel detectors were bump-bonded to the readout wafers. Both the mean threshold and mean noise were observed to shift to higher values after bump-bonding, with an average shift in the mean threshold of $275\pm30$ e$^-$ and the mean noise of $32\pm5$ e$^-$ for the pre-production batch of anodes, where e$^-$ is the equivalent number of electrons. The pixel threshold and noise of the HPD anodes after bump-bonding were found
to exceed specifications. The quality-control programme included a test of the
tegrity of the anode bump-bonds. This test was applied to a representative
sample of anodes which had undergone accelerated ageing, designed to simulate
ten years of LHCb operation. No degradation in the performance of the anodes
was observed.

The performance of the HPDs was investigated in a beam test, using a
demonstrator RICH detector consisting of an aerogel radiator, a spherical mirror
and three prototype HPDs. In addition, this provided vital information on the
Cherenkov angle resolution and photon yield that could be achieved with the
aerogel radiator. The measurements presented in Chapter 4 of the Cherenkov
angle resolution and photon yield meet the requirements for the LHCb detector.
The resolution in data was degraded with respect to the expected resolution
from the Monte Carlo simulation. The typical sigma of a Gaussian fit to the
Cherenkov angle distribution in data was 3.2 mrad, whereas in Monte Carlo it
was 2.4 mrad. This discrepancy was due to geometrical misalignments of the
HPDs, uncertainties in the electron optics corrections and the parameterisation
of the aerogel dispersion curve, highlighting the need for a good understanding
of these experimental parameters.

A number of neutral $B$ meson decay modes provide the opportunity for pre-
cision measurements of the CP violating observables, the phases of the CKM
matrix, through the measurement of time-dependent decay asymmetries. Ex-
perimentally this requires a knowledge of the flavour of the $B^0$ at production,
achieved by tagging the $B^0$. The tag provides an estimate of the flavour of the
selected $B^0$ in the decay of interest, and can be provided from the decay products
of this $B^0$ or from the decay of the other $B$ in the event. Experimental errors
in the tagging give rise to a mistag fraction which directly contributes to the
systematic error on the asymmetry measurement. Therefore, this mistag fraction must be reliably measured in order to understand the impact on the CP violation measurement, for example in comparison with the statistical precision.

The mistag fraction in data will be determined from self-tagging control channels, where the charge of one of the final-state particles unambiguously determines the flavour of the $B$. However, the assumption that this measurement is also valid for the CP channel where the asymmetry measurement is made is actually not trivial. Differences in the triggering and selection of the two channels can affect the tagging performance, and thus the mistag fraction. In Chapter 5 the effects of trigger bias were investigated for the channel $B_s^0 \rightarrow D_s \pi$, providing a control for the CP channels $B_s^0 \rightarrow D_s K$ and $B_s^0 \rightarrow J/\Psi (\mu \mu) \Phi$, and the channel $B_d^0 \rightarrow J/\Psi (\mu \mu) K^* 0$, providing a control for the CP channels $B_d^0 \rightarrow J/\Psi (\mu \mu) K_s$ and $B_d^0 \rightarrow \pi \pi$. Using Monte Carlo data, a strategy was developed to estimate the mistag fraction in real data by correcting for the systematic bias introduced by the trigger. Using the transverse momentum distribution of the selected $B$, the control channel distribution could be weighted to estimate the mistag fraction in the CP channel. The strategy was demonstrated for the $B_s^0$ channels. For example, for events triggered on the decay products of the selected $B_s^0$ at Level-0 and Level-1, the mistag fraction in the $B_s^0 \rightarrow D_s \pi$ control channel was $33.66 \pm 0.73$ and in the CP channel $B_s^0 \rightarrow J/\Psi (\mu \mu) \Phi$ the mistag fraction was $36.25 \pm 0.28$. After applying the weighting method, the estimate of the CP channel mistag rate was $34.84 \pm 1.06$, thus consistent within errors with the true mistag fraction. The strategy was also applied to the $B_d^0$ channels, where the results were limited by the Monte Carlo statistics available. Nevertheless, the method to control systematic bias was shown to be promising and a further programme of work was suggested.
Bibliography


[DEP] Delft Electronic Products B.V., Dwazziewegen 2, P.O. Box 60, NL-9300 AB Roden, The Netherlands.


[VTT] VTT Information Technology, Microelectronic, P.O. Box 1208, FIN-02044 VTT, Finland.


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