Slow-light optical bullets in arrays of nonlinear Bragg-grating waveguides

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We demonstrate how to control independently both spatial and temporal dynamics of slow light. We reveal that specially designed nonlinear waveguide arrays with phase-shifted Bragg gratings demonstrate the frequency-independent spatial diffraction near the edge of the photonic bandgap, where the group velocity of light can be strongly reduced. We show in numerical simulations that such structures allow a great flexibility in designing and controlling dispersion characteristics, and open a way for efficient spatiotemporal self-trapping and the formation of slow-light optical bullets.

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Nonlinear response is the fundamental property of optical materials that leads to interaction of propagating optical waves and allows all-optical switching for various applications. Efficiency of nonlinear interactions can be greatly enhanced in the regime of light propagation with small group velocities [1]. The slow-light regime can be realized when the frequency is tuned close to the edge of a photonic bandgap where optical waves experience strong Bragg scattering from a periodically-modulated dielectric structure. These effects were studied extensively in the structures where the propagation direction is fixed by the waveguide geometry, including experimental observations of pulse propagation in fibers [2] and AlGaAs ridge waveguides [2] with Bragg gratings, or coupled-defect waveguides in photonic crystals with two- or three-dimensional modulation of the refractive index (see, e.g., Refs. [1, 2, 3, 4]). In particular, it was demonstrated that the group velocity and the corresponding pulse delay can be tuned all-optically in nonlinear photonic structures, and at the same time nonlinearity may compensate for pulse broadening due to dispersion [2].

All-optical control of the spatial beam dynamics becomes possible in periodic two- (2D) or three-dimensional (3D) photonic structures, where light can propagate in various directions. The possibility for beam steering is inherently linked to the effect of beam spreading due to diffraction. Similar to pulses, nonlinearity can suppress beam spreading and support the formation of nondiffracting spatial optical solitons [5]. Recent studies have emphasized many unique properties of spatial solitons in periodic structures such as waveguide arrays or photonic lattices [5, 6], where modulation in the transverse spatial dimension fundamentally affects the nonlinear wave dynamics. There appear multiple angular band gaps supporting the formation of spatial gap solitons, which possess tunable steering properties [11]. Recent experiments [11] demonstrated the effect of simultaneous spatial and temporal self-trapping of optical pulses in the form of optical light bullets [12].

In this Letter, we address an outstanding key problem and demonstrate how to perform dynamical tunability over both the magnitude and direction of the speed of light through all-optical control in the slow-light regime. In order to realize an effective nonlinear control, it is necessary to balance the effects of temporal dispersion in the propagation and spatial diffraction in the transverse directions. For this purpose, instead of usually studied symmetric photonic crystals [13], here we suggest to consider the photonic structures with multiple-scale modulations in different directions. This overcomes the limitations of the previously studied 2D or 3D defect-free...
photonic-crystal structures where diffraction usually increases for smaller group velocities due to a shrinkage of the isofrequency contours [14], thus greatly restricting the efficiency of spatial self-focusing of slow light. Indeed, only quasi-localized nonlinear states with large spatial extent were predicted for a slab 2D waveguide with a 1D Bragg grating [12]. In a sharp contrast, in this Letter we show that it is possible to realize the frequency-independent diffraction near the edge of the bandgap in specially designed Bragg-grating waveguide arrays. This allows to engineer independently the strength of diffraction and dispersion in the slow-light regime, and thus provides optimal conditions for the nonlinear control of spatiotemporal dynamics. In particular, we predict and demonstrate numerically the formation of strongly localized slow-light optical bullets in such structures.

The arrays of nonlinear optical waveguides which are homogeneous in the propagation direction have been extensively studied in recent years, and many possibilities for spatial beam control were demonstrated experimentally [8,9]. We reveal that new regimes of the spatiotemporal dynamics can be achieved in the waveguide arrays modulated in the propagation direction with the period satisfying the Bragg-resonance condition at the operating frequency. Such structures can be fabricated in AlGaAs samples, with accessible fast nonlinearity.

In the vicinity of the Bragg resonance, the evolution of optical pulses can be modeled by a set of coupled-mode nonlinear equations [10] for the slowly varying envelopes of the forward ($u_n$) and backward ($v_n$) propagating fields in each of $n$-th waveguide. In the normalized form, these equations can be written as follows,

\begin{equation}
\begin{aligned}
    i\frac{\partial u_n}{\partial t} + i\frac{\partial u_n}{\partial z} + C(u_{n-1} + u_{n+1}) &+ \rho_n u_n + \gamma(|u_n|^2 + 2|v_n|^2)u_n = 0, \\
    i\frac{\partial v_n}{\partial t} - i\frac{\partial v_n}{\partial z} + C(v_{n-1} + v_{n+1}) &+ \rho_n^* u_n + \gamma(|v_n|^2 + 2|u_n|^2)v_n = 0,
\end{aligned}
\end{equation}

where $t$ and $z$ are the dimensionless time and propagation distance, respectively, $C$ is the coupling coefficient for the modes of the neighboring waveguides, $\rho_n$ characterizes the efficiency of scattering from the Bragg grating, $\gamma$ is the nonlinear coefficient, and the group velocity far from the Bragg resonance is normalized to unity.

We reveal that both diffraction and dispersion can be precisely tailored by introducing a phase shift between the otherwise equivalent waveguide gratings, as illustrated in Figs. 1(a,b). Only two- and three-waveguide nonlinear couplers with in-phase gratings were analyzed before [10,13]. Here, we consider the effect of linear phase shift of the gratings across the array characterized by the scattering coefficients $\rho_n = \rho \exp(-i\varphi n)$. With no loss of generality, we can take $\rho$ to be real and positive. Then, the wave propagation in the linear regime (at small intensities) can be fully defined through the properties of Floquet-Bloch eigenmodes,

\begin{equation}
\begin{aligned}
    u_n = u_0 \exp[iK n + i\beta z - i\omega t], \\
    v_n = v_0 \exp[i(K + \varphi)n + i\beta z - i\omega t],
\end{aligned}
\end{equation}

where $K$ and $\beta$ are the transverse and longitudinal components of the Bloch wavevector. After substituting Eqs. (2) into the linearized equations (1) (with $\gamma = 0$), we obtain the following relations,

\begin{equation}
\begin{aligned}
    u_0[\omega - \beta + 2C \cos(K)] + v_0 \rho = 0, \\
    u_0\rho + v_0[\omega + \beta + 2C \cos(K + \varphi)] = 0.
\end{aligned}
\end{equation}

These are the eigenmode equations which define the dispersion properties of the Bloch waves, $\omega(K, \beta)$. Since Eqs. (3) represent a square $2 \times 2$ matrix, there will appear two branches of the dispersion curves. These dependencies determine the key features of the wave spectrum. First, the gaps may appear for a certain frequency range, where the propagating waves with real $K$ and $\beta$ are absent. We notice that quasi-2D spectral gaps can indeed appear for the modes localized in the high-index waveguides, i.e. below the light-line of the substrate. Second, the spatial beam refraction and diffraction are defined by the shape of the corresponding isofrequency contours. The propagation angle is defined by the value of $\alpha(\omega, K) = -\partial\beta / \partial K$, and the effective diffraction experienced by the beam depends on the curvature, $D(\omega, K) = -\partial^2 \beta / \partial K^2$. Below, we analyze how these fundamental characteristics can be controlled by selecting the phase shift $\varphi$ between the Bragg gratings.

For the in-phase gratings [i.e., when $\varphi = 0$; see Fig. 1(a)], the dispersion relation becomes $\omega(K, \beta) = -2C \cos(K) \pm (\rho^2 + \beta^2)^{1/2}$. It follows that the 2D gap appears only when the waveguide coupling is weak, $C < \rho/2$. On the other hand, the shape of the isofrequency contours is strongly frequency-dependent as the transmission band edge is approached, see Fig. 1(c). This happens because the position of the one-dimensional frequency gap depends on the propagation direction.

We find that the nature of wave dispersion is fundamentally altered for the waveguide structure with out-of-phase shift of the neighboring gratings [i.e., when $\varphi = \pi$; see Fig. 1(b)]. The corresponding dispersion relation has a different form, $\omega(K, \beta) = \pm(\rho^2 + [\beta - 2C \cos(K)]^2)^{1/2}$. Moreover, for any propagation angle defined by the transverse Bloch wavevector component $K$, the width and position of the one-dimensional frequency gap remains the same, $|\omega| < \rho$. This unusual property leads to remarkable spectral features. First, the 2D (quasi-)gap is always present in the spectrum irrespectively to the grating strength ($\rho$) and coupling between the waveguides ($C$). Second, the shape of isofrequency contours does not depend on frequency in the transmission band, see
FIG. 2: (colour online) Family of the stationary (zero-velocity) light bullets characterized by (a) energy and (b) width along the transverse (solid) and longitudinal (dashed) directions vs. the frequency tuning inside the spectral gap. (c-d) Intensity profiles of slow-light optical bullets for (c) $\omega = 0.8$ and (d) $\omega = 0.995$; brighter shading marks higher intensity.

FIG. 3: (colour online) Moving slow-light optical bullets with the frequency detuning $\omega = 0.5$. Notations are the same as in Fig. 2 and the velocity is normalized to the speed of light away from the Bragg resonance. Profiles are shown for the velocities (c) $V = 0.1$ and (d) $V = 0.2$. This means that the beam diffraction remains the same even when the band edge is approached.

The dependence of diffraction on frequency strongly affects the shaping of short pulses with broad frequency spectra. We perform numerical modeling of the pulse dynamics, when the input beam is focused into a single waveguide. The pulse duration is $T = 10$, and the central frequency is $\omega_0 = 1$. The snapshots of the light intensity after the pulse enters the photonic lattice are shown in Figs. 3(e,f). For the structure with the in-phase gratings [Figs. 3(e)], the spatial diffraction strongly depends on frequency, creating a colored pattern similar to the superprism effect in photonic crystals [19]. For the phase-shifted gratings, all the components inside the frequency band experience exactly the same diffraction and propagate together. As a result, the intensity profile [Figs. 3(e)] has precisely the same shape as for the discrete diffraction of monochromatic beams [8], with well-pronounced field minima and peak intensities at the outer wings.

The unique features of linear spectrum in arrays with phase-shifted gratings suggest that these structures provide optimal conditions for a nonlinear control of the pulse dynamics. In particular, since the 2D gap appears for any values of the grating strength and waveguide coupling, it is possible to choose these parameters independently in order to balance the rates of dispersion and diffraction. This allows for simultaneous compensation of the pulse broadening in space and time through the nonlinear self-trapping effect. Indeed, we find numerically localized solutions of Eqs. (1) for stationary and moving light bullets of the form of solitary waves $\{u, v\}_n(z, t) = \{\tilde{u}, \tilde{v}\}_n(z - V t) \exp(-i \omega t)$, where $V$ is the propagation velocity. We confirm that localization is possible in both the cases of positive ($\gamma = +1$) and negative ($\gamma = -1$) nonlinear response, since anomalous or normal dispersion regimes can be accessed on either edges of the photonic bandgap. The soliton solutions for different signs of $\gamma$ can be mapped by changing $\omega \to -\omega$ and making a corresponding transformation of pulse profiles. In Fig. 4 we show the characteristics of immobile solitons such as energy and width vs. the frequency detuning inside the bandgap for self-focusing nonlinearity. These solitons have a well-pronounced X-shape near the upper edge of the gap, becoming more localized inside the gap as the pulse energy is increased. We notice however that these are fully localized states that should not be confused with the so-called X-waves [20] which remain quasi-localized only over a finite propagation distance. The gap solitons can also propagate along the waveguides, and we present the characteristic of moving solitons in Fig. 5. These slow-light bullets become more extended in both the transverse and longitudinal directions as the propagation velocity is increased.

Finally, we perform numerical simulations of the spatiotemporal pulse dynamics in these structures. In the linear regime, the pulse broadens in both transverse and
FIG. 4: Snapshots of field intensities for an optical pulse propagating in a waveguide array structure show in Fig. 1(d): (a-d) Linear broadening due to spatial diffraction and temporal dispersion; (e-h) Nonlinear self-trapping in space and time and formation of an optical bullet.

longitudinal directions [Figs. 4(a-d)]. Nonlinear self-action results in the pulse self-trapping in both space and time [Figs. 4(e-h)]. In this example, the velocity of the generated light bullet is below 30% of the speed of light in the absence of the Bragg grating, and smaller velocities can be accessed as well by controlling the frequency of the input pulse. The profile of the self-trapped state shown in Fig. 4(h) closely resembles the shape of the exact soliton solution shown in Fig. 3(d).

In conclusion, we have revealed that both spatial diffraction and temporal dispersion can be engineered independently near the edge of the photonic bandgap in the waveguide arrays with phase-shifted Bragg gratings. We have shown that such structures possess quasi-two-dimensional photonic bandgaps and provide optimal conditions for self-localization of pulses in nonlinear media.

We have demonstrated that these waveguide arrays can be employed for shaping and control of optical pulses simultaneously in space and time, and allow the formation of optical bullets propagating with slow group velocities. Such slow-light optical bullets offer novel possibilities for all-optical switching, steering, and control of short pulses.

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