Standard Model expectations on $\sin 2\beta(\phi_1)$ from $b \to s$ penguins

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Recent results of the standard model expectations on $\sin 2\beta_{\text{eff}}$ from penguin-dominated $b \to s$ decays are briefly reviewed.

I. INTRODUCTION

Although the Standard Model is very successful, New Physics is called for in various places, such as neutrino-oscillation, dark matter (energy) and baryon-asymmetry. Possible New Physics beyond the Standard Model is being intensively searched via the measurements of time-dependent CP asymmetries in neutral $B$ meson decays into final CP eigenstates defined by

$$\frac{\Gamma(B(t) \to f) - \Gamma(B(t) \to f)}{\Gamma(B(t) \to f) + \Gamma(B(t) \to f)} = S_f \sin(\Delta m t) + A_f \cos(\Delta m t),$$

where $\Delta m$ is the mass difference of the two neutral $B$ eigenstates, $S_f$ monitors mixing-induced CP asymmetry and $A_f$ measures direct CP violation. The CP-violating parameters $A_f$ and $S_f$ can be expressed as

$$A_f = -\frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad S_f = \frac{2 \text{Im} \lambda_f}{1 + |\lambda_f|^2},$$

where

$$\lambda_f^2 = \frac{g_B^2 A(B^0 \to f)}{p_B^2 A(B^0 \to f)}.$$

In the standard model $\lambda_f \approx \eta_i e^{-2i\beta}$ for $b \to s$ penguin-dominated or pure penguin modes with $\eta_i = 1$ ($-1$) for final $CP$-even (odd) states. Therefore, it is expected in the Standard Model that $-\eta_i S_f \approx \sin 2\beta$ and $A_f \approx 0$ with $\beta$ being one of the angles of the unitarity triangle.

The mixing-induced CP violation in $B$ decays has been already observed in the golden mode $B^0 \to J/\psi K_S$ for several years. The current world average the mixing-induced asymmetry from tree $b \to c\bar{c}s$ transition is

$$\sin 2\beta = 0.687 \pm 0.032.$$

However, the time-dependent $CP$ asymmetries in the $b \to sqq\bar{q}$ induced two-body decays such as $B^0 \to (\phi, \omega, \pi^0, \eta', f_0)K_S$ are found to show some indications of deviations from the expectation of the Standard Model (SM) (see Fig. 1). In the SM, CP asymmetry in all above-mentioned modes should be equal to $S_{J/\psi K}$ with a small deviation at most $O(0.1)$ [2]. As discussed in [2], this may originate from the $O(\lambda^2)$ truncation and from the subdominant (color-suppressed) tree contribution to these processes. Since the penguin loop contributions are sensitive to high virtuality, New Physics beyond the SM may contribute to $S_f$ through the heavy particles in the loops. In order to detect the signal of New Physics unambiguously in the penguin $b \to s$ modes, it is of great importance to examine how much of the deviation of $S_f$ from $S_{J/\psi K}$,

$$\Delta S_f \equiv -\eta_i S_f - S_{J/\psi K},$$

is allowed in the SM.

The decay amplitude for the pure penguin or penguin-dominated charmless $B$ decay in general has the form

$$M(B^0 \to f) = V_{ub}^* V_{us}^* F^u + V_{cb}^* V_{cs}^* F^c + V_{tb}^* V_{ts}^* F^t.$$ (6)

![FIG. 1: Experimental results for $\sin 2\beta_{\text{eff}}$ from $b \to s$ penguin decays](image)
Unitarity of the CKM matrix elements leads to
\[
M(B^0 \rightarrow f) = V_{ub} V_{us} A_f^u + V_{cb} V_{cd} A_f^c \\
\approx A_f^u R_b e^{-i \gamma_f} + \lambda^2 A_f^c,
\] (7)
where \( A_f^u = F^u - F^t \), \( A_f^c = F^c - F^t \), \( R_b \equiv |V_{ub} V_{us}|/(V_{cd} V_{cb}) | = \sqrt{\rho^2 + \eta^2} \). The first term is suppressed by a factor of \( \lambda^2 \) relative to the second term. For a pure penguin decay such as \( B^0 \rightarrow \phi K^0 \), it is naively expected that \( A_f^u \) is in general comparable to \( A_f^c \) in magnitude. Therefore, to a good approximation \( -\gamma_f S_f \approx \sin 2\beta \approx S_f/\omega K \). For penguin-dominated modes such as \( \omega K_S, \rho^0 K_S, \pi^0 K_S \), \( A_f^u \) also receives tree contributions from the \( b \rightarrow u d u s \) tree operators. Since the Wilson coefficient for the penguin operator is smaller than the one for the tree operator, \( A_f^u \) could be significantly larger than \( A_f^c \). As the first term carries a weak phase \( \gamma \), it is possible that \( S_f \) is subject to a significant “tree pollution”. To quantify the deviation, it is known that to the first order in \( r_f \equiv (\lambda_u A_f^u)/(\lambda_c A_f^c) \) \cite{2,10},
\[
\Delta S_f = 2 |r_f| \cos 2\beta \sin \gamma \cos \delta_f, \quad A_f = 2 |r_f| \sin \gamma \sin \delta_f,
\]
with \( \delta_f = \arg(A_f^u/A_f^c) \). Hence, the magnitude of the CP asymmetry difference \( \Delta S_f \) and direct CP violation are both governed by the size of \( A_f^u/A_f^c \). However, for the aforementioned penguin-dominated modes, the tree contribution is color suppressed and hence in practice the deviation of \( S_f \) is expected to be small \cite{2}. It is useful to note that \( \Delta S_f \) is proportional to the real part of \( A_f^u/A_f^c \) as shown in the above equation.

Below \( \Sf \) will review the results of the SM expectations on \( \Delta S_f \) from short-distance and long-distance calculations. Recent reviews of results obtained from the SU(3) approach can be found in \cite{17}.

II. \( \Delta S_f \) FROM SHORT-DISTANCE CALCULATIONS

There are several QCD-based approaches in calculating hadronic B decays \cite{12,19,26}. \( \Delta S_f \) from calculations of QCDF \cite{8,10}, pQCD \cite{11}, SCET \cite{12} are shown in Table 1. The QCD calculations on \( PP, VV \) modes are from \cite{9,25}, while those in \( SP \) modes are from \cite{10}. It is interesting to note that (i) \( \Delta S_f \) are small and positive in most cases, while experimental central values for \( \Delta S_f \) are all negative, except the one from \( f_0 K_S \); (ii) QCDF and pQCD results agree with each other, since the main difference of these two approach is the (penguin) annihilation contribution, which hardly affects \( S_f \); (iii) The SCET results involve some non-perturbative contributions fitted from data. These contributions affect \( \Delta S_f \) and give results in the \( \eta K_S \) mode different from the QCDF ones.

It is instructive to understand the size and sign of \( \Delta S_f \) in the QCDF approach \cite{8}, for example. Recall that \( \Delta S_f \) is proportional to the real part of \( A_f^u/A_f^c \). We follow \( \bar{x} \) to denote a complex number \( x \) by \( |x| \) if \( \text{Re}(x) > 0 \). In QCDF the dominant contributions to \( A_f^u/A_f^c \) are basically given by \cite{8,21}:
\[
\begin{align*}
A_{f_0 K_S}^u & \sim \left[-(a_4^u + r_x a_6^u)\right] \sim -[P^u], \\
A_{\phi K_S}^u & \sim \left[-(a_4^u + r_x a_6^u)\right] \sim -[P^u], \\
A_{\phi K_S}^c & \sim \left[\frac{1}{3} a_4^c + r_x a_6^c\right] + |a_2^c R| \sim [P^u] + [C], \\
A_{\phi K_S}^c & \sim \left[\frac{1}{3} a_4^c + r_x a_6^c\right] + |a_2^c R| \sim [P^u] + [C], \\
A_{\phi K_S}^c & \sim \left[-(a_4^u + r_x a_6^u)\right] \sim -[P^u], \\
A_{\phi K_S}^c & \sim \left[-(a_4^u + r_x a_6^u)\right] \sim -[P^u], \\
A_{\eta K_S}^u & \sim \left[-(a_4^u + r_x a_6^u)\right] + |a_2^c R| \sim -[P^u], \\
A_{\eta K_S}^u & \sim \left[-(a_4^u + r_x a_6^u)\right] + |a_2^c R| \sim -[P^u], \\
A_{\eta K_S}^c & \sim \left[-(a_4^u + r_x a_6^u)\right] \sim -[P^u], \\
A_{\eta K_S}^c & \sim \left[-(a_4^u + r_x a_6^u)\right] \sim -[P^u],
\end{align*}
\]
where \( a_n^u \) are effective Wilson coefficients \cite{26}, \( r_x = O(1) \) are the chiral factors and \( R^{(u,v)} \) are (real and positive) ratios of form factors and decay constants.

From Eq.(8), it is clear that \( \Delta S_f > 0 \) for \( \phi K_S, \omega K_S, \pi^0 K_S \), while their \( \text{Re}(A_f^u/A_f^c) \) can only be positive. Furthermore, due to the cancellation between \( a_4 \) and \( r_x a_6 \) in the \( \omega K_S \) amplitude, the corresponding penguin contribution is suppressed. This leads to a large and positive \( \Delta S_{S_{\omega K_S}} \) as shown in Table 1. For the cases of \( \rho^0 K_S \) and \( \eta \rho K_S \), there are chances for \( \Delta S_f \) to be positive or negative. The different signs in front of \( |P| \) in \( \rho^0 K_S \) and \( \eta \rho K_S \) are originated from the second term of the wave functions \( (u \bar{u} \pm d \bar{d})/\sqrt{2} \) of \( \omega \) and \( \rho^0 \) in the \( B^0 \rightarrow \omega \) and \( B^0 \rightarrow \rho^0 \) transitions, respectively. The \( |P| \) in \( \rho^0 K_S \) is also suppressed as the one in \( \omega K_S \), resulting a negative \( \Delta S_{S_{\rho^0 K_S}} \). On the other hand, \( -|P| \) in \( \eta \rho K_S \) is not only unsuppressed (no cancellation in the \( a_4 \) and \( a_6 \) terms), but, in fact, is further enhanced due to the constructive interference.
of various penguin amplitudes. This enhancement is responsible for the large $\eta/K_S$ rate and also for the small $\Delta S_{\eta/K_S}$. 

### III. FSI Contributions to $\Delta S_f$

Evidence of direct CP violation in the decay $B^0 \rightarrow K^-\pi^+$ is now established, while the combined BaBar and Belle measurements of $B^0 \rightarrow \rho^\pm\pi^\mp$ imply a sizable direct CP asymmetry in the $\rho^+\pi^-$ mode. In fact, direct CP asymmetries in these channels are much bigger than expectations of many people and may be indicative of appreciable LD rescattering effects, in general, in $B$ decays. The possibility of final-state interactions in bringing in the possible tree production sources to $S_f$ are considered. Both $A_f^+$ and $A_f^-$ will receive long-distance tree and penguin contributions from rescattering of some intermediate states. In particular there may be some dynamical enhancement of light $u$-quark loop. If tree contributions to $A_f^+$ are sizable, then final-state rescattering will have the potential of pushing $S_f$ away from the naive expectation. Take the penguin-dominated decay $B^0 \rightarrow \omega K^0$ as an illustration. It can proceed through the weak decay $B^0 \rightarrow K^+\pi^-$ followed by the rescattering $K^+\pi^- \rightarrow \omega K^0$. The tree contribution to $B^0 \rightarrow K^+\pi^-$ is a pure penguin process at short distances, it does receive contributions via long-distance rescattering. Note that in addition to these charmless final states contributions, there are also contributions from charmful $D_s^{(*)}D^{(*)}$ final states, see Fig. 2. These final-state rescatterings provide the long-distance $u$- and $c$-penguin contributions.

An updated version of results in Table II are shown in Table III. Several comments are in order. (i) $\phi K_S$ and $\eta/K_S$ are the theoretical and experimental cleanest modes for measuring $\sin 2\beta_{\text{eff}}$ in these penguin modes. The constructive interference behavior of penguins in the $\eta/K_S$ mode is still hold in the LD case, resulting a tiny $\Delta S_{\eta/K_S}$. (ii) Tree contributions in $\omega K_S$ and $\rho^0 K_S$ are diluted due to the LD $c$-penguin contributions.

It is found that LD tree contributions are in general not large enough in producing sizable $\Delta S_f$, since their contributions are overwhelmed by LD $c$-penguin contributions from $D_s^{(*)}D^{(*)}$ rescatterings. On the other hand, while it may be possible to have a large $\Delta S_f$ from rescattering models that enhance the contributions from charmless states, a sizable direct CP violation will also be generated. Since direct CP violations are sensitive to strong phases generated from FSI, these approaches will also give a sizable direct CP violation at the same time when a large $\Delta S_f$ is produced. The present data on the $\phi K_S$ and $\eta/K_S$ modes do not support large direct CP violations in these modes. Consequently, it is unlikely that FSI will enlarge their $\Delta S_f$. In order to constrain or to
IV. $\Delta S_f$ IN KKK MODES

$B^0 \rightarrow K^+ K^- K_{S,L}$ and $B^0 \rightarrow K_S K_S K_S$ are penguin-dominated and pure penguin decays, respectively. They are also used to extract sin$2\beta_{\text{eff}}$ with results shown in Fig. 1. Three-body modes are in general more complicated than two-body modes. For example, while the $K_S K_S K_S$ mode remains as a CP-even mode, the $K^+ K^- K_{S(L)}$ mode is not a CP-eigenstate [27]. Furthermore, the mass spectra of these modes are in general complicated and non-trivial.

A factorization approach is used to study these KKK modes [14]. In the factorization approach, the $B^0 \rightarrow K^+ K^- K_S$ amplitude, for example, basically consists of two factorized terms: $(B^0 \rightarrow K_S) \times (0 \rightarrow K^+ K^-)$ and $(B^0 \rightarrow K^+ K_S) \times (0 \rightarrow K^-)$, where $(A \rightarrow B)$ denotes a $A \rightarrow B$ transition matrix element. The dominant contribution is from the $(B^0 \rightarrow K_S) \times (0 \rightarrow K^+ K^-)$ term, which is a penguin induced term, while the sub-leading $(B^0 \rightarrow K^+ K_S) \times (0 \rightarrow K^-)$ term contains both tree and penguin contributions. In fact, $B^0 \rightarrow K^+ K_S$ transition is a $b \rightarrow u$ transition, which has a color allowed tree contribution.

Results of CP asymmetries for these modes are given in Table III. The first uncertainty is from hadronic parameter in $B^0 \rightarrow K^+ K^- K_{S,L}$ transition in $K^+ K^- K_{S,L}$ mode (and a similar term in $K_S K_S K_S$ mode), the second uncertainty is from other hadronic parameters, while the last uncertainty is from the uncertainty in $\gamma$.

To study $\Delta S_f$ and $A_f$, it is crucial to know the size of the $b \rightarrow u$ transition term ($A_f^u$). For the pure-penguin $K_S K_S K_S$ mode, the smallness of $\Delta S_{K_S K_S K_S}$ and $A_{K_S K_S K_S}$ can be easily understood. For the $K^+ K^- K_S$ mode, there is a $b \rightarrow u$ transition in the $\langle B^0 \rightarrow K^+ K_S \rangle \otimes \langle 0 \rightarrow K^- \rangle$ term. It has the potential of giving large tree pollution in $\Delta S_{K^+ K^- K_S}$. It requires more efforts to study the size and the impact of this term.

It is important to note that the $b \rightarrow u$ transition term in the $K^+ K^- K_S$ mode is not a CP self-conjugated term, since under a CP conjugation, this term will be turned into a $\langle B^0 \rightarrow K^- K_S \rangle \times \langle 0 \rightarrow K^+ \rangle$ term, which is, however, missing in the original amplitude. Hence, this term contributes to both CP-even and CP-odd configurations with similar strength. Therefore, information in the CP-odd part can be used to constrain its size and its impact on $\Delta S_f$ and $A_f$. Indeed, it is found recently [24] that the CP-odd part is highly dominated by $\phi K_S$, where other contributions (at $m_{K^+ K^-} \neq m_{\phi}$) are highly suppressed. Since the $\langle B^0 \rightarrow K^+ K_S \rangle \times \langle 0 \rightarrow K^- \rangle$ term favors a large $m_{K^+ K^-}$ region, which is clearly separated from the $\phi$-resonance region, the result of the CP-odd configuration strongly constrains the contribution from this $b \rightarrow u$ transition term. Consequently, the tree pollution is constrained and the $\Delta S_{K^+ K^- K_S}$ should not be large. Note that results shown in Table III were obtained without fully incorporating these information. The first uncertainty in Table III will be reduced, if the CP-odd result is taken into account. To further refine the results it will be very useful to perform a detail Dalitz-plot analysis.

TABLE III: Mixing-induced and direct CP asymmetries $\Delta S_f$ (top) and $A_f$ (in %, bottom), respectively, in $B^0 \rightarrow K^+ K^- K_S$ and $K_S K_S K_S$ decays. Results for $(K^+ K^- K_L)_{CP\pm}$ are identical to those for $(K^+ K^- K_S)_{CP\pm}$.

<table>
<thead>
<tr>
<th>Final State</th>
<th>$\Delta S_f$</th>
<th>Expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(K^+ K^- K_S)_{\phi K_S}$ excluded</td>
<td>$0.03^{+0.08+0.02+0.00}_{-0.01-0.01-0.02}$</td>
<td>$-0.12^{+0.18}_{-0.17}$</td>
</tr>
<tr>
<td>$(K^+ K^- K_S)_{CP\pm}$</td>
<td>$0.05^{+0.11+0.04+0.00}_{-0.03-0.02-0.01}$</td>
<td>$0.00^{+0.09+0.02+0.00}_{-0.00-0.00-0.02}$</td>
</tr>
<tr>
<td>$(K^+ K^- K_L)_{\phi K_L}$ excluded</td>
<td>$0.03^{+0.02+0.00+0.01}_{-0.01-0.01-0.02}$</td>
<td>$0.60 \pm 0.34$</td>
</tr>
<tr>
<td>$K_S K_S K_S$</td>
<td>$0.02^{+0.00+0.00+0.01}_{-0.00-0.00-0.02}$</td>
<td>$0.19 \pm 0.23$</td>
</tr>
<tr>
<td>$K_S K_S K_L$</td>
<td>$0.02^{+0.00+0.00+0.01}_{-0.00-0.00-0.02}$</td>
<td>$0.00^{+0.00+0.00+0.01}_{-0.00-0.00-0.02}$</td>
</tr>
<tr>
<td>$A_f(%)$</td>
<td>Expt.</td>
<td></td>
</tr>
<tr>
<td>$(K^+ K^- K_S)_{\phi K_S}$ excluded</td>
<td>$0.2^{+0.1+0.3+0.0}_{-0.1-0.3-0.0}$</td>
<td>$-8 \pm 10$</td>
</tr>
<tr>
<td>$(K^+ K^- K_S)_{CP\pm}$</td>
<td>$-0.1^{+0.7+0.2+0.0}_{-0.0-0.3-0.0}$</td>
<td>$-0.1^{+0.7+0.2+0.0}_{-0.0-0.3-0.0}$</td>
</tr>
<tr>
<td>$(K^+ K^- K_L)_{\phi K_L}$ excluded</td>
<td>$0.2^{+0.1+0.3+0.0}_{-0.1-0.3-0.0}$</td>
<td>$-54 \pm 24$</td>
</tr>
<tr>
<td>$K_S K_S K_S$</td>
<td>$0.7^{+0.0+0.0+0.0}_{-0.0-0.0-0.0}$</td>
<td>$31 \pm 17$</td>
</tr>
<tr>
<td>$K_S K_S K_L$</td>
<td>$0.8^{+0.1+0.1+0.0}_{-0.1-0.1-0.0}$</td>
<td>$31 \pm 17$</td>
</tr>
</tbody>
</table>

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[23] Results obtained agree with those in [13].

[24] In general, we have Re($a_2$) > 0, Re($a_6$) < Re($a_4$) < 0.

[25] However, it is found that $K^+ K^- K^0_S$ is dominated by the CP-even part and hence it is still useful in extracting sin2$\beta_{\text{eff}}$. 

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