KILLING HORIZONS KILL HORIZON DEGREES

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Frequently it is argued that the microstates responsible for the Bekenstein–Hawking entropy should arise from some physical degrees of freedom located near or on the black hole horizon. In this Essay we elucidate that instead entropy may emerge from the conversion of physical degrees of freedom, attached to a generic boundary, into unobservable gauge degrees of freedom attached to the horizon. By constructing the reduced phase space it can be demonstrated that such a transmutation indeed takes place for a large class of black holes, including Schwarzschild.

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1. Introduction

Of all the problems quantum gravity is beset with, the most serious is the absence of experimental data — even if we had the correct theory how would we know? Would its Beauty alone reveal its Truth? Depending on philosophical prejudices sometimes this is answered affirmatively, but more cautious people invoke Nature as the ultimate arbitrator. What can be done to satisfy or at least appease the latter in the absence of experiments? Fortunately, there are some aspects of classical and semi-classical gravity that serve as selection criteria. For instance, a hypothetical theory of quantum gravity failing to reproduce Einstein’s equations, at least in some limit, would be considered as perverse. Let us now search for some semi-classical selection criterion, because reproducing the Einstein equations certainly is laudable but hardly a milestone. According to[1] the “closest thing to experimental data” we
have at our disposal is the Bekenstein–Hawking law,

\[ S_{BH} = \frac{1}{4} A, \]

which states that the black hole (BH) entropy \( S_{BH} \) equals to a quarter of the horizon surface area \( A \). This law has been checked by many independent methods and seems to be so stable against variations in the underlying assumptions that few people would take a putative theory of quantum gravity seriously if it failed to reproduce (1). In this sense it provides a semi-classical litmus test for any approach to quantum gravity. Among the plethora of them essentially all retrodict the Bekenstein–Hawking law. As Carlip pointed out lucidly 1:

“In a field in which we do not yet know the answers, the existence of competing models may be seen as a sign of health. But the existence of competing models that all agree cries out for a deeper explanation.”

2. Gauge-to-physics conversion?

A possible explanation for the universality of the Bekenstein–Hawking law might stem from near horizon conformal symmetry. This line of arguments led to numerous ways 2 of recovering (1) by use of the Cardy formula 3 which yields a direct relation between entropy and the central charge. Its presence reflects breaking of the conformal symmetry, so the corresponding gauge degrees of freedom should be converted, in a Goldstone-like manner, into physical degrees of freedom counted by Cardy’s formula. Thus, although no particular assumption has been made what the microstates actually are, one is able to establish (1) in a rather model independent way.

Without a concrete implementation this is just yet another speculative idea in quantum gravity. Gratifyingly, Carlip has found a way to make this program work 4 in the context of 2D dilaton gravity 5. We cannot avoid to present the action 6,

\[ S_{2DG} = - \int \left[ X^+(d-\omega) \wedge e^- + X^-(d+\omega) \wedge e^+ + X d\omega + e^+ \wedge e^- V \right], \]

but we will need very little of it for the purpose of this Essay. The basic idea now is to ask questions of the form “If a BH is present, what is the probability of this particular physical process?”. Thereby, the BH horizon is implemented as an effective boundary of space-time, which requires specific constraints defining its characteristics. But as we know since Dirac, constraints change the structure of phase space. Carlip could show 4 that due to the presence of “stretched horizon constraints” the algebra of the gauge constraints is changed, as the classical Virasoro

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*For sake of a self-contained text we present some of our notation 5. \( e^\pm \) are dyad 1-forms and \( \omega \) is essentially the spin-connection. The free function in 6, \( V = X^+ X^- U(X) + V(X) \), defines the model and depends on the dilaton \( X \) and the auxiliary fields \( X^\pm \). The Schwarzschild BH arises for \( U = -1/(2X) \) and \( V = \text{const} \); the Witten BH 7 for \( U = -1/X \) and \( V \propto X \).
algebra acquires a central charge proportional to the stretching parameter (i.e., it vanishes in the limit of approaching a “sharp horizon”). In this way a noteworthy agreement with (1) is found and Carlip’s attractive proposal appears to work.

But there is something very puzzling about this result: a “stretched horizon” does not differ essentially from some generic boundary, so it is not clear in what sense a BH horizon is special. Deriving (1) certainly is commendable, but deriving (1) for an arbitrary surface “cries out for some deeper explanation”. In order to even address this issue it is mandatory to impose sharp horizon constraints rather than stretched ones.

3. Physics-to-gauge conversion!

Exactly this task has recently been attacked and led to a complementary picture. Subsequently the surprising result will be recovered and explained, though we impose a simpler variational principle than the Gibbons-Hawking prescription applied in 8.

Variation of the action (2) leads not only to the equations of motion, but also to the boundary requirements

\[ X^\mp \delta e_\parallel^\pm = 0, \quad X \delta \omega_\parallel = 0, \]  

where \( \parallel \) denotes the parallel component along the boundary. This is seen immediately from (2) by tracing all partial integrations. Generically

\[ \delta e_\parallel^\pm = \delta \omega_\parallel = 0 \]  

is the only choice possible, but there exists a crucial exception: one can prove that a Killing horizon emerges if either \( X^+ \) or \( X^- \) vanish, regardless of the gauge. Thus, a consistent set of sharp horizon boundary conditions is given by

\[ X^- = \delta e_\parallel^- = \delta \omega_\parallel = 0. \]  

It is remarkable that horizons differ from generic boundaries already at the level of the variational principle.

The cases (4) and (5) are implemented via boundary constraints besides the primary and secondary bulk constraints generating gauge symmetries. Quite generically, this effects the bulk constraints to become second class. However, it is found that a horizon entails several first class constraints and consequently gauge symmetries. A study of the latter reveals diffeomorphisms along the boundary and local Lorentz transformations to be unrestricted, concurrent with our intuition about null hyper-surfaces. This provides another hint that sharp horizons behave profoundly different from generic boundaries. Still one should be careful as the relevant contributions to the Poisson brackets between the constraints have support only at the boundary. Therefore naive counting arguments cannot be invoked to get the respective numbers of physical degrees of freedom.

Instead we have to construct the reduced phase space explicitly as described detailed in 8. It is illuminating to pinpoint the pivotal technical observations: during
the construction most boundary constraints are found to be obsolete as they simply
represent the analytic continuation of the bulk gauge fixing to the boundary. In this
way all boundary constraints disappear for a generic boundary, leaving two free
functions which comprise the (physical) boundary phase space, in accordance with
a well-known result by Kuchař [9]. They can be interpreted as mass and its conjugate,
the “proper time” at the boundary. Indeed, all 2D dilaton gravity models [2] exhibit
a constant of motion [5] directly related to the ADM mass. In order to reduce its
boundary value to the core part,
\[ M = X^+ X^- \bigg|_{\text{boundary}}, \]
the trivial shift and scaling ambiguities inherent to any mass definition have been
fixed by choosing a convenient ground state and energy scale, respectively. From
[4] there is no restriction on \(X^+ X^-\) at the boundary, so \(M\) may fluctuate off-shell.
However, the sharp horizon boundary conditions [5] uniquely determine the con-
straint \(M = 0\) which is first class and thus represents a boundary gauge degree of
freedom. The “proper time” is pure gauge as well because of residual Lorentz trans-
formations. Consequently the physical boundary phase space is empty for horizons.

The conclusion is inevitable: The physical degrees of freedom present on a generic
boundary are converted into gauge degrees of freedom on a horizon.

4. Reconciliation and interpretation

At first glance our results seem to contradict earlier findings, as apparent already
from the respective section titles. In order to reconcile them it may be tempting to
interpret the disaccord as a manifestation of BH complementarity [11] for the outside
observer the horizon is not accessible anyhow and thus a stretched horizon could
provide an adequate description consistent with simple thermodynamical consider-
ations. However, by the same token one could argue that there should be something
wrong with “sharp” asymptotic conditions because no physical observer can get
access to the infinite far region.

Evidently a more convincing interpretation is needed. Since the reduced phase
space is non-empty for any generic boundary it seems unlikely to us that the corre-
sponding physical degrees of freedom bear a direct relation to BH entropy [11]. As
we have shown this information is lost on a horizon by transmutation due to en-
hanced symmetry. Because the very notion of entropy is built upon (non-accessible)
information it is plausible at a qualitative level why [11] emerges. To obtain it quan-
titatively one can still employ the Cardy formula and proceed e.g. by analogy to [3],
but our results put that procedure in a quite different perspective. Though entropy
appears to stem from Goldstone modes due to breaking of conformal symmetry, our
analysis demonstrates that they actually mimic physical modes which were there in
the first place, but which have been converted on a horizon into something unob-
servable, namely gauge degrees of freedom. This resonates strikingly with a proposal
by ’t Hooft [10].
“A more subtle suggestion is that, although we do have fields between the [brick] wall and the horizon, which do carry degrees of freedom, these degrees of freedom are not physical. They could emerge as a kind of local gauge degrees of freedom, undetectable by any observer.”

It is reassuring that we could establish the validity of this conjecture for BHs permitting an effective 2D description as in [2], including Schwarzschild. Since also more complicated scenarios allow such a treatment, at least in the vicinity of the horizon, the features discussed in this Essay ought to be universal for BHs.

To summarize our findings in a single sentence:

Entropy arises because approaching the black hole horizon does not commute with constructing the physical phase space.

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