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UNITARITY CONSTRAINTS ON HELICITY FLIP AMPLITUDES
IN NUCLEON-NUCLEON SCATTERING

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A B S T R A C T

From direct channel unitarity we derive upper bounds on the Pomeron contributions to the $NN \rightarrow NN$ helicity flip amplitudes in terms of the non-flip amplitudes.

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It has been suggested by several authors ¹⁾ that the s channel helicities at the NN vertex may be conserved for Pomeron exchange. We want to investigate this hypothesis for the case of elastic NN scattering. If we believe in duality the imaginary part of all amplitudes should be given by the Pomeron exchange alone, since the NN channel is exotic. For example, we find for the imaginary parts of the s channel helicity amplitudes

$$\text{Im } \varphi\left(\frac{1}{2} \frac{1}{2}, \frac{1}{2} \frac{1}{2}\right) = \frac{\sqrt{s}}{2k} \sum_{j=0}^{\infty} (2j+1) (a_0^j + a_{11}^j) P_j(z_s) \sim \beta_{++}(t)^2 S^{\alpha_P(t)} \quad (1)$$

$$\text{Im } \varphi\left(-\frac{1}{2} \frac{1}{2}, \frac{1}{2} \frac{1}{2}\right) = \frac{\sqrt{s}}{2k} \sum_{j=0}^{\infty} (2j+1) (a_{11}^j - a_0^j) P_j(z_s) \sim t \beta_{+-}(t)^2 S^{\alpha_P(t)} \quad (2)$$

$$\text{Im } \varphi\left(-\frac{1}{2} \frac{1}{2}, -\frac{1}{2} \frac{1}{2}\right) = \frac{\sqrt{s}}{2k} \sum_{j=1}^{\infty} (2j+1) (a_{22}^j + a_1^j) d_{11}^j(z_s) \sim \beta_{++}(t)^2 S^{\alpha_P(t)} \quad (3)$$

$$\text{Im } \varphi\left(\frac{1}{2} \frac{1}{2}, -\frac{1}{2} \frac{1}{2}\right) = \frac{\sqrt{s}}{2k} \sum_{j=1}^{\infty} (2j+1) (a_{22}^j - a_1^j) d_{1-1}^j(z_s) \sim t \beta_{+-}(t)^2 S^{\alpha_P(t)} \quad (4)$$

$$\text{Im } \varphi\left(\frac{1}{2} \frac{1}{2}, \frac{1}{2} -\frac{1}{2}\right) = \frac{\sqrt{s}}{2k} \sum_{j=1}^{\infty} (2j+1) a_{21}^j d_{01}^j(z_s) \sim \sqrt{t} \beta_{++}(t) \beta_{+-}(t) S^{\alpha_P(t)} \quad (5)$$

Here $a_i^J \sqrt{i} = 0, 1, 11, 22, 21$ denote the absorptive parts of the s channel partial wave amplitudes f_i^J with definite parity π ²⁾

$$\pi = (-1)^J \quad \text{for } f_i^J \quad i = 11, 22, 21 \quad (6)$$

$$\pi = (-1)^{J+1} \quad \text{for } f_i^J \quad i = 0, 1 \quad (7)$$

The tu crossing symmetry exhibits some restrictions on these partial wave amplitudes

$$f_i^{J I_s} = (-1)^{J+I_s+1} f_i^{J I_s} \quad i = 0, 11, 22, 21 \quad (8)$$

$$f_1^{J I_s} = (-1)^{J+I_s} f_1^{J I_s} \quad (9)$$

where I_s is the total isotopic spin of the NN system. Furthermore, we have the following unitarity inequalities

$$\alpha_i^J - |f_i^J|^2 \geq 0 \quad i = 0, 1 \quad (10)$$

$$\alpha_i^J - |f_i^J|^2 - |f_{21}^J|^2 \geq 0 \quad i = 11, 22 \quad (11)$$

It is the purpose of this paper to find the constraints on the factorized Pomeron residues $\beta_{++}(t)$ and $\beta_{+-}(t)$ which result from the unitarity inequalities (10)-(11).

A first bound can be seen directly from

$$\begin{aligned} A_0 &= \left[\Im \varphi\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) - \Im \varphi\left(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \right]_{t=0} \\ &= \frac{\sqrt{s}}{k} \sum_{J=0}^{\infty} (2J+1) a_0^J \end{aligned} \quad (12)$$

and

$$\begin{aligned} A'_0 &= \frac{d}{dt} \left[\Im \varphi\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) - \Im \varphi\left(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \right]_{t=0} \\ &= \frac{\sqrt{s}}{k} \sum_{J=0}^{\infty} (2J+1) a_0^J \frac{J(J+1)}{s-4M^2} \end{aligned} \quad (13)$$

Combining Eqs. (12) and (13) with the crossing relation (8) and the unitarity inequality (10), one easily derives ³⁾⁻⁵⁾ a lower bound for A'_0

$$A'_0 \geq \frac{1}{2s} A_0^2 \quad (14)$$

This implies an upper bound for the first derivative of the helicity double flip amplitude at $t = 0$

$$\left| \frac{\frac{d}{dt} \Im \varphi\left(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)}{\frac{d}{dt} \Im \varphi\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)} \right|_{t=0} \leq 1 - \frac{1}{2s} \frac{\left[\Im \varphi\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \right]^2}{\frac{d}{dt} \Im \varphi\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)} \bigg|_{t=0} \quad (15)$$

On the right-hand side, the imaginary part of the non-flip amplitude at $t = 0$ can be taken from the total cross-section. For its derivative at $t = 0$, let us take a convenient choice, which is suggested by the slope of the diffraction peak in the elastic pp scattering

$$a = \frac{\frac{d}{dt} \operatorname{Im} P\left(\frac{1}{2} \frac{1}{2}, \frac{1}{2} \frac{1}{2}\right)}{\operatorname{Im} P\left(\frac{1}{2} \frac{1}{2}, \frac{1}{2} \frac{1}{2}\right)} \bigg|_{t=0} = 4 \quad (16)$$

With this choice one gets for the bound (15) a value of about 0.75. The bound (15) can be considerably improved (by a factor of 5) if we have an upper bound on the quantity

$$\Sigma_0 = \frac{4\pi}{k^2} 2 \sum_{J=0}^{\infty} (2J+1) |a_0^J|^2 \quad (17)$$

For example, if $\Sigma_0 / \sigma_{\text{tot}} \leq 2/3$, it has been shown by MacDowell and Martin ⁵⁾ that the lower bound of A_0' is given by

$$\frac{A_0'}{A_0} \geq \frac{1}{9} \frac{\sigma_{\text{tot}}^2}{4\pi \Sigma_0} \quad (18)$$

However, the quantity Σ_0 alone is not accessible experimentally. We only know the total elastic cross-section which is related to the sum

$$\sigma_{\text{el}} = \frac{\pi}{k^2} 2 \sum_{J=0}^{\infty} (2J+1) \{ |f_0^J|^2 + |f_1^J|^2 + |f_{11}^J|^2 + |f_{22}^J|^2 + 2|f_{21}^J|^2 \} \quad (19)$$

In order to isolate the contribution $\sum_0$ to the total elastic cross-section, let us make use of the factorization property of the Pomeron residues [cf. Eqs. (1)-(5)] in the following sense

$$\int_0^1 dz_s \left\{ \left(\Im \varphi\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \right)^2 + \left(\Im \varphi\left(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \right)^2 \right\} \quad (20)$$

$$= \int_0^1 dz_s \left\{ \left(\Im \varphi\left(-\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right) \right)^2 + \left(\Im \varphi\left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \right)^2 \right\}$$

and

$$\begin{aligned} & \int_0^1 dz_s \Im \varphi\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \Im \varphi\left(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \\ &= - \int_0^1 dz_s \left(\Im \varphi\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right) \right)^2 \end{aligned} \quad (21)$$

where $z_s = 1 + \frac{2t}{s-4M^2}$.

Insertion of the partial wave expansions (1)-(5) leads to the equalities

$$\sum_{j=0}^{\infty} (2j+1) \{ |a_0^j|^2 + |a_{11}^j|^2 \} = \sum_{j=0}^{\infty} (2j+1) \{ |a_1^j|^2 + |a_{22}^j|^2 \} \quad (22)$$

$$\sum_{\beta=0}^{\infty} (2\beta+1) \{ |a_{0\beta}|^2 - |a_{1\beta}|^2 \} = \sum_{\beta=0}^{\infty} (2\beta+1) |a_{2\beta}|^2 \quad (23)$$

Thus it turns out that

$$\Sigma_0 \leq \sigma_{el} \quad (24)$$

Finally, the inequality (18) can be rewritten as an upper bound for the first derivative of the double flip amplitude

$$\left| \frac{\frac{d}{dt} \Im \varphi(-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2})}{\frac{d}{dt} \Im \varphi(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})} \right|_{t=0} \leq 1 - \frac{1}{9} \frac{\sigma_{tot}^2}{4\pi \sigma_{el} \cdot a} \quad (25)$$

where a is defined in Eq. (16).

Taking the experimental data of the total and total elastic cross-sections ⁶⁾ at $P_{Lab} = 19.6$ GeV/c

$$\sigma_{tot} = 39 \text{ mb} \quad \sigma_{el} = 10 \text{ mb} \quad (26)$$

and the derivative of the non-flip amplitude, as assumed in Eq. (16), we get for the right-hand side of the inequality (25) a value of about 0.14.

It should be emphasized that the inequalities (15) and (25) do not represent absolute bounds on the double flip amplitude since the non-flip amplitude with its first derivative is involved. They imply, however, non-trivial conditions on the Regge residue parameters, which are imposed by direct channel unitarity.

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R E F E R E N C E S

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