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ENERGY DEPENDENCE OF COHERENT PRODUCTION

CROSS-SECTION IN NUCLEI

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A B S T R A C T

A very simple explanation of the energy dependence of the coherent production cross-section of 3π and 5π systems in nuclei by 16-200 GeV/c incident pions is presented. The decrease of the longitudinal momentum transfer with increasing energy results in an increase of the coherent cross-section.

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Recently new data on the coherent production of three and five pions in nuclear emulsions by high-energy pions at 45 and 60 GeV/c have been published ¹⁾⁻⁵⁾. These data, together with existing results in the momentum range 14-17 GeV/c ⁶⁾⁻¹⁰⁾, and from a Cracow group in a cosmic ray experiment at about 200 GeV/c ^{2),11),12)}, indicate that the coherent production cross-section of three and five pions in nuclei increases considerably with the incident pion energy. The bubble chamber data ¹³⁾ also show an increase of the 3π production cross-section in 6 to 18 GeV/c π^- interactions with nuclei. The increase of the 5π production cross-section is even stronger than that of the 3π cross-section, if we combine the bubble chamber ¹⁴⁾ and emulsion data. This behaviour of the coherent nuclear cross-section differs from the dependence of the corresponding cross-section for production on hydrogen. In the latter case, the cross-section starts from zero at threshold, reaches a maximum at an energy of several GeV and slowly decreases thereafter ¹⁵⁾.

Two possible explanations of this phenomenon are frequently quoted [see Refs. 2),3)].

- 1) The heaviest mass $m_{\max}$ which can be produced coherently increases with momentum p

$$m_{\max} = \sqrt{m_{\pi}^2 + \frac{2p}{R}} \quad (1)$$

where R is the nuclear radius. This approximate relation comes from the so-called coherence condition

$$q_{\min} R \lesssim 1 \quad (2)$$

$q_{\min}$ is the minimum momentum transfer to the nucleus in the production of a pion system having mass m

$$q_{min} = \frac{m^2 - m_{\pi}^2}{2p}, \quad (3)$$

which decreases with increasing momentum.

- 2) The cross-section for the interaction of the produced 3π and 5π systems with nucleons inside the nucleus is a decreasing function of the incoming pion energy.

The second possibility is closely related to the determination of the A_1 nucleon cross-section. In Ref. 16) limits on the total A_1 nucleon cross-section have been deduced from the data for the production of the $\rho\pi$ system in the mass interval 0.96-1.20 GeV in a freon bubble chamber at 16 GeV/c

$$\sigma(A_1 N) \leq (0.5^{+0.3}_{-0.2}) \sigma(\pi N). \quad (4)$$

Lately preliminary results for the coherent production cross-section $\pi^- + A \rightarrow \pi^+ \pi^- \pi^- + A$ at 15.1 GeV/c for nuclei A ranging from Be to Pb were presented¹⁷⁾. If " A_1 " is defined as the coherently produced system in the mass range 1.0-1.2 GeV, the A dependence of the $(3\pi)^-$ production cross-section gives a value of the " A_1 " nucleon total cross-section which is very close to the pion-nucleon total cross-section at the same energy [see also Ref. 18)]. The $\pi^+ \pi^- \pi^-$ mass distribution produced in nuclei is very broad (the width being about 400 MeV), with the maximum at about 1.1 GeV, while the $\pi^+ \pi^-$ mass distribution is strongly dominated by the ρ peak.

The purpose of this paper is to describe the energy dependence of the coherent 3π production cross-section for incoming pion energies from 10 to several hundred GeV. At present, we only have the nuclear emulsion data over this wide energy range. Because of the large experimental errors these data do not allow us to study the energy dependence of the 3π nucleon cross-section.

We can try, however, to examine the influence of the minimum momentum transfer on the 3π mass distribution produced in nuclei. It was observed [see Refs. 12), 19)] that the 3π distributions for coherent production in nuclei ²⁰⁾ are much less spread out than those obtained from interactions with hydrogen at the same energy ^{21), 22)}. As we shall see, this effect can be related to the energy dependence of the coherent production cross-section.

In our analysis we assume that the 3π system is produced "diffractively" on the nucleons inside the nucleus, and that the cross-section corresponding to this process is independent of energy. For simplicity we have taken into account only the "one-step" coherent production process. This means that the production process takes place only on one nucleon inside the nucleus. In addition, we have elastic scattering of the incoming pion and the outgoing 3π system on the remaining nucleons. Multi-step contributions to the A_1 production cross-section have been studied by von Bochmann and Margolis ²³⁾, and the corrections to the A_1 production cross-section were found to be small [see also Ref. 24)]. At high energy we can apply the optical model formula for the coherent production differential cross-section:

$$\frac{d^2\sigma_c}{dt dm} = \frac{d^2\sigma_0}{dt dm}(t=0) |G(p, m)|^2 \quad (5a)$$

$$G(p, m) = A \int d^2t dz e^{i\vec{q}_T \cdot \vec{t}} e^{iq_L z} g(t, z) \cdot \\ \exp \left[-\frac{1}{2} \sigma_1 (1 - i\alpha_1) A \int_0^{\infty} g(t, z') dz' - \frac{1}{2} \sigma_2 (1 - i\alpha_2) A \int_z^{\infty} g(t, z') dz' \right] \quad (5b)$$

where

$$\frac{d^2\sigma_0}{dt dm}(t=0)$$

is the forward production cross-section on a free nucleon, q_T and

q_L are transverse and longitudinal components of the momentum transfer to the nucleus, $\vec{b}$ is the impact parameter vector, the z axis is chosen along the incident pion momentum, and $t = -(q_T^2 + q_L^2)$. A is the number of nucleons in the nucleus, $\rho(b, z)$ is the nuclear density distribution, σ_1 and σ_2 are the total cross-sections for the incident and produced particles, α_1 and α_2 denote the corresponding ratios of the real to imaginary parts of the forward scattering amplitudes on nucleons. In the above formula the angular dependence of the $d^2\sigma_0/dt dm$ cross-section is neglected because the nuclear angular distribution is much more forward peaked than that for production on a nucleon. If the produced mass m is significantly heavier than the incoming pion mass m_π , we can approximately take $q_L \approx q_{\min} \approx m^2/2p$ ²⁵⁾.

In order to calculate the 3π mass spectrum produced from the nucleus we integrate over t the cross-section given by Eqs. (5a,b)

$$\frac{d\sigma_c(p, m)}{dm} = \tilde{\sigma}_c(p, m) W(m), \quad (6a)$$

where

$$\tilde{\sigma}_c(p, m) = \frac{d\sigma_0}{dt}(t=0) \int dt |G(p, m)|^2, \quad (6b)$$

$$\frac{d\sigma_0}{dt}(t=0) = \int dm \frac{d^2\sigma_0}{dt dm}(t=0) \quad (6c)$$

and

$$W(m) = \frac{d^2\sigma_0}{dt dm}(t=0) / \frac{d\sigma_0}{dt}(t=0) . \quad (6d)$$

The quantity $W(m)$ is the normalized to unity mass distribution of the 3π system produced in the forward direction on the target nucleon according to the selection rules for the coherent processes in the 3π channel. Because of our assumptions the dependence of the cross-section $(d\sigma_c/dm)(p,m)$ on the momentum p comes solely from the q_L dependence of Eq. (5b). The momentum dependence of $W(m)$ can be neglected at sufficiently high energy.

The general features of the function $W(m)$ could, in principle, be obtained from the experimental data on the 3π production on hydrogen at high energy after subtraction of the incoherent channels such as, for example, A_2 production. The contributions of these channels decrease with increasing energy and we expect that at high energy diffractive channels dominate in the reaction $\pi^- p \rightarrow \pi^- \pi^+ \pi^- p$. On the other hand, we can construct the mass distribution function $W(m)$ starting from some theoretical model. In our calculations we use the simple diffraction model formulated by Stenger²⁶⁾. In this model the incoming pion scatters quasi-elastically on the nucleon and then dissociates into the $g\pi$ system. The model gives a good description of the effective mass distribution of the 3π system produced on hydrogen at 20 GeV/c²⁷⁾, where the diffraction dissociation mechanism probably dominates. At present, we are interested only in the main features of the 3π mass spectrum produced in nuclei; the details of this spectrum influence but little the total production cross-section $\sigma_c(p)$

$$\sigma_c(p) = \int dm \frac{d\sigma_c(p,m)}{dm} . \quad (7)$$

Using the formulae (5)-(7) we calculate the total cross-sections for the emulsion nuclei: Ag, Br, C, N and O. For the heavy nuclei Ag and Br we take the Saxon-Woods density

$$\rho(r) = \rho(0) \left[1 + \exp\left(\frac{r-R}{c}\right) \right]^{-1} \quad (8)$$

with the nuclear radius $R = 1.12 A^{1/3}$ ²⁸⁾ and diffuseness $c = 0.545$ fm. For the light nuclei C, N and O the harmonic oscillator density is used

$$\rho(r) = 4A^{-1} \pi^{-3/2} R^{-3} \left[1 + \frac{A-4}{6} \frac{r^2}{R^2} \right] \exp\left(-\frac{r^2}{R^2}\right) \quad (9)$$

with the radii taken from high-energy electron scattering data ²⁹⁾.

Figure 1 shows the momentum dependence of the coherent production cross-section $\tilde{\sigma}_c(p, m)$ for different masses produced in silver. We see that as the produced mass increases the cross-section $\tilde{\sigma}_c(p, m)$ progressively becomes a more steeply rising function of energy. This energy dependence has an obvious influence on the mass distribution. In Fig. 2 the 3π mass spectra are shown at 10, 16, 60 GeV/c and infinite momentum. The distributions are normalized in such a way that the total production cross-section for each momentum is proportional to the area under the curves. In the infinite energy limit the effective mass distribution is the same for the production of the 3π system on hydrogen and in the complex nucleus, but at lower energies we observe that the distributions are much less spread out. At the same time, the total production cross-section appreciably increases with increasing π momentum. For the parameters indicated in Fig. 2 the cross-section at 230 GeV/c is almost three times larger than at 16 GeV/c, and more than four times larger than at 10 GeV/c. Similar dependence appears for the case of Br, and is also prominent for the lighter components of the nuclear emulsion.

Knowing the nuclear emulsion composition we can calculate the total cross-section for the coherent production of three pions and compare the results with the existing data (Fig. 3). In these calculations we have assumed that the total cross-section for the interaction of the 3π system with nucleons is the same as for the pion, and that the amplitudes are purely imaginary. The forward production cross-section on nucleons was obtained from the assumed exponential t dependence of the production amplitude

$$\frac{d\sigma_0}{dt}(t=0) = \sigma_0 t \quad (10)$$

where the slope parameter $b = 9 \text{ GeV}^{-2}$, and $\sigma_0 = 0.65 \text{ mb}$ [Veillet has quoted the value $\sigma_0 = 0.65 \pm 0.13 \text{ mb}$ in Ref. 30)]. The theoretical curve shown in Fig. 3 gives a good description of the present experimental results. This figure does not include the bubble chamber data because one cannot directly combine data taken in materials of different composition when the nuclear cross-section has a complicated A dependence. The bubble chamber data¹³⁾ are compared with our results in Table 1.

Our analysis indicates that the main contribution to the coherent production cross-section for nuclear emulsion comes from the heavy nuclei Ag and Br (about 70 % at 16 GeV/c and 75 % at 60 GeV/c). Recently new data on 3π production at 60 GeV/c in special emulsion enriched with lighter elements were published⁵⁾. The total production cross-section deduced from these data $\sigma_c = (6.8^{+3.0}_{-1.6}) \text{ mb}$ is in good agreement with the calculated cross-section $\sigma_c = 6.1 \text{ mb}$ per emulsion nucleus.

Some remarks on the energy dependence of the 5π coherent production cross-section may be in order. It is known, however, that the mass of the 5π system is widely spread out with an enhancement at about 1.9 GeV¹⁴⁾. We may see from Fig. 1 that for such high masses the energy dependence of the cross-section is even stronger than in the case of the 3π system described above.

Therefore we expect that the 5π production cross-section will be an even more rapidly changing function of energy, and this qualitative observation is supported by the existing data.

In conclusion, we state that the existing data on high-energy coherent production of 3π and 5π systems in nuclei are reasonably well described by the optical model under the assumption that the produced pion systems interact with the target nucleons similarly to the incoming pion. More accurate experimental data on coherent production of pions in nuclei with definite mass number A [as in Ref. 17] are needed over wide energy range.

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T A B L E 1

3π TOTAL PRODUCTION CROSS-SECTION

bubble chamber	π^- momentum in GeV/c	σ_{exp} (mb)	σ_{theor} (mb)
C ₂ F ₅ Cl	16	24 ± 6	21
C ₄ F ₃ Br	17	22 ± 6	22
C ₂₂ F ₃ Br	18	55 ± 15	57
C ₂₂ F ₃ Br	6	26 ± 7	21

R E F E R E N C E S

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FIGURE CAPTIONS

- Figure 1: π^- momentum dependence of the quantity $\tilde{\sigma}(p,m)$ defined by Eqs. (6a,b) for the 3π coherent production in Ag nucleus. The numbers refer to the 3π mass in GeV.
- Figure 2: Distribution of the 3π mass system produced coherently in Ag nucleus. The numbers are the incoming pion momenta in GeV/c. The area under each curve is proportional to the total coherent cross-section.
- Figure 3: Momentum dependence of the 3π coherent production cross-section in nuclear emulsions. The crossed experimental points at 45 GeV/c and 60 GeV/c refer to the reaction $\pi^- \rightarrow \pi^- \pi^+ \pi^- + \text{eventually } 2\pi^0$. Experimental points are taken from Refs. 1), 2), 6), 8), 9).

Ag

$$\sigma_1 = \sigma_2 = 25 \text{ mb}$$

$$\alpha_1 = \alpha_2 = 0$$

$$R = 5.331 \text{ fm } c = 0.545 \text{ fm}$$

$\tilde{\sigma}_c(p, m)$
mb

FIG.1

FIG.2

FIG.3