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BOOTSTRAPS AND PARTONS ⁺⁾
(A FIELD GUIDE TO COMPOSITE HADRONS)

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A B S T R A C T

In a bootstrap theory of infinitely composite hadrons, it is shown that parton-like behaviour for inclusive reactions arises naturally. The question is raised : might the bootstrap be equivalent to theories with explicitly local entities, such as partons or currents, playing a fundamental role? We suggest the answer is no, and point out several crucial experiments which will distinguish between the bootstrap and theories of composite hadrons with elementary constituents.

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I. INTRODUCTION

An impressive amount of experimental evidence suggests that hadrons are composite objects. Much of the recent theoretical effort to understand the strong interactions has been directed toward models of composite hadrons built out of unhadron-like elementary objects, or, more abstractly, toward theories in which local entities such as current operators play a fundamental role. Perhaps the most interesting question in strong interaction physics which can be experimentally settled during the next decade is whether a description of hadrons based on localized elementary objects, be they partons or currents, or a true bootstrap description of infinitely composite hadrons will be more successful.

Having posed this question, an interesting theoretical question naturally arises: might the two descriptions, in fact, be equivalent? The aim of this paper, which also contains a review of our bootstrap theory of hadrons, is to show that the two descriptions are inequivalent, and to outline the most crucial experimental tests which will allow the former question to be settled.

During the past several years we have developed and explored the consequences of a bootstrap theory of hadronic interactions. There are four features of our theory which are worth emphasizing at the outset:

1. The hadrons are described as infinitely composite objects, made up out of hadrons. This concept will be made mathematically precise in the next section.
2. We have characterized the leading asymptotic behaviour of N particle scattering amplitudes satisfying exact unitarity and crossing in the asymptotic limit in which all the Mandelstam variables are large in absolute value.
3. Form factors and wide-angle elastic amplitudes predicted by this bootstrap are in excellent agreement with experiment.
4. Despite the absence of elementary constituents or localized currents, a "parton-like" behaviour for certain inclusive reactions is predicted by the theory. This will be discussed in detail below.

Since the early days of bootstrapping, the exploration of this concept has primarily been directed toward the calculation of low energy hadronic parameters such as masses and coupling constants ¹⁾ and toward the extraction of information about symmetries and broken symmetries from the non-linear bootstrap constraints ²⁾. While the compatibility of these constraints with an underlying symmetry group was established, the effort to actually calculate self-consistent masses and couplings has bogged down, primarily because of the

ambiguities associated with the unknown asymptotic behaviour of the hadronic interactions. Valiant efforts along these lines continue, especially in the direction of including more sophisticated multi-reggeon interactions ³⁾, but the most compelling evidence for the bootstrap, from low energy parameter determinations, still probably comes from the Chew-Low model.

While the non-linear constraints imposed by unitarity and the bootstrap are exceedingly ugly in low energy calculations, we began our work with the hope that their simplicity might lie in asymptopia. Thus we took a quite different approach and asked whether it was possible to bootstrap the asymptotic properties of amplitudes ⁴⁾. Here we shall review the results of this work, compare this bootstrap theory and its predictions with other models and theories of hadrons, and point out the crucial future experiments which will provide tests of the asymptotic bootstrap.

II. BOOTSTRAP THEORY OF THE HADRONS - A REVIEW

Consider the collection of all hadronic scattering amplitudes. The constraints of unitarity can be imposed by writing an infinite set of Bethe-Salpeter-type equations which relate the amplitudes to each other and to irreducible kernels. The general form of such equations can be written :

$$\begin{aligned} \text{amplitude} = & \text{irreducible kernel} + \\ & + (\text{irreducible kernel}) \times (\text{amplitude}') \end{aligned} \quad (1)$$

where $\times$ denotes a Feynman integration which, for the l particle irreducible kernel, is over an l partial intermediate state. A more complete discussion of the cluster decomposition implicit here, of the exact definition of the kernels, and of the treatment of the propagators is given in Ref. 5).

Upon these equations we can impose the bootstrap constraints. What we first require is that each of the particles identified with external lines in the unitarity equations also appears as a dynamical pole in channels with appropriate quantum numbers. As discussed in Ref. 5), a dynamical pole is one which can never appear in an irreducible kernel; this statement is a translation, into the language of singularity structure, of the $Z=0$ condition of field theory and in Fig. 1 this concept is elucidated. To complete the implementation of the bootstrap constraint, we invoke crossing and require that the residue of a pole term in an n to m particle amplitude satisfying the unitarity equation, factor into the product of appropriately crossed $n+1$ and

$n+1$ particle amplitudes which, of course, also satisfy the unitarity constraints. Thus crossing allows us to identify dynamical poles with external lines and we guarantee that every hadron is infinitely composite.

We have found a characterization of asymptotic scattering amplitudes and kernels satisfying the above-mentioned constraints in the asymptotic limit in which all of the Mandelstam variables describing an n particle amplitude are large in absolute value. Where asymptopia actually lies can only be determined, at the present level of understanding, by recourse to experiment; we shall return to this question in Section IV.

Our result takes the form of a tree representation of the asymptotic amplitudes. But the tree graphs are not the Feynman graphs of a field theory, for the vertices in the trees are exponentially decreasing functions of the invariant four-momenta and the internal lines in the trees do not correspond to single particle states. We call such a tree graph a composite tree graph (CTG).

The CTG representation of an amplitude contains all topologically possible tree configurations with trilinear vertices. The irreducible kernels are described asymptotically by a subset of all CTG's which are manifestly irreducible [see Ref. 5]. Quantum number constraints do not restrict the trees as we do not identify internal lines with single particle states.

To complete our description of the CTG representation we must characterize the vertex function appearing at each vertex in the CTG's. Consider an arbitrary vertex connecting lines with momenta $p_1, p_2, p_3 = p_1 + p_2$. The vertex function, Γ , is prescribed as follows :

$$1. \quad \Gamma(p_1^2, p_2^2, p_3^2) \equiv P(p_1^2, p_2^2, p_3^2) e^{-\alpha g(p_1^2, p_2^2, p_3^2)} \quad (2.a)$$

$$2. \quad g(p_1^2, p_2^2, p_3^2) = 0 \quad (2.b)$$

when at least one of its arguments vanishes.

3. If one of its arguments, say p_1^2 , grows asymptotically large in absolute value, then

$$g(p_1^2, p_2^2, p_3^2) \xrightarrow{|p_1^2| \rightarrow \infty} (-p_1^2)^\gamma f_{23}(p_2^2, p_3^2). \quad (2.c)$$

4. If two of its arguments, say p_1^2, p_2^2 , grow asymptotically, then

$$g(p_1^2, p_2^2, p_3^2) \xrightarrow{|p_1^2|, |p_2^2| \rightarrow \infty} (-p_1^2)^\gamma (-p_2^2)^\gamma h_3(p_3^2). \quad (2.d)$$

5. If all three of its arguments are large, then

$$g(p_1^2, p_2^2, p_3^2) \xrightarrow{|p_1^2|, |p_2^2|, |p_3^2| \rightarrow \infty} (-p_1^2)^\gamma (-p_2^2)^\gamma (-p_3^2)^\gamma. \quad (2.e)$$

6. $P(p_1^2, p_2^2, p_3^2)$ and its inverse are asymptotically power bounded as the $|p_i^2| \rightarrow \infty$.

For the moment, γ in Eqs. (2.c)-(2.e) is only restricted to the range

$$0 < \gamma \leq 1/2 \quad (3)$$

but we shall give four independent arguments, shortly, which all point to the value

$$\gamma = 1/4 \quad (4)$$

as the correct one. The constant, a , in Eq. (2.a) is a universal constant ⁶⁾. It is, in principle, determined in terms of the masses of the bootstrapped hadrons.

The result derived in Ref. 5) is that if the function Γ describes each vertex in the CTG's, then the CTG representation asymptotically satisfies the unitarity and bootstrap constraint. The proof of this assertion consists in showing that Feynman integrated products of CTG's yield CTG's. To carry this through, one first makes a Wick rotation on the contours of integration and replaces all four-momenta by Euclidean variables. Then the method of steepest descent (MOSD) is used to determine the asymptotic behaviour of the integrals. It is found that Eqs. (2) force the saddle points in each four-dimensional integral to lie at the values $k_i^2 = 0$ where the k_i are the four-momenta of the internal lines in the product of CTG's appearing in the unitarity integrals. From each of these saddle points, the MOSD yields an asymptotic result which just exactly reproduces the exponential behaviour of one of the possible CTG's appearing on the left-hand side of the unitarity equation. When the contributions from all saddle points are summed, all CTG's appear and the exponential behaviour of the asymptotic amplitudes is consistent

with the full set of unitarity equations. Details of the proof are given in Ref. 5). It should be emphasized that if g vanished not at $p_i^2=0$ but at some other values, or if g were simply minimum, but not zero, at $p_i^2=0$, then asymptotic self-consistency would not be obtained.

We conclude this section with three comments :

1. The Wick rotation appears essential in our derivation. We have proved ⁵⁾ that a necessary, though by no means sufficient, condition on γ for this rotation to be possible is

$$\gamma \leq 1/4. \quad (5)$$

Thus a vertex function $\Gamma(t) \sim e^{-a(-t)^{1/4}}$ represents a lower bound. If we postulate saturation of this bound, then we have considerably narrowed the ambiguity in our result. Interestingly, we shall see that three other theoretical and phenomenological arguments also point to the value $\gamma = 1/4$.

2. The behaviour of the function g in Eqs. (2) for vanishing values of its arguments is crucial to the application of the MOSD, and is partially restricted by the unitarity equations. One allowed form for g , for all values of the p_i^2 , is $g(p_1^2, p_2^2, p_3^2) = (-p_1^2)^\gamma (-p_2^2)^\gamma (-p_3^2)^\gamma$, but of course the singularities that would then appear in scattering amplitudes at $p_i^2=0$ would be unphysical as there are no massless hadrons. In fact, g must vanish like an even power of the p_i^2 as $p_i^2 \rightarrow 0$. It can also be easily shown that the derivatives of g on the mass shell are restricted. For some n (odd) it must be true that

$$\frac{\partial^{(n)} g(p_1^2, p_2^2, p_3^2)}{\partial p_i^{(n)}} \bigg|_{p_i^2 = m_i^2} = 0 \quad (6)$$

and

$$\frac{\partial^{(n+1)} g(p_1^2, p_2^2, p_3^2)}{\partial p_i^{(n+1)}} \bigg|_{p_i^2 = m_i^2} > 0 \quad (7)$$

for $j=1, \dots, n$ and $i=1, 2$, or 3 . In these equations, m_i is the mass of an external hadron coupling, with momentum p_i , to the vertex described by the function g . These constraints on g , at $p_i^2=0$ and at $p_i^2=m_i^2$, follow simply from the requirement that the values $p_i^2=0$ are points of relative

maximum integrand rather than relative minimum integrand in the integration over internal lines in the unitarity equations. The possibility of exploiting Eqs. (6) and (7) to restrict the hadron mass spectrum remains to be explored, but we note an amusing self-consistency property of Eq. (7) : if as implied by Eq. (7), g is a relation minimum on a particular mass shell, this implies that the effective interaction in a bound state equation for that particular mass state is a relative maximum, and the bound state may indeed be produced.

3. The exponential behaviour of the asymptotic n particle amplitudes satisfies the self-consistency constraint imposed by the unitarity plus bootstrap constraints. A multiplicative power dependent factor, P , in Eq. (2.a), is as yet undetermined. In a simple model bootstrap with spinless particles the leading power of P has been determined and we find, taking $\gamma = 1/4$,

$$\Gamma(p^2, m^2, m^2) \xrightarrow{|p^2| \rightarrow \infty} (-p^2)^{-5/4} e^{-a(-p^2)^{1/4}} f(m^2, m^2) \quad (8)$$

but this is clearly uninteresting. In the real world with many, possibly an infinite number of, spinning particles being bootstrapped, the determination of the leading power would be a prodigious task.

III. PARTON-LIKE BEHAVIOUR FROM THE BOOTSTRAP

Having sketched the basic result of the bootstrap, the CTG representation for asymptotic amplitudes, we now point out an unusual property of this representation. It follows immediately from the asymptotic behaviour of CTG's that scattering amplitudes are, in general, rapidly decreasing functions of the Mandelstam variables when all these variables are large. Let us look, however, in more detail at the asymptotic vertex function $\Gamma(p_1^2, p_2^2, p_3^2)$. From Eqs. (2) we can write, for $|p_1^2| \rightarrow \infty$,

$$\Gamma(p_1^2, p_2^2, p_3^2) \sim e^{-a(-p_1^2)^{\gamma}} f_{23}(p_2^2, p_3^2) \quad (9)$$

If p_2^2 and p_3^2 are fixed, say on the mass shell of a hadron, then we obtain the rapidly decreasing behaviour expected for the form factor of an infinitely composite particle :

$$F(p^2) \sim e^{-b(-p^2)^\alpha} \quad (10)$$

where

$$b = a f_{2,3}(m^2, m^2). \quad (11)$$

However, suppose that $p_2^2 = 0$, then, by Eqs. (2.b) and (2.c)

$$\Gamma(p_1^2, p_2^2, p_3^2) \sim e^{-a(-p_1^2)^\alpha} f_{2,3}(0, p_3^2) = 1 \quad (12)$$

because

$$f_{2,3}(0, p_3^2) = 0. \quad (13)$$

Thus, if a zero four-momentum squared line is attached to a tree graph, then the vanishing of the function f at $p^2 = 0$ kills the exponent of the vertex function; it couples as if it were a point-like object to a line with a large four-momentum squared. Recall that the vanishing of the function g , for vanishing values of its arguments, was crucial to the asymptotic consistency of the CTG representation.

We shall discuss in the next section several experimental consequences of this parton-like phenomenon and in Section V point out a number of similarities and, for certain reactions, a few crucial differences between the parton-like behaviour induced in the asymptotic bootstrap by zero invariant mass states and the behaviour expected in theories based upon fundamental, point-like, entities.

IV. EXPERIMENTAL CONSEQUENCES OF THE ASYMPTOTIC BOOTSTRAP

In this section we shall systematically list and discuss the experimental consequence of the asymptotic bootstrap. Some of the predictions are rigorously forced upon us, once we accept the CTG representation, while others are only plausible consequences of the theory but require additional assumptions for their support. We shall attempt here to point out our level of confidence in the various results discussed below, emphasizing those predictions which, if wrong, would nullify the entire theory.

A. Exclusive Reactions

1. Elastic electromagnetic form factors

The large momentum transfer electromagnetic form factor of a hadron of mass m is predicted to behave like (see Section II) :

$$F(t) \sim e^{-a f(m^2, m^2) (-t)^\gamma} \quad (14)$$

The fit ⁷⁾ of Eq. (14) to the proton magnetic form factor, F_{pM} , gives excellent agreement (see Fig.2) in the range

$$1 \text{ GeV}^2 \leq -t \leq 25 \text{ GeV}^2 \quad (15)$$

with

$$\gamma = 1/4 \quad (16)$$

and

$$a f(m_p^2, m_p^2) = 5 (\text{eV})^{-1/2} \quad (17)$$

A value as large as $\gamma = \frac{1}{2}$ gives a very poor fit. If Eq. (16) is indeed correct, then in the time-like region the proton form factor should behave like

$$F_{pM}(t) \sim e^{-5 (+t)^{1/4} (1+i)/\sqrt{2}} \quad (18)$$

which would be nearly as rapidly decreasing as in the space-like region.

Preliminary colliding beam data are consistent with this predicted rapid fall-off, but the prediction remains to be tested.

We predict a sizeable two-photon exchange contribution to elastic e^-p scattering at very large momentum transfers. This result is derived, from the asymptotic bootstrap, in Ref. 8) but can also be motivated on very general grounds : note that the imaginary part of the two-photon amplitude is a product of the amplitudes for deep inelastic production. Since these are slowly varying in Q^2 , application of a fixed t dispersion relation suggests our result ⁹⁾. A power, rather than exponential, dependence of A_2 on t is expected. The two-photon contribution must eventually dominate that from one photon if the asymptotic bootstrap is correct.

The pion form factor is predicted to be

$$F_{\pi}(t) \sim e^{-a f(m_{\pi}^2, m_{\pi}^2) (-t)^{1/4}} \quad (19)$$

Since $f(0,0)=0$, we might expect, if f is slowly varying, that a $f(m_{\pi}^2, m_{\pi}^2)$ will be small, and therefore the pion will be more point-like than the proton. However, we cannot find adequate justifications for this smoothness assumption for f .

2. Wide angle elastic scattering

In the kinematical region $s, -t, -u \rightarrow \infty$, the CTG representation leads to the following form for the proton-proton scattering amplitude :

$$A_{pp}(s, t, u) \sim F_p^2(s) + F_p^2(t) + F_p^2(u) \quad (20)$$

where each term on the right-hand side corresponds to one of the three CTG's contributing to the asymptotic four-point function and is the square of the proton form factor in the channel variable. If we assume the value $\gamma = 1/4$, then an excellent fit ⁷⁾ (see Fig.3) of Eq. (20) to the proton-proton data for $|t| \geq 4 \text{ GeV}^2$, $s \geq 10 \text{ GeV}^2$ is obtained. $F_p(p^2)$ is taken from Eqs. (14)-(18).

The "break" in the 90° data is easily accounted for here by the fact that, since $s \approx -2t = -2u$ at 90° , the $F_p^2(s)$ and $F_p^2(t)$ or $F_p^2(u)$ terms in Eq. (20) have different slopes. Above the break $F_p^2(t)$ dominates while, below, $F_p(s)$ dominates; we expect no more breaks [with the variable $(s)^{1/4}$] at 90° . We emphasize the sensitivity of such a fit to the value of γ ; for $\gamma = \frac{1}{2}$, the $F_p^2(s)$ term in Eq. (20) would give a constant contribution to the cross-section at fixed angle and increasing s .

The wide angle π -p amplitude is predicted to be

$$A_{\pi p}(s, t, u) \sim F_{\pi p}^2(s) + F_{\pi p}^2(u) + F_{\pi}(t) F_p(t) \quad (21)$$

Adequate large angle π -p data and data on the pion form factor do not exist as yet to allow a test of this result.

Wide angle Compton scattering with real photons offers an amusing opportunity to test Eq. (2.b) since $m_{\gamma}^2 = 0$ implies that the CTG's shown in Fig. 4 will give rise to a non-rapidly decreasing contribution to the fixed angle cross-section.

Finally, we should point out that the wide angle $p\bar{p}$ and $K^\pm p$ cross-sections should also be of the general form of Eqs. (20) and (21). Asymptopia may be further off in non-exotic channels than in exotic ones but the role of quantum numbers should diminish as asymptopia is reached. The fits to wide angle pp scattering suggest that space-like, non-exotic channel asymptopia is at $t \leq -4 \text{ GeV}^2$, while time-like, exotic channel asymptopia is at $s \geq 10 \text{ GeV}^2$. What we do not know is where time-like, non-exotic channel asymptopia lies.

All the results of this section, except the possible slow decrease of $F_\pi(t)$ suggested by $f(m_\pi^2, m_\pi^2) \approx f(0,0)$, are forced upon us by the CTG representation; their violation would signal fundamental troubles with the theory.

B. Inclusive Reactions

The behaviour of the cross-sections for various inclusive reactions is governed by the possibility of the presence of zero invariant masses (zim's) attached onto the CTG's. When a zim appears and couples to a massive photon line, it contributes terms to the amplitude which, like the parton contribution in the parton model, do not fall rapidly for increasing photon mass. We shall see, because of kinematical restrictions, that not all deep inelastic inclusive reactions allow the existence of zim's attached to tree graphs and thus for some processes we will make predictions which are quite similar to those of the parton model, while for others our predictions differ markedly.

1. Deep inelastic e^-p scattering

The leading contribution for inelastic e^-p scattering in the Bjorken limit is expected ¹⁰⁾ to come from contributions of the form shown in Fig. 5. In general, the contribution of Fig. 5 will include a factor

$$e^{-\alpha f(t, m_z^2) (Q^2)^{1/4}} \quad (22)$$

and for this expression to be slowly varying in Q^2 , we require $f(t, m_z^2) \approx 0$ or $t \approx 0$. The condition that the value $t = 0$ be included in the physical region leads, in the Bjorken limit, to the constraint that

$$t_{min.} = \frac{m_p^2}{\omega} - \frac{m_i^2}{\omega - 1} \geq 0 \quad (23)$$

where $\omega = 2m\nu/Q^2 \geq 1$.

From Eq. (23) we deduce that $m_1^2 < m_p^2$ and thus the parton-like exchanged object in Fig. 5 has baryon number one as is generally the case in parton models. Note that Fig. 5 is not a CTG, the CTG representation would not be valid here, for the Mandelstam variable, t , is not asymptotic. On the other hand, the square vertex may be resolved into CTG's and will include the contribution of Eq. (22).

It is expected that meson exchange, rather than baryon exchange, will dominate in the limit $\omega \rightarrow \omega$, Q^2 fixed, as then $t_{\min} \rightarrow 0$ even if $m_1 = m_p$. The actual shape of the structure functions will, in general, be a result of both contributions and is difficult to reliably calculate. But certain general features stand out. $R(\omega) \equiv \sigma_L/\sigma_T$ should increase and the ratio of the neutron to proton structure functions should approach one in the limit $\omega \rightarrow \omega$, Q^2 fixed. If ω is fixed and very large, and Q^2 increases, then a breakdown of scaling should be observed because $(Q^2)^{1/4} t_{\min}^2$ will increase and exponential damping of the meson exchange contribution will result. The scaling observed at moderate ω , Q^2 would be restored when Q^2 increased sufficiently so that the baryon exchange term again dominated.

Note finally, that in deep inelastic $e^- \pi$ scattering, Eq. (23) with $m_p \rightarrow m_\pi$ can never be satisfied and so the pion structure function should not scale in the limit $Q^2 \rightarrow \omega$, ω fixed.

2. e^+e^- annihilation

The process $e^+e^- \rightarrow p+X$ is qualitatively different from the e^-p reaction. Now, a $t=0$ line can never attach to the massive photon (in lowest order, at least, with no external real photons) and scaling should not occur. In fact, we expect¹⁰⁾ that $\sigma_{e^+e^- \rightarrow \text{hadrons}}(Q^2)$ should not approach a point-like limit of c/Q^2 , but fall more rapidly. Each final state should fall faster than any power, and it is unlikely that a sum over final states could raise this to a c/Q^2 behaviour, for if it did, then the same sum would lead to increasing e^-p structure functions because each definite final state in the e^-p reaction has only power dependence on Q^2 .

3. μ -pair production

Here we expect¹⁰⁾ the contribution shown in Fig. 6 to dominate. Note that a light meson must be produced in order to kinematically allow an internal $t=0$ line to couple to the massive photon. The argument parallels that following Eq. (22). We are unable to make detailed predictions for the shapes of the cross-section for this process but we do predict

$$\frac{\frac{d\sigma}{dQ^2}(P+P \rightarrow N^+N^-+P+P)}{\frac{d\sigma}{dQ^2}(P+P \rightarrow N^+N^-+\pi+P+P)} \xrightarrow{Q^2 \rightarrow \infty} 0 \quad (24)$$

4. Purely hadronic inclusive reactions

The asymptotic bootstrap provides a model for fireball decay and yields ¹¹⁾ the Boltzmann spectrum

$$E \frac{d\sigma}{d^3p} \approx e^{-\beta E_{fb}^c} \quad (25)$$

for the momentum distribution in the process $a+b \rightarrow c+X$. Here β is a constant, given by $\beta = ah(m_\pi^2)$ where a and the function h are defined in Eqs. (2) and E_{fb}^c is the energy of the particle c in the rest frame of the fireball. Equation (25) is derived under the assumption that CTG's of the form shown in Fig. 7 describe fireball decay and that $\gamma = 1/4$. Remarkably, only this value of γ results in the constant temperature Boltzmann distribution. In Ref. 11) the derivation of Eq. (25) and a discussion of its experimental consequences are given. It is shown that Feynman scaling occurs, for $p_{\perp}^{c.m.} \gg p_{\perp}$, if the fireball is produced diffractively as in the models of Hwa ¹²⁾, and Jacob and Slansky ¹³⁾. We differ from these authors only in our picture of fireball decay and in the multiplicity growth. We expect ¹¹⁾ that the multiplicity growth per fireball of mass M will be $n(M) \sim \log M$, leading to $n(s) \sim \sim \text{constant}$ if the excitation spectrum $\rho(M) \sim M^{-2}$. A $n(s) \sim \log s$ behaviour can then only arise if the fireballs are produced, say by a multiperipheral mechanism, with multiplicity growth: $N(s) \sim \log s$.

V. HOW TO DISTINGUISH BETWEEN INFINITELY AND FINITELY COMPOSITE HADRONS

Our purpose in this section is to compare the asymptotic bootstrap with the salient features of a variety of hadron models based upon explicitly elementary entities such as partons or current operators. Before discussing such models, however, we briefly compare our results with those of the, in spirit, related work of Hagedorn ⁴⁾.

Hagedorn's thermodynamic approach to the bootstrap, and the subsequent reformulation and refinement by Frautschi ¹⁴⁾ and Frautschi and Hamer ¹⁵⁾ contain two especially important predictions: one is the exponentially growing level density formula for the hadron mass spectrum

$$\rho(m) \sim m^{-3} e^{+bm} \quad (26)$$

and the second is the Boltzmann distribution [Eq. (25)] for the momentum spectrum in inclusive hadron collisions. These models have not, as yet, yielded information about form factors or wide-angle elastic cross-sections, while the asymptotic bootstrap has not yet revealed a level density formula. The thermodynamical or statistical bootstrap model overlaps the asymptotic bootstrap in that both lead to a constant temperature Boltzmann distribution and this encourages the belief that further analysis will reveal more similarities.

We turn now to theories of hadrons involving fundamental, point-like entities.

A. Parton Models

Parton models ¹⁶⁾ have generated a large number of predictions, many in the form of sum rules, which we have not been able to derive from the bootstrap. It is not our purpose to review parton models here, and so we will restrict our discussion to those parton predictions which overlap results from the bootstrap.

For the inclusive reactions $e^-p \rightarrow e^- + X$ and $pp \rightarrow \mu^+ \mu^- + X$ there are no fundamental differences between the parton and bootstrap models. The former primarily yields information about the inclusive cross-section while the latter makes its most definitive predictions for exclusive final states. In both models the inclusive structure functions are expected to hold up in Q^2 , and bootstrap analysis suggests the result

$$R \equiv \frac{\sigma_L}{\sigma_T} \approx 0 \quad (27)$$

at moderate energy where a small number of final states should dominate. Until parton models are able to yield predictions about exclusive deep inelastic e^-p scattering, it seems unlikely that any fundamental disagreement will be encountered here.

The angular distributions in deep inelastic exclusive reactions such as $e^-p \rightarrow e^- + p + \rho^0$ are predicted by the asymptotic bootstrap to peak ever more sharply near $u=0$ or $t=0$ as Q^2 increases. Interestingly, the currently

popular notion ¹⁷⁾ that the photon's "radius" decreases as Q^2 increases is in conflict with our predicted angular distribution, for it would imply a broadening t distribution as Q^2 increases. A test of this behaviour, for $Q^2 \geq 2 \text{ GeV}^2$ where we expect asymptotic statements to be valid, would be useful.

In muon-pair production, the asymptotic bootstrap results agree with the field theoretic analysis ¹⁸⁾ that graphs of the form shown in Fig. 6, representing what is called parton-antiparton annihilation, should dominate the bremsstrahlung graph shown in Fig. 8.

The most striking disagreement between partons and bootstraps is over the reaction $e^+e^- \rightarrow p+X$. Drell, Levy and Yan ¹⁹⁾ predict a scaling behaviour for this reaction, a result which is claimed to follow from an analytical continuation of their results for deep inelastic e^-p scattering. As discussed in Section IV, we differ markedly, predicting a violent scaling breakdown for this process.

In inclusive hadron-hadron collisions, there are no fundamental differences between parton models and the asymptotic bootstrap, although it should be emphasized that while the origin of Bjorken scaling is similar (in the sense of the correspondence between partons and zim's) in the two approaches, the origins of Feynman scaling are quite different (in the sense that zim's play no role in the fireball mechanism described in Section IV).

A recent parton theory ²⁰⁾ of wide angle elastic processes leads to the prediction that the 90° proton-proton cross-section should decrease as the sixth, rather than the fourth, power of the proton form factor (assumed to be power dependent). Furthermore, this parton approach leads to wide angle cross-sections which are strongly quantum number dependent; for example, the $p\bar{p}$ cross-section is predicted to be highly skewed about 90° whereas, from the near equality of the s and t, u channel CTG's in the fit to the pp cross-section, we conclude that the skewing will be gentle. In the asymptotic bootstrap, the influence of quantum numbers is obliterated when all Mandelstam variables are asymptotic.

Another prediction of Ref. 20) is based on the hypothesis that there is something special about virtual photons, as opposed to large-invariant-mass hadronic states, which leads to point-like matrix elements of the electromagnetic current between parton states. Thus, the fixed angle cross-section for (high Q^2 photon)+ $p \rightarrow p+p$ at $t=u$ is predicted to fall relatively slowly compared, say, to the $pp \rightarrow pp$ cross-section, whereas the asymptotic bootstrap predicts an even more rapid rate of decrease, in the energy variable, of this $t=u$ cross-section as Q^2 increases.

This parton model viewpoint (which, of course, involves assumptions that go beyond the original parton model) also has led to speculation ²¹⁾ that electromagnetic effects will eventually show up in inclusive hadron-hadron reactions when $p_{\perp}$ of the observed final state particle is large. The reasoning behind this is that such production involves a large momentum transfer and this will ordinarily damp out rapidly unless this momentum transfer is effected by a high Q^2 photon which, through a parton interaction, kicks out the large $p_{\perp}$ particle. We predict no such electromagnetic effects; our zim's will give rise to undamped vertices whether a virtual photon or a large t hadronic state kicks out the particle with large $p_{\perp}$.

The Drell-Lee ²²⁾ Bethe-Salpeter model of the nucleon, like our work, leads to rapidly falling elastic form factors and approximate scaling in deep inelastic e^-p scattering. The Drell-Lee form factors have power, rather than exponential, fall off in Q^2 , a result which follows from their choice of the interaction kernel binding the point-like constituents. It is not clear whether measurements will ever be sensitive enough to distinguish exponential fall-off from power fall-off, and so it will be of interest to see what predictions emerge from a Drell-Lee analysis of e^+e^- annihilation and of wide-angle, elastic $p-p$ scattering.

B. Light Cone Analysis, Scale Invariance, and Current Algebra

For both muon pair production and electron-positron annihilation, the results of light cone analysis ²³⁾ differ from the predictions of the asymptotic bootstrap. The graph shown in Fig. 8 is expected to provide the dominant light cone contribution while Fig. 6 shows the leading term in the bootstrap analysis.

Experimentally, distinguishing between these mechanisms for muon pair production may be difficult, but fortunately the predictions of the theories differ more dramatically for electron-positron annihilation.

Light cone analysis leads to the result that $\sigma(e^+e^- \rightarrow \text{hadrons}) \sim c/Q^2$ while the asymptotic bootstrap (see Section IV) predicts a more rapid decrease in Q^2 , provided higher order electromagnetic effects can be experimentally excluded. The forthcoming measurements of e^+e^- annihilation at SPEAR should provide a crucial means of distinguishing between the asymptotic bootstrap and light cone or parton ideas.

Castell ²⁴⁾ has applied conformal invariance to wide-angle elastic reactions and has found that in the limit of large s , $|t|$, $|u|$, $d\sigma/dt$ ought to depend upon a power of the invariants times a non-trivial function only of the dimensionless variables s/t , u/t . In contrast, our result [Eqs. (14) and (20)] explicitly involves a parameter, $af(m^2, m^2)$, with dimensions, and does not even approximately scale. A conformal invariance analysis of the three-point function by Migdal ²⁵⁾ leads to a prediction of power fall-off for the hadron form factor even in the $Z=0$ limit; our exponential behaviour would be ruled out in theories with finite dimension. In fact, scaling in the asymptotic bootstrap stems from an interplay of both zero (zim's) and very large invariant four momenta, and thus can never occur for exclusive hadronic reactions.

The algebra of currents, whether on the light cone or at equal times, appears to involve locality assumptions which a theory of infinitely composite objects might not respect. The clearest manifestation of this is the Dashen, Gell-Mann, Fubini sum rule ²⁶⁾

$$\frac{1}{\pi} \int d\nu A(\nu, Q_1^2, Q_2^2, t) = F(t) \quad (28)$$

This sum rule cannot be satisfied in our theory because, for increasing Q_1^2 , Q_2^2 , and fixed $t \neq 0$, the left-hand side decreases faster than any power while the right-hand side is constant. The only kinematical point at which Eq. (28), or its first derivative with respect to t , can be valid is at $t=0$, for there the $t=0$ line acts like a parton, killing the Q_1^2 , Q_2^2 dependence. Interestingly, current algebra sum rules have been tested only at $t=0$, any test of this sum rule away from the point $t=0$ would provide another crucial test of our bootstrap.

C. Dual models

It is not clear yet whether dual models will become theories of infinitely or finitely composite particles. That the mass spectrum turns out to be the same as in the thermodynamic bootstrap is suggestive, but it appears difficult to obtain rapidly falling form factors from dual models.

The wide angle, high energy elastic hadron-hadron cross-section obtained from the beta function representation differs considerably from our result, Eqs. (14), (16) and (20), and decreases instead like $e^{-csf(\cos\theta)}$. We would be surprised if unitarity corrections to this model, could they be

carried out, did not alter this prediction considerably. In the kinematical region where the unitarity correction may be small, e.g., the Mandelstam strip region, there the asymptotic bootstrap is unable to yield predictions. There is no incompatibility of our bootstrap ideas with duality; perhaps one can think of the results of dual models in the strip regions and of the asymptotic bootstrap at large $|s|$, $|t|$, $|u|$, as boundary conditions to be imposed upon further extensions of the other.

VI. CONCLUSIONS

We have seen that the predictions of the asymptotic bootstrap span a wide range of exclusive and inclusive processes involving asymptotic kinematics. A number of crucial tests of this theory have been reviewed. The most important of these are :

1. measurement of hadronic electromagnetic form factors at large space-like and time-like Q^2 . Search for predicted large 2γ effects.
2. wide-angle hadron-hadron scattering at NAL energies. No new breaks should appear in the 90° pp cross-section when plotted in the variable $(s)^{1/4}$.
3. annihilation experiments to look for scaling in $e^+e^- \rightarrow p+X$ and measurement of $\sigma(e^+e^- \rightarrow \text{hadrons})$. We predict no scaling and non-point-like σ .
4. search for virtual electromagnetic effects in large $p_\perp$ inclusive production. We predict no such effects.
5. measurement of deep inelastic e^-p exclusive angular distributions and dependence of the photon radius on Q^2 . We predict the effective radius should increase with increasing $Q^2 > 2 \text{ GeV}^2$.
6. measurement of wide-angle Compton scattering. A large wide-angle cross-section is predicted.
7. test of current algebra sum rules away from $t=0$. Breakdown is predicted.

Electron-positron annihilation appears to offer the most dramatic and conclusive means of testing the bootstrap. The absence of zim's in this reaction (to lowest order in α) leads to a violent breakdown of scaling in $e^+e^- \rightarrow p+X$ and a non-point-like total cross-section $\sigma(e^+e^- \rightarrow \text{hadrons})$. Perhaps annihilation experiments will be as important in revealing the composition of the hadrons as the Rutherford experiment was in revealing the composition of the atom.

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FIGURE CAPTIONS

- Figure 1 : Illustration of the connection between $Z=0$ and the concept of a dynamical pole as one which does not occur in an irreducible part.
- Figure 2 : Fit of Eq. (14) with $\gamma = 1/4$ to the proton's magnetic form factor.
- Figure 3 : Fit of Eq. (20) to the wide-angle, elastic proton-proton cross-section. The parameters in the fit are discussed in detail in Ref. 7).
- Figure 4 : CTG's contributing to wide-angle Compton scattering and giving non-exponentially decreasing contributions.
- Figure 5 : Contribution to deep inelastic electron-proton scattering which should dominate for $t \approx 0$.
- Figure 6 : Contribution to muon pair production which should dominate for $t \approx 0$.
- Figure 7 : A CTG representing a typical decay chain of a fireball of mass M_1 into n final state hadrons.
- Figure 8 : Bremsstrahlung contribution to muon pair production.

FIG.1

FIG.2

FIG.3

FIG. 4

FIG. 5

FIG. 6

FIG.7

FIG.8