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S' AND D STATE EFFECTS IN ^3He FORM FACTORS
AND IN $p-^3\text{He}$ ELASTIC SCATTERING CROSS-SECTION

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A B S T R A C T

The charge and magnetic form factors obtained by the Stanford group are fitted with a Gaussian wave function taking into account the S' and D states of the ^3He . Probabilities of 2 and 6% respectively are obtained for these states. The result is used to calculate the differential cross-section for $p-^3\text{He}$ elastic scattering, which is compared with experimental data.

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The ^3He charge and magnetic form factors obtained from the $e - ^3\text{He}$ experimental results of McCarthy et al. ¹⁾ have been well reproduced by Khanna ²⁾ using a sum of Gaussians as the wave function and taking into account only the S state of ^3He . However, Lim ^{3),4)}, using the three-body wave function obtained from a perturbation solution of the tri-nucleon system with the Eikemeier-Hackenbroich potential, has found a small admixture of the S' state necessary to fit the minimum in the charge form factor. He has also found it necessary to consider the magnetic exchange contributions.

We have fitted the Stanford data using a ground state wave function comprising S, S' and D states given in terms of Gaussian functions. Following the formalism of Schiff ^{5),6)} and using a superposition of Gaussians for the S state, one can write :

$$\psi = \psi_S + \psi_{S'} + \psi_D = u_2 \eta_1 - u_1 \eta_2 ,$$

where

$$\psi_S = (\psi_2 \eta_1 - \psi_1 \eta_2) f_1(S_5) ,$$

$$\psi_{S'} = (\psi_2 \eta_1 + \psi_1 \eta_2) f_2(S_1) + (\psi_2 \eta_2 - \psi_1 \eta_1) f_3(S_2) ,$$

$$\psi_D = [(5D_5 S_2 - 2D_2 S_5) \eta_1 - (5D_5 S_1 - 2D_1 S_5) \eta_2] f_6(S_5)$$

$$+ (D_2 S_5 \eta_1 - D_1 S_5 \eta_2) f_7(S_5)$$

$$+ [(D_2 S_1 + D_1 S_2) \eta_1 - (D_2 S_2 - D_1 S_1) \eta_2] f_8(S_5)$$

with

$$\begin{aligned}
 f_1(S_3) &= A e^{-\frac{1}{2}a^2(\tau_{12}^2 + \tau_{13}^2 + \tau_{23}^2)} [1 - C e^{-\frac{1}{2}b^2(\tau_{12}^2 + \tau_{13}^2 + \tau_{23}^2)}], \\
 f_2(S_4) &= \frac{1}{\sqrt{6}} [g(12,3) + g(13,2) - 2g(23,1)], \\
 f_3(S_2) &= \frac{1}{\sqrt{2}} [g(12,3) - g(13,2)], \\
 f_6(S_5) &= \mu f, \quad f_7(S_5) = \nu f, \quad f_8(S_5) = f, \\
 g(12,3) &= B e^{-\frac{1}{2}\alpha^2(\tau_{13}^2 + \tau_{23}^2) - \frac{1}{2}\beta^2\tau_{12}^2}, \\
 f &= D e^{-dS_5},
 \end{aligned}$$

where notation from 5), 6) is used. The wave function is normalized using the probabilities $P_S, P_{S'}, P_D$ of the three states to give :

$$\begin{aligned}
 A^2 &= P_S \frac{3\sqrt{3}}{\pi^3} \left[\frac{1}{a^6} - \frac{2C}{(a^2 + \frac{1}{2}b^2)^3} + \frac{C^2}{(a^2 + b^2)^3} \right]^{-1}, \\
 B^2 &= P_{S'} \frac{1}{2\pi^3} \left[\frac{1}{\alpha^3(\alpha^2 + \beta^2)^{3/2}} - \frac{8}{(\alpha^2 + \beta^2)^{3/2}(5\alpha^2 + \beta^2)^{3/2}} \right]^{-1}, \\
 D^2 &= P_D \frac{48}{\sqrt{3}\pi^3} \left[(105\mu^2 + 30\nu^2 + 14) \int_0^\infty e^{-2dR^2} R^{13} dR \right]^{-1}
 \end{aligned}$$

with $P_S + P_{S'} + P_D = 1$. The values of μ and ν are taken as $\mu = \sqrt{2/15}$, $\nu = \sqrt{7/15}$ to give equal probabilities for the three D states. The value of parameter d is determined from the result of 6) :

$$0.05 \pm 0.01 = \frac{P_D(6\mu + 7\nu)}{240\mu^2 + 60\nu^2 + 28} \frac{\int_0^\infty f^2 R^{15} dR}{6 \int_0^\infty f^2 R^{13} dR},$$

where, in the calculations, the lower sign was taken because of our choice of the neutron charge form factor.

The ${}^3\text{He}$ form factors are then given by :

$$F_{ch} = F_{ch}^{(p)} F_L + \frac{1}{2} F_{ch}^{(n)} F_0 ,$$

$$\mu_{3\text{He}} F_{mag} = \mu_n F_{mag}^{(n)} F_0 + \frac{2}{3} \mu_p F_{mag}^{(p)} (F_0 - F_L) + (\mu_{3\text{He}} - \mu_n) F_x ,$$

where

$$F_L = \frac{1}{2} (3F_a + F_b), \quad F_0 = 2F_b ,$$

$$F_a = \int e^{i\vec{q}\cdot\vec{r}_1} u_1^* u_1 d_3 r_1 ,$$

$$F_b = \int e^{i\vec{q}\cdot\vec{r}_2} u_2^* u_2 d_3 r_2 .$$

In our calculation, the terms f_2^2 and f_3^2 have been neglected. F_x is the normalized magnetic exchange form factor. We use :

$$F_{ch}^{(p)} = (1 + q^2/a_0^2)^{-2}, \quad a_0^2 = 0.71(\text{GeV}/c)^2,$$

$$F_{ch}^{(n)} = 0.02 q^2 ,$$

$$\mu_p = 2.7927, \quad \mu_n = -1.9132, \quad \mu_{3\text{He}} = -2.1276.$$

We first fitted the charge form factor only, with a^2 , b^2 , $\alpha - \beta$, $\alpha + \beta$, C , P_S , and P_D as parameters. The χ^2 value of 42 for 35 degrees of freedom indicated a reasonable fit. The values of parameters are :

$$a^2 = 0.216 \text{ fm}^{-2}, \quad b^2 = 0.486 \text{ fm}^{-2},$$

$$\alpha - \beta = -8.5 \cdot 10^{-7} \text{ fm}^{-1}, \quad \alpha + \beta = 0.588 \text{ fm}^{-1},$$

$$C = 1.56, \quad P_S = 0.02, \quad P_D = 0.06.$$

The Coulomb energy and the charge radius of the ${}^3\text{He}$ calculated with these parameters are $E_c = 0.784 \text{ MeV}$ and $r_{\text{rms}}^{\text{ch}} = 1.83 \text{ fm}$ respectively, with a good agreement with the experimental data $^{(1)}$ ($E_c = 0.76 \text{ MeV}$ and $r_{\text{rms}}^{\text{ch}} = 1.88 \pm 0.05 \text{ fm}$, respectively).

The amplitudes of the S and S' states have opposite signs and $\beta > \alpha$ in agreement with 3). The result for F_{ch} is presented in Fig. 1, together with results of Lim⁴⁾ and Khanna²⁾.

Using these values of the parameters, one can predict F_{mag} . In Fig. 2, we have presented curves for the magnetic form factor with $F_x = 1.0$ and $F_x = e^{-\gamma q^2}$, $\gamma = 0.248 \text{ fm}^{-2}$. This is the value obtained from the fit to F_{mag} with only γ as a free parameter and others taken from the fit to F_{ch} . It turns out that to obtain a reasonable F_{mag} it is necessary to take into account the exchange effects described by F_x , as experienced by Lim⁴⁾ (only the result with F_x in the exponential form agrees reasonably with experimental data). The value of the magnetic radius obtained with $\gamma = 0.248 \text{ fm}^{-2}$ is $r_{rms}^{mag} = 1.93 \text{ fm}$, in good agreement with the experimental value 1.94 fm from 1).

Having obtained the wave function as a sum of Gaussians, one can use now Glauber theory to study the elastic scattering of hadrons on ^3He . Although the effect of the D state is negligible as indicated in 7), the S' state does make a noticeable change to the differential cross-section. Unfortunately, the only available data on the scattering of ^3He are with protons^{8),9)} and the complicated spin structure of the nucleon-nucleon amplitude makes a full calculation rather tedious. We are then compelled to use the effective amplitude for NN scattering

$$f_{p(n)} = \frac{k_L \sigma_{p(n)}}{4\pi} (i + e_{p(n)}) e^{-\beta_{p(n)} q^2}.$$

With this amplitude we calculated the $p-^3\text{He}$ differential cross-sections for different sets of NN parameters and compared it with experimental data for $k_L = 1.2 \text{ GeV/c}$ ⁸⁾ (Fig. 3). One can observe that the results are very sensitive to the NN parameters, which are themselves not well known at this momentum. On the other hand, as one can see in Fig. 4, there is quite a difference in the results using our parameters for the ^3He wave function (with S' and D states used only in single scattering) and those using Khanna's²⁾.

One can also calculate the $p-^3\text{H}$ differential cross-section using the same formalism. The results are very similar to those for $p-^3\text{He}$ scattering, the only differences are in the region of the minimum, due to the differences in pp and pn scattering amplitudes. As for our values of the NN parameters, the differences are very small, we do not indicate them here.

Owing to the importance of the spin structure of the NN amplitude, we do not attempt to deduce anything from the $p\text{-}^3\text{He}$ data. However, we have shown that the wave function obtained by us gives reasonable results and is suitable for more detailed calculations including the spin and isospin structure of the NN amplitude. This wave function describes the ^3He form factors rather well if one includes the momentum transfer dependent exchange effects in F_{mag} and it also gives good values for both Coulomb energy and charge and magnetic radii of ^3He . To test the validity of this wave function for hadron-nucleon scattering, experimental data on $\pi\text{-}^3\text{He}$ and $\pi\text{-}^3\text{H}$ scattering would be welcome as the elementary amplitudes are well determined in this case and calculations are much easier to handle.

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FIGURE CAPTIONS

Figure_1 ^3He charge form factor :
 ——— our fit with parameters as given in the paper,
 ----- from 4) with S and S' states,
 -.-.- from 2) with S state only.

Figure_2 ^3He magnetic form factor :
 ——— our fit with $\gamma = 0.248 \text{ fm}^{-2}$,
 -.-.- our predictions with $\gamma = 0$,
 ----- predictions from 4) with S and S' states without magnetic exchange effects.

Figure_3 Differential cross-sections $d\sigma/d\Omega_L$ for $p - ^3\text{He}$ elastic scattering with different sets of parameters for NN amplitude and our wave function, $\sigma_p = 36.6 \text{ mb}$, $\sigma_n = 36.2 \text{ mb}$,

	$\beta_p = \beta_n (\text{GeV}/c)^{-2}$	e_p	e_n
.-.-.-.-.	2.15	0.78	-0.68
-----	2.15	0.11	-0.17
————	2.15	0.43	0.43
-...-...-	3.0	0.43	0.43.

Figure_4 Differential cross-sections $d\sigma/d\Omega_L$ for $p - ^3\text{He}$ elastic scattering with $\sigma_p = 36.6 \text{ mb}$, $\sigma_n = 36.2 \text{ mb}$, $\beta_p = \beta_n = 2.15 (\text{GeV}/c)^{-2}$, $e_p = e_n = 0.43$:
 ——— with our wave function,
 ----- with wave function from 2).

FIG.1

FIG. 2

FIG. 3

FIG. 4

E R R A T U M

p. 5 : Replace acknowledgements by :

We wish to thank Professor L. Bertocchi for many discussions and Professor T.E.O. Ericson for a critical reading of the manuscript and useful comments. One of us (G.W.) would like to thank the CERN Theoretical Study Division for its hospitality and the other (V.K.S.) the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

p. 9 : At the end of the caption for Fig. 3, add :

Experimental points are from Ref. 8).