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A Modification of Glauber Theory
for Diffraction Dissociation of Hadrons from Nuclei

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ABSTRACT

The formulation of the Glauber theory for diffraction dissociation of hadrons from heavy nuclei is modified to take into account the fact that the dissociated system D can oscillate between many internal states as it propagates through nuclear matter. The set of internal states of D is approximated by a continuum and the successive D -nucleon collisions are represented by a scattering amplitude which is taken to be a smooth function of the internal parameters of D . This smoothness property is argued to result from the very short time between successive D -nucleon collisions; it is therefore an off-shell effect. The physical implications of the modified scheme are an increased transparency of nuclear matter and a A -dependent modification in the distribution of internal D states, mainly a narrowing of the mass distribution. These effects are demonstrated by the numerical treatment of a highly simplified example.

1. Introduction

The Glauber theory of multiple scattering [1] has found one of its most interesting applications in the interpretation of inelastic reactions where a high energy particle collides with a nucleus and gets dissociated in the course of the reaction, with small transfer of momentum and energy to the nucleus. The latter remains in its groundstate (coherent process) or gets itself excited (incoherent process). The best studied case concerns photoproduction of φ^0 mesons by high energy photons [2]. Next comes diffraction dissociation of hadrons, particularly of pions [3,4]. Glauber theory describes such processes in terms of high energy forward amplitudes on single nucleons. In φ^0 photoproduction it leads to the experimental determination of the forward amplitude for $\varphi^0 N$ elastic scattering (N = nucleon); it is found to be about equal to the corresponding πN amplitude as expected from the quark model.

A recent experiment of the CERN-Zurich-London-Milano group [4] measures diffraction dissociation of 15 GeV/c negative pions into 3 and 5 charged pions from various complex nuclei. Analysis of the data by means of Glauber theory gives a forward $D N$ elastic amplitude about equal to the πN value when the dissociated system D contains 3 pions, for all D effective masses up to 1.5 GeV. Even smaller values are found when D contains 5 pions, but they are affected by large errors.

At first sight, these results suggest that D behaves like a single meson rather than like a composite system, e.g. a $\pi \varphi$ system. In fact, for a $\pi \varphi$ system an amplitude of about 0.85 times the sum of the πN and φN values is expected theoretically under neglect of the strong $\pi \varphi$ interactions [5]. This type of interpretation should be re-examined, however, if one visualizes D as a composite system which has strong interactions at work between its constituents but is not dominated by one or a few well separated resonances. The system D can then occur in a number of quantum states $|\alpha\rangle$, the rapid succession of $D N$ collisions

inside the nucleus causing repeated transitions of D among the states $|\alpha\rangle$.

The aim of the present paper is to explore the implications of Glauber theory under such circumstances, concentrating on the limiting situation of very high energy and large number of participating states $|\alpha\rangle$ of D . In this limit one can neglect the longitudinal momentum transfers resulting from mass differences between the pion and the various D states. In addition, the mass of the dissociated system can then range over a considerable interval, which suggests an approximation where the family of states $|\alpha\rangle$ is treated as a continuum (continuum model). The problem is greatly simplified under these asymptotic approximations. We make use of them in order to bring out as clearly as possible the qualitative consequences of the fact that the dissociated system D is expected to oscillate rapidly between many quantum states as it propagates through nuclear matter. The actual situation in the 15 GeV π^- experiment [4] is of course not asymptotic and may lie somewhere between our continuum model and the conventional Glauber model.

The main features revealed by the continuum model are an increased transparency of nuclear matter for D propagation and a new type of interference patterns. The latter are diffraction-like patterns in the continuous parameters characterizing the D states. The two phenomena are related to each other in the sense that the interference patterns occur because particular coherent superpositions of D states propagate through nuclear matter with small absorption. We believe that this effect can provide a qualitative explanation for the small DN amplitudes found in ref. [4] on the basis of the conventional Glauber analysis.

The paper is organized as follows. The continuum model is presented in the two next sections. Its physical implications are indicated in section 4 and are illustrated by the analytic and numerical

treatment of a very simple example. In the last section we come back to the basic physical assumption of the continuum model and we end with some concluding remarks. Throughout the paper we consider the coherent process of diffraction dissociation of pions on nuclei, but our treatment would apply unchanged to any other hadronic dissociation and the extension to incoherent processes is straightforward.

The idea of repeated transitions of the dissociated system between various states as it propagates through the nucleus is also embodied in a recent study of C. Wilkin and C. Rogers who consider ρ^0 photoproduction on nuclei. In contrast with our treatment, these authors approximate the $\pi^+\pi^-$ mass continuum by a few (in fact three) discrete states, so that their model cannot bring out the characteristic consequences of the continuum treatment, especially the new interference patterns [6]. One must keep in mind in addition that the ρ meson seems to be mostly in a single quantum state, so that the physical idea of repeated transitions in the course of propagation is perhaps less natural.

2. Eikonal Equations and DN Effective Scattering Amplitude

We work in the simple eikonal approximation where the incident pion and the dissociated mesonic system D are assumed to propagate through the nucleus along a rectilinear ray parallel to the z -axis at fixed value of the impact parameter b , as illustrated in Fig. 1. The propagation equations can be written as

$$\left(\frac{d^2}{dz^2} + E^2 - m_0^2 \right) \psi(z) = u \psi(z) + \int v(m) dm \chi(m, z) \quad (2.1)$$

$$\left(\frac{d^2}{dz^2} + E^2 - m^2 \right) \chi(m, z) = v(m) \psi(z) + \int w(m, m') dm' \chi(m', z) \quad (2.2)$$

m_0 is the pion mass and $\psi(z)$ is the pion wave function. We denote by $\chi(m, z)$ the wave function of the dissociated system D . For concreteness the continuum of D states is labelled by the parameter m representing its mass. Other continuous parameters could be used and the integrals in (2.1) and (2.2) could be multiple ones. Both ψ and χ also depend on the impact parameter b , but we do not write this dependence explicitly.

Neglecting nuclear recoil, we call E the common laboratory energy of π and D , so that the momenta are respectively

$$k = (E^2 - m_0^2)^{\frac{1}{2}}, \quad k(m) = (E^2 - m^2)^{\frac{1}{2}} \quad (2.3)$$

In our high energy approximation we set

$$k(m) = k \quad (2.4)$$

The matrix elements u, v, w of the optical potential operator are related by the equations

$$u = -4\pi g f_{e1}, \quad v(m) = -4\pi g f_{di}(m), \quad w(m, m') = -4\pi g f_{sc}(m, m') \quad (2.5)$$

to the density of nucleons ρ and various forward amplitudes f on single nucleons. Here f_{el} refers to forward elastic pion scattering, $f_{di}(m)$ to the forward pion dissociation $\pi \rightarrow D(m)$ into a state of mass m , and $f_{sc}(m, m')$ to forward transitions $D(m) \rightarrow D(m')$ between D states, all on a single nucleon. All amplitudes f are supposed to be the same on protons and neutrons. Time-reversal invariance gives

$$f_{sc}(m, m') = f_{sc}(m', m) \quad (2.6)$$

and ensures that $f_{di}(m)$ also describes the forward recombination $D(m) \rightarrow \pi$ on a single nucleon. The optical theorem gives

$$4\pi \text{Im } f_{el} = k \sigma_{\pi} \quad (2.7)$$

σ_{π} being the total pion nucleon cross section. All f depend on E . The nucleon density is a function of z and b , its integral over the whole nucleus being the total nucleon number A . Here, we work at constant b , on the ray of Fig. 1. Neglecting surface effects, we take ρ constant in the interval $z_0 \leq z \leq z_1$ and zero outside.

The eikonal equations (2.1-2) are no more than a continuum version of the well known Glauber equations for coupled channels. The new features of the continuum model originate from our basic assumption which is that $f_{sc}(m, m')$ can be approximated by a smooth function of the continuous parameters. The conventional Glauber model is recovered when

$$f_{sc}(m, m') = F \delta(m - m') \quad (2.8)$$

with F independent of m .

If the transitions $\pi N \rightarrow DN$ and $DN \rightarrow DN$ are regarded to be reactions on single nucleons in free space, one can easily discuss the functions f_{di} , f_{sc} in terms of theoretical models for high energy hadron

collisions. A Deck calculation of f_{di} , for example, would make use of diagrams of the type given in Fig. 2, where the rectangular boxes describe diffractive collisions (we take the simplest case of two constituents of D). But the constituents of D are strongly interacting and are produced within each other's range of force, so that strong final state interactions must also be at work. They are described by diagrams as the one in Fig. 3 with the triangular box representing the final state interaction.

If $f_{sc}(m, m')$ describes $DN \rightarrow DN$ collisions on a single nucleon in free space, one thinks of diagrams as illustrated in Figs. 4 to 6, the boxes having the same meaning as in Figs. 2 and 3. The diagram of Fig. 4 is disconnected and gives to f_{sc} a contribution of type (2.8). The delta function is a consequence of energy and momentum conservation. The diagrams of Figs. 5 and 6 have two effects: they give contributions which are smooth functions of m and m' , and they modify the $\delta(m - m')$ term contributed by Fig. 4 (one way of seeing this is to think of a Glauber theory approach to the scattering of N on the composite system D). Consequently, the general form of f_{sc} is then expected to be

$$f_{sc}(m, m') = F'(m)\delta(m - m') + f'(m, m') \quad (2.9)$$

F' and f' being smooth functions of their arguments. As to $f_{di}(m)$, of course, it is a smooth function of m .

The basic assumption mentioned above is that the delta function present in eq. (2.9) for DN scattering on a single nucleon in free space cannot manifest itself for very high energy propagation of D through nuclear matter. In this case one must take for $f_{sc}(m, m')$ an effective amplitude without delta singularity. The physical reason for this is the short time available to the composite system D between successive collisions on nucleons in nuclear matter preventing energy conservation to establish itself. Even if at first D would have a specific mass m ,

it has no time to settle to any special mass value between successive collisions, and this means that the effective $f_{sc}(m,m')$ to be used in the Glauber treatment cannot have a delta singularity.

3. Formal Treatment

We first simplify eqs. (2.1 - 2) by well known manipulations. At high energy one can restrict oneself to waves propagating in one direction, say the positive z -direction, which amounts to replacing (2.1 - 2) by first order equations

$$\left(\frac{d}{dz} - i k\right) \psi(z) = (2ik)^{-1} \left[u \psi(z) + \int v(m) dm \chi(m, z) \right] \quad (3.1)$$

$$\left(\frac{d}{dz} - i k(m)\right) \chi(m, z) = (2ik)^{-1} \left[v(m) \psi(z) + \int w(m, m') dm' \chi(m', z) \right] \quad (3.2)$$

The π dissociation on nucleons constitutes only a small part of the πN cross section σ_π . We therefore treat the equations to first order in $v(m)$. Eq. (3.1) has then for $z_0 \leq z \leq z_1$ the solution

$$\psi(z) = \exp(i k z) \exp[(i k - \kappa_0)(z - z_0)] \quad (3.3)$$

with the definition

$$\kappa_0 = i u / 2k \quad (3.4)$$

(3.3) corresponds to an incident pion wave

$$\psi(z) = \exp(i k z), \quad z < z_0 \quad (3.5)$$

Note that from (2.7) and (3.4)

$$\text{Re } \kappa_0 = \frac{1}{2} \rho \sigma_\pi, \quad (3.6)$$

so that (3.3) shows the normal absorption determined by the total πN cross section. The first order treatment in $v(m)$ is equivalent to neglect of inelastic shadowing of the incident pion.

Substituting (3.3) into (3.2), this equation reduces for $z_0 \leq z \leq z_1$ to

$$\frac{d}{dz} \xi(m, z) = \kappa(m) \exp[-\kappa_0(z - z_0)] - \int \lambda(m, m') dm' \xi(m', z) \quad (3.7)$$

with the following definitions

$$\xi(m, z) = \exp(-i k z) \chi(m, z) \quad (3.8)$$

$$\kappa(m) = i v(m)/2k \quad (3.9)$$

$$\lambda(m, m') = i w(m, m')/2k \quad (3.10)$$

$\chi(m, z)$ and $\xi(m, z)$ vanish at $z = z_0$.

Eq. (3.7) can be studied by introducing the eigenfunctions $g_\alpha(m)$ and eigenvalues λ_α of the kernel $\lambda(m, m')$. They are defined by

$$\int \lambda(m, m') dm' g_\alpha(m') = \lambda_\alpha g_\alpha(m) \quad (3.11)$$

We take the eigenvalues to be discrete; generalization to a continuous spectrum is obvious (see the example treated in section 4). In general the $g_\alpha(m)$ can be taken to form an orthonormal set in the sense of

$$\int g_\alpha(m) g_\beta(m) dm = \delta_{\alpha\beta} \quad (3.12)$$

This is true in particular for real $\lambda(m, m')$, which corresponds to purely imaginary, i.e. purely diffractive $DN \rightarrow DN$ transitions (remember also eq. (2.6)). Expanding (3.7) in the g_α we obtain

$$\left(\frac{d}{dz} + \lambda_\alpha\right) \xi_\alpha(z) = \kappa_\alpha \exp[-\kappa_0(z - z_0)], \quad \xi_\alpha(z_0) = 0$$

where we have put

$$\xi(m, z) = \sum_{\alpha} \xi_{\alpha}(z) g_{\alpha}(m), \quad \kappa(m) = \sum_{\alpha} \kappa_{\alpha} g_{\alpha}(m) \quad (3.13)$$

The solution is for $z_0 \leq z \leq z_1$

$$\xi_{\alpha}(z) = \kappa_{\alpha} \{ \exp[-\kappa_0(z - z_0)] - \exp[-\lambda_{\alpha}(z - z_0)] \} / (\lambda_{\alpha} - \kappa_0) \quad (3.14)$$

The scattering amplitude for coherent dissociation on the nucleus is related to $\xi(m, z)$ by

$$F(m, q) = ik \int_0^R b db J_0(qb) \xi(m, z_1) \quad (3.15)$$

where $q = (-t)^{\frac{1}{2}}$ is the momentum transfer. Collecting the preceding equations one is led to the expression

$$F(m, q) = ik \sum_{\alpha} \frac{\kappa_{\alpha} g_{\alpha}(m)}{\lambda_{\alpha} - \kappa_0} \int_0^R b db J_0(qb) [\exp(-\kappa_0 z_b) - \exp(-\lambda_{\alpha} z_b)] \quad (3.16)$$

with R the nuclear radius. We have put

$$z_b = 2(R^2 - b^2)^{\frac{1}{2}} \quad (3.17)$$

In the normalization here used the differential cross section is given by

$$\frac{d^2\sigma}{dt dm} = \frac{d^2\sigma}{dq^2 dm} = \frac{\pi}{k^2} |F(m, q)|^2 \quad (3.18)$$

The partially integrated cross sections $d\sigma/dt$ and $d\sigma/dm$ are readily derived by using (3.12) and the identity

$$\int dq^2 J_0(qb) J_0(qb') = 2b^{-1} \delta(b - b')$$

They are given by

$$\frac{d\sigma}{dt} = \sum_{\alpha} \frac{|\kappa_{\alpha}|^2}{|\lambda_{\alpha} - \kappa_0|^2} \left| \int_0^R b db J_0(bq) [\exp(-\kappa_0 z_b) - \exp(-\lambda_{\alpha} z_b)] \right|^2 \quad (3.19)$$

$$\frac{d\sigma}{dm} = 2\pi \int_0^R b db \left| \sum_{\alpha} \frac{\kappa_{\alpha} g_{\alpha}(m)}{\lambda_{\alpha} - \kappa_0} [\exp(-\kappa_0 z_b) - \exp(-\lambda_{\alpha} z_b)] \right|^2 \quad (3.20)$$

4. Discussion and Simple Example

The formal solution (3.16) allows us to discuss the qualitative behaviour of the dissociation process under various assumptions. The conventional application of Glauber theory corresponds to the case (2.8), all eigenvalues λ_α then have a common value λ and eq. (3.16) then reduces to the familiar form

$$F(m,q) = ik \frac{\kappa(m)}{\lambda - \kappa_0} \int_0^R b \, db \, J_0(qb) [\exp(-\kappa_0 z_b) - \exp(-\lambda z_b)] \quad (4.1)$$

Its important characteristic for our discussion is that the distribution in mass m is independent of the nuclear size (remember that we work at very high energy and neglect the longitudinal momentum transfer; at finite energy the effect of this transfer is to suppress high masses of the dissociated system D).

The situation is different in the continuum model here considered. When $f_{sc}(m,m')$ and $\lambda(m,m')$ are smooth functions, the eigenvalue problem (3.11) leads to a spectrum of eigenvalues λ_α ranging from some finite value down to zero. While this fact can of course be formulated in more precise terms by specifying more restrictive mathematical conditions on $\lambda(m,m')$, a matter not pursued here, the existence of small eigenvalues can be intuitively understood by the following qualitative consideration. We take $\lambda(m,m')$ to be real and smooth. The function

$$\bar{g}(m) = \int \lambda(m,m') dm' g(m')$$

is very small whenever $g(m')$ is a function of m' which oscillates rapidly around zero, i.e. when the Fourier analysis of $g(m)$ in m is dominated by high frequency components. Suppose $g(m)$ real and consider its eigenvector expansion

$$g(m) = \sum_{\alpha} c_{\alpha} g_{\alpha}(m)$$

We have

$$\bar{g}(m) = \sum_{\alpha} c_{\alpha} \lambda_{\alpha} g_{\alpha}(m) \approx 0$$

and hence

$$\int [\bar{g}(m)]^2 dm = \sum_{\alpha} c_{\alpha}^2 \lambda_{\alpha}^2 \approx 0$$

But

$$\int [g(m)]^2 dm = \sum_{\alpha} c_{\alpha}^2$$

is certainly not small, so that λ_{α} must be small for most α contributing to the last sum.

Returning to (3.16), we consider in this equation the terms containing a small λ_{α} . For them, the absorption described by $\exp(-\lambda_{\alpha} z_b)$ is weak, corresponding to high transparency of nuclear matter for the corresponding components $g_{\alpha}(m)$ of the amplitude. This leads to two characteristic predictions of the continuum model: increased transparency of nuclear matter for part of the amplitude and variation in shape of the mass distribution with nuclear size.

To get a more concrete idea of this general behaviour, we now consider a very crude but simple model of $D \rightarrow D$ transitions for which the eigenvalue problem (3.11) is soluble by Fourier transform. It is certainly not realistic but may be useful for a first discussion of the qualitative consequences of our model. It supposes the $D \rightarrow D$ transition amplitude $f_{sc}(m, m')$ to depend only on the difference $m - m'$:

$$\lambda(m, m') = \Lambda(m - m'), \quad \Lambda(-m) = \Lambda(m) \quad (4.2)$$

The unrealistic nature of this ansatz is obvious for small values of m, m' because it neglects threshold effects. As we shall see, however, this should not seriously affect the conclusions. The eigenvalue problem (3.11) is solved by

$$g_{\alpha}(m) = (2\pi)^{-1} \exp(i\alpha m), \quad \lambda_{\alpha} = \int \Lambda(m) \exp(-i\alpha m) dm \quad (4.3)$$

the index α being a continuous variable. Similarly, κ_{α} of eq. (3.13) is given by

$$\kappa_{\alpha} = \int \kappa(m) \exp(-i\alpha m) dm \quad (4.4)$$

For $\Lambda(m)$ we adopt a real gaussian

$$\Lambda(m - m') = (\lambda_0 / 2\sqrt{\pi}\mu) \exp[-\frac{1}{4}(m - m')^2 / \mu^2] \quad (4.5)$$

and we have accordingly

$$\lambda_{\alpha} = \lambda_0 \exp(-\mu^2 \alpha^2) \quad (4.6)$$

The case of small width μ corresponds to the limiting case (4.1) with $\lambda = \lambda_0$ and is equivalent to the conventional treatment. It holds whenever λ_{α} remains constant over the relevant range of α , which is the range where κ_{α} is sizable. We are interested in the case of larger μ and it is natural to consider values such that the width of (4.5) is comparable to the width of the A bump produced in diffractive dissociation of a pion on a proton, i.e. μ of order 0.1 or 0.2 GeV. For the value of λ_0 we shall consider the range $0 < \lambda_0 \leq 2\kappa_0$, the upper bound corresponding to a maximum eigenvalue of the absorption coefficient equal to twice the pion absorption.

As to $\kappa(m)$, one could take a dissociation amplitude of Deck type. This gives essentially a real $\kappa(m)$ such that $[\kappa(m)]^2$ simulates the mass distribution of the A bump as produced on protons. Alternatively, one could also adopt for $\kappa(m)$ a linear superposition of Breit-Wigner functions. We examined both choices and found similar results. We here report on a simplified version of the former case in which $\kappa(m)$ is also given a gaussian form

$$\kappa(m) = (\bar{\kappa}/2\pi\gamma) \exp\left[-\frac{1}{4}(m - m_0)^2/\gamma^2\right] \quad (4.7)$$

This roughly simulates the $A_1 - A_2$ part of the A bump if $m_0 = 1.15$ GeV, $\gamma = 0.2$ GeV. $\bar{\kappa}$ is a scale parameter which we need not worry about. The Fourier transform is

$$\kappa_\alpha = \bar{\kappa} \exp(-i\alpha m_0 - \alpha^2 \gamma^2) \quad (4.8)$$

The function $\xi(m, z)$ of section 3 is now found to be

$$\xi(m, z) = (\bar{\kappa}/\pi) X(m - m_0, z - z_0) \quad (4.9)$$

with

$$X(M, z) = \int_0^\infty d\alpha \cos(\alpha M) \exp(-\alpha^2 \gamma^2) \frac{\exp(-\kappa_0 z) - \exp(-\lambda_\alpha z)}{\lambda_\alpha - \kappa_0} \quad (4.10)$$

λ_α is given by (4.6). From eq. (3.15) the scattering amplitude F becomes

$$F(m, q) = (ik\bar{\kappa}/\pi) \int_0^R b db J_0(qb) X(m - m_0, z_b) \quad (4.11)$$

z_b being defined by (3.17). In the case here taken, all functions of m

are symmetric around m_0 , as seen from the fact that X of (4.10) is even in $M = m - m_0$.

We now proceed with numerical illustrations based on the above equations. We shall express the masses m, M as well as α^{-1} in units of γ (as mentioned above, γ is of order 0.2 GeV). Similarly, we shall express R, b, z and q^{-1} in units of κ_0^{-1} which we suppose to be real (using (3.6) with $R = 1.12 A^{1/3}$ fm and $\sigma_\pi = 26$ mb one finds $\kappa_0^{-1} = 4.5$ fm). The strength and width of the function $\Lambda(m - m')$, eq. (4.5), are then expressed by the two dimensionless parameters

$$L = \lambda_0 / \kappa_0, \quad N = \mu / \gamma \quad (4.12)$$

As mentioned above the characteristics of the continuum model stem from the increased transparency of nuclear matter for rapidly oscillating components of the amplitude as function of m . These are in our case the Fourier components (4.3) of high $|\alpha|$, corresponding to small λ_α , eq. (4.6). We illustrate this effect by considering the α -distribution in eq. (4.10) for various values of z , the penetration depth. The function in question is

$$g(\alpha, z) = \exp(-\alpha^2 \gamma^2) [\exp(-\kappa_0 z) - \exp(-\lambda_\alpha z)] / (\lambda_\alpha - \kappa_0) \quad (4.13)$$

Since it becomes very small both at small and large z , we rescale it as follows

$$G(\alpha, z) = z^{-1} \exp(\kappa_0 z) g(\alpha, z) \quad (4.14)$$

The function G is plotted in Fig. 7 for $\kappa_0 z = Z$ very small, $Z = 1$ and $Z = 2$. The values adopted for the parameters (4.12) are $L = 2$, $N = 1$. The abscissa is $\alpha\gamma = A$. The curve for $Z \approx 0$ is of course the gaussian $\exp(-A^2)$. For increasing Z one sees that the small α region is extinguished more rapidly than the one of large α .

The function $A_f(M)$ represented in Fig. 8 is the amplitude (4.11) in the forward direction ($q = 0$), i.e.

$$A_f(M) = F(m, 0)$$

calculated from (4.10-11). It is plotted as function of $M = (m - m_0)/\gamma$ for positive M (remember that it is even in M) and we have chosen again $L = 2$, $N = 1$. Taken from bottom to top at $M = 0$, the three curves correspond to the nuclear radii

$$R_1 = 0.5 \kappa_0^{-1}, \quad R_2 = \kappa_0^{-1}, \quad R_3 = 1.5 \kappa_0^{-1} \quad (4.15)$$

The A values of the corresponding nuclei are approximately

$$A_1 = 8, \quad A_2 = 64, \quad A_3 = 216$$

The vertical scale of Fig. 8 is arbitrary but the same for all curves. In the conventional case (λ_α independent of α) all curves of Fig. 8 would be of type $\text{const} \times \exp(-M^2/4)$. In our case the change of shape, in particular the occurrence of the zero which moves to smaller M as A increases, is the consequence of the greater transparency of nuclear matter for higher Fourier components $g_\alpha(m)$.

Fig. 9 gives the mass distribution integrated over the coherent peak, i.e. the quantity

$$I(M) = \int \frac{d^2\sigma}{dq^2 dm} dq^2 = \frac{d\sigma}{dm}$$

It is an even function of $M = (m - m_0)/\gamma$ and is plotted for positive M . As before, we have chosen $L = 2$, $N = 1$. Taken from bottom to top at $M = 0$ the three lower curves refer to the nuclei of radii (4.15). The vertical scale is arbitrary but the same for the three curves. The top

curve, given for comparison, is proportional to the unperturbed mass spectrum $\exp(-\frac{1}{2} M^2)$. The insert of Fig. 9 gives the three curves for $I(M)$ on an enlarged scale in the region $M \gtrsim 2$.

The qualitative behaviour of $I(M)$ as the nuclear radius increases is interesting. The main part of the distribution gets narrower but small outer wings appear which have small secondary maxima (remember the symmetry in M). This looks like a diffraction-like interference pattern in M and is due to the occurrence of zeros in the amplitude as illustrated in Fig. 8. As ordinary diffraction this effect is a coherent one; if one would just allow D to scatter on one single nucleon, the effect on the mass distribution would only be to replace the unperturbed gaussian $\exp(-\frac{1}{2} M^2)$ by the broader one $\exp(-M^2/4)$.

Since the main effect of nuclear absorption on $I(M)$ is to narrow the mass peak, we expect that the neglect of threshold effects in the ansatz (4.2) is not too bad an approximation. Only the small outer wing for $M < 0$ ($m < m_0$) is expected to be strongly modified by these effects.

The total coherent cross section integrated over both m and q^2 is given in Fig. 10 as function of nucleon number A for various choices of the parameters (4.12). The vertical scale is arbitrary but the same for all curves. The effect of increasing N , i.e. the width of $\Lambda(m - m')$, while keeping L fixed is illustrated by the three curves for $L = 2$: larger N means larger transparency. The remaining curve, with $L = 1.3$ and $N = 0$, shows that the conventional theory ($N = 0$) for smaller L has a A dependence analogous to the continuum model ($N > 0$) for larger L .

5. Concluding Remarks

The example studied in the previous section demonstrates the main physical implications of the continuum model, which we recall to be the following: increased transparency of nuclear matter for propagation of the dissociated system D , and occurrence of diffraction-like interference patterns in the distribution of the continuous parameter(s) describing the internal state of D (the parameter m in our case). The two phenomena have a single cause: the transparency is increased for the higher frequency components of the D amplitude when Fourier-analysed in m .

These qualitative properties will undoubtedly hold true in more realistic versions of the model which should incorporate the following improvements: smooth boundaries for the density distribution of nuclear matter (a Saxon - Wood form for example), threshold effects in the function $f_{sc}(m, m')$, correction for non-vanishing longitudinal momentum transfers. In view of the large freedom which results from our a priori ignorance of $f_{sc}(m, m')$ and to some extent of $f_{di}(m)$, one expects that it should not be too difficult to fit existing data by selecting appropriate forms for these functions and by including the above corrections, but the significance of such fits would be modest. It will be of greater interest to see whether future high precision measurements of the mass distribution of diffractively dissociated hadrons from various nuclei will support the main qualitative predictions of our model. The energy should be taken as high as possible in order to reduce the distortion effect of longitudinal momentum transfers on the mass distribution. Additional tests can be devised if diffraction dissociation on deuterium becomes measurable in the q^2 region where double scattering dominates.

As to the physical ideas underlying the continuum model, we are sure that further thought will be needed to assess their theoretical validity. We recall the main point: each of the successive DN collisions

in the nucleus has only a very short time available to it before the next such collision takes place. This, we have assumed, prevents energy-momentum conservation to set in and reflects itself in the smoothness of $f_{sc}(m,m')$. In other words, our smoothness assumption for $f_{sc}(m,m')$ is a manifestation of off-shell scattering effects in Glauber theory. The possible relevance of such effects has been noted in general terms by Kantor [7]^{*)} and recently again by Białas, Czyż and Kotański [8]. We have proposed here a concrete model incorporating off-shell effects for diffraction dissociation and we have shown that it has interesting observable consequences. Whether the model is valid we cannot say. We believe, however, that the basic idea is correct and that it introduces a significant modification in the principles of Glauber theory.

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FIGURE CAPTIONS

- Figure 1 : Rectilinear propagation path of the pion and the dissociated system through the nucleus.
- Figure 2 : Deck diagrams for diffraction dissociation.
- Figure 3 : Deck diagram with final state interaction.
- Figure 4 : Simplest diagram for DN scattering.
- Figure 5 : Diagram of Fig. 4 with interactions in initial and final states of D.
- Figure 6 : Double scattering diagram with interactions in initial and final states of D.
- Figure 7 : Function (4.14) for various values of $\kappa_0 z = Z$. The abscissa is $\alpha\gamma = A$.
- Figure 8 : Amplitude (4.11) at $q = 0$ (forward direction) for various nuclear radii, eq. (4.15). The abscissa is $M = (m-m_0)/\gamma$. The vertical scale is arbitrary.
- Figure 9 : Mass distribution integrated over the coherent peak for various nuclear radii, and unperturbed mass distribution. See text for details.
- Figure 10 : Total coherent cross section versus nucleon number for various values of the parameters (4.12).

FIG.1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG. 7

FIG. 8

FIG. 9

FIG. 10