Two-Jet Inclusive Cross Sections in Heavy-Ion Collisions in the Perturbative QCD

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In the framework of perturbative QCD, double inclusive cross sections for high $p_t$ parton production in nucleus-nucleus collisions are computed with multiple rescattering taken into account. The induced long-range correlations between numbers of jets at forward and backward rapidities are found to reach $10 \div 20\%$ for light nuclei at $\sqrt{s} = 200\text{ GeV/c}$ and to be suppressed for heavy nuclei and LHC energies.

I. INTRODUCTION

Particle production in high-energy heavy ion collisions is now at the center of experimental efforts to discover the quark-gluon plasma \cite{1,2}. The observed particle spectra are the result of different mechanisms which are responsible for the creation of initial high-$p_t$ partons, their propagation and subsequent hadronization. (Experimentally the initially produced partons can be traced as jets of hadrons, so that in the following we shall often use the term 'jet' for the produced parton in the pure theoretical sense, neglecting all the subtleties related to its actual determination in the experiment.) To be able to see the formation of the quark-gluon plasma against the background of more conventional effects, such as as gluon emission due to bremsstrahlung and multiple hard collisions in the nuclear medium, one has to fully study the consequences of the latter. Jet quenching due to gluon emission has been studied in considerable detail (see e.g. \cite{3,4}). Also the effects of rescattering in single particle inclusive spectra initiated by the observation of \cite{5} has been considered in a series of articles for multiple soft \cite{6,7} and multiple hard pQCD scatterings \cite{8,9,10,11,12,13}. This paper generalizes the study of multiple hard collisions to double inclusive cross-sections and the following long-range correlations between the secondaries. Observation of such correlations has always served a very precise instrument for analyzing the dynamics of the interaction.

Following the framework introduced in \cite{11}, we consider only relatively hard collisions which allow for the perturbative QCD approach. The colliding nuclei are assumed to contain a variable number of partons with initially small transverse momenta, which become large as a result of hard collisions between partons belonging to the projectile and target. The latter are assumed to move fast along the collision axis in the opposite directions. In the present work we restrict ourselves to the study of high $p_t$ parton production at rapidities well separated from central rapidity region. Here longitudinal momenta of partons in the final state are much larger than the transverse ones. So we neglect attenuation of the former during the collisions and assume them to be conserved throughout the nucleus-nucleus interaction in agreement with the standard Glauber treatment. We neglect intrinsic correlations between partons inside the colliding nuclei which correspond to nuclear shadowing and take the partonic distributions just as a product of such distributions for the nucleons smeared out with the standard nuclear profile functions. This implies that we correctly describe quantum evolution of these distributions except for effects coming from interaction of partons between different nucleons in the nucleus, which should be studied from the DGLAP
evolution equation for the nucleus as a whole or, at very small \( x \), in the framework of the BFKL evolution (e.g. in the Colour Glass Condensate approach, see [14] and references therein). Jet quenching and hard parton contents of the participants are some of other important phenomena to be taken into account in the full treatment of jet production. Thus the physical phenomenon we study is restricted to multiple hard elastic scattering between partons. Our study is then to be considered as a baseline calculation to be followed by inclusion of all the above-mentioned effects.

Our formalism closely follows [11] to which paper we refer the reader for details. As in [11], our basic approximations in the study of multiple partonic collisions are 1) purely elastic partonic collisions with conservation of longitudinal momenta and 2) factorization of the \( S \)-matrix into the product of elementary partonic \( S \)-matrices (the Glauber approximation). We shall also use the notations of [11]. To make our presentation more self-contained we reproduce some basic notations below.

The initial states of the colliding nuclei A and B are represented as a superposition of states with different number of partons, having specific values of scaling variables and transverse positions with respect to the target nucleus center. We combine them into a single argument \( z = \{v, b\} \) for nucleus A and \( u = \{w, c\} \) for nucleus B. The (variable) number of partons will be denoted correspondingly by \( n \) and \( l \). In these notations the initial state of the nucleus A is

\[
|A\rangle = \sum_{n=1}^{\infty} \int d\tau_A(n) \Psi_{A,n}(z_1, \ldots, z_n)|n, z_i\rangle,
\]

where \( d\tau_A(n) = \prod_{i=1}^{n} d^3z_i \) stands for the phase space volume of the configuration. Wave functions \( \Psi_{A,n} \) of the \( n \)-parton configuration, symmetric in their arguments, are normalized to fulfill \( \langle A|A\rangle = 1 \):

\[
W_n \equiv \int d\tau(n) |\Psi_{A,n}(z_1, \ldots, z_n)|^2, \quad \sum_n W_n = 1.
\]

We will assume a Poissonian distribution for the number of partons: \( w_n = e^{-\langle n \rangle}/n! \) and, as we neglect intrinsic partonic correlations, a factorization property for the wave function

\[
|\Psi_{A,n}(z_i)|^2 = c_n \prod_{i=1}^{n} \rho_A(z_i).
\]

The Poissonian distribution immediately implies

\[
c_n = \frac{1}{n!} e^{-\langle n \rangle}, \quad \langle n \rangle = \int d^3 z \rho_A(z),
\]

where \( \langle n \rangle \) corresponds to the average number of partons in nucleus A. The same notations are used also for nucleus B.

The paper is organized as follows. In sections II and III we derive expressions for 2-parton inclusive cross section for two different cases. The case when both partons belong to the same forward rapidity region is examined in the section II derivation for the forward-backward case is presented in section III. The form of the two-parton cross sections implies that non-trivial forward-backward correlations emerge in high-\( p_t \) parton production. In section IV a quantity measuring the strength of correlations,
correlation coefficient, is proposed which appears to be expressed in a simple way in terms of the derived cross sections. Numerical values for the correlation coefficient followed by discussion are presented in section VI for light and heavy nuclei interactions at RHIC and LHC energies. The interpretation of our results and conclusions follow in section V.

II. TWO JETS IN THE SAME DIRECTION

The double inclusive cross-section to produce two jets in the same (forward) direction is obtained as a direct generalization of the single inclusive cross-section in \[11\]. As indicated above, here we limit our studies to the production of partons at rapidities well separated from zero and neglect changes in the parton longitudinal momenta during the interaction. This implies that both observed partons originate from the same nucleus (projectile A).

We have to fix the final state \( A' \) of the nucleus A to describe the two produced partons. Since the total wave function is symmetric in all the partons we choose them to be the first and the second one and account for other possibilities by factor \( \sqrt{n(n-1)} \):

\[
\Psi_{A',n}(z_1, z_2, z_3, \ldots z_n) = \sqrt{n(n-1)}\psi_{\alpha_1}(z_1)\psi_{\alpha_2}(z_2)\tilde{\Psi}_{A',n-2}(z_3, \ldots z_n)
\]

where \( \tilde{\Psi}_{A',n-2}(z_3, \ldots z_n) \) is the symmetrized wave function of the unobserved \( n-2 \) partons and \( \psi_{\alpha_i}(z_i), i = 1, 2 \) are the wave functions of the observed partons in the final state. \( \alpha_i \) combine scaling variables and transverse momenta \( p_1 \) and \( p_2 \) of the latter.

The probability to observe the two partons is given by the modulus squared of \( \Psi \) of the nucleus A to describe the two produced partons. In our approximation (purely elastic collisions) the number of partons is not changed by the interaction. So the S-matrix is diagonal in the basis \( \{n, z_i; l, u_i\} \):

\[
\langle n', z'_i, l', u'_j | S | n, z_i; l, u_j \rangle = \delta_{nn'}\delta_{ll'}\prod_{i=1}^{n}\delta^{(3)}(z_i - z'_i)\prod_{j=1}^{l}\delta^{(3)}(u_i - u'_j)S_{nl}(z_1, \ldots, z_n|u_1, \ldots, u_l).
\]

Following ref. \[11\] we take the square modulus of (6) making use of the specific form for the final-state wave function of the projectile nucleus \( A \). The double differential cross-section at a fixed overall impact parameter \( \beta \) which follows reads

\[
\frac{d\sigma_{\alpha_1\alpha_2}}{d^2\beta} = \sum_{n=2}^{\infty} \sum_{l\geq0} n(n-1) \int dz_1 dz_2 dz'_1 dz'_2 \psi_{\alpha_1}(z_1')\psi_{\alpha_2}(z_1)\psi_{\alpha_1}^*(z_2')\psi_{\alpha_2}^*(z_2)
\]

\[
\int d\tau_A(n-2)d\tau_B(l)\Psi_{A,n}^*(z'_1, z'_2, z_3, \ldots z_n)\Psi_{A,n}(z_1, z_2, z_3, \ldots z_n)|\Psi_{B,l}(u_1, \ldots, u_l)|^2
\]

\[
\left[ S_{nl}(z_1, z_2, z_3, \ldots z_n|u_1, \ldots u_l) - 1 \right] \left[ S_{nl}^*(z'_1, z'_2, z_3, \ldots z_n|u_1, \ldots u_l) - 1 \right]
\]

The product of square brackets in (7) gives four terms. The double inclusive cross-section for transverse momenta \( p_1, p_2 \gg 1/R_{A,B} \) corresponds only to the term which
originates from the product of the $S$-matrices. Indeed in the Glauber approximation we assume that the $S$-matrix is a product of $S$-matrices for pair parton collisions. Then terms linear in $S$ or $S^*$ diagrammatically correspond to cutting the forward scattering amplitude either to the extreme left or to the extreme right of all partonic interactions. In both cases the momenta of intermediate partons in the cut coincide with their values in the initial colliding nuclei, so that their transverse momenta are small, of the order of typical nuclear scale $1/R_{A,B}$ (see [11] for more detail).

We present each of the $S$-matrices as a product of the elementary partonic ones,

$$S_{nl}(z_1, \ldots, z_n | u_1, \ldots, u_l) = \prod_{i=1}^{n} \prod_{j=1}^{l} s_{ij}$$  \hspace{1cm} (8)

Here $s_{ij} = 1 + ia(z_i, u_j)$ and $a(z_i, u_j)$ are the parton-parton scattering matrix and amplitude respectively. Since only elastic parton scattering are considered, unitarity of the partonic $s$-matrices means $s_{ij}s^*_{ij} = 1$. So we obtain that

$$S^*_{nl}(z'_1, z'_2, z_3\ldots z_n | u_1, \ldots u_l)S_{nl}(z_1, z_2, z_3\ldots z_n | u_1, \ldots u_l)$$

$$= \prod_{i=1,2} \prod_{j=1}^{l} [1 + ia(z'_i, u_j)]^*[1 + ia(z_i, u_j)].$$  \hspace{1cm} (9)

and does not depend on $z_3, \ldots z_n$. This allows to integrate over these variables and sum over $n$ to produce the density matrix $\rho_A$ of the nucleus $A$ for a pair of partons:

$$\sum_n n(n-1) \int \tau_A(n-2)\Psi_{A,n}^* z'_1, z'_2, z_3, \ldots z_n \Psi_{A,n} z_1, z_2, z_3, \ldots z_n = \rho_A(z_1, z_2 | z'_1, z'_2).$$  \hspace{1cm} (10)

Factor $n(n-1)$ again accounts the possibility that any two of the partons entering the symmetric wave function can be chosen. Thus we get

$$\frac{d\sigma_{a_1 a_2}}{d^2\beta} = \sum_l \int dz_1 dz_2 dz'_1 dz'_2 \psi_{a_1}(z'_1)\psi_{a_2}(z'_2)\psi^*_{a_1}(z_1)\psi^*_{a_2}(z_2) \rho_A(z_1, z_2 | z'_1, z'_2)$$

$$\int d\tau_B(l)|\Psi_{B,l}(u_j)|^2 \left\{ \prod_{i=1,2} \prod_{j=1}^{l} [1 + ia(z'_i, u_j)]^*[1 + ia(z_i, u_j)] - 1 \right\}.$$  \hspace{1cm} (11)

Making use of the factorization of the wave function \hspace{1cm} (3) we have

$$\rho_A(z_1, z_2 | z'_1, z'_2) = \rho_A(z_1 | z'_1)\rho_A(z_2 | z'_2) \quad \text{and} \quad |\Psi_{B,l}(u_j)|^2 = \frac{1}{l!}e^{-l} \prod_{j=1}^{l} \rho_B(u_j | u_j).$$  \hspace{1cm} (12)

Assuming factorization of partonic transverse and longitudinal degrees of freedom, for equal scaling variables $v_i = v'_i = v$, we have

$$\rho_A(v, b_i | b'_i) = P_A(v)\tilde{\rho}_A(b_i | b'_i),$$  \hspace{1cm} (13)

where $P_A(v)$ is the mean parton number distribution and $\tilde{\rho}_A(b_i | b'_i)$ is the transverse part of single parton density matrix. For equal arguments it goes into the standard
profile function of the projectile nucleus $\tilde{\rho}_A(b_i|b_i) = T_A(b_i - \beta)$ (recall that the origin in the transverse plane is in the center of the target nucleus B). Similarly for the target nucleus $\rho_B(u_j|u_j) = P_B(u_j)T_B(c_j)$.

As a result of this factorization the inclusive cross section transforms into

$$I_{AA}(\beta, y_1, p_1, y_2, p_2) \equiv \frac{(2\pi)^4d\sigma}{dy_1 dy_2 dp_1 dp_2 d^2/} \bigg|_{y_1, y_2 > 0}$$

$$= \sum_l \frac{1}{l!} e^{(-l)} \int \prod_{i=1,2} \left( d^2b_i d^2b_i' e^{ip_i(b_i - b_i')} P_A(v_i) \tilde{\rho}_A(b_i|b_i') \right) \int \prod_{j=1}^{l} d^2c_j dw_T B(w_j) T_B(c_j) \left\{ \prod_{i=1,2} \prod_{j=1}^{l} [1 + i\alpha(z_i', u_j)]^* [1 + i\alpha(z_i, u_j)] - 1 \right\}$$

(14)

where the rapidities $y_1$ and $y_2$ correspond to the scaling variables $v_1$ and $v_2$. The subscript $AA$ for the inclusive cross-section indicates that both jets are produced from the nucleus $A$, although the collision is between $A$ and $B$.

The non-trivial integral over $u_j = \{w_j, c_j\}$ factorizes to give the $l$-th power of

$$J(v_1, v_2, b_1, b_1', b_2, b_2') = \int d^2cdw T_B(c) P_B(w) \prod_{i=1,2} [1 + i\alpha(v_i, w; b_i - c)]^* [1 + i\alpha(v_i, w, b_i - c)]$$

(15)

so that after summation over $l$ we get

$$I_{AA}(\beta, y_1, p_1, y_2, p_2) = \int \prod_{i=1,2} \left( d^2b_i d^2b_i' e^{ip_i(b_i - b_i')} \tilde{\rho}_A(b_i|b_i') \right) \left( e^{J(v_1, v_2, b_1, b_1', b_2, b_2') - (l - 1)} - 1 \right)$$

(16)

We are left with a problem of calculation $J$, which is now more complicated than for the single inclusive cross-section. From the 16 terms in the product in (7) we separate a part

$$1 + \sum_{i=1,2} \left( [i\alpha(v_i, w; b_i - c)]^* + i\alpha(v_i, w; b_i - c) + [i\alpha(v_i, w; b_i' - c)]^* i\alpha(v_i, w; b_i - c) \right)$$

Physically it corresponds to the case when either their is no partonic interaction at all or only one parton from nucleus A interacts with a given parton from nucleus B. Integration over $c$ and $w$ of this part repeats that for the single inclusive case and gives

$$J^{(1)} = \langle l \rangle + \sum_{i=1,2} T_B \left( \frac{1}{2} (b_i + b_i') \right) \left( F_B(v_i, b_i' - b_i) - F_B(v_i, 0) \right)$$

(17)

where

$$F_B(x, b) = \int dw P_B(w) \int d^2p \frac{d\sigma(x, w)}{d^2p} e^{ipb}$$

(18)

Putting (9) into (8) we find that the corresponding part of the double inclusive cross-section factorizes into a product of two single inclusive ones:

$$I_{AA}^{(1)}(\beta, y_1, p_1, y_2, p_2) = I_A(y_1, p_1) I_A(y_2, p_2)$$

(19)
where \( I_A(y, p) \) is the inclusive cross-section to produce a parton from nucleus \( A \) in its collision with nucleus \( B \) at impact parameter \( \beta \) (the latter dependence implicit):

\[
I_A(y, p) = \frac{(2\pi)^2 d\sigma}{dy dp d^2 \beta} = P_A(v) \int d^2 b d^2 r T_A(b - \beta) e^{ipr} \left( e^{T_B(b)(F_b(v, r) - F_B(0))} - e^{-T_B(b)F_B(0)} \right)
\]

(20)

So the part \( I_{AA}^{(1)} \) corresponds to independent production of the two partons as expected.

The rest 9 terms correspond to the case when both the observed partons interact with the same parton from nucleus \( B \). Evidently in this case the two observed partons have to be located close to each other, since the partonic interactions are short-ranged.

As a result we shall find under a single integral over the impact parameter \( b \) a square of the profile function \( T^2(b) \). Thus this part of the inclusive cross-section will have the order \( \sim 1/A^{2/3} \) as compared to the independent production part. Having in mind calculation of correlations, we can neglect this part in the first approximation, as we shall see in the following.

### III. TWO JETS IN OPPOSITE DIRECTIONS

In this case the wave functions of the observed partons \( \psi_{\alpha_1, \alpha_2} \) from nucleus \( A \) (moving along the \( z \)-axis) and nucleus \( B \) (moving in opposite direction) will be denoted by \( \psi_p(z) \) and \( \psi_q(u) \) respectively, where \( p \) and \( q \) combine their scaling variables (\( v \) and \( w \) respectively) and transverse momenta. The \( n \)-particle wave function for the final state of the nucleus \( A \) takes the form:

\[
\Psi_{A', n}(z_1, \ldots, z_n) = \sqrt{n} \psi_p(z_1) \psi_{A', n-1}(z_2, \ldots, z_n)
\]

and similarly for the nucleus \( B \).

Again, writing down the relevant matrix element for the transition amplitude and again following [11], we obtain the double inclusive cross section at a fixed overall impact parameter \( \beta \):

\[
I_{AB}(\beta, y_1, p, y_2, q) = \sum_{n, n' \geq 1} \sqrt{n n'} \sum_{A'B'} \int d\tau_A'(n') d\tau_B'(l') d\tau_A(n) d\tau_B(l) \psi_p(z_1') \psi_q(u_1') \Psi_{A, n'}^*(z_1' \ldots z_n') \Psi_{B, l'}^*(u_1' \ldots u_l') \Psi_{A', n-1}(z_2' \ldots z_n') \bar{\Psi}_{B, l-1}(u_2' \ldots u_l')
\]

\[
[S_n^*(z_1' \ldots z_n'|u_1' \ldots u_l) - 1][S_n(z_1 \ldots z_n|u_1 \ldots u_l) - 1]
\]

(21)

where the set \( z_i = \{ v_i, b_i \} \) of longitudinal momenta and transverse positions corresponds to the nucleus \( A \), whereas the set of \( u_j = \{ w_j, c_j \} \) to nucleus \( B \). The subscript \( AB \) indicates that the two jets are now produced from different nuclei.

Summing over all unobserved states of final nuclei \( A' \) and \( B' \), we can perform integrations over \( z_2' \ldots z_n' \) and \( u_2' \ldots u_l' \) to obtain

\[
I_{AB}(\beta, y_1, p, y_2, q) = \sum_{n \geq 1} n! \int d\tau_A(n - 1) du_1 da_1 d\tau_B(l - 1) \Psi_{A,n}^*(z_1', z_2 \ldots z_n) \Psi_{B,l}(u_1 \ldots u_l)[S^*(z_1', z_2 \ldots z_n, u_1', u_2, \ldots u_l) - 1] \psi_p(z_1') \psi_q(u_1')
\]
\[ \psi_p^*(z_1)\psi_q^*(u_1)[S(z_1...z_n, u_1...u_t) - 1]\Psi_{A,n}(z_1...z_n)\Psi_{B,t}(u_1...u_t) \] (22)

The product of the amplitude and its conjugate gives three terms

\[ [S^* - 1][S - 1] = [S^* S - 1] - [S^* - 1] - [S - 1] \] (23)

For the same reason as for the case of two jets in the same direction discussed in the previous section, the second and third terms in (23) give the partonic spectrum only at very small values of transverse momenta and can be neglected. To be more explicit, consider terms with no more than one partonic collision with the projectile and/or target. Inserting transverse parts of the wave functions of observed particles in the form \( \psi \sim e^{ikb} \) we arrive at the integral (for the term with \( S \))

\[ \int d^2b_1 d^2b_2 c_1 d^2b_2 c_1 d^2b_1 e^{ipb_1 + iqc_1 - ipb_1' - iqc_1'} X(b, b_1, c, c_1) \]

\[ \Psi_A(b_1...b..)\Psi_B(c_1...c..)\Psi_A^*(b_1'...b..)\Psi_B^*(c_1'...c..) \] (24)

where

\[ X = ia(b_1 - c)ia(c_1 - b), \text{ or } ia(b_1 - c), \text{ or } ia(b - c_1) \] (25)

The scale at which the wave functions of nuclei change is much greater than \( 1/p \) or \( 1/q \) so all of these terms after integrations in \( b_1', c_1' \) give contributions proportional to the product of delta-functions \( \delta(p)\delta(q) \). If we are interested in production of particles with large transverse momenta all these contributions can be neglected, so that in (23) only the first term contributes.

We present the total \( S \)-matrix elements as

\[ S_{nl}(z_1...z_n|u_1...u_t) = \prod_{j=1}^l s_{l_j} \prod_{i=2}^n s_{i1} \prod_{k,m=2}^{n,l} s_{km} \]

Due to unitarity of the partonic \( s \)-matrix, \( s_{ij}s_{ij}^* = 1 \), the product \( S^* S \) takes the form

\[ S_{nl}^*(z'_1, z_2...z_n|u'_1, u_2...u_t)S_{nl}(z_1, z_2...z_n|u_1, u_2...u_t) = (1 + ia(z'_1, u'_1))^*(1 + ia(z_1, u_1)) \]

\[ \prod_{j=2}^l (1 + ia(z'_1, u_j))^* \prod_{i=2}^n (1 + ia(z_i, u'_1))^* \prod_{j=2}^l (1 + ia(z_1, u_j)) \prod_{i=2}^n (1 + ia(z_i, u_1)) \] (26)

To move further, in contrast to the derivation in [11], we are forced from the start to make the assumption of factorization of the nuclear wave functions.

\[ \Psi_A^*(z'_1, z_2...z_n)\Psi_A(z_1, z_2...z_n) = \rho_A(z'_1|z_1)\rho_A(z_2|z_2)\rho_A(z_n|z_n) \frac{e^{-(n)}}{n!} \] (27)

and similarly for nucleus B, where the density matrices \( \rho_A(z_i|z_i') \) have the factorization property [13] and normalization [1].
Assuming for simplicity a central collision (in other case the argument in all \(T_A\)’s must be shifted by the impact parameter \(\beta\)) we get

\[
I_{AB}(\beta, y_1, p, y_2, q) = \sum_{n \geq 1} n \int dz_1 dz_1' du_1' \psi_p(z_1') \psi_q(u_1') \psi_p^*(z_1) \psi_q^*(u_1) \rho_A(z_1 | z_1') \rho_B(u_1 | u_1') \\
\left\{ [1 + \text{i}a(z_1', u_1')] [1 + \text{i}a(z_1, u_1)] \left( \prod_{j=2}^l du_j \rho_B(u_j | u_j) [1 + \text{i}a(z_1', u_j)] [1 + \text{i}a(z_1, u_j)] \right) \right\} \\
\left( \prod_{i=2}^n dz_i \rho_A(z_i | z_i) [1 + \text{i}a(z_i, u_1')] [1 + \text{i}a(z_i, u_1)] \right) - \int \prod \rho_B \prod \rho_A \frac{e^{-(n-l)}}{l! n!} \tag{28}
\]

Already at this stage it is convenient to separate from the total inclusive cross-section its part which does not contain interactions between the observed partons, that is the term which comes with unity from the product of the first two square brackets in (28). Comparing with (11) we find it to be:

\[
I_{AB}^{(1)}(\beta, y_1, p, y_2, q) = \left\{ \sum_{n \geq 1} \frac{e^{-n}}{(n-1)!} \int dz_1 dz_1' \psi_p(z_1') \psi_p^*(z_1) \rho_A(z_1 | z_1') \right\} \\
\left( \prod_{j=2}^l du_j \rho_B(u_j | u_j) [1 + \text{i}a(z_1', u_j)] [1 + \text{i}a(z_1, u_j)] - \int \prod_{j=2}^l du_j \rho_B(u_j | u_j) \right) \right\} \\
\left( \prod_{i=2}^n dz_i \rho_A(z_i | z_i) [1 + \text{i}a(z_i, u_1')] [1 + \text{i}a(z_i, u_1)] - \int \prod_{i=2}^n dz_i \rho_A(z_i | z_i) \right) - \int \prod \rho_B \prod \rho_A \frac{e^{-(n-l)}}{l! n!} \tag{29}
\]

where \(I_{A(B)}\) is the inclusive cross-section to produce a single jet from nucleus \(A(B)\) in \(AB\) collisions at impact parameter \(\beta\) (see (20)). So this term factorizes into a product of two independent single inclusive cross-sections to produce each of the observed partons. It is important that this result is exact and not based on the smallness of the parton interaction range on the nuclear scale. In particular, its validity is not spoiled by corrections of the order \(1/A^{2/3}\).

Now we turn to the rest terms in (28). In them the integrals in round brackets can be done exactly as in the standard Glauber derivation, taking into account that the space range of the partonic interaction is much smaller than the nuclear scale (see (11)). After that summations in \(n\) and \(l\) can be easily performed and we get

\[
I_{AB}^{(2)}(\beta, y_1, p, y_2, q) = \int d^2 b_1 d^2 b_1' d^2 c_1 d^2 c_1' \\
e^{i \nu (b_1 - b_1')} \rho_A(v, b_1 | b_1') \rho_B(w, c_1 | c_1')
\]
\[
\left\{ \left[ 1 + \imath a(b'_1 - c'_1, v, w) \right]^* \left[ 1 + \imath a(b_1 - c_1, v, w) \right] - 1 \right\} e^{E_B(v, b_1, b'_1) + E_A(w, c_1, c'_1) - 1} \tag{30}
\]

where
\[
E_B(v, b_1, b'_1) = T_B((b_1 + b'_1)/2)(F_B(v, b_1 - b'_1) - F_B(v, 0)) \tag{31}
\]

\(E_A\) is defined by a similar formula for nucleus A and \(y_1, y_2, p_1, p_2\) are the rapidities and transverse momenta of the observed particles (partons), and \(v, w\) are longitudinal momentum fractions corresponding to \(y_1\) and \(y_2\) respectively. Functions \(F_B(v, b)\) is the Fourier transform of the transverse momentum distributions \(I(v, w, p)\) in the elastic scattering of two partons with scaling variable \(v\) and \(w\), averaged over the longitudinal momenta of nucleus B partons (see [11]):

\[
F_B(v, b) = \int dw P(w) \int \frac{d^2 p}{(2\pi)^2} I(p, v, w) e^{\imath p b} \tag{32}
\]

It is convenient to introduce new variables for integration. Define

\[
\begin{align*}
& r_1 \equiv b_1 - b'_1; \\
& r_2 \equiv c_1 - c'_1; \\
& b \equiv (b_1 + b'_1)/2; \\
& c \equiv (c_1 + c'_1)/2
\end{align*}
\]

In these variables the cross section is rewritten as

\[
I^{(2)}_{AB}(\beta, y_1, p, y_2, q) = \int d^2 r_1 d^2 r_2 d^2 b d^2 c e^{\imath p r_1 + \imath q r_2} \rho_A(v, b_1 | b'_1) \rho_B(w, c_1 | c'_1)
\]

\[
\left\{ \left[ 1 + \imath a(b - c - \frac{r_1 - r_2}{2}, v, w) \right]^* \left[ 1 + \imath a(b - c + \frac{r_1 - r_2}{2}, v, w) \right] - 1 \right\} e^{E_B(v, b, r_1) + E_A(w, c, r_2) - 1} \tag{33}
\]

where in the new variables
\[
E_B(v, b, r_1) = T_B(b)(F_B(v, r_1) - F_B(v, 0)) \tag{34}
\]

and similarly for \(E_A\).

Assuming that \(E(..., b, c...)\) and density matrices change significantly only when \(b\) and \(c\) suffer macroscopic shifts of about nucleus radius, we can perform integrations in \(c\). Take the terms in (20) containing single amplitudes \(\imath a(b - c \pm (r_1 - r_2)/2)\). On the nuclear scale they can be effectively substituted as

\[
\imath a(b - c \pm (r_1 - r_2)/2) \to \imath \tilde{a}(0) \delta^2(b - c \pm (r_1 - r_2)/2) \simeq \imath \tilde{a}(0) \delta^2(b - c)
\]

where \(\tilde{a}\) is the amplitude in the transverse momentum space and we have used that \((r_1 - r_2)/2\) is small on the nuclear scale. In the term with the product

\[
a(b - c + (r_1 - r_2)/2) a^*(b - c - (r_1 - r_2)/2)
\]

we pass from the integration variable \(c\) to \(r = b - c + (r_1 - r_2)/2\) in which this product takes the form

\[
a(r) a^*(r - r_1 + r_2)
\]
Obviously $r$ is also small on the nuclear scale, so that $c \simeq b$ and we may take all the $c$-dependent function out of the integral over $r$ at $c = b$. The integral over $r$ takes the form

$$\int d^2r a(r) a^*(r - r_1 + r_2) = \int d^2p e^{i p (r_1 - r_2)} \frac{d \sigma(p, v, w)}{d^2p}$$

In this way we finally get for the second part of the inclusive cross-section

$$I_{AB}^{(2)}(\beta, y_1, p, y_2, q) = P_A(v) P_B(w) \int d^2b T_A(b) T_B(b) \int d^2r_1 d^2r_2 e^{i p r_1 + i q r_2}$$

$$\left( \int d^2l e^{i l (r_1 - r_2)} \frac{d \sigma(l, v, w)}{d^2l} - \sigma_{\text{tot}}(v, w) \right)$$

$$\left( e^{T_A(b)(F_A(w, r_2) - F_A(w, 0)) + T_B(b)(F_B(b, v, r_1) - F_B(v, 0))} - 1 \right)$$

(35)

This part of the cross-section corresponds to the case when the two observed partons interact with each other. Obviously this requires the two partons to be produced at the same point in the transverse space ($b = c$). As a result, this part is smaller than $I_{AB}^{(1)}$, corresponding to independent production part, by $\sim 1/A^{2/3}$. It is important to recall that the independent production part has been found without expansion in powers of $1/A$, so that $I_{AB}^{(2)}$ fully represents the difference from independent production.

To make the integrals over $r_1$ and $r_2$ convergent we change unity in the last bracket to $\exp(-T_A(b) F_A(w, 0) - T_B(b) F_B(v, 0))$ since this does not produce terms which could contribute at $p, q \neq 0$ and present the resulting cross-section in the form

$$I_{AB}^{(2)}(\beta, y_1, p, y_2, q) = P_A(v) P_B(w) \int d^2b T_A(b) T_B(b) \int d^2r_1 d^2r_2 e^{i p r_1 + i q r_2}$$

$$\left( \int d^2l e^{i l (r_1 - r_2)} \frac{d \sigma(l, v, w)}{d^2l} - \sigma_{\text{tot}}(v, w) \right)$$

$$\left\{ e^{T_A(b)(F_A(w, r_2) - F_A(w, 0))} - e^{-T_A(b) F_A(w, 0)} \right\} \left( e^{T_B(b)(F_B(v, r_1) - F_B(v, 0))} - e^{-T_B(b) F_B(v, 0)} \right)$$

$$+ e^{-T_A(b) F_A(w, 0)} \left( e^{T_B(b)(F_B(v, r_1) - F_B(v, 0))} - e^{-T_B(b) F_B(v, 0)} \right) + e^{-T_B(b) F_B(v, 0)}$$

$$\left( e^{T_A(b)(F_A(w, r_2) - F_A(w, 0))} - e^{-T_A(b) F_A(w, 0)} \right) + e^{-T_A(b) F_A(w, 0) - T_B(b) F_B(v, 0)}$$

$$\sum_{i=1}^4 I_{AB}^{(2i)}$$

(36)

The most non-trivial correlations are given by the first term. The simplest correlation is expressed by the last term. Indeed, dropping terms proportional to $\delta^2(p)$ or $\delta^2(q)$,

$$I_{AB}^{(24)} = (2\pi)^4 P_A(v) P_B(w) \delta^2(p + q) \frac{d \sigma(p, v, w)}{d^2p} \int d^2b T_A(b) T_B(b) e^{-T_A(b) F_A(w, 0) - T_B(b) F_B(v, 0)}$$

(37)

and shows back-to-back correlations. The second and third terms are expressed via partonic and nuclear single inclusive cross-sections:

$$I_{AB}^{(22)} = (2\pi)^2 P_B(w) \frac{d \sigma(q, v, w)}{d^2q} \int d^2b T_B(b) e^{-T_A(b) F_A(w, 0)} I_A(p + q, v, b)$$

(38)
where we defined the single cross-section at fixed \( b \) as the integrand in (20) (for the central collision, \( \beta = 0 \))

\[
I_A(p, v, b) = P_A(v)T_A(b) \int d^2r e^{ir} \left( e^{TB(b)(F_b(v,r) - F_B(v,0))} - e^{-TB(b)F_B(v,0)} \right)
\] (39)

The third term corresponds to \( v, p \leftrightarrow w, q \)

\[
I_{AB}^{(23)} = (2\pi)^2 P_A(v) \frac{d\sigma(p, v, w)}{d^2p} \int d^2b T_A(b) e^{-TB(b)F_B(v,0)} I_B(p + q, w, b)
\] (40)

Finally in terms of \( I_{A,B}(p + q, v(w), b) \)

\[
I_{AB}^{(21)} = \int d^2b \left( \int d^2l \frac{d\sigma(l, v, w)}{d^2l} I_A(p + l, v, b) I_B(q - l, w, b) - \sigma^{tot}(v, w) I_A(p, v, b) I_B(q, w, b) \right)
\] (41)

**IV. CORRELATIONS**

We restrict ourselves to the study of the forward-backward multiplicity correlations. Since the correlations are in any case small we may restrict ourselves to a linear dependence of the average multiplicity in the backward window at a fixed multiplicity in the forward window as a function of the latter;

\[
\frac{\langle n_B \rangle_{n_F}}{\langle n_B \rangle} = a + b n_F
\] (42)

Here \( \langle n_B \rangle \) is the overall average of the multiplicity in the backward window (at all \( n_F \)). Thus defined coefficient \( b \) shows the relative deviation of the conditional average \( \langle n_B \rangle_{n_F} \) when the number of jets observed in the forward rapidity window \( n_F \) changes by unity.

It can be expressed via averages of linear and bilinear products of multiplicities:

\[
b = \frac{1}{\langle n_B \rangle} \frac{\langle n_B n_F \rangle - \langle n_B \rangle \langle n_F \rangle}{\langle n_F^2 \rangle - \langle n_F \rangle^2}
\] (43)

One gets this expression by multiplying (42) first by \( p(n_F) \) then by \( n_F p(n_F) \), summing over \( n_F \) and solving the arising system of linear equations for \( a \) and \( b \).

To compute the mentioned bilinear products we first point out that \( n! \) in the denominator in the phase space volume for \( n \) identical particles should refer only to particles within the same phase space volume. For our problem it implies that identical particles produced in different rapidity windows can be considered as different (see Appendix for details). This allows to immediately obtain simple expressions for the squares of multiplicities coming from independent pair production described by the product of single inclusive cross-sections. Consider first emission of two particles into the forward rapidity window. At a fixed overall impact parameter \( \beta \) we find

\[
\langle n_F \rangle = c \int \frac{dy d^2p}{(2\pi)^2} I_A(y, p)
\] (44)
and
\[ \langle n_F(n_F - 1) \rangle = c \int \frac{dy_1 d^2 p_1 dy_2 d^2 p_2}{(2\pi)^4} I_{AA}(\beta, y_1, p_1, y_2, p_2) \] (45)
where integrations over \( y \) are restricted to the forward rapidity window and \( c = 1/(d^2\sigma_{AB}(\beta)/d^2 \beta) \) where \( d^2\sigma_{AB}(\beta)/d^2 \beta \) is the AB inelastic cross-section at fixed impact parameter \( \beta \). The latter is practically unity for \( \beta < R_A + R_B \). As shown in section II (Eq. (19)), in the first approximation, the double inclusive cross-section in (45) is just a product of two single inclusive ones. This gives a relation
\[ \langle n_F(n_F - 1) \rangle = \frac{1}{c} \langle n_F \rangle^2 \] (46)
which, with \( c \simeq 1 \) leads to
\[ \langle n_F^2 \rangle - \langle n_F \rangle^2 = \langle n_F \rangle \] (47)
(effectively the distribution seems to be Poissonian). Note that the dispersion is different from zero. This is the reason why for the production of the pair into the same (forward) rapidity window we can limit to the first approximation in powers of \( 1/A \) or \( 1/B \).

Passing to the emission of jets into different rapidity windows, we may consider the jets different. So instead of (45) we shall find
\[ \langle n_F n_B \rangle = c \int \frac{dy_1 d^2 pdy_2 d^2 q}{(2\pi)^4} I_{AB}(\beta, y_1, p, y_2, q) \] (48)
From independent production (part \( I_{AB}^{(1)} \)) we shall get just the product of average multiplicities, so that
\[ \langle n_F n_B \rangle - \langle n_F \rangle \langle n_B \rangle = \int \frac{dy_1 d^2 pdy_2 d^2 q}{(2\pi)^4} I_{AB}^{(2)}(\beta, y_1, p, y_2, q) \] (49)
where we used \( c \simeq 1 \).

This gives for the correlation coefficient \( b \)
\[ b = \frac{\int dy_1 d^2 pdy_2 d^2 q I_{AB}^{(2)}(\beta, y_1, p, y_2, q)}{\int dy_1 d^2 p I_A(y_1, p) \int dy_2 d^2 q I_A(y_2, q)} \] (50)
where integrals over \( y_1, p \) and \( y_2, q \) go over the forward and backward rapidity windows, respectively. We see that in the numerator of the expression for the correlation coefficient the leading terms in powers of \( 1/A \) and \( 1/B \) cancel and only subleading terms of the relative order \( 1/A^{2/3} \) or \( 1/B^{2/3} \) remain. This means that in any case forward-backward multiplicity correlations at a fixed \( \beta \) have the order \( 1/A^{2/3} \) or \( 1/B^{2/3} \). So for their observation collision of comparatively light nuclei is preferable. To calculate the correlations one has to evaluate the integrals in (50).

Our definition of the correlation coefficient differs from the conventional one which reads \( b_0 = \text{Cov}(n_B, n_F)/\sqrt{\text{Var}(n_F)\text{Var}(n_B)} \) and for equal dispersions of forward and backward multiplicities is \( b_0 = \langle (n_F n_B) - \langle n_F \rangle \langle n_B \rangle \rangle / \langle (n_F^2) - \langle n_F \rangle^2 \rangle \). As one can see comparing \( b_0 \) to (43) the difference is by factor \( \langle n_F^2 \rangle - \langle n_F \rangle \langle n_B \rangle \). As this factor is proportional to the backward rapidity window width \( \Delta y_2 \) (see (17)), the correlation coefficient (43) has an advantage compared to \( b_0 \) having no explicit dependence on the chosen rapidity intervals. However the value of correlation coefficient \( b \) is not limited from above by unity as for \( b_0 \).
V. NUMERICAL RESULTS

In the general case calculation of (50) presents a formidable numerical task. To simplify it we limit ourselves to central collisions of identical nuclei \( A = B \) and \( \beta = 0 \). We also assume the two rapidity windows narrow in rapidity and magnitudes of the transverse momenta \(|p|\) and \(|q|\) centered around \( y_1, p \) and \( y_2, q \), so that in (50) the inclusive cross-sections can be taken out of the integrals in \( y_{1,2} \) and \(|p|\) and \(|q|\) at these points. As to the azimuthal angle \( \phi \) between the jet momenta \( p \) and \( q \) it may be chosen differently in the experimental setup. If one takes into account pairs of jets with an arbitrary \( \phi \) then after integration over the angles we will get a correlation coefficient depending only on the chosen \( y_{1,2} \) and \(|p|\) and \(|q|\):

\[
b(y_1, p, y_2, q) = \frac{1}{\pi} \int d\phi \frac{I^{(2)}_{AB}(\beta = 0, y_1, p, y_2, q, \phi)}{I_A(y_1, p)I_B(y_2, q)}
\]

We restricted our calculations to the case \( y_1 = y_2 \) and \( p = q \).

Note that the elementary processes taken into account in \( I^{(2)}_{AB} \) formally include the contribution from a single hard rescattering accompanied by multiple soft rescatterings. Physically it corresponds to smearing of the lowest order back-to-back correlation by soft interactions both before and afterwards. In our formalism soft interactions are suppressed by a cutoff in the elementary parton-parton cross-section. However one can see that at large jet transverse momenta the total correlations become dominated by the above mentioned contribution with soft rescatterings taken at the transferred momenta of the order of the cutoff. Obviously such contribution cannot be described correctly in our perturbative formalism. To eliminate the effect of soft-smeared back-to-back correlations, especially pronounced at high momenta, we choose to restrict the experimental range of azimuthal angles \( \phi \) between the jets, excluding from it angles close to the back-to-back configuration. We introduce a “veto angle” for the final momenta, that is we demand that the azimuthal angle \( \phi \) should not be larger than \( \pi - \phi_{\text{veto}}/2 \). This excludes the undesirable interval of \( \phi \) around \( \pi \) of length \( \phi_{\text{veto}} \). The value \( \pi - \phi_{\text{veto}}/2 \) then serves as the upper limit of integration in the numerator of (51) and the denominator is modified by a factor of \( (\pi - \phi_{\text{veto}}/2)/\pi \).

For the partonic cross-sections we have taken the same expression as in [11]. Namely we assumed the effective partonic distributions with the gluon-gluon cross-section as a dynamical input. The lowest order cross-section was multiplied by the so-called \( K \) factor to take into account higher order corrections. The infrared regularization was realized by a non-zero gluon mass \( p_0 \). The scale in the partonic distributions was taken as \( p_0^2 + p_1^2 \) where \( p_1 \) was the transferred transverse momentum.

We studied S-S and Pb-Pb collisions at two c.m. energies 200 and 6000 GeV. In accordance with [13] we set \( p_0 = 1 \) GeV and \( K = 1.04 \) at 200 GeV and \( p_0 = 2 \) GeV and \( K = 2 \) at 6000 GeV. In our calculations we take \( \phi_{\text{veto}} = \pi/6 \). We also present results for 200 GeV·A Cu-Cu and Au-Au collisions and for 200 GeV·A S-S with \( \phi_{\text{veto}} = \pi/3 \).

In figures 1 and 2 we show the correlation coefficient \( b \) as a function of \( p \) at \( y_1 = y_2 = 1, 2 \) and \( 3 \) for S-S and Pb-Pb collisions at c.m. energy 200 GeV. Figures 3 and 4 show the same correlation coefficients at 200 GeV for Cu and Au nuclei and figures 5 and 6 represent the correlation coefficient for S and Pb at 6000 GeV.

The found correlation coefficients drop with atomic numbers and energy and grow with jet momenta. For Su-Su collisions at 200 GeV they have a sharp rise for momenta around \( 7 \div 8 \) GeV/c and rapidity \( y_B = y_F = 3 \). One of the reasons for this was already
discussed in this section, another is due to kinematics: the single inclusive cross-sections in the denominator of (43) tend to zero at the limits of the phase space volume. As expected, for Pb-Pb collisions the correlation coefficients are an order of magnitude smaller than for S-S collisions.
VI. CONCLUSIONS

Our calculations show that in spite of the fact that in heavy nuclei long range multiplicity correlations between jets are small, of the order $A^{-2/3}$ for identical nuclei, they are of observable magnitude if nuclei are not too heavy. They also visibly grow with the transverse momentum and for S-S collisions at 200 GeV. For transverse momenta of jets of about $10 \div 12$ GeV/$c$ they stay large ($10 \div 20\%$) even if the back-to-back correlations are totally excluded by $\pi/3$ veto angle. Their observation and mea-
surement for non-zero veto angles will favor a hypothesis that multiple hard collisions indeed occur before the fragmentation of jets into hadrons and are described by the perturbative QCD mechanism. However at the supposed LHC energies of 6 TeV these correlations are strongly suppressed reaching 0.5% for S-S collisions at jet momenta of about 15 GeV/c.

Of course we understand that other effects may somewhat change our predictions based on the simple Glauber approach. Among them the most prominent is quenching of jets as they propagate through the nuclear medium. Also if the jet energy hap-
pens to be insufficiently high the formation length may shorten to allow for the jets hadronization inside the nucleus, which will spoil our simple picture of hard rescattering. Another important point is the jet fragmentation which in general can lead to correlation pattern for hadrons different from that for partons. These questions are under our consideration at present. In any case these other effects will produce certain corrections to the results presented in this paper, which thus may serve as a natural starting point.

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VIII. APPENDIX. IDENTICAL PARTICLES IN DIFFERENT PARTS OF THE PHASE VOLUME (RELATED TO EQ. (47)

Consider two particles characterized, say, by their rapidities only \( y_1 \) and \( y_2 \) which take values in the interval \([0,1]\). The integrals over intermediate state of two identical particles, between which no distinction is made, in general have the form:

\[
I = \int_0^1 dy_1 dy_2 \frac{1}{2!} f(y_1, y_2)
\]  

(52)

where \( f(y_1, y_2) \) is some function symmetric in \( y_1 \) and \( y_2 \). Now we split the single particle phase volume in two, say \([0,1]=\[0,1/2]+\[1/2,1]\) and assume that particle with its rapidity in \([0,1/2]\) is particle 1 and that with its rapidity in \([1/2,1]\) is particle 2. Now instead of a single intermediate state we have 3 different ones: with two particles 1 (state A), with two particles 2 (state B) and with one particle 1 and one 2 (state C). The same integral will now be given by a sum

\[
I_A + I_B + I_C = \int_0^{1/2} dy_1 dy_2 \frac{1}{2!} f(y_1, y_2) + \int_{1/2}^1 dy_1 dy_2 \frac{1}{2!} f(y_1, y_2) + \int_0^{1/2} dy_1 \int_{1/2}^1 dy_2 f(y_1, y_2)
\]  

(53)

As we see this sum is equal to \( I \) exactly. So considering particles in \([0,1/2]\) and \([1/2,1]\) as different gives the correct result.