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RAPID SCALING AND INCLUSIVE SUM RULES

L. Caneschi

CERN - Geneva

A B S T R A C T

The influence of s channel quantum numbers on inclusive distributions is investigated. The way chosen by nature to enforce charge conservation is proposed as a test for production models.

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The ten-year old prediction ¹⁾ of scaling behaviour of the one-particle inclusive distributions ²⁾ is currently experiencing a period of remarkable popularity. It has been established that only very general properties of the multiperipheral model (MPM) are necessary to lead to scaling ³⁾⁻⁵⁾, which holds also in some non-multiperipheral models ⁶⁾⁻⁸⁾.

An experimental test of the validity of scaling, however, is not straightforward, because most data available at present have only moderate values of s ($\lesssim 50 \text{ GeV}^2$), and non-asymptotic corrections can in principle be sizeable: in particular, in the MPM ³⁾ and in the Müller ⁵⁾ model, non-leading terms of the form $s^{-1/2}$ are expected, and correspond to the analogous terms in the behaviour of the total cross-sections.

Following the hint provided by the latter schemes, we assume for the inclusive reaction $a + b \rightarrow c + X$ (all variables are measured in the centre-of-mass system, and $x = 2p_L/\sqrt{s}$ as usual) :

$$E_c \frac{d^2\sigma}{dp_L dp_T^2}(a, b, c, p_L, p_T, s) = \bar{f}_{ab}^c(x, p_T^2) + \left(\frac{s_0}{s}\right)^{1/2} \bar{C}_{ab}^c(x, p_T^2) + O(s^{-1}) \quad (1)$$

with $s_0 = 1 \text{ GeV}^2$ from now on ⁹⁾. Assuming that the corrections of order s^{-1} can already be neglected for $s \gtrsim 30 \text{ GeV}^2$, the scaling limit should be exhibited in the present data whenever $\bar{C}_{ab}^c(x, p_T^2)$ happens to be small.

We want to point out in this note that the structure of the functions $\bar{C}_{ab}^c(x, p_T^2)$ must depend crucially on the (ab) quantum numbers. In fact, using the energy conservation sum rule recently proposed by several authors ¹⁰⁾⁻¹²⁾, it is straightforward to obtain :

$$2 \sigma_{ab}(s) = \int_{-1}^{+1} f_{ab}(x) dx + \frac{1}{\sqrt{s}} \int_{-1}^{+1} C_{ab}(x) dx + O(s^{-1}) \quad (2)$$

with

$$f_{ab}(x) = \sum_c \int \bar{f}_{ab}^c(x, p_T^2) dp_T^2$$

and an analogous definition for $C_{ab}(x)$.

We see from (2) that

$$\int_{-1}^{+1} C_{ab}(x) dx = \sigma_{ab}(s) - \sigma_{ab}(\infty) \quad (3)$$

and in particular the integral on the left-hand side vanishes if the (ab) channel is exotic.

To examine how Eq. (2) can hold for all possible choices of a and b, and b, let us use a Regge parametrization of $\sigma_{ab}(s)$

$$\sigma_{ab}(s) = \gamma_a^P \gamma_b^P + \frac{1}{\sqrt{s}} \sum_M \gamma_a^M \gamma_b^M \quad (4)$$

where the index M labels the set of Regge trajectories with intercept close to 0.5.

Let us also use the factorization properties of f_{ab} ¹³⁾:

$$\begin{aligned} f_{ab}(x) &= \gamma_a^P F_b(x) && \text{for } x < 0 \\ &= \gamma_b^P F_a(x) && \text{for } x > 0 \end{aligned} \quad (5)$$

and the fact that, from longitudinal momentum conservation and the limitation on p_π ,

$$\int_{-1}^0 f_{ab}(x) dx - \int_0^1 f_{ab}(x) dx = O(s^{-1})$$

The consistency of the asymptotic behaviour of (2) yields

$$\int_0^1 F_a(x) dx = \gamma_a^P \quad ; \quad \int_{-1}^0 F_b(x) dx = \gamma_b^P \quad (6)$$

and Eq. (3) becomes :

$$2 \sum_M \gamma_a^M \gamma_b^M = \int_{-1}^1 C_{ab}(x) dx \quad (7)$$

Equation (7) can hold for all choices of a and b only if the function $C_{ab}(x)$ also factorizes in a form analogous to (4) and (5) :

$$\begin{aligned} C_{ab}(x) &= \sum_M \gamma_a^M C_b^M(x) && x < 0 \\ &= \sum_M \gamma_b^M C_a^M(x) && x > 0 \end{aligned} \quad (8)$$

with

$$\int_0^1 C_a^M(x) dx = \gamma_a^M \quad ; \quad \int_{-1}^0 C_b^M(x) dx = \gamma_b^M \quad (9)$$

The set of equations (5) to (9) are very natural in the MPM or in the Müller framework : we find them here as the only reasonable way to satisfy the energy conservation constraint [however, remark that assumption (1) relies heavily on the above mentioned models].

The sum rules (5) and (9) have a very natural physical significance. Consider for instance a $\lambda \varphi^3$ model in which a Regge pole in the imaginary part of the elastic amplitude is generated through the sum of ladder diagrams ¹⁾. The position of the resulting Regge pole is determined by the behaviour of the production cross-sections in the limit $n \rightarrow \infty$, i.e., is related to the divergence of $d^2\sigma/dx dp_T^2$ at $x = 0$. On the contrary, the residue of the Regge pole is determined by the behaviour of intermediate states with any n . Whereas it was to be expected that this residue was related to an integral over $d^2\sigma/dx dp_T^2$, and that a factor had to be introduced to de-emphasize the $x \simeq 0$ region, it was not obvious a priori that the factor was simply E , and that the residues of the Regge poles were in the simple relations (6) and (9) with the scaling functions.

Let us now examine how the integral in Eq. (3) can vanish in the case of (ab) exotic. An obviously tempting possibility is that $C_{ab}(x)$ vanishes identically in this case, i.e., that the exchange degeneracy mechanism that is responsible for the vanishing of the right-hand side of (3) works locally at fixed x . This is the case if the functions $C_a^M(x)$, $C_b^M(x)$ factorize in their dependence on M and x . We would in this case obtain the elegant form

$$\begin{aligned} \sum_c \int E_c \frac{d^2\sigma(a, b, c, p_L, p_T^2)}{dp_L dp_T^2} dp_T^2 = \\ = \sigma_{ab}(\infty) \tilde{F}_a(x) + (\sigma_{ab}(s) - \sigma_{ab}(\infty)) \tilde{C}_a(x) + O(s^{-1}) \end{aligned} \quad (10)$$

for $x > 0$, and the symmetric with $a \rightarrow b$ for $x < 0$. In Eq. (10) $\tilde{F}_a(x) = F_a(x)/\gamma_e^P$ and $\tilde{C}_a(x) = C_a^M(x)/\gamma_a^M$, independent of M by assumption. Equation (10) shows a local compensation between the various channels c , in such a way that the sum over c of the inclusive distribution $a + b \rightarrow c + X$ exhibits rapid scaling if the ab channel is exotic : an even stronger assumption could be $\tilde{F}_a(x) = \tilde{C}_a(x)$, which would lead to the rapid scaling of $(\sum_c d^2\sigma_{ab}^c/dp_L dp_T^2) \times \sigma_{ab}^{-1}(s)$ for all a, b . To examine the validity of the assumed factorization between the x and the M dependence, let us consider the problem in the Müller framework. Then $C_a^M(x)$ is the sum over c of the imaginary part of the forward scattering amplitude $a + c \rightarrow a + c$ with the

emission of a "spurion" of quantum numbers M at the $c\bar{c}$ vertex. The energy of the reaction under consideration is a function of x ($s_{ac} \simeq m_c^2/x + 4m_a^2x$). The Regge pole expansion of $C_a^M(x)$

$$C_a^M(x) = \sum_{M'} \sum_c \gamma_c^{M'} \Gamma_{cM}^{M'} s_{ac}^{\alpha_{M'}(0)} \quad (11)$$

where $\Gamma_{cM}^{M'}$ is the residue of the forward coupling of the M' Regge pole to the particle c with the emission of the spurion M , shows that the dependence on M and x decouples in general only if a set of exchanges M' with the same intercept can be considered dominant over the whole x range under consideration, a rather unrealistic situation. We conclude, therefore, that the averaging to zero of the non-leading term in (2) when (ab) is exotic is more likely to happen through the x integration than as a local (fixed x) compensation between different channels. If this is the case, the function $C_{ab}(x)$ must be negative somewhere in the x integration range for (ab) exotic.

A comparison with experimental data is difficult in view of the lack of information on neutral produced particles. However, a comparison of the 8 and 16 GeV data ¹⁴⁾ on $\pi^+p \rightarrow \pi^\pm + X$ seems to indicate that $C(x)$ is likely to be negative for small values of (x) even for this non-exotic channel. Of course, this conclusion is not a sharp one, because the terms of order s^{-1} (essentially phase space effect) that we boldly discarded can be crucial at 8 GeV.

A negative value of $C_{ab}(x)$ is a very puzzling feature in the framework of the MPM, in which this term is generated by the non leading term of the (incoming particle)-(exchanged particle) total cross-section, and is, therefore, expected to be positive or null. For the dual model, the behaviour of $C_{ab}(x)$ at small x is related to the discontinuity of the one-loop forward six-point function and there is no obvious reason why $C_{ab}(x)$ cannot be negative at small values of $|x|$ in this model. In the limit $|x| \rightarrow 1$ the analysis of Ref. 15) should apply, leading to a positive definite value of $C_{ab}(x)$.

A similar type of analysis can be performed by giving a prescription to identify one and only one particle in the final state of the reaction $a + b \rightarrow \text{anything}$ (e.g., the fastest particle in the laboratory system). Obviously

$$\sigma_{ab}(s) = \int \frac{d^2 \sigma^{(F)}}{d p_L^2 d p_T^2} d p_L^2 d p_T^2$$

where $d^2\sigma^{(F)}/dp_T^2 dp_L$ is the spectrum of the fastest particles. Within the MPM, the spectrum $d^2\sigma^{(F)}$ should to a large extent coincide with the spectrum $d^2\sigma^{(I)}$ of the first particle of the MP chain. However, also the possibility that there is a crossing along the chain, in such a way that sometimes the second particle is actually faster than the first one, can be easily by-passed by considering the integral of $d^2\sigma^{(I)}$. As in the MPM there is one and only one first particle for each event

$$\int \frac{d^2\sigma^{(I)}}{dp_L dp_T^2} dp_L dp_T^2 = \sigma_{ab}(s) \quad (12)$$

Remark now that also when (ab) is exotic, there are always particles c (e.g., a itself) such that abc is non-exotic. Their contribution to the integral (12) produces a positive $1/\sqrt{s}$ term³⁾ and $\sigma_{ab}(s)$ defined by (12) decreases with s also when (ab) is exotic. We see here that we are asking too much to the MPM : the constancy of the exotic cross-sections is related to the lack of (ab) resonances in exotic channels, i.e., to the analytic properties in s , an information not available within the MP scheme. From another point of view, this problem is related to the well-known difficulty of inserting signature and exchange degeneracy in a multi Regge bootstrap scheme.

In the dual model we cannot reach any strict contradiction : the analogue to considering the first particles of the MP chain would be to consider the spectrum $d^2\sigma^{(B)}/dp_T^2 dp_L$ of the particles that are described by the Born term of the six-point function, in the way proposed by Chan et al.¹⁵⁾

In the integral

$$\int \frac{d^2\sigma^{(B)}}{dp_L dp_T^2} dp_L dp_T^2$$

all channels c with (abc) not exotic contribute a positive $1/\sqrt{s}$ term, and, therefore, the integral under consideration has a positive $1/\sqrt{s}$ term also for (ab) exotic. Several ways out are possible here :

- a) the procedure of squaring the amplitudes and then summing is not acceptable¹⁶⁾,
- b) even if this is acceptable, not all the events contain one particle, the spectrum of which is described by the Born term of the six-point function, i.e., the integration over intermediate states composed of one scalar and one resonant state do not exhaust the unitarity relation.

To finish with a less intriguing remark, let us remember that the sum rule of the form of Eq. (2) can be obtained from any additive conserved quantity¹⁰⁾. In particular, charge conservation yields, in the limit $s \rightarrow \infty$:

$$Q_a = \sum_c \int_0^1 \frac{dx}{x} F_a(x) \varphi_c(x)$$

where the function $Q_c(x)$ defined by

$$\frac{Q_c}{\sigma_{ab}(\infty)} \int \bar{f}_{ab}^c(x, p_T^2) d p_T^2 = F_a(x) \varphi_c(x)$$

represents the probability that a particle detected at a given value of x is of the type c , times the charge of c , Q_c .

Considering together the contributions from a particle c and its antiparticle $\bar{c}$, and defining $\delta_c(x) = Q_c(x) + Q_{\bar{c}}(x)$, we obtain

$$Q_a = \sum_{c \text{ (but not } \bar{c})} \int_0^1 F_a(x) \delta_c(x) \frac{dx}{x}$$

If $F_a(x) \neq 0$, $\delta_c(x)$ must vanish¹⁷⁾ in the limit $x \rightarrow 0$. In the multi-peripheral model and in the Müller model $\delta_c(x) \simeq x^{\frac{1}{2}}$; on the contrary, in a model of diffractive production like the one considered in Ref. 7), the charge of the particle a should be shared uniformly amongst its fragmentation products, and δ_c should be of the order of $1/\langle n \rangle$ independently of x ¹⁸⁾. The behaviour of $\delta_c(x)$ constitutes, therefore, a very interesting test of the models of particle production, which, however, cannot be carried out with the present data¹⁴⁾, which still show a large value of $\delta_c(0)$ (e.g., in $\pi^+p \rightarrow \pi^+ + X$).

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- 9) Diffractive processes are hard to fit in the scheme of Eq. (1). Consider, for instance, in the case $c = a$, the contribution from the diffractive process $a + b \rightarrow a + b^*$ to Eq. (1). If this process contributes to $f_{ab}^a(x, p_T^2)$ (i.e., if the triple pomeron vertex is non-vanishing), the contribution is of the form ³⁾ $(1-x)^{1-2\alpha_P(t)}$, and produces a divergence of the $a - b$ total cross-section when inserted in Eq. (2) ; the divergence is of the form $\ln s$ if $\alpha_P' = 0$, $\ln \ln s$ if $\alpha_P' \neq 0$, in agreement with the behaviour of the $2 \rightarrow 3$ production cross-section with double Pomeron exchange [J. Finkelstein and K. Kajantie, Nuovo Cimento 56A, 659 (1968) ; L. Caneschi, to be published in Nuclear Physics]. If on the contrary the sum over the set of resonances b^* produces only a $(\text{missing mass})^{\frac{1}{2}}$ behaviour, as expected in dual model, then diffractive processes would contribute a term of the form $(1-x)^{1/2-2\alpha_P(t)}/\sqrt{s}$. This term does not force the integral in Eq. (2) to diverge in the $s \rightarrow \infty$ limit, because the upper limit of integration is actually $1 - O(s^{-1})$; however, this contribution cannot be lumped in the function $C_{ab}(x)$, because it would become not normalizable. This corresponds to the fact that diffraction dissociation is not a non-asymptotic feature. In the following, we will assume that diffractive processes (i.e., inelastic Pomeron exchanges in the multi-Regge language) are small, or that they can be separately studied in exclusive reactions and their contribution subtracted away from the inclusive spectrum.
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- 13) A nice property of the sum rule (3) is that the point $x = 0$ is a regular point of the integrand : therefore, all the integrals can be taken between -1 and $-\epsilon$ and $+\epsilon$ and $+1$, and the $\epsilon \rightarrow 0$ limit performed afterwards. Therefore, in this integral, we are always dealing with fragmentation, and not pionization, properties.
- 14) See, for instance, the review paper of D. Horn, CERN Preprint TH.1387, to be published in Phys.Reports.
- 15) H.M. Chan, C. Quigg and J.M. Wang, Phys.Rev.Letters 26, 672 (1971).
- 16) The same is true in the MPM, in principle, but in the MPM, the interference terms are a function of the coupling constant and vanish in the weak coupling (strong ordering) limit.
- 17) A possible cancellation between different c can be ruled out using other constraints, like strangeness and baryon number conservation.
- 18) Remember that the model of Ref. 7) scales only up to $\ln s$ terms.