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THE ALGEBRA OF VERTEX STRENGTHS AND THE LIGHT CONE

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A B S T R A C T

On the basis of light cone current algebra it is shown that tensor meson trajectories couple to hadrons through the matrix elements of scalar densities. Some consequences of this fact are discussed.

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It has been pointed out a few years ago ¹⁾ that certain regularities observed in high energy meson-baryon and baryon-baryon scattering ²⁾ can be accounted for by the hypothesis that Regge residues are proportional to matrix elements of operators belonging to an $U(12)$ algebra. In particular, the Regge exchanges relevant in forward scattering, i.e., those associated with the nonets of vector ($j^P = 1^-$) and tensor ($j^P = 2^+$) mesons were related to a subalgebra $U(3) \times U(3)$ of $U(12)$ generated by two nonets of scalar and vector densities.

In Cabibbo-Horwitz-Ne'eman (CHN), however, the relation between the algebraic structure of vertex strengths and the $U(12)$ algebra of hadron currents had not been completely clarified. In this paper, we show how the two algebraic structures can indeed be related within a wider frame, that of the operator light-cone expansion ³⁾⁻⁵⁾. Our treatment is a natural extension of that given in a recent paper by one of us ⁶⁾, which established the desired result in the case of vector trajectories. We will assume as a basic hypothesis the canonical light cone algebra of the quark model without elementary scalar fields, involving the following elementary currents and densities :

$$\begin{aligned} g_\mu^i(x) &= : \bar{\psi}(x) \lambda^i \gamma_\mu \psi(x) : \\ q_\mu^i(x) &= : \bar{\psi}(x) \lambda^i \gamma_\mu \gamma_5 \psi(x) : \\ u^i(x) &= : \bar{\psi}(x) \lambda^i \psi(x) : \\ v^i(x) &= : \bar{\psi}(x) \lambda^i \gamma_5 \psi(x) : \end{aligned} \quad (1)$$

where λ^i are the Gell-Mann matrices with $\lambda^0 = \sqrt{\frac{2}{3}} I$. We assume these quantities to obey the equal time commutation relations implied by the canonical formalism ⁷⁾.

In order to establish the connection between tensor Regge exchanges and scalar densities, we consider the light cone commutator ⁸⁾ of $g_\mu^i(x)$ and $v^j(0)$:

$$\begin{aligned} [g_\mu^i(x), v^j(0)]_{x^0 \rightarrow 0} &\xrightarrow{x^0 \rightarrow 0} \partial_\mu (\epsilon(x_0) \delta(x^1)) \{ i d^{ijk} [\theta^k(x,0) + \theta^k(0,x)] + \\ &+ \int^{ik} [\theta^k(x,0) - \theta^k(0,x)] + \epsilon(x_0) \delta(x^1) \theta_\mu^j(x,0) \} \end{aligned} \quad (2)$$

where $O^k(x,0)$ and $O_{\mu}^{ij}(x,0)$ are bilocal operators, regular when $x^2 \rightarrow 0$.

By taking the equal time limit of Eq. (2) we find

$$O^k(0,0) = \frac{1}{4\pi} u^k(0) \quad (3)$$

In Ref. 6), the light cone commutator of $j_{\mu}^i(x)$ and $j_{\nu}^k(0)$ was considered in order to establish the relation between vector currents and the couplings of vector trajectories. The implications of the full algebra, which also contains information on unnatural parity trajectories, will be discussed elsewhere.

Consider now the "forward scattering amplitude"

$$\begin{aligned} \int e^{iq \cdot x} \theta(x_0) \langle p | [q_{\mu}^i(x), v_{\nu}^j(0)] | p \rangle d^4x = \\ = A^{ij}(q^2, \nu) q_{\mu} + B^{ij}(q^2, \nu) p_{\mu} \end{aligned} \quad (4)$$

where $\nu = 2p \cdot q$.

In the Bjorken limit ⁹⁾, $[q^2 \rightarrow -\infty, \omega = -(\nu/q^2) \text{ fixed}]$ the light cone expansion given in Eq. (2) implies that the non-spin flip amplitude $A^{ij}(q^2, \nu)$ scales in the following way :

$$\begin{aligned} \nu A^{ij}(q^2, \nu) \xrightarrow{B_j} G^{ij}(\omega) = \\ = d_{ijk} \sum_{h=0}^{\infty} a_{2h}^k \omega^{2h+1} + f_{ijk} \sum_{h=0}^{\infty} a_{2h+1}^k \omega^{2h+2} \end{aligned} \quad (5)$$

On the other hand, the partial wave decomposition in the t channel of $A^{ij}(q^2, \nu)$ reads :

$$A^{ij}(q^2, \nu) = \sum_{n=0}^{\infty} b_n^{ij}(q^2) P_n(\cos \theta_t) \quad (6)$$

In the Bjorken limit, we have :

$$\nu A^{ij}(q^2, \nu) \xrightarrow{B_j} \sum_{n=0}^{\infty} b_n^{ij}(q^2) \frac{(\sqrt{-q^2})^{n+2}}{(2n)^n} \omega^{n+1} \quad (7)$$

Comparing with Eq. (5), we find

$$\lim_{q^2 \rightarrow -\infty} b_n^{ij}(q^2) \frac{(\sqrt{-q^2})^{n+2}}{(2n)^n} = \begin{cases} d_{ijk} a_n^k & (n \text{ even}) \\ f_{ijk} a_n^k & (n \text{ odd}) \end{cases} \quad (8)$$

If we now assume that in this limit the n dependence of b_n^{ij} is still dominated (for small n) by Regge poles, the same will be true of the n dependence of the a_n^k . A Regge pole with intercept $\alpha(0)$ will give to the structure function an asymptotic contribution of the form $G(\omega) \xrightarrow{\omega \rightarrow \infty} \omega^{\alpha(0)+1}$.

The leading contribution for even n should be given by the nonet of tensor trajectories and by the Pomeron. We can then write

$$a_n^k \approx \frac{\beta_{gv}^{(k)} \beta_p^{(k)}}{n - \alpha_k(0)} \left(\frac{1 + e^{i\pi n}}{2} \right) \quad (k=1, \dots, 8)$$

$$a_n^{(0)} \approx \left[\frac{\beta_{gv}^{(0)} \beta_p^{(0)}}{n - \alpha_0(0)} + \frac{\beta_{gv}^{(P)} \beta_p^{(P)}}{n - \alpha_P(0)} \right] \left(\frac{1 + e^{i\pi n}}{2} \right) \quad (9)$$

where $\beta_{gv}^{(\ )}$ and $\beta_p^{(\ )}$ denote the factorized couplings to the currents and to the target respectively.

In the spirit of pole dominance, possible contributions from background or lower lying poles can be neglected for $n \approx \alpha(0)$. The intercepts of the nonet of tensor meson trajectories are near $\frac{1}{2}$ while the Pomeron is near 1, so that we can apply the above equation to the case $n = 0$. On the other hand, from Eqs. (2) and (3) it follows via an analysis which is by now standard, that

$$a_0^k = \langle p | u^k | p \rangle \quad (10)$$

Comparing Eqs. (9) and (10), we find :

$$\beta_p^{(k)} \approx - \langle p | u^k | p \rangle \frac{\alpha_k(0)}{\beta_{gv}^{(k)}} \quad (k=1, \dots, 8) \quad (11)$$

$$\beta_P^{(0)} + \beta_P^{(P)} \left(\frac{\beta_{gv}^{(P)}}{\beta_{gv}^{(0)}} \frac{\alpha_0(0)}{\alpha_P(0)} \right) \approx - \langle P | U^0 | P \rangle \frac{\alpha_0(0)}{\beta_{gv}^{(0)}} \quad (12)$$

The result is much simplified if we assume that on the light cone the couplings present a nonet symmetry so that $\beta_{gv}^{(k)} = \beta_{gv}$ ($k=0, \dots, 8$). If we also put $\alpha_k(0) = \frac{1}{2}$ ($k=0, \dots, 8$) the same factor appears on the right-hand side in Eqs. (11) and (12).

We thus recover the CHN hypothesis, except for the vacuum trajectories. In this last case the CHN treatment has anyway to be modified since it assumed the presence of only one vacuum trajectory.

To conclude, let us discuss some possible consequences of these results.

First of all, if we assume that scalar densities are relevant to the breaking of $SU(3)$, i.e.,

$$\delta m(p) \propto \langle p | U^8 | p \rangle \quad (13)$$

we find that for each octet of hadrons the couplings of the octet of tensor trajectories are related to the mass differences within the octet, and in particular should have the same F/D ratio. This conclusion seems to be in approximate agreement with the experimental situation ¹⁰⁾.

Secondly, let us consider the vacuum channel, Eq. (12). It is an experimental fact that the non-diagonal Pomeron couplings are, in hadron processes, much smaller than the diagonal ones. It is interesting to speculate on the possibility that the same happens in the case of current-hadron amplitudes, i.e., that

$$\beta_{gv}^{(P)} \ll \beta_{gv}^{(0)} \quad \left(\begin{array}{c} \text{diagonal} \\ \text{Pomeron} \end{array} \right) \quad (14)$$

With this hypothesis we fully recover the CHN prescription for the nonet of tensors, but, at the same time, lose any information on the Pomeron couplings to hadrons. Finally we recall that in the model according to which $SU(3) \times SU(3)$ symmetry is broken by a $(3, \bar{3}) \oplus (\bar{3}, 3)$ term of the form ¹¹⁾

$$\mathcal{L}' = u^0 + c u^8 \quad (15)$$

the σ term in πN scattering is given by

$$\sigma_{NN} = -\frac{1}{3} \left(\frac{\sqrt{2} + c}{c} \right) \left(1 + \sqrt{2} \frac{\langle u^0 \rangle}{\langle u^8 \rangle} \right) c \langle u^8 \rangle \quad (16)$$

Under the hypothesis of a diagonal Pomeron [Eq. (14)], Eqs. (11) and (12) imply that $\langle u^0 \rangle$ and $\langle u^8 \rangle$ are of the same order of magnitude implying $\sigma_{NN} \sim 20 \div 30$ MeV if $c \approx -1.25$. In order to escape this conclusion one should either abandon the diagonal Pomeron hypothesis, or invoke another strongly coupled $SU(3)$ singlet scalar trajectory ¹²⁾ which would spoil Eq. (12).

In closing, we note that the hypothesis of Regge dominance on the light cone can give rise to certain conditions on the structure of the operators appearing in the light cone expansion. As an example, we could, by the same procedure, relate the residues of vector meson trajectories to matrix elements of $(\partial_0(x,0)/\partial x^\mu)|_{x=0}$. Current algebra alone does not give any information about these operators, such as Eq. (3). On the other hand, these new relations have to be consistent with those derived in Ref. 6) which relate the vector couplings to matrix elements of the vector currents.

We propose to give elsewhere a further discussion on this problem.

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