Lepton flavour violation in future linear colliders
in the long-lived stau NLSP scenario

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Abstract

We analyze the prospects of observing lepton flavour violation in future $e^-e^-$ and $e^+e^-$ colliders in scenarios where the gravitino is the lightest supersymmetric particle, and the stau is the next-to-lightest supersymmetric particle. The signals consist of multilepton final states with two heavily ionizing charged tracks produced by the long-lived staus. The Standard Model backgrounds are absent and the supersymmetric backgrounds can be kept well under control by the use of suitable kinematical cuts. We discuss in particular the potential of the projected International Linear Collider to discover lepton flavour violation in this class of scenarios, and we compare the estimated sensitivity with the constraints stemming from the non-observation of rare decays.

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I. INTRODUCTION

The discovery of flavour violation in neutrino oscillations [1] has opened a new era for flavour physics in the leptonic sector. This crucial discovery also encourages the search for flavour violation in the charged lepton sector, and if low energy supersymmetry is discovered, in the slepton sector.

The scalar sector of supersymmetric theories contains many new flavour violating couplings, stemming from the off-diagonal elements of the soft breaking terms. Namely, in the mass eigenstate basis for the charged leptons, the soft-breaking Lagrangian reads:

$$- L_{\text{soft}} = \left( m_{iL}^2 \right)_{ij} \bar{l}_i L_j + \left( m_{iR}^2 \right)_{ij} \bar{l}_i R_j + \left( Y_l l_{ii} \bar{l}_i R_j H_d + h.c. \right),$$

(1)

where $i, j = 1, 2, 3$ are generational indices. $\bar{l}_{iL,R}$ denote the left and right-handed charged sleptons, $H_d$ is the down-type Higgs doublet, $Y_l$ is the charged-lepton Yukawa coupling, and $m_{iL,R}^2$ and $A_l$ are the soft scalar mass matrix squared and the soft trilinear matrix, respectively. The resulting $6 \times 6$ charged-slepton mass matrix reads:

$$M^2 = \left( \begin{array}{cc}
  m_{iL}^2 + m_l^2 - \left( \frac{1}{2} - \sin^2 \theta_W \right) m_Z^2 \cos 2\beta & m_l (A_l - \mu \tan \beta)^+ \\
  m_l (A_l - \mu \tan \beta) & m_{iR}^2 + m_l^2 - \sin^2 \theta_W m_Z^2 \cos 2\beta
\end{array} \right).$$

(2)

It is customary to express this mass matrix in the form [2]:

$$M^2 = \left( \begin{array}{cc}
  m_{iL}^{\text{av}} (1 + \delta_{LL}) & m_l (A_l^{\text{av}} - \mu \tan \beta)^* + m_{iR}^{\text{av}} m_{R}^{\text{av}} \delta_{LR} \\
  m_l (A_l^{\text{av}} - \mu \tan \beta) & m_{iR}^{\text{av}} (1 + \delta_{RR}) + m_{iL}^{\text{av}} m_{R}^{\text{av}} \delta_{RL}
\end{array} \right),$$

(3)

so that the amount of flavour violation is parametrized by the $3 \times 3$ matrices $\delta_{LL}, \delta_{RR}, \delta_{LR}$ and $\delta_{RL}$. In this expression, $m_{iL}^{\text{av}}, m_{iR}^{\text{av}}$ are an average of the masses of the left-handed and right-handed sleptons, respectively, and $A_l^{\text{av}}$ is an average of the diagonal elements of the soft trilinear matrix.

The size of these flavour violating terms is model dependent, although very rarely strictly vanishing [3]. They arise when the mechanism of supersymmetry breaking distinguishes among flavours, thus producing flavour violation in the soft breaking terms already at tree level. For example, in the framework of the Froggatt-Nielsen mechanism [4], the flavon fields not only generate the mass hierarchies and the mixings, but in general contribute to the breaking of supersymmetry, inducing off-diagonal elements in the soft terms [5]. Besides, in weakly coupled string constructions where the Kähler moduli fields participate in the breaking of supersymmetry, if the matter metric is non-diagonal also flavour violating
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couplings in the scalar sector are generated \[6, 7\]. Furthermore, even if the supersymmetry breaking mechanism is flavour blind, so that the soft terms are flavour diagonal at the cut-off scale, radiative effects might spoil this diagonal structure. Many well motivated models predict the existence of particles in the desert with flavour violating couplings to the Minimal Supersymmetric Standard Model (MSSM) superfields, that would generate off-diagonal entries in the soft-breaking matrices through quantum corrections. Renowned examples are the MSSM extended with right-handed neutrinos \[8\] and Grand Unified models \[9\].

Flavour violation in the scalar sector is propagated through loop effects to the charged lepton sector, inducing rare decays. The non-observation of these processes impose very strong constraints on the flavour violating couplings, that are summarized in table I for the supersymmetric spectrum of the benchmark point \(\epsilon\) of \[10\] (see details in the text).

Table I: Constraints on the flavour violating parameters \(\delta_{LL}, \delta_{RR}, \delta_{LR}\) and \(\delta_{RL}\), for the supersymmetric spectrum of the benchmark point \(\epsilon\) of \[10\] (see details in the text).

<table>
<thead>
<tr>
<th>sector</th>
<th>(\delta_{LL})</th>
<th>(\delta_{RR})</th>
<th>(\delta_{LR})</th>
<th>(\delta_{RL})</th>
</tr>
</thead>
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<tr>
<td>12</td>
<td>(4 \times 10^{-4})</td>
<td>(3 \times 10^{-4})</td>
<td>(4 \times 10^{-6})</td>
<td>(4 \times 10^{-6})</td>
</tr>
<tr>
<td>13</td>
<td>0.1</td>
<td>0.1</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>23</td>
<td>0.07</td>
<td>0.05</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

1 This benchmark point belongs to the class of scenarios that we would like to study in this paper, namely when the gravitino is the lightest supersymmetric particle and the stau is the next-to-lightest supersymmetric particle.
depend crucially on the particular supersymmetric scenario considered. We can differentiate roughly two main classes of scenarios, according to the nature of the lightest supersymmetric particle (LSP). A popular choice for the LSP is the neutralino, although scenarios with superweakly interacting LSP, such as the gravitino or the axino, are also compatible with all the collider experiments and cosmology. In this paper we will concentrate on the latter class of scenarios, focusing for definiteness in the case with gravitino LSP although the analysis and the conclusions for the case with axino LSP are completely analogous. These scenarios have received a lot of attention recently and their properties have been studied in a number of papers \cite{10,13,14,15} One of the most remarkable features of these scenarios is the longevity of the next-to-lightest supersymmetric particle (NLSP). The NLSP could only decay to gravitinos and Standard Model particles through gravitational interactions, with a decay rate strongly suppressed by the Planck mass. This translates into lifetimes that could be long enough to allow the NLSP to traverse the detector.

Under the assumption of universality of the soft breaking scalar, gaugino and trilinear parameters at some cut-off scale (the so-called constrained MSSM) the NSLP can be either a neutralino or a stau, although in more general scenarios other candidates could also be possible. In the case that the NLSP is the neutralino, the signals for lepton flavour violation will be identical to the case with neutralino LSP, which have been discussed extensively in the literature \cite{16}. On the other hand, the signals for the case with stau NLSP could be very different. Whereas the neutralino escapes detection and is identified as missing energy, the stau would produce a heavily ionizing track in the vertex detector. This signature is very unique and is not produced by any Standard Model particle, hence the observation of heavily ionizing tracks would give strong support to this scenario and would allow the search for lepton flavour violation without any Standard Model background.  

The longevity of the staus would also allow their collection and the subsequent detection of their decay products with reduced backgrounds. This possibility has been discussed in \cite{10,13}, and in particular the possibility of detecting lepton flavour violation was studied in \cite{15}.

\footnote{The staus are very slow and they might reach the detector typically after a few bunch crossing times. This could complicate the reconstruction of the event, although in principle it can be done \cite{10}. Alternative ways to search for lepton flavour violation that circumvent this technical difficulty will be presented elsewhere \cite{17}.}
This paper is organized as follows. In Section II, we present the notation and the setup for our analysis, and discuss the different signatures for lepton flavour violation at future $e^-e^-$ and $e^+e^-$ colliders. In Section III we particularize the analysis for the case of the projected International Linear Collider (ILC) and estimate the sensitivity reach of this experiment to lepton flavour violation in scenarios where the gravitino is the LSP and the stau the NLSP. We also compare this sensitivity with the present and future constraints on lepton flavour violation stemming from the non-observation of rare leptonic decays. Finally, in Section IV we present our conclusions.

II. FLAVOUR VIOLATING SIGNATURES AT FUTURE ELECTRON COLLIDERS

Throughout this paper we will assume that the gravitino is the LSP and the NLSP is mainly a right-handed stau, although it could have some admixture of left-handed stau or other leptonic flavours. We will denote the mass eigenstates by the dominant flavour, so that the NLSP will be denoted by $\tilde{\tau}_1$. Motivated by the low energy spectrum of the constrained MSSM, we will also assume that next to the $\tilde{\tau}_1$, the lightest superparticles are the two combinations of right-handed selectron and smuon, also with a very small admixture of left-handed states (due to the Yukawa suppression of the left-right mixing) and some admixture of stau. The mass splitting between them is expected to be very small, and the absolute values of their masses are expected to be not very different from the NLSP mass. We will denote these states by $\tilde{e}_R$ and $\tilde{\mu}_R$, the former being the mass eigenstate with largest right-handed selectron component and the latter with largest right-handed smuon component. Next in mass in the supersymmetric spectrum are the lightest neutralino and the rest of the sparticles. Schematically the spectrum reads

$$m_{3/2} < m_{\tilde{\tau}_1} < m_{\tilde{e}_R, \tilde{\mu}_R} < m_{\chi^0_1, m_{\tilde{e}_L, \tilde{\mu}_L, m_{\tilde{\tau}_2}...}$$

(4)

It is important for our analysis that the NLSP decays outside the detector, so that it is detected as a heavily ionizing track. If R-parity is conserved, the NLSP can only decay into a gravitino and a charged lepton with total decay rate

$$\Gamma \simeq \frac{m_{\tilde{\tau}_1}^5}{48\pi m_{3/2}^2 M_P^2} \left(1 - \frac{m_{3/2}^2}{m_{\tilde{\tau}_1}^2}\right)^4,$$

(5)
where \( M_P = (8\pi G_N)^{-1/2} \) is the reduced Planck mass. Therefore, the requirement that the NLSP decay length is larger than ten centimeters, to guarantee that the NLSP traverses a few layers in the vertex detector leaving a heavily ionizing track, translates into the constraint on the gravitino mass \( m_{3/2} \gg 0.1 \text{ keV} \times (m_\tau/100 \text{ GeV})^{5/2} \).

After discussing the set-up for this analysis, let us discuss the possible signals and backgrounds for the detection of lepton flavour violation. Despite the fact that the discussion is very similar for the \( e^- e^- \) and for the \( e^+ e^- \) collider, let us analyze, for the sake of clarity, each case separately.

A. \( e^- e^- \) collider

When lepton flavour is conserved, only left and right-handed selectrons would be produced in this mode, to be precise in the \( t \)-channel by neutralino exchange. Since right-handed selectrons are lighter than left-handed selectrons, the largest production cross section would correspond to the process \( e^- e^- \rightarrow \tilde{e}_R \tilde{e}_R \). The signatures for this process depend crucially on the mass splitting between the right-handed selectron and the NLSP. If the mass splitting is sufficiently large (\( \sim 15 - 20 \text{ GeV} \)), the right-handed selectron would decay mainly into charged leptons and a NLSP before reaching the detector, in a process mediated by neutralinos. The signature for this lepton flavour conserving process would be the detection of two heavily ionizing tracks, two electrons and two taus

\[
(6) \quad e^- e^- \rightarrow \tilde{e}_R \tilde{e}_R \rightarrow (e^- \tau^{\pm} \bar{\tau}_1^{\mp})(e^- \tau^{\pm} \bar{\tau}_1^{\mp}).
\]

If the mass splitting between the right-handed selectron and the NLSP is smaller, the charged leptons could be too soft to be detected, and only the two heavily ionizing tracks would be observed. Finally, if the mass splitting is very small, the decay channel into charged leptons would be kinematically closed. Selectrons could only decay into NLSPs and neutrinos by a process mediated by charginos, with a decay rate suppressed by the small electron Yukawa coupling:

\[
(7) \quad e^- e^- \rightarrow \tilde{e}_R \tilde{e}_R \rightarrow (\nu_e \bar{\nu}_\tau \bar{\tau}_1)(\nu_e \bar{\nu}_\tau \bar{\tau}_1).
\]

\( ^3 \) To be precise, in the final state one could find two taus, two antitaus or a pair of tau-antitau, depending on the charges of the outgoing NLSPs. However, and for simplicity in the notation, we will denote as “tau” either the tau or the antitau, and analogously for the electron and the muon.
If this is the case, again only two heavily ionizing tracks would be detected, corresponding to the NLSPs or perhaps to the right-handed selectrons, if these are long lived enough to traverse the detector. Therefore, production of two right-handed selectrons would result in the detection of two heavily ionizing tracks, two electrons and two taus; or the detection of just two heavily ionizing tracks.

Less likely than the production of two right-handed selectrons is the associated production of one left-handed selectron and one right-handed selectron. Left-handed selectrons can decay via neutralino exchange either into charged leptons and a NLSP, or via chargino exchange into neutrinos and a NLSP, with comparable decay rates. Therefore, in the detector two heavily ionizing tracks would be observed, with either two electrons and two taus, or one electron and one tau, or no charged leptons:

\[
e^{-} e^{-} \rightarrow \tilde{e}_{L} \tilde{e}_{R} \rightarrow (e^{-} \tau^{\pm} \tilde{\tau}_{1}^{\mp})(e^{-} \tau^{\pm} \tilde{\tau}_{1}^{\mp}),
\]

(8)

\[
e^{-} e^{-} \rightarrow \tilde{e}_{L} \tilde{e}_{R} \rightarrow (\nu_{e} \bar{\nu}_{\tau} \tilde{\tau}_{1}^{-})(e^{-} \tau^{\pm} \tilde{\tau}_{1}^{\mp}),
\]

(9)

\[
e^{-} e^{-} \rightarrow \tilde{e}_{L} \tilde{e}_{R} \rightarrow (\nu_{e} \bar{\nu}_{\tau} \tilde{\tau}_{1}^{-})(\nu_{e} \bar{\nu}_{\tau} \tilde{\tau}_{1}^{-}).
\]

(10)

(The two first processes would have comparable cross sections, whereas the third would be suppressed by the small electron Yukawa coupling in the decay of $\tilde{e}_{R}$.) Similar signatures would follow from the production of two left-handed selectrons, in this case with comparable cross sections for the three processes. As we will see, processes associated with the production of left-handed selectrons constitute the most important source of background for the detection of lepton flavour violation.

If lepton flavour violation exists in nature at an observable level, novel possibilities arise, namely the associated production of a right-handed selectron and a smuon, or a right-handed selectron and a NLSP. The associated production of a right-handed smuon and a NLSP could also be possible, although it would require two flavour violating vertices and is therefore very suppressed.

The associated production of $\tilde{e}_{R}$ and $\tilde{\mu}_{R}$ would give rise to the observation of two heavily ionizing tracks, two taus, one muon and one electron:

\[
e^{-} e^{-} \rightarrow \tilde{e}_{R} \tilde{\mu}_{R} \rightarrow (e^{-} \tau^{\pm} \tilde{\tau}_{1}^{\mp})(\mu^{-} \tau^{\pm} \tilde{\tau}_{1}^{\mp}).
\]

(11)

No other process in an $e^{-}e^{-}$ collider yields this same signal, and if particle identification is sufficiently good, the observation of this process would represent a clear signal for lepton
flavour violation in the selectron-smuon sector, parametrized by the off-diagonal element of the right-handed slepton mass matrix \((m_{l_R}^2)_{12}\).

On the other hand, the associated production of \(\tilde{e}_R\) and \(\tilde{\tau}_1\) would give rise to the observation of two heavily ionizing tracks (when the mass splitting between the right-handed selectron and the NLSP is small), or two heavily ionizing tracks, one electron and one tau (when the mass splitting is sufficiently large). The former process cannot be distinguished from the production of two right-handed selectrons and cannot be used for the search of lepton flavour violation. However, the latter process could constitute a strong signal of lepton flavour violation:

\[
e^- e^- \rightarrow \tilde{e}_R \tilde{\tau}_1 \rightarrow (e^- \tau^\pm \tilde{\tau}_1^\mp) \tilde{\tau}_1^-.
\]

(12)

The final state in this processes could be mimicked by the production of two right-handed selectrons, and the subsequent decay of one of them into neutrinos and a NLSP by an interaction mediated by higgsinos, while the other decays into charged leptons and a NLSP: \(e^- e^- \rightarrow \tilde{e}_R \tilde{e}_R \rightarrow (e^- \tau^+ \tilde{\tau}_1^- \nu_e \bar{\nu}_\tau + e^- \tau^- \tilde{\tau}_1^+ \nu_e \bar{\nu}_\tau)\). However, the decay mode \(\tilde{e}_R \rightarrow \nu_e \bar{\nu}_\tau \tilde{\tau}_1^-\) is highly suppressed due to the presence of the electron Yukawa coupling, which implies decay rates of the order of \(10^{-19}\) GeV. Hence, this source of background can be neglected in general.

A more important background could follow from the associated production of a left-handed and a right-handed selectron, or two left-handed selectrons, where the left-handed selectron decays into neutrinos and a NLSP with unsuppressed decay rate, Eq.(9). One should note here that these background events includes two neutrinos in the final state which give rise to imbalance in momentum. On the other hand, the signal events, Eq.(12), do not have any neutrinos in the final state and hence there is no missing transverse momentum \(p_T\). Therefore, if we demand that the final state should have a \(p_T < 5\) GeV then most of the background could be eliminated. This background could be further reduced using a right-polarized electron beam\(^4\), or completely eliminated (with a previous knowledge of the spectrum) tuning the center of mass energy of the collider to be below the threshold for left-handed selectron production.

Additional signals for lepton flavour violation in the right-handed sector follow from the lepton flavour violating decays of the right-handed selectrons. Depending on which particular

\(^4\) The use of right-polarized electron beams also enhances the signal strength.
sector violates lepton flavour, right-handed selectrons could decay into two electrons and a NSLP or two taus and a NLSP, if the violation occurs in the right-handed selectron-stau sector, i.e. \((m_{l_R}^2)_{13} \neq 0\); one muon, one tau and a NLSP if it occurs in the right-handed selectron-smuon sector, i.e. \((m_{l_R}^2)_{12} \neq 0\); or one electron, one muon and a NLSP if it occurs in the right-handed smuon-stau sector, i.e. \((m_{l_R}^2)_{23} \neq 0\). The corresponding lepton flavour violating processes read:

\[
e^{-} e^{-} \rightarrow \tilde{e}_R^- \tilde{e}_{Ri}^{-} \rightarrow (e^{-} e^{\mp} \tilde{\tau}_1^\mp) (e^{-} \tau^{\pm} \tilde{\tau}_1^\mp) \quad \text{if} \quad (m_{l_R}^2)_{13} \neq 0, \quad (13)
\]

\[
e^{-} e^{-} \rightarrow \tilde{e}_R^- \tilde{e}_R^- \rightarrow (\tau^- \tau^{\mp} \tilde{\tau}_1^\mp) (e^{-} \tau^{\pm} \tilde{\tau}_1^\mp) \quad \text{if} \quad (m_{l_R}^2)_{13} \neq 0, \quad (14)
\]

\[
e^{-} e^{-} \rightarrow \tilde{e}_R^- \tilde{e}_R^- \rightarrow (\mu^{-} \tau^{\mp} \tilde{\tau}_1^\mp) (e^{-} \tau^{\pm} \tilde{\tau}_1^\mp) \quad \text{if} \quad (m_{l_R}^2)_{12} \neq 0, \quad (15)
\]

\[
e^{-} e^{-} \rightarrow \tilde{e}_R^- \tilde{e}_R^- \rightarrow (e^{-} \mu^{\pm} \tilde{\tau}_1^\mp) (e^{-} \tau^{\pm} \tilde{\tau}_1^\mp) \quad \text{if} \quad (m_{l_R}^2)_{23} \neq 0. \quad (16)
\]

None of these processes suffers from important backgrounds, provided particle identification is sufficiently reliable, and constitute important probes of lepton flavour violation in the right-handed slepton sector.

B. \(e^+ e^-\) collider

Production of sleptons at the \(e^+ e^-\) collider proceeds via \(t\)-channel by neutralino exchange and via \(s\)-channel by photon and \(Z\) boson exchange. In the \(t\)-channel the possible processes are analogous to those for the \(e^- e^-\) collider, with the appropriate changes in the electric charges of the particles. Namely, when lepton flavour is conserved, only selectrons will be produced, and when lepton flavour is violated, the analogous processes to Eqs. (11)-(16) will occur, with analogous backgrounds.

On the other hand, in the \(s\)-channel all types of sleptons can be produced:

\[
e^+ e^- \rightarrow \tilde{l}_{Ri}^+, \tilde{l}_{Rj}^-; \quad (17)
\]

\[
e^+ e^- \rightarrow \tilde{l}_{Ri}^+, \tilde{l}_{Lj}^-; \quad (18)
\]

\[
e^+ e^- \rightarrow \tilde{l}_{Li}^+, \tilde{l}_{Lj}^-; \quad (19)
\]

where \(\tilde{l}_i, \tilde{l}_j = \tilde{e}, \tilde{\mu}, \tilde{\tau}\).

If lepton flavour is conserved, sleptons would be produced in pairs with opposite lepton family number. The production of a pair of NLSPs with opposite charges would be detected as two heavily ionized tracks travelling in opposite directions, whereas production of pairs of
right-handed smuons or selectrons would be detected as two heavily ionized tracks, together
with two taus and two muons or electrons, respectively. Right-handed smuons and selectrons
could also decay into neutrinos via higgsino exchange, yielding a signature consisting on
just two heavily ionizing tracks plus missing energy. Nevertheless, these processes are very
suppressed by the small electron and muon Yukawa couplings and can be usually neglected,
even in the large $\tan\beta$ regime.

The experimental signatures are qualitatively different when left-handed sfermions are
produced. These could decay via gaugino exchange either into charged leptons or into
neutrinos with comparable decay rates, yielding signatures with two heavily ionizing tracks
and four, two or no charged leptons in the final state. For the case of selectron production,
the possible final states are

$$
e^+ e^- \rightarrow \tilde{e}_R^+ \tilde{e}_L^- \rightarrow (e^+ \tau^+ \tilde{\tau}_1^+) (e^- \tau^- \tilde{\tau}_1^-), \quad (20)
$$

$$
e^+ e^- \rightarrow \tilde{e}_R^+ \tilde{e}_L^- \rightarrow (e^+ \tau^+ \tilde{\tau}_1^+) (\nu_\tau \tilde{\nu}_\tau \tilde{\tau}_1^-), \quad (21)
$$

$$
e^+ e^- \rightarrow \tilde{e}_L^+ \tilde{e}_L^- \rightarrow (e^+ \tau^+ \tilde{\tau}_1^+) (e^- \tau^- \tilde{\tau}_1^-), \quad (22)
$$

$$
e^+ e^- \rightarrow \tilde{e}_L^+ \tilde{e}_L^- \rightarrow (e^+ \tau^+ \tilde{\tau}_1^+) (\nu_\tau \tilde{\nu}_\tau \tilde{\tau}_1^-), \quad (23)
$$

$$
e^+ e^- \rightarrow \tilde{e}_L^+ \tilde{e}_L^- \rightarrow (\nu_\tau \tilde{\nu}_\tau \tilde{\tau}_1^-) (\nu_\tau \tilde{\nu}_\tau \tilde{\tau}_1^-). \quad (24)
$$

As for the case of the $e^- e^-$ linear collider, the left-handed slepton decays into neutrinos will
represent the most important source of background for the search of lepton flavour violation.

Lepton flavour violation could manifest itself either in the production of sleptons or
in their decays. Concentrating just on lepton flavour violation in the right-handed slepton
sector, the lepton flavour violating photon or $Z$-boson vertices would give rise to the following
processes:

$$
e^+ e^- \rightarrow \tilde{e}_R^+ \tilde{\mu}_R^- \rightarrow (e^+ \tau^+ \tilde{\tau}_1^+) (\mu^- \tau^- \tilde{\tau}_1^-) \quad \text{if} \quad (m^2_{l_R})_{12} \neq 0, \quad (25)
$$

$$
e^+ e^- \rightarrow \tilde{\mu}_R^- \tilde{\tau}_1^- \rightarrow (\mu^+ \tau^- \tilde{\tau}_1^+) \tilde{\tau}_1^- \quad \text{if} \quad (m^2_{l_R})_{23} \neq 0, \quad (26)
$$

$$
e^+ e^- \rightarrow \tilde{e}_R^+ \tilde{\tau}_1^- \rightarrow (e^+ \tau^+ \tilde{\tau}_1^+) \tilde{\tau}_1^- \quad \text{if} \quad (m^2_{l_R})_{13} \neq 0. \quad (27)
$$

It is important to note that the lepton flavour violating photon and $Z$-boson vertices appear
only at the one loop level. Consequently the cross sections for these flavour violating
processes in the $s$-channel are suppressed compared to flavour violating processes in the
t-channel, and therefore will not be considered in our analysis. Additional flavour violating
signatures could stem from the pair production of two neutralinos in the s-channel, followed by the flavour violating decay of one of them into a charged lepton and the NLSP, for example, $e^+ e^- \rightarrow \chi^0 \chi^0 \rightarrow (e^{\pm} \bar{\tau}_1^R) (\tau^{\pm} \bar{\tau}_1^R)$. The cross section for this process is also smaller than that of the flavour violating processes proceeding in the t-channel, since in this scenario neutralinos are heavier than sleptons, and will not be considered either.

Lepton flavour violation signals could also stem from the decay of right-handed selectrons, similarly to the case for the $e^- e^-$ collider, Eqs. (13-14): 

$$e^+ e^- \rightarrow e^+_R e^-_R \rightarrow (e^+ e^{\pm} \bar{\tau}_1^R) (e^- \tau^{\pm} \bar{\tau}_1^R) \quad \text{if} \quad (m^2_{l_R})_{13} \neq 0,$$

$$e^+ e^- \rightarrow e^+_R e^-_R \rightarrow (\tau^+ \tau^{\pm} \bar{\tau}_1^R) (e^- \tau^{\pm} \bar{\tau}_1^R) \quad \text{if} \quad (m^2_{l_R})_{13} \neq 0,$$

$$e^+ e^- \rightarrow e^+_R e^-_R \rightarrow (\mu^+ \tau^{\pm} \bar{\tau}_1^R) (e^- \tau^{\pm} \bar{\tau}_1^R) \quad \text{if} \quad (m^2_{l_R})_{12} \neq 0,$$

$$e^+ e^- \rightarrow e^+_R e^-_R \rightarrow (e^+ \mu^{\pm} \bar{\tau}_1^R) (e^- \tau^{\pm} \bar{\tau}_1^R) \quad \text{if} \quad (m^2_{l_R})_{23} \neq 0,$$

and analogous processes when it is $e^-_R$ the particle that decays violating flavour instead of $e^+_R$. On the other hand, production and lepton flavour violating decay of right-handed smuons would yield the following signals:

$$e^+ e^- \rightarrow \tilde{\mu}^+_R \tilde{\mu}^-_R \rightarrow (\mu^+ \mu^{\pm} \bar{\tau}_1^R) (\mu^- \tau^{\pm} \bar{\tau}_1^R) \quad \text{if} \quad (m^2_{l_R})_{23} \neq 0,$$

$$e^+ e^- \rightarrow \tilde{\mu}^+_R \tilde{\mu}^-_R \rightarrow (\tau^+ \tau^{\pm} \bar{\tau}_1^R) (\mu^- \tau^{\pm} \bar{\tau}_1^R) \quad \text{if} \quad (m^2_{l_R})_{23} \neq 0,$$

$$e^+ e^- \rightarrow \tilde{\mu}^+_R \tilde{\mu}^-_R \rightarrow (\tau^+ \tau^{\pm} \bar{\tau}_1^R) (\mu^- \tau^{\pm} \bar{\tau}_1^R) \quad \text{if} \quad (m^2_{l_R})_{12} \neq 0,$$

$$e^+ e^- \rightarrow \tilde{\mu}^+_R \tilde{\mu}^-_R \rightarrow (\mu^+ e^{\pm} \bar{\tau}_1^R) (\mu^- \tau^{\pm} \bar{\tau}_1^R) \quad \text{if} \quad (m^2_{l_R})_{13} \neq 0,$$

and analogously when $\tilde{\mu}^-_R$ decays violating flavour instead of $\tilde{\mu}^+_R$. No other process in the $e^+ e^-$ collider yields the same signals, therefore, if particle identification is sufficiently good, the observation of these processes would constitute robust evidences for lepton flavour violation in the right-handed slepton sector.

III. DISCOVERY POTENTIAL OF THE ILC

In this section we analyze the prospects to observe lepton flavour violation in the projected International Linear Collider (ILC), both for the $e^- e^-$ mode and the $e^+ e^-$ mode. For definiteness, we will assume a center of mass energy $\sqrt{s} = 500$ GeV, and an integrated
luminosity of 500 fb$^{-1}$. We will also assume in our numerical analyses that the beams are unpolarized, although as discussed in the previous section, the use of polarized beams would enhance the strength of the lepton flavour violating signals. The cross sections of the signal events have been calculated in the narrow width approximation. We have calculated the relevant $2 \rightarrow 2$ differential cross section $d\sigma/d\cos\theta$ and then folded into it the appropriate branching fractions of the corresponding decay channels to get the various final states described earlier.

We select the signal events according to the following criteria:

- The transverse momentum of the electrons, the positrons and $\mu^\pm$ must be large enough: $p_T^{e\pm,\mu^\pm} > 5$ GeV.

- Slightly stronger selection criterion has been set on the transverse momentum of the $\tau$s: $p_T^\tau > 10$ GeV.

- The transverse momentum of the $\tilde{\tau}_1$s must satisfy $p_T^{\tilde{\tau}_1} > 10$ GeV.

- The electrons, the positrons, the $\mu^\pm$ and the $\tau$s and both the staus must be relatively central, i.e. their pseudorapidities must fall in the range $|\eta^{e\pm,\mu^\pm,\tilde{\tau}_1,\tau}| < 2.5$.

- The electrons, the positrons, the $\mu^\pm$, the $\tau$s and the staus must be well-separated from each other: i.e. the isolation variable $\Delta R \equiv \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2}$ (where $\eta$ and $\phi$ denote the separation in rapidity and the azimuthal angle, respectively) should satisfy $\Delta R > 0.4$ for each combination.

- The missing transverse momentum $p_T^\not{E} < 5$ GeV.

To illustrate the discovery potential of lepton flavour violation in scenarios with stau NLSP at the ILC we will show contour plots of constant cross section for a variant of the $\epsilon$ benchmark point of [10]. We will take all the supersymmetric parameters as in the $\epsilon$ benchmark point, but we will vary the NLSP mass between 144 GeV and 167 GeV (recall that in this benchmark point the mass of the next-to-NLSP, the right-handed selectron, is

\footnote{The International Linear Collider is likely to operate in two phases; the first with $\sqrt{s} = 500$ GeV and the second with $\sqrt{s} = 1$ TeV, with an integrated luminosity of 1ab$^{-1}$ for the $e^+e^-$ mode, and smaller for the $e^-e^-$ mode.}
169 GeV). We will also admit some small amount of lepton flavour violation in the right-handed slepton sector, parametrized by $\delta_{R_R}^{ij}$.

### A. \(e^-e^-\) collider

As discussed in previous sections, in the \(e^-e^-\) mode there are mainly two channels where lepton flavour violation in the right-handed selectron-stau sector could be discovered, namely processes with lepton flavour violating production of sleptons:

\[
e^- e^- \rightarrow \tilde{e}_R \tilde{\tau}_1^- \rightarrow (e^- \tau^\pm \tilde{\tau}_1^\mp) \tilde{\tau}_1^{-}, \tag{36}
\]

or with lepton flavour violating decay of selectrons:

\[
e^- e^- \rightarrow \tilde{e}_R \tilde{e}_R \rightarrow (e^- e^\pm \tilde{\tau}_1^\mp) (e^- \tau^\pm \tilde{\tau}_1^\mp), \tag{37}
\]

\[
e^- e^- \rightarrow \tilde{e}_R \tilde{e}_R \rightarrow (\tau^- e^\pm \tilde{\tau}_1^\mp) (e^- \tau^\pm \tilde{\tau}_1^\mp). \tag{38}
\]

In Fig. 1 we show contours of constant cross section for the process with lepton flavour violating production of sleptons, that manifest itself as two heavily ionizing tracks and two charged leptons. We show the results in the $\delta_{R_R}^{13}$–$\Delta m$ plane with the cuts mentioned above (here, $\Delta m \equiv m_{\tilde{e}_R} - m_{\tilde{\tau}_1}$.) As discussed in the text, this process suffers from backgrounds stemming from the associated production of $\tilde{e}_L\tilde{e}_R$ and the subsequent decay of the left-handed selectron into neutrinos. Nevertheless, imposing our cuts these background processes have very small cross sections and can be neglected in general, as can be realized from Fig. 2 where we show an upper bound on the background cross section, calculated assuming for simplicity $BR(\tilde{e}_L \rightarrow \nu_e \bar{\nu}_\tau \tilde{\tau}_1^-) = 1$.

Demanding that the signal significance (defined as $S/\sqrt{S+B} \approx \sqrt{S}$, where $S$ is the number of signal events and $B$ is the number of background events) is larger than 5, we find that searching for this decay channel in the $e^-e^-$ mode, the ILC could be sensitive to lepton flavour violation down to the level $\delta_{R_R}^{13} \sim 0.02$, when the supersymmetric spectrum is as in the $\epsilon$ benchmark point (i.e. with a mass splitting between the right-handed selectron and the NLSP of $\sim 20$ GeV). The sensitivity would improve when the mass splitting increases, whereas for smaller mass splittings, the outgoing charged leptons would be too soft and the cross section would be significantly reduced. For the $\epsilon$ benchmark point, the sensitivity of the ILC to lepton flavour violation is better than the present sensitivity of experiments.
searching for the rare decay $\tau \rightarrow e\gamma$, $\delta_{RR}^{13} \sim 0.1$, and comparable to the projected sensitivity of present $B$-factories \cite{13}, $\delta_{RR}^{13} \sim 0.02$. On the other hand, future super-$B$ factories could produce of the order of $10^{10}$ $\tau$ pairs at a luminosity of $10\, ab^{-1}$, allowing to probe branching ratios for the rare $\tau$ decays down to the level of $10^{-8} - 10^{-9}$ \cite{20}, which would translate into a sensitivity reach of $\delta_{RR}^{13} \sim 5 \times 10^{-3}$.

Despite the future sensitivity to lepton flavour violation in rare decay experiments is better than at the ILC in the $\epsilon$ benchmark point, there could be other regions in the parameter space where the decay rate for $\tau \rightarrow e\gamma$ is suppressed, due to cancellations in the loops, but not the tree-level lepton flavour violating production of sleptons. Furthermore, whereas the observation of rare decays does not shed any light on whether the source of lepton flavour violation is in the left-handed sector, the right-handed sector or the trilinear soft terms, the observation of the process $e^- e^- \rightarrow (e^- \tau^\pm \tilde{\tau}_1^\mp) \tilde{\tau}_1^-$ at the linear collider would pinpoint the right-handed sector as one of the sources of lepton flavour violation. Complementing this information with the one from rare decays could help to identify the sources of flavour violation in the leptonic sector, providing invaluable information about the soft-breaking Lagrangian. To be more precise, assuming that the process $e^- e^- \rightarrow (e^- \tau^\pm \tilde{\tau}_1^\mp) \tilde{\tau}_1^-$ is observed at the ILC, the quantity $\delta_{RR}^{13}$ inferred from experiments could be used to predict the rate for $\tau \rightarrow e\gamma$. If the observed rate for this rare decay is larger than the predicted one, it would follow that there are necessarily additional sources of flavour violation in the leptonic Lagrangian (either in $\delta_{LL}^{13}$, $\delta_{RL}^{13}$ or $\delta_{LR}^{13}$); if they are comparable, it would follow that the right-handed sector is the dominant source of lepton flavour violation; and if it is smaller, it would follow that different contributions to the decay amplitude are cancelling each other in order to produce a suppressed decay rate.

In this class of scenarios, lepton flavour violation could also be studied at future colliders using stopped staus \cite{13}. At the LHC, cascade decays of squarks and gluinos could produce of the order of $10^6$ NLSPs per year if particle masses are close to the current experimental limit. A fraction of them would be stopped in the walls of the detector, \cite{6} and decay at late times producing very energetic particles that would spring from inside the detector. If there is no LFV, all the NLSPs would decay into taus and gravitinos, $\tilde{\tau} \rightarrow \tau \psi_{3/2}$. If on the contrary

\textsuperscript{6} A larger number of staus could be trapped by placing 1-10 kton massive material around the LHC detectors, to be precise around $O(10^3 - 10^4)$. Similarly, at the ILC up to $O(10^3 - 10^5)$ could be collected and studied.
LFV exists in nature, some of the NLSPs would decay into electrons and muons. Therefore the detection of very energetic electrons and muons coming from inside the detector would constitute a signal of lepton flavour violation. Using this technique, it has been estimated that at the LHC or the ILC it would be possible to probe down to $\delta_{13}^{RR} \sim 3 \times 10^{-2} \ (9 \times 10^{-3})$ if $3 \times 10^3 \ (3 \times 10^4)$ staus could be collected [15].

On the other hand, in Fig. 3 we consider the lepton flavour violating signals in the decays of the right-handed selectrons, Eqs. (37, 38), that can be detected as two heavily ionizing tracks and four charged leptons. We see from these two figures that the lepton flavour violating signal from $e^- e^- \rightarrow \tilde{e}_R \tilde{e}_1^- \rightarrow e^- \tau^+ \tilde{\tau}_1^- + e^- \tau^- \tilde{\tau}_1^+ \tilde{\tau}_1^-$ could be used to probe a larger region in the relevant parameter space compared to that from $e^- e^- \rightarrow \tilde{e}_R \tilde{e}_R \rightarrow \tau^- \tau^- \tilde{\tau}_1^+ \tilde{\tau}_1^- \ (e^- \tau^- \tilde{\tau}_1^- \tilde{\tau}_1^-)$. This is due to the fact that the final states in these two processes are different and we have used different $p_T$ cuts on the $\tau$s and the electrons (positron).

Although the source of lepton flavour violation is $\delta_{13}^{RR}$ in both Figs. (11) and (3), the region in the parameter space explored by the process in Fig. (11) is larger than that in Figs. (3(a)) or (3(b)). There are two reasons for this feature. In the case when the lepton flavour violation is in the decays of the right selectron, due to the presence of $(m_{1 R}^2)_{13}$ there is a branching
FIG. 2: Background cross section (as discussed in the text) $\sigma_B$ in fb, with $\sqrt{s} = 500$ GeV, as a function of the mass-difference between the right-handed selectron and the NLSP, $\Delta m$. Other parameters choices are the same as the $\epsilon$ point. Both the $e^-$ beams are unpolarized.

ratio suppression in different channels shown in Eq.(13) and in Eq.(14). Also, since the final states are different when one considers lepton flavour violation in production, the effects of the cuts are also different.

Other lepton flavour violating decays of the right-handed selectron which could be discovered in an $e^-e^-$ collider are:

$$e^- e^- \rightarrow \tilde{e}_R^- \tilde{e}_R^- \rightarrow (\mu^- \tau^\pm \tilde{\tau}_1^\mp) (e^- \tau^\pm \tilde{\tau}_1^\mp)$$ if $(m_{l_R}^2)_{12} \neq 0$,

$$e^- e^- \rightarrow \tilde{e}_R^- \tilde{e}_R^- \rightarrow (e^- \mu^\pm \tilde{\tau}_1^\mp) (e^- \tau^\pm \tilde{\tau}_1^\mp)$$ if $(m_{l_R}^2)_{23} \neq 0$.  

In Figs.(4a) and (4b), we have shown contours of constant cross sections of these two processes in the plane $(\delta^{12}_{RR} - \Delta m)$ for the process in Eq.(39) and similarly in the plane $(\delta^{23}_{RR} - \Delta m)$ for the process in Eq.(40). Obviously, the reach in the process in Eq.(40) is larger since it contains only one $\tau$ and hence less affected by the $p_T$ cuts. On the other hand, if one compares the process in Eq.(39) with the process in Eq.(40), then one can observe that though these two processes look very similar (since we impose similar $p_T$ cuts for the $e^\pm$ and the $\mu^\pm$), the behaviours are different for higher values of the corresponding parameters $\delta_{RR}^{ij}$. 


FIG. 3: Contours of constant (a) $\sigma(e^{-}e^{−} → \tilde{e}_{R} \tilde{e}_{R} \rightarrow e^{-}e^{-}e^{±}τ^{±}\tilde{\tau}^{±}_{1})$ and (b) $\sigma(e^{-}e^{-} → \tilde{e}_{R} \tilde{e}_{R} \rightarrow \tau^{−}e^{−}\tau^{±}\tilde{\tau}^{±}_{1})$ in fb with $\sqrt{s} = 500$ GeV. Other parameters choices are the same as the $\epsilon$ point. Both the $e^{-}$ beams are unpolarized.

and $\Delta m$. This is again due to the fact that there is a branching ratio suppression in the process in Eq.(37) for relatively large values of the $\delta_{RR}^{ij}$ and $\Delta m$. For very low values of $\delta_{RR}^{ij}$ the nature is very similar in both the figures. Again, for low values of $\Delta m$ and large values of $\delta_{RR}^{ij}$ these two figures show quite similar behaviour since the branching ratio suppression in the process in Eq.(37) is less pronounced in this region and the lower limits in $\Delta m$ are determined by the $p_{T}$ cuts employed.

As before, if we demand that the signal significance is greater than or equal to 5, for the $\epsilon$ benchmark point the ILC could probe lepton flavour violation down to the level $\delta_{RR}^{12} \sim 0.04$, $\delta_{RR}^{23} \sim 0.03$, i.e. for a mass splitting between the right-handed selectron and the NLSP of $\sim 20$ GeV. For this benchmark point, the sensitivity to lepton flavour violation in the smuon-stau sector is slightly better than the present sensitivity from the rare decay $\tau \rightarrow \mu \gamma$, and in the smuon-selectron sector, is much worse than from the decay $\mu \rightarrow e\gamma$.

For the sake of completeness, let us now discuss the process $e^{-}e^{-} → \tilde{e}_{R} \tilde{e}_{R} \rightarrow (e^{−}τ^{±}\tilde{\tau}^{±}_{1}) (μ^{−}τ^{±}\tilde{\tau}^{±}_{1})$. The cross section for this process should be slightly smaller than for the process $e^{-}e^{-} → \tilde{e}_{R} \tilde{\tau}^{−}_{1} \rightarrow (e^{−}τ^{±}\tilde{\tau}^{±}_{1}) \tilde{\tau}^{−}_{1}$, due to the larger multiplicity of the final
FIG. 4: Contours of constant (a) $\sigma(e^-e^- \rightarrow \tilde{e}_R\tilde{e}_R \rightarrow \mu^-e^-\tau^+\tau^-\tilde{\tau}^+\tilde{\tau}^-)$ and (b) $\sigma(e^-e^- \rightarrow \tilde{e}_R\tilde{e}_R \rightarrow e^-e^-\mu^+\tau^+\tilde{\tau}^+\tilde{\tau}^-)$ in fb with $\sqrt{s} = 500$ GeV. Other parameters choices are the same as the $\epsilon$ point. Both the $e^-$ beams are unpolarized.

From the above discussion we can see that different regions in the corresponding parameter space can be explored in the lepton flavour violating production and decays of the right-selectron in a future $e^-e^-$ collider.
FIG. 5: Contours of constant cross sections \( \sigma(e^+e^- \rightarrow \tilde{e}_R^+\tilde{\tau}_1^- + \tilde{e}_R^-\tilde{\tau}_1^+ \rightarrow e^+\tau^+\tilde{\tau}_1^- + e^-\tau^+\tilde{\tau}_1^+) \) in fb with \( \sqrt{s} = 500 \text{ GeV} \). Other parameters choices are the same as the \( \epsilon \) point. Both the \( e^+ \) and the \( e^- \) beams are unpolarized.

B. \( e^+ e^- \) collider

Next, we will discuss the situation in an \( e^+e^- \) collider. As discussed in Section IIB, lepton flavour violating production processes are similar to the ones in the \( e^-e^- \) collider with the corresponding changes in the electric charge of the particles. We would also like to reiterate that only the \( t \)-channel neutralino mediated diagrams are taken into account for the calculation of these production processes, since the \( s \)-channel contributions through photon and \( Z \)-boson exchange are loop-suppressed. Let us first consider the case when only \( \delta_{RR}^{13} \) is non-vanishing. In this scenario the lepton flavour violating production process in an \( e^+e^- \) collider looks like

\[
e^+ e^- \rightarrow \tilde{e}_R^+\tilde{\tau}_1^- + \tilde{e}_R^-\tilde{\tau}_1^+ \rightarrow (e^+\tau^+\tilde{\tau}_1^- + e^-\tau^+\tilde{\tau}_1^+) \tilde{\tau}_1^+.
\]

(41)

The contours of constant cross sections of the process in Eq. (41) have been plotted in Fig. 5 in the \( (\delta_{RR}^{13} - \Delta m) \) plane for other parameter choices as in Fig. 1 and with \( \sqrt{s} = 500 \text{ GeV} \). The background to this process can come from the associated production \( e^+e^- \rightarrow \tilde{e}_R^+ e_L^- \) as shown in Eq. (21) (with the corresponding modifications for the charge conjugate process).
FIG. 6: Background cross section (as discussed in the text) $\sigma_B$ in fb, with $\sqrt{s} = 500$ GeV, as a function of the mass-difference between the right-handed selectron and the NLSP, $\Delta m$. Other parameters choices are the same as the $\epsilon$ point. Both the $e^+$ and $e^-$ beams are unpolarized.

The cross section of the background process is plotted in Fig.6 as a function of the mass difference between the right-selectron and the NLSP. Comparing Fig.1 and Fig.5 and looking at the background cross section we observe that the parameter region which can be explored in an $e^+e^-$ collider with $5\sigma$ significance is slightly smaller than in the case of an $e^-e^-$ collider, to be precise, $\delta_{RR}^{13} \gtrsim 0.03$ against $\delta_{RR}^{13} \gtrsim 0.02$ for $\Delta m \lesssim 20$ GeV. This is due to the fact that the production cross section of the lepton flavour violating process in an $e^+e^-$ collider is smaller compared to the corresponding process in an $e^-e^-$ collider. On the other hand, the smaller cross sections in the $e^+e^-$ for the lepton flavour violating processes could be compensated with a bigger luminosity, as will presumably occur since the electrons in the $e^-e^-$ collider repel each other translating into a decrease in the luminosity with respect to the $e^+e^-$ mode. Also, using right-polarized electron and positron beams the signal strength could be enhanced, thus improving the sensitivity to lepton flavour violation.

In Fig.7 we consider lepton flavour violation in right-handed selectron decays as in the case of an $e^-e^-$ collider. These signals are generated from the $\tilde{e}_R^+\tilde{e}_R^-$ pair production followed by their lepton flavour violating decays and can be detected as four charged leptons and two
FIG. 7: Contours of constant (a) $\sigma(e^+e^- \rightarrow \tilde{e}_R^+\tilde{e}_R^- \rightarrow e^+e^-\tau^+\tau^-\tau_1^+\tau_1^-)$ and (b) $\sigma(e^+e^- \rightarrow \tilde{e}_R^+\tilde{e}_R^- \rightarrow \tau^+e^-\tau^+\tau^-\tau_1^+\tau_1^- + \tau^-e^+\tau^+\tau^-\tau_1^+\tau_1^-)$ in fb with $\sqrt{s} = 500$ GeV. Other parameters choices are the same as the $\epsilon$ point. Both the $e^+$ and the $e^-$ beams are unpolarized.

heavily ionizing charged tracks. We can see from Fig. 7(b) that the process $e^+e^- \rightarrow \tilde{e}_R^+\tilde{e}_R^- \rightarrow (\tau^+\tau^+\tau_1^+\tau_1^-)$ ($e^-\tau^+\tau^-\tau_1^+\tau_1^-$) is not an efficient way to search for lepton flavour violation. On the other hand, Fig. 7(a) shows that the sensitivity of the $e^+e^-$ collider for the lepton flavour violating process $e^+ e^- \rightarrow \tilde{e}_R^+\tilde{e}_R^- \rightarrow (e^+ e^\pm \tau_1^\mp) (e^- e^\pm \tau_1^\mp)$ is comparable to the present sensitivity of the experiments, $\delta_{13}^{13} \sim 0.1$ when $\Delta m \sim 20$ GeV. It should also be noted that the corresponding sensitivity in an $e^-e^-$ collider is much better (see Fig. 8). This is again because of the fact that the right-selectron pair production cross section is larger in an $e^-e^-$ collider since both $t$- and $u$-channel diagrams are present and they interfere constructively.

In order to find out the sensitivity to $\delta_{12}^{12}$ and $\delta_{23}^{23}$ in an $e^+e^-$ collider, we have plotted in Fig. 8 the contours of constant cross sections for the processes

$$e^+ e^- \rightarrow \tilde{e}_R^+\tilde{e}_R^- \rightarrow (\mu^+ \tau^\pm \tau_1^\mp) (e^- \tau^\pm \tau_1^\mp), \quad (42)$$

$$e^+ e^- \rightarrow \tilde{e}_R^+\tilde{e}_R^- \rightarrow (e^+ \mu^\pm \tau_1^\mp) (e^- \tau^\pm \tau_1^\mp), \quad (43)$$

respectively. Demanding that the signal significance is greater or equal to 5, we obtain that the ILC in the $e^+e^-$ mode could be sensitive to $\delta_{12}^{12} \sim 0.09$ and $\delta_{23}^{23} \sim 0.07$ when $\Delta m \sim 20$
FIG. 8: Contours of constant (a) $\sigma(e^+ e^- \to \tilde{e}_R^+ \tilde{e}_R^- \to \mu^+ e^- \tau^+ \tau^- \tilde{\tau}_1^\mp \tilde{\tau}_1^\pm + \mu^- e^+ \tau^+ \tau^- \tilde{\tau}_1^\mp \tilde{\tau}_1^\pm)$ and (b) $\sigma(e^+ e^- \to \tilde{\tau}_1^+ \tilde{\tau}_1^- \to e^+ e^- \mu^+ \tau^\pm \tilde{\tau}_1^\pm \tilde{\tau}_1^\mp)$ in fb with $\sqrt{s} = 500$ GeV. Other parameters choices are the same as the $\epsilon$ point. Both the $e^+$ and the $e^-$ beams are unpolarized.

GeV. Once again, the sensitivity to these flavour violating quantities are slightly poorer than at the $e^- e^-$ collider.

As we also argued in the case of $e^- e^-$ collider, the process $e^+ e^- \to \tilde{e}_R^+ \tilde{e}_R^- \to (e^+ \tau^\pm \tilde{\tau}_1^\mp) (\mu^- \tau^\pm \tilde{\tau}_1^\mp)$ should have a cross section slightly smaller than the process $e^+ e^- \to \tilde{\tau}_1^+ \tilde{\tau}_1^- \to (e^+ \tau^\pm \tilde{\tau}_1^\pm) \tilde{\tau}_1^-$ and hence the sensitivity to $\delta_{12}^{\text{RR}}$ through the observation of the former process is slightly poorer than the sensitivity to $\delta_{13}^{\text{RR}}$ obtained from the observation of the latter. However, as pointed out in the $e^- e^-$ case, the final state of the process with associated production of a right-handed selectron and a right-handed smuon is identical to that in Eq. (42) and combining the two cross sections should improve the sensitivity of $\delta_{12}^{\text{RR}}$ to a significant extent.

Finally, let us discuss briefly the processes with pair production of two right-handed smuons, followed by the lepton flavour violating decay of one of them, Eqs. (32)–(35). The production cross section of a pair of right-handed smuons is suppressed approximately by a factor of 5 compared to the right-handed selectron pair production. In consequence, the constraints on the different $\delta_{ij}^{\text{RR}}$ from the non-observation of these processes are approximately...
a factor of five smaller than the constraints coming from processes with pair production of right-handed selectrons, and are therefore poorer probes of lepton flavour violation.

IV. CONCLUSIONS

In this paper we have estimated the sensitivity of future $e^+e^-$ and $e^−e^−$ colliders to lepton flavour violation, in scenarios where the gravitino is the lightest supersymmetric particle (LSP) and the stau is the next-to-lightest supersymmetric particle (NLSP). Since the NLSP can only decay gravitationally into gravitinos and charged leptons, the decay rate is very suppressed and the NLSP could traverse several layers of the vertex detector before decaying or even being stopped and trapped in it. This peculiar signature would be a clear signal for this class of scenarios and in particular would allow the clean search for lepton flavour violation, as the Standard Model backgrounds are absent and the supersymmetric backgrounds can be kept under control by using suitable kinematic cuts.

The signals of lepton flavour violation would consist of two heavily ionizing tracks accompanied by two or four charged leptons. Final states with two heavily ionizing tracks and two charged leptons correspond to the lepton flavour violating production of a right-handed selectron and a NLSP, followed by the decay of the right-handed selectron into the NLSP and two charged leptons, and would constitute a signal for lepton flavour violation in the right-handed selectron-stau sector. On the other hand, final states with two heavily ionizing tracks and four charged leptons correspond to the pair production of two right-handed selectrons (and also smuons, in the case of the $e^+e^−$ collider), followed by one lepton flavour violating decay and one lepton flavour conserving decay into the NLSP and two charged leptons. These signals arise when there exists mixing between any two generations of the right-handed sector. Nevertheless, to search for lepton flavour violation in the selectron-stau sector, we find that signals with two charged leptons in the final state are a more sensitive probe than signals with four charged leptons. We also find that the sensitivity to lepton flavour violation is slightly better at the $e^−e^−$ collider than at the $e^+e^−$ collider, due to the slightly larger production cross-section of sleptons at the $e^−e^−$ collider (either in a lepton flavour conserving or in a lepton flavour violating mode), which is due to the constructive interference of the $t$- and the $u$-channel production amplitudes.

To illustrate the sensitivity reach of this experiment, we have analyzed in detail a variant
of the $\epsilon$ benchmark point presented in [10], taking all the supersymmetric parameters as in the $\epsilon$ benchmark point, but varying the NLSP mass between 144 GeV and 167 GeV and admitting some small amount of lepton flavour violation in the right-handed slepton sector, parametrized by $\delta_{RR}^{ij}$. In particular, when the mass splitting between the NLSP and the next-to-NLSP is 20 GeV, we find that the International Linear Collider with a center of mass energy of $\sqrt{s} = 500$ GeV and an integrated luminosity of 500 fb$^{-1}$ could probe lepton flavour violation down to the level $\delta_{RR}^{13} \sim 0.02$, $\delta_{RR}^{12} \sim 0.04$, $\delta_{RR}^{23} \sim 0.03$ at 5$\sigma$ in the $e^-e^-$ mode using unpolarized beams, and slightly worse in the $e^+e^-$ mode. Furthermore, the use of right-polarized electron and positron beams could enhance the signal strength, improving the sensitivity to lepton flavour violation. This sensitivity is competitive with the present and projected sensitivities to lepton flavour violation from rare tau decays, although not from rare muon decays where the non observation of the process $\mu \rightarrow e\gamma$ still gives the most stringent constraints. Finally, we would like to remark that whereas the origin of the lepton flavour violation cannot be determined just by the observation of rare decays, the observation of the tree level production and/or decay of sleptons at the International Linear Collider would pinpoint the right-handed slepton sector as one of the sources of lepton flavour violation. Complementing this information with the one from rare decays could help to identify the sources of flavour violation in the leptonic sector, providing invaluable information about the soft-breaking Lagrangian.

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