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SINGLE PARTICLE SPECTRUM OF THE DUAL POMERON AMPLITUDE

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A B S T R A C T

We investigate the single-particle inclusive spectrum of the dual Pomeron amplitude. Scaling is obtained [if $\alpha_p(0)=1$] but no pionization if $\alpha_p(0)<1$. The momentum transverse cut-off turns out to be $\exp(-4k^2)$, which is the same as the one obtained from tree diagrams. An intuitive interpretation of these results is given and the dual amplitude which will be responsible for pionization is indicated.

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1. INTRODUCTION

Dual resonance models have recently been used to calculate one-particle distributions and two-particle correlations in high-energy inclusive reactions¹⁾. They provide us with an explicit expression for an arbitrary N-particle amplitude, and are therefore well suited to the use of the Mueller technique for the study of inclusive distributions²⁾. Indeed, a specific discontinuity of a 2N-particle amplitude is supposed to give the probability distribution for a reaction of the form 2 particles $\rightarrow$ (N-2) particles + anything.

Dual models describe any multiparticle reaction as mediated by the production and subsequent decay of a large number of zero-width resonances which are just a complete set of states giving the spectrum of the model. Consider the simplest inclusive reaction one can look at, $a + b \rightarrow$ anything; that is to say, the total cross-section for $a + b$ scattering. In lowest order of dual perturbation theory the total cross-section is given by the sum of the probabilities for producing one or two resonances, which we call σ_R and σ_P , respectively, and can be represented by

$$\sigma_R = \sum_X \left| \begin{array}{c} b \\ \nearrow \\ a \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} X \right|^2 \quad (1)$$

$$\sigma_P = \sum_{X, X'} \left| \begin{array}{c} b \\ \nearrow \\ a \end{array} \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} X \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} X' \right|^2 \quad (2)$$

Clearly σ_R is proportional to the absorptive part of the forward four-point function, and has consequently a $s^{\alpha_R(0)}$ Regge behaviour, $\alpha_R(0)$ being the intercept of the input linear trajectory of the model. On the other hand, σ_P is known to give an $s^{\alpha_P(0)}$ behaviour, where $\alpha_P(0)$ is the intercept of the dual Pomeron singularity. Indeed, σ_P is proportional to the absorptive part of the non-planar loop diagram that gives the dual Pomeron. We remind the reader that $\alpha_P(0)$ is given by the specific structure of the model and is not related directly to $\alpha_R(0)$. In the conventional model $\alpha_P(0) = 1/3$ and in addition the Pomeron has an undesirable cut behaviour³⁾. In the ghost-free model [$\alpha_R(0) = 1$] describing a fake 26-dimensional space-time world the Pomeron is a pole with $\alpha_P(0) = 2$ ⁴⁾. We will circumvent the difficulty of not having a correct dual model to describe the real

world by using the conventional model and considering $\alpha_R(0)$ and $\alpha_P(0)$ as parameters. We shall only compute dual amplitudes in asymptotic regions where fine details are not expected to be very relevant, and consequently we believe that the results of our calculations are related more to the different origins of the Reggeon and Pomeron singularities than to the specific properties of the dual model in use.

As far as one-particle inclusive distributions are concerned, they are given -- again to lowest order in dual perturbation theory -- by the sum of the probabilities of producing one, two, or three resonances, and the corresponding contributions are proportional to the missing-mass discontinuity of zero-, one-, and two-loop amplitudes, respectively⁵⁾. All inclusive calculations using dual amplitudes done up to now were performed by considering the forward six-point function only in the tree approximation¹⁾. Such a contribution can be interpreted as the spectrum of the first (or last) particle of a dual multiperipheral chain formed by the decay of the intermediate resonance produced by the incoming particle. This is easily seen if we write

$$\text{Disc}_{M^2} A_6 = \sum_X \left| \begin{array}{c} b \quad c \\ \nearrow \quad \nearrow \\ a \quad \quad X \end{array} \right|^2 = \sum_X \left| \begin{array}{c} b \quad c \\ \nearrow \quad \nearrow \\ a \quad \quad X \end{array} \right|^2. \quad (3)$$

This one-particle distribution exhibits a scaling behaviour of the form

$$E_c \frac{d\sigma}{dp_c} = s^{\alpha_R(0)-1} f(x, k_T^2), \quad (4)$$

where $x = 2p_L^c/\sqrt{s}$ is Feynman's parameter, and k_T is the transverse momentum of particle c. Moreover, for large k_T^2 the function $f(x, k_T^2)$ behaves as¹⁾:

$$f(x, k_T^2) \simeq (1 - x^2)^3 \left[\frac{4x}{(1+x)^2} \right]^{1-\alpha_R(0)} k_T^{-5} \exp \left[-\frac{2k_T^2}{x} \log \left(\frac{1+x}{1-x} \right) \right]. \quad (5)$$

It is interesting to observe that if one chooses $\alpha_R(0) = 1$ -- so as to have asymptotically a constant σ_R given by Regge exchange -- then the six-point dual amplitude in question gives a non-vanishing contribution in the pionization region ($x \simeq 0$); therefore from the exponential cut-off

in k_T^2 one can also conclude, by the usual arguments, that the multiplicity of secondaries grows like $\log s$. At first sight it is rather surprising that the spectrum of the first particle in the dual multiperipheral chain does contribute to pionization. One rather expects that the particles contributing to pionization are the ones emitted in the central region of a long multiperipheral chain. However, in the dual resonance model this only happens for $\alpha_R(0) = 1$, and for such a value of the intercept we know that long-range correlations in the multiperipheral chain are to be expected anyhow.

These considerations make it very appealing to investigate the contribution to one-particle distributions of diagrams containing a dual Pomeron, where the constancy of the total cross-section is not given by the Regge exchanges but rather by the large number of particles produced (the mechanism that gives rise to the Pomeron singularities in dual models). Indeed, in the process shown in Eq. (2), the number of particles produced grows exponentially with $\sqrt{s}$, while in processes (1) or (3) it grows only with a power of s . The purpose of this paper is to consider some of the one-particle distributions in which the particle detected is the decay product of one of the two resonances produced in a Pomeron-type process (2). The different dynamical origin of the Pomeron singularity could lead us to expect that the multiplicity of secondaries as well as spectra should be drastically different from the ones predicted by the process described by Eq. (3). We shall show, however, that this is not the case, and that a very similar behaviour in multiplicity and transverse momentum distribution is indeed obtained.

2. POMERON CONTRIBUTION TO ONE-PARTICLE DISTRIBUTIONS

As discussed in the Introduction, we are interested in studying inclusive one-particle distributions that exhibit the Pomeron asymptotic behaviour. In particular, we want to calculate in this section the contribution of the "dragon-fly" diagram of Fig. 1, where the wavy line represents a Pomeron exchange, and we are clearly looking at the kinematical region in which c is a fragment of b . The case c fragment of a is completely analogous and will not be considered any further.

At the one-loop level, the only diagrams contributing to the fragmentation region of b are the ones exhibited in Figs. 2a and 2b, where the particle c is seen to be the first particle coming from the decays of resonances X and X' , respectively. The modulus square of the graph shown in Fig. 2a summed over X and X' and integrated over phase space is the discontinuity across the singularity in $M^2 = (p_a + p_b - p_c)^2$ of the non-planar dual-loop diagram of Fig. 3. The contribution of the diagram of Fig. 2b is obtained from the same loop by interchanging the indices b and c .

Strictly speaking, the initial and final energies $s = (p_a + p_b)^2$ and $\bar{s} = (p_{\bar{a}} + p_{\bar{b}})^2$ must be continued to different sheets; and they must therefore be distinguished from each other in the course of the calculation. However, the missing-mass discontinuity of the dual loop we are looking at does not have singularities in s and $\bar{s}$ -- particles a and b do not resonate -- and the sheet condition is therefore irrelevant, so we can forget about this problem and safely identify s and $\bar{s}$.

The non-planar dual loop of Fig. 3 can immediately be computed from the prescriptions of Gross et al.⁶⁾. With the convention $\alpha_R(s) = \alpha_R(0) + s$, the amplitude for Fig. 3 has, in terms of Chan variables, the form

$$A = \int d\mu e^{-2p_a \cdot p_b V_1} e^{-2p_a \cdot p_c V_2} e^{-2p_b \cdot p_c V_3}, \quad (6)$$

where

$$\begin{aligned} d\mu &= 4\pi^2 g^6 [\psi(x_1)\psi(x_3x_4x_5)\psi(x_4)]^{-2\alpha_R(0)} (1-\omega^2) \frac{f^{-4}(\omega)}{(\log \omega)^2} \omega^{-\alpha_R(0)-1} \\ &\quad \times [(1-x_1)(1-x_2)(1-x_4)(1-x_6)(1-x_3x_4x_5)(1-x_5x_6x_1x_2x_3)]^{\alpha_R(0)-1} \prod_{i=1}^6 dx_i \\ \omega &= \prod_{i=1}^6 x_i, \end{aligned} \quad (7)$$

$$V_1 = \log \left[\frac{\psi_T(x_3) \psi_T(x_5)}{\psi_T(x_3x_4) \psi_T(x_4x_5)} \right] \quad (8a)$$

$$V_2 = \log \left[\frac{\psi_T(x_2x_3) \psi_T(x_5x_6)}{\psi_T(x_2x_3x_4) \psi_T(x_4x_5x_6)} \right] \quad (8b)$$

$$V_3 = \log \left[\frac{\psi(x_2) \psi(x_6)}{\psi(x_1x_6) \psi(x_1x_2)} \right] \quad (8c)$$

$$f(\omega) = \prod_{n=1}^{\infty} (1 - \omega^n) , \quad (9)$$

and the functions $\psi(x)$ and $\psi_T(x)$ are the usual one-loop functions defined in Ref. 6 in terms of Jacobian theta functions.

The integration region that avoids periodicities runs over positive x_i such that the six factors raised to the power $\alpha_R(0)-1$ in Eq. (7) are also positive. This restriction is much more transparent if one introduces as independent variables⁶⁾

$$q = \exp \frac{2\pi^2}{\log \omega} \quad (10)$$

as well as five angular differences [that is to say, six angles θ_i ($i = 1, \dots, 6$), one of which can be arbitrarily fixed] defined by

$$\theta_{i+1} - \theta_i = 2\pi \frac{\log x_i}{\log \omega} \quad i = 1, \dots, 5 \quad (11a)$$

$$2\pi \frac{\log x_6}{\log \omega} = 2\pi \left(1 - \frac{\log x_1x_2x_3x_4x_5}{\log \omega} \right) = 2\pi - (\theta_6 - \theta_1) . \quad (11b)$$

In terms of these variables, it is well known that the momentum-dependent part of the loop integrand is simply the exponential of the total heat generated by incoming currents proportional to the external moments in an annulus of radii q^2 and 1 ⁷⁾. The currents are attached as shown in Fig. 4, and the angular variables determine their position on the boundaries. The integration region is now easily visualized. The

angles are integrated over all possible configurations that keep the cyclic order of the currents on each boundary unchanged. One of them -- which we will take to be θ_6 , on the outer boundary with four currents -- is held fixed. In addition, q is integrated over the range $0 \leq q \leq 1$.

Let us now consider the asymptotic behaviour of the loop amplitude (7) in the fragmentation region of b , namely $s \rightarrow \infty$, $M^2 \rightarrow \infty$, keeping the Feynman parameter x fixed and positive. Introducing the longitudinal mass $m_T = \sqrt{\mu^2 + k_T^2}$, where k_T is the transverse component of $\vec{p}_c$, the scalar products that occur in the loop integrand are given by

$$2p_a \cdot p_b = s - 2\mu^2 \simeq s \quad (12a)$$

$$2p_b \cdot p_c = -xs - \left(\frac{m_T^2}{x} - x\mu^2 \right) \simeq -xs \quad (12b)$$

$$2p_a \cdot p_c = -\frac{m_T^2}{x} - x\mu^2. \quad (12c)$$

We remind the reader that, in terms of these variables, $M^2 \simeq (1-x)s$. We can immediately see that the integrand of the loop amplitude will depend on s through the factor

$$e^{-s(V_1 - xV_2)}, \quad (13)$$

and the asymptotic behaviour will be given by the contribution of all critical points of the function $(V_1 - xV_2)$. A critical point means either a saddle point in all variables or a point where the hypersurface $(V_1 - V_2x) = \text{const}$ becomes tangent to some bounding hypersurface⁸⁾. From the explicit form (8) of the V_i functions their individual critical points are easily found:

$$\begin{aligned} V_1 &\equiv 0 & \text{for } q = 0 & \text{ or } \theta_2 = \theta_5 \\ V_2 &\equiv 0 & \text{for } q = 0 & \text{ or } \theta_1 = \theta_6 \\ V_3 &\equiv 0 & \text{for } \theta_2 = \theta_5 & \text{ or } \theta_1 = \theta_6 \end{aligned} \quad (14)$$

and then we can conclude that $V_1 - xV_2$ has critical points for $q = 0$ and $\theta_1 - \theta_6 = \theta_2 - \theta_5 = 0$. Their contribution to the asymptotic formula for A_2 has to be added. However, as we shall see, only the first one gives an $s^{\alpha_P(0)}$ behaviour. The second one gives rise to an $s^{\alpha_R(0)}$ behaviour,

and it can be interpreted as a renormalization to the tree diagram calculation⁸⁾. We shall thus concentrate from now on on the $q = 0$ critical point, and compute the leading term of the asymptotic series by expanding the integrand in powers of q and carrying out explicitly the q integration.

With the angular variables defined by Eqs. (11) we can therefore write

$$\begin{aligned}
 A = & 8\pi g^6 (-2)^{2\mu^2} \int_0^1 dp \frac{q^{-\alpha_P(0)-1}}{(\log q)^3} \int_0^{2\pi} d\sigma \int_0^{2\pi} d\eta \int_{0 \leq \theta_1 \leq \theta_2 \leq \theta_5 \leq \theta_6 = 2\pi} d\theta_1 d\theta_2 d\theta_5 \times \\
 & \times [(\theta_1 - \theta_6)(\theta_2 - \theta_1)(\theta_5 - \theta_2)(\theta_6 - \theta_5)\eta(2\pi - \eta)]^{\alpha_R(0)-1} \times \\
 & \times \left[\sin \frac{\theta_1}{2} \sin \frac{\theta_5 - \theta_2}{2} \sin \frac{\eta}{2} \right]^{-2\alpha_R(0)} \times \\
 & \times \exp [-f(x, m_T) V_3(q = 0)] \exp \left\{ -sq \left[\frac{\partial}{\partial q} (V_1 - xV_2) \right]_{q=0} \right\}, \quad (15)
 \end{aligned}$$

where we have replaced the inner angles θ_3, θ_4 by σ, η (see Fig. 3) defined by

$$\eta = \theta_4 - \theta_3 \quad (16)$$

$$\sigma = \frac{\theta_1 + \theta_2 + \theta_5 + \theta_6}{2} - \frac{\theta_3 + \theta_4}{2}. \quad (17)$$

The variable σ can be interpreted as the angle between the "centres of mass" of the two boundaries of the annulus, and is the only variable that correlates them. Moreover, from the expansion in q of the $\psi(x)$ and $\psi_T(x)$ functions,

$$V_3(q = 0) = \log \left[\frac{\sin \frac{\theta_5 - \theta_1}{2} \sin \frac{\theta_6 - \theta_2}{2}}{\sin \frac{\theta_2 - \theta_1}{2} \sin \frac{\theta_6 - \theta_5}{2}} \right] \quad (18)$$

$$\begin{aligned}
 \frac{\partial}{\partial q} [V_3 - xV_2]_{q=0} = & -8 \sin \frac{\eta}{2} \left[\sin \left(\frac{\theta_5 - \theta_2}{2} \right) \cos \left(\frac{\theta_1}{2} - \sigma \right) - \right. \\
 & \left. - x \sin \frac{\theta_1}{2} \cos \left(\frac{\theta_2 - \theta_5}{2} - \sigma \right) \right]. \quad (19)
 \end{aligned}$$

The factor $q^{-\alpha_P(0)-1}$ [$\alpha_P(0) = 1/3$ in the conventional model] comes from the development of the function

$$\frac{4\pi^2}{(\log \omega)^2} f^{-4}(\omega) = \omega^{1/6} q^{-1/3} f^{-4}(q) . \quad (20)$$

The q integration is now easily done and, neglecting $\log q$ factors in the integrand, that is to say the cut nature of the Pomeron, it gives

$$\begin{aligned} \int_0^1 dp \, q^{-\alpha_P(0)-1} \exp \left\{ -sq \left[\frac{\partial}{\partial q} (V_1 - xV_2)_{q=0} \right] \right\} = \\ = \Gamma \left[-\alpha_P(0) \right] \left[s \frac{\partial}{\partial q} (V_1 - xV_2)_{q=0} \right]^{\alpha_P(0)} . \end{aligned} \quad (21)$$

Once the q -integration is done, then η and σ integrations are also easily done. The η integration decouples from all the others and gives rise to the usual Pomeron form factor⁸⁾

$$f_P(0) = \int_0^{2\pi} d\eta \left[\eta(2\pi - \eta) \right]^{\alpha_R(0)-1} \left(\sin \frac{\eta}{2} \right)^{\alpha_P(0)-\alpha_R(0)-1} \quad (22)$$

which describes the coupling of the Pomeron to the $a\bar{a}$ system (see Fig. 1). The integral over σ is done by writing the right-hand side of Eq. (19) in the form $A \sin \sigma + B \cos \sigma$ and using

$$\begin{aligned} \int_0^{2\pi} d\sigma \left[A \sin \sigma + B \cos \sigma \right]^{\alpha_P(0)} = B_4 \left(\frac{\alpha_P(0)+1}{2}, \frac{1}{2} \right) \times \\ \left[\left(\sqrt{A^2 + B^2} \right)^{\alpha_P(0)} + \left(-\sqrt{A^2 + B^2} \right)^{\alpha_P(0)} \right] . \end{aligned} \quad (23)$$

We can recognize from Eq. (23) that, exactly as it happened for the usual Pomeron amplitude⁸⁾ the σ integration is the one responsible for the Pomeron signature. Indeed, A and B are linear functions of the kinematical variables s and $(-xs)$ and change their sign when we change the sign of these two variables. The two terms in the right-hand side of Eq. (23)

show up because we split the σ -integration range into two parts in which $(A \sin \sigma + B \cos \sigma)$ is always positive or negative. Therefore, they will transform into each other, interchanging their roles when we change the signs of s and $M^2 = (1 - x)s$. One of them will then give the singularities for the positive values and the other one for the negative values of s , $-xs$, and M^2 . The inclusive one-particle distribution is the discontinuity across the positive M^2 cut starting at $M^2 = \mu^2$. We shall therefore keep only one of the two terms of the right-hand side of Eq. (23), namely, the one that by convention has a negative sign for positive s , $-sx$ and M^2 . Collecting all results, we obtain for the inclusive distribution

$$A = 8\pi^2 g^6 (-2)^{2\mu^2} f_P(0) (8s)^{\alpha_P(0)} \Gamma[-\alpha_P(0)] B_4\left[\frac{\alpha_P(0) + 1}{2}, \frac{1}{2}\right] \times \text{Disc}_{M^2}(I) , \quad (24)$$

where

$$I = \int_{0 \leq \theta_1 \leq \theta_2 \leq \theta_5 \leq \theta_6 \leq 2\pi} d\theta_1 d\theta_2 d\theta_5 \left[(\theta_1 - \theta_6)(\theta_2 - \theta_1)(\theta_5 - \theta_2)(\theta_6 - \theta_5) \right]^{\alpha_R(0)-1} \times$$

$$\times \left[\sin\left(\frac{\theta_6 - \theta_1}{2}\right) \sin\left(\frac{\theta_5 - \theta_2}{2}\right) \right]^{-2\alpha_R(0)} \times$$

$$\times U^{\alpha_P(0)/2} \left[\frac{\sin \frac{\theta_2 - \theta_1}{2} \sin \frac{\theta_6 - \theta_5}{2}}{\sin \frac{\theta_5 - \theta_1}{2} \sin \frac{\theta_6 - \theta_2}{2}} \right]^{f(x, m_T)} \quad (25)$$

$$U = \sin^2 \frac{\theta_5 - \theta_2}{2} + x^2 \sin^2 \frac{\theta_1 - \theta_6}{2} - 2x \sin \frac{\theta_1 - \theta_6}{2} \sin \frac{\theta_5 - \theta_2}{2} \times$$

$$\times \cos \left(\frac{\theta_5 + \theta_2 - \theta_1 - \theta_6}{2} \right) \quad (26)$$

and we have the convention $U^{\alpha_P(0)/2} \equiv (-\sqrt{U})^{\alpha_P(0)}$.

We now turn to the calculation of the M^2 discontinuity of I . In order to do this it is first necessary to understand how the different

singularities in s and M^2 of the integral I are generated. It is convenient at this stage to introduce the variables

$$\psi = \frac{\theta_1 - \theta_6}{2} \quad (27a)$$

$$\phi = \frac{\theta_2 - \theta_5}{2} \quad (27b)$$

$$\xi = \frac{\theta_5 + \theta_2 - \theta_1 - \theta_6}{2} \quad (27c)$$

which simplify considerably the limits of integration. The integration range in Eq. (25) is now given by

$$\int_{0 \leq \theta_1 \leq \theta_2 \leq \theta_5 \leq \theta_6 \leq 2\pi} d\theta_1 d\theta_2 d\theta_5 = \int_0^\pi d\psi \int_0^\psi d\phi \int_{\phi-\psi}^{\psi-\phi} d\xi = 2 \int_0^\pi d\psi \int_0^\psi d\theta \int_0^{\psi-\phi} d\xi, \quad (28)$$

where we have used the fact that the integrand is an even function of ξ .

Since all the s, M^2 dependence of the integrand is in U , singularities of I in s and M^2 will be generated by pinches of the singularity surface:

$$U = \sin^2 \phi + x^2 \sin^2 \psi - 2x \sin \psi \sin \phi \cos \xi = 0 \quad (29)$$

with the surfaces giving the boundaries of the integration region. For x real, Eq. (29) can be satisfied in the integration range only for $\xi = 0$. For $\xi \neq 0$, the solutions of Eq. (29) become complex. Let us therefore restrict ourselves to $\xi = 0$. In this case, Eq. (29) becomes

$$\sin \phi - x \sin \psi = 0. \quad (30)$$

The region of integration and the singularity surface $U = 0$ for $\xi = 0$ are shown in Fig. 5. In Fig. 5a we show the two branches of the surface $U = 0$ for some value of x in the range $0 < x < 1$. When $x \rightarrow 0$, they collapse into two straight lines $\phi = 0, \pi$ pinching the boundary surface $\phi = 0$ and giving rise to a singularity. When $x \rightarrow 1$, the two branches of the surface collapse into two straight lines $\phi = \psi$ and $\phi = \pi - \psi$ pinching the surface $\phi - \psi = 0$ and giving rise again to a singularity. Finally, when $x > 1$ we show in Fig. 5b the two branches

of the surface $U = 0$. As $x \rightarrow \infty$, they collapse into the straight lines $\psi = 0, \pi$ pinching the boundary $\psi = \pi$ and giving rise to a third singularity.

In order to discuss the physical meaning of these three singularities it is convenient to include the factor $s^{\alpha_p(0)}$ in the definition of U and rewrite the equation $U = 0$ as

$$s(\sin \phi - \sin \psi) + M^2 \sin \psi = 0 . \quad (31)$$

Then we see that the three singularities at $x = 0, 1, \infty$ are singularities at $M^2 = s$, $M^2 = 0$, and $s = 0$ generated by pinches of $U = 0$ with the boundaries $\phi = 0$, $\phi = \psi$, and $\psi = 0$, respectively. The branch point at $s = 0$ simply corresponds to normal thresholds in the $(a + b)$ channel. For fixed s the forward three-body amplitude is supposed to have a cut in the region $\mu^2 \leq M^2 \leq (\sqrt{s} - \mu)^2$ corresponding to the inclusive physical region, and a cut in the region $(\sqrt{s} + \mu)^2 \leq M^2 \leq \infty$ corresponding to the three-particle scattering physical region. Since an asymptotic formula never describes the origin of the branch points correctly, we have our branch points at $M^2 = 0$ and $M^2 = s$ (which follow at once from the previous formulas by neglecting the mass μ of the external scalar particles).

We must therefore compute the discontinuity across the cut in M^2 starting at $M^2 = 0$. The first thing we shall do is to restrict the region of integration in the (ϕ, ψ) plane to a small strip near the $\psi = \phi$ boundary as shown in Fig. 6. This strip is cut near the corner $\phi = \psi = 0$ by a curve that coincides with the singularity surface $\sin \phi - x' \sin \psi = 0$ ($0 < x' < 1$) (arc AB in Fig. 6) and also near the corner $\phi = \psi = \pi$ by another curve that coincides with the singularity surface $\sin \phi - x'' \sin \psi = 0$ ($x'' > 1$) (arc A'B' in Fig. 6). Clearly, this has several advantages:

- a) The region of integration left over does not contain the end-point $\phi = \psi$ and therefore has no $M^2 = 0$ singularity. All the $M^2 = 0$ singularity is contained in the restricted region of integration.
- b) The new region of integration does not contain the $\phi = 0$ and $\psi = 0$ end-points, so it does not have singularities at $s = 0$ or $M^2 = s$.

The new region of integration has spurious singularities at $M^2 = M'^2 = (1 - x'')s < 0$ and $M^2 = M'^2 = (1 - x')s > 0$ due to pinches of $U = 0$ with the arcs $A'B'$ and AB , respectively. However, as M^2 moves across $M^2 = 0$ without reaching these extreme values the correct singularity is generated, and its discontinuity coincides with the discontinuity of the full integral. If we want to compute the discontinuity of the integral for some positive value of $M^2 = (1 - x)s$ we can then use this restricted region of integration, and since M' must be bigger than M but is otherwise arbitrary, we can let x' approach x as much as we want so that in practice we can replace x' by x . As far as ξ is concerned, it is integrated from 0 to some arbitrary value ϵ .

Of course, there are general techniques available for the calculation of the discontinuity of a multiple integral⁹⁾. The integral we are looking at proves to be complicated to analyse because of the absence of the so-called "vanishing cycles" in the language of Ref. 9. This difficulty can be traced back to the non-planarity of the diagram we started from. The discontinuity of the integral will be given by the integral of the discontinuity of the integrand but extended over some region of integration that will not depend on our arbitrary restricted region and will involve complex hypercontours. We believe it will only contain, in common with the real contour, the edge $\phi = \psi$, $\xi = 0$. Anyway, we shall not need to use these more powerful methods for our purposes.

With the integration range now restricted to give only the $M^2 = 0$ singularity, we go back to Eq. (24). It shows already the usual scaling behaviour in the sense that, up to the factor $s^{\alpha_P(0)}$, it is only a function of x and k_T^2 . In order to further investigate the properties of our amplitude we next examine the behaviour for large k_T^2/x . This implies either small x (pionization) or large k_T^2 for arbitrary x . We shall let therefore $k_T^2/x \rightarrow \infty$ in Eq. (24), but keeping in mind that this implies $k_T^2/xs \rightarrow 0$, namely k_T^2/x large with respect to particle masses but small compared to s or xs .

The large k_T^2/x behaviour will be determined by the critical points of the function $V_3(q = 0)$ inside our restricted region of integration. If we introduce four complex variables on the unit circle $Z_i = e^{i\theta_i}$ ($i = 1, 2, 5, 6$) it follows at once from Eq. (18) that $V_3(q = 0)$ is the

logarithm of the anharmonic ratio (Z_1, Z_2, Z_5, Z_6) of the four points Z_i on the external boundary of the annulus; then we can conclude that $V_3 \leq 0$. In terms of the ξ , ϕ , and ψ , $V_3(q = 0)$ can be written as

$$V_3 = \log \left[\frac{\sin^2 \frac{\phi - \psi}{2} - \sin^2 \xi}{\sin^2 \frac{\phi + \psi}{2} - \sin^2 \xi} \right], \quad (32)$$

and we can see that V_3 can possibly attain its maximum value zero only when $\sin \phi = 0$ or $\sin \psi = 0$, that is to say, at the points A and A' (see Fig. 6) of our region of integration. Let us start by considering the point A. With our region of integration we have that if $\phi \rightarrow 0$, ψ also tends to zero, and the value that V_3 attains at that point depends on the way in which we approach it. Indeed, it is easy to check that V_3 is maximum when $\xi = 0$ and when we approach the point A along the boundary $\sin \phi = x \sin \psi$. This maximum value of V_3 controls the asymptotic behaviour in k_T^2/x .

We then compute the asymptotic behaviour of I by expanding the integrand around the critical point. Introducing the variable

$$\eta = \frac{\phi}{\psi} \quad (33)$$

the critical point under consideration is

$$\eta \simeq x, \quad \xi \simeq 0, \quad \phi \simeq 0, \quad (34)$$

and then the leading term in k_T^2/x is

$$I = \int_0^\pi d\psi \int_0^1 d\eta \int_0^{\psi(1-\eta)} d\xi \psi^{\alpha_P(0)-3\alpha_R(0)} \eta^{-\alpha_R(0)-1} \left[(1-\eta)^2 \psi^2 - \xi^2 \right]^{\alpha_R(0)-1} \\ \left[(\eta - x)^2 + x\eta\xi \right]^{\alpha_P(0)/2} \left[\frac{\psi^2(1-\eta)^2 - \xi^2}{\psi^2(1+\eta)^2 - \xi^2} \right]^{-k_T^2/x}, \quad (35)$$

which can be rewritten as

$$I = \exp \left[- \frac{2k_T^2}{x} \log \left(\frac{1+x}{1-x} \right) \right] (1-x)^{2\alpha_R(0)-1} x^{\alpha_P(0)-\alpha_R(0)} \int_0^\pi d\psi \psi^{\alpha_P(0)-\alpha_R(0)-1} \\ \int_0^{1-x/x} dv \int_0^1 d\rho \left[v^2 + \psi^2 (1-x)^2 \rho^2 \right]^{\alpha_P(0)/2} \exp \left[- \frac{4k_T^2}{1+x} (v + \rho^2) \right], \quad (36)$$

where

$$v = \frac{\eta - x}{x}, \quad \rho = \frac{\xi}{\psi(1-\eta)}. \quad (37)$$

The right-hand side of Eq. (36) has a well-defined limit when $x \rightarrow 0$ and then we obtain in the pionization limit

$$A \propto \lim_{x \rightarrow 0} e^{-4k_T^2/x} x^{\alpha_P(0)-\alpha_R(0)} g(k_T^2). \quad (38)$$

This result shows that if $\alpha_P(0) = 1$ the amplitude vanishes as $x^{1-\alpha_R(0)}$, so that it does not contribute to pionization unless, of course, we choose the Regge intercept $\alpha_R(0) = 1$.

Let us now consider the large k_T^2 behaviour for arbitrary x . The asymptotic behaviour in k_T^2 of the right-hand side of Eq. (36) can be readily computed with the result

$$I \simeq \exp \left[- \frac{2k_T^2}{x} \log \left(\frac{1+x}{1-x} \right) \right] x^{\alpha_P(0)-\alpha_R(0)} (1-x)^{2\alpha_R(0)+\alpha_P(0)} \\ \times \left(\frac{1+x}{4k_T^2} \right)^{[\alpha_P(0)+3]/2} \Gamma \left(\frac{\alpha_P(0)+1}{2} \right) \Gamma \left[2\alpha_P(0) - \alpha_R(0) + 1 \right]. \quad (39)$$

The factor $(1-x)^{\alpha_P(0)}$ combines with $s^{\alpha_P(0)}$ to give $(M^2)^{\alpha_P(0)}$ so that the missing-mass discontinuity of A is simply $\sin \pi \alpha_P(0) A$. Then, collecting all results, disregarding numerical coefficients and making $\alpha_P(0) = 1$, we find

$$A \simeq f_p(0) s \exp \left[- \frac{2k_T^2}{x} \log \frac{1+x}{1-x} \right] x^{1-\alpha_R(0)} (1-x)^{1+2\alpha_R(0)} \left(\frac{k_T^2}{1+x} \right)^{-2}. \quad (40)$$

The behaviour of A when $x \approx 0$ and the exponential cut-off in k_T^2 imply that the dual amplitude we are considering predicts a finite average multiplicity; it is only for the choice $\alpha_R(0) = 1$ that one recovers the usual $\log s$ behaviour of the multiplicity. We shall discuss the physical reason for such a result later on.

A few comments are still in order. First of all, this contribution of the point A' to the asymptotic behaviour in k_T^2/x can be shown to be proportional to

$$\exp \left[- \frac{2k_T^2}{x} \log \left(\frac{x'' + 1}{x'' - 1} \right) \right], \quad (41)$$

but since x'' can be taken as close to 1 as we wish, we can see that this contribution is non-leading and disappears completely in the pionization limit. Even better, the asymptotic contribution of the point A' has no singularity for positive M^2 . Then we must also remember that the amplitude we have calculated corresponds to the dual diagram of Fig. 2a. The diagram of Fig. 2b gives a similar result. The calculation parallels the preceding one exactly, except that some angles are interchanged so that the point A' , instead of A , is the one that controls the asymptotic behaviour in k_T^2/x .

3. DISCUSSION OF RESULTS

We found that the dual Pomeron contribution to the one-particle inclusive distribution predicts an exponential cut-off in the square transverse momentum. The cut-off (slope) -- as a function of x -- is the same as the one obtained in the tree approximation. The mechanism that gives the exponential cut-off in k_T^2 is also the same one as in the tree approximation¹⁾. The forward three-body amplitude is given by an integral representation that always exhibits a Regge-like power behaviour in k_T^2 for large values of k_T^2 . However, one is not interested in the forward three-body amplitude but rather in its missing-mass discontinuity. The calculation of this discontinuity restricts the region of integration and introduces new boundaries that give the desired exponential behaviour.

We found no pionization [if $\alpha_R(0) < 1$], which is reasonable if we think that the amplitudes of Fig. 2a and 2b we have studied represent the analysis of the spectrum of the first -- or last -- stable particle

produced in the decay of one of the two resonances in the final state.

An $s^{\alpha_P(0)}$ behaviour is obtained in this way, but the spectrum is reasonably depleted in the central region. We expect that the central region will be dominated by the contribution of the dual diagram of Fig. 7 which represents the spectrum of stable particles that are neither the first nor the last in the decay of X and X' . This implies the calculation of the M^2 discontinuity of a two-loop diagram that contains a three-Pomeron coupling. Work is in progress in this direction and we hope to present it elsewhere.

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Figure captions

- Fig. 1 : Pomeron contribution to one-particle distributions.
- Fig. 2 : Dual amplitudes containing a Pomeron asymptotic behaviour.
- Fig. 3 : Dual loop considered in the text.
- Fig. 4 : Analogue interpretation of the dual loop amplitude.
- Fig. 5 : Singularity surfaces of Pomeron integrand: a) for $x < 1$
and b) for $x > 1$.
- Fig. 6 : The domain of integration in the (ϕ, ψ) plane contributing
to the M^2 discontinuity.
- Fig. 7 : Two-loop amplitude expected to contribute in the central
region.

Fig.1

(a)

(b)

Fig.2

Fig.3

Fig.4

(a)

(b)

Fig.5

Fig. 6

Fig. 7