CM-P00058873

Archives

Ref.TH.1528-CERN

A PHENOMENOLOGICAL TREATMENT OF DUAL LOOP CONTRIBUTIONS
TO INCLUSIVE REACTIONS

D. Ebert *)
CERN - Geneva

A B S T R A C T

A phenomenological B_6 type amplitude is constructed which contains Regge singularities of twisted loop graphs such as ordinary trajectories, Regge cuts and Pomeron trajectories. With this amplitude possible implications of the different one and two-loop graphs for single particle distributions are derived for different kinematical regions including the fixed angle limit. The results emphasize the importance of dual loop contributions to single particle distributions.

*) On leave from the Department of Physics of the Humboldt University, DDR, Berlin.

1. - INTRODUCTION

Most recently, the dual resonance model has been applied to a qualitative and quantitative investigation of inclusive processes $a + b \rightarrow c + X$ (anything). Several interesting features of single particle distributions such as scaling behaviour, exponential cut-off in transverse momentum and triple Regge behaviour have been demonstrated by several groups ¹⁾. Numerical calculations of inclusive spectra ²⁾ have also been shown to be in rough agreement with experiment. Further interesting results have been obtained by studying the regions of double Regge expansion with fixed missing mass ³⁾ as well as the region of fixed angle scattering ⁴⁾.

In all these cases one observes, however, important deviations from experiment which cannot be explained within the usual B_6 model. Thus, the predicted exponential fall-off in transverse momentum is too strong. The same holds for the results of the double Regge expansion. Finally, the data deviate from the scaling law derived in the fixed angle limit ⁴⁾. At this point one has to keep in mind that the contribution of the Pomeron singularity has not been included into this approach. Since in the dual loop theory the Pomeron is described by twisted loop graphs ⁵⁾ one has to investigate dual loop amplitudes. Because detailed mathematical investigations of one and two-loop graphs in the fixed angle region are lacking at present, we attempt a phenomenological description of loop diagrams using modified B_6 functions. These functions are constructed in such a way that they reproduce the Regge behaviour of the loops predicted by the Reggeon calculus ⁶⁾. In order to simulate the Regge cut background which is dual to the Pomeron in accordance with the Harari-Freund conjecture, we use a smoothing prescription which avoids poles and introduces cuts in the corresponding Mandelstam channels ⁷⁾. It cannot be expected, of course, that such a smoothing integration may reproduce all the details of dual loop integrations which lead, e.g., to a special type of Pomeron form factor ⁵⁾. Nevertheless, it has been shown for special cases ⁵⁾ that a loop may have a duality structure very similar to that of a tree function, at least in some kinematical region.

In Section 2, we define the phenomenological amplitudes which will be investigated in Section 3 in different kinematical regions such as the region of double Regge expansion with fixed missing mass, the fixed angle and the pionization region. Studying the loop graphs proposed in Ref. 8), as well as the diffraction dissociation graphs, one can remove, at least phenomenologically, some of the deficiencies of the tree B_6 model mentioned at the beginning.

2. - GENERAL RELATIONS

We consider the inclusive process

$$a + b \rightarrow c + X(\text{anything}) \quad (1)$$

where c is the observed particle. The invariant variables are defined according to Fig. 1 by

$$\begin{aligned} s &= (p_a + p_b)^2 & s + t + u &= M^2 + m_a^2 + m_b^2 + m_c^2 \\ u &= (p_a - p_c)^2 \\ t &= (p_b - p_c)^2 \\ M^2 &= (p_a + p_b - p_c)^2 \end{aligned} \quad (2)$$

As is well known, the inclusive differential cross-section can be related by Mueller's generalized optical theorem ⁹⁾ to the discontinuity in the missing mass variable of the three-to-three forward scattering amplitude $a + b + \bar{c} \rightarrow a + b + \bar{c}$, i.e.,

$$\begin{aligned} s \frac{d^2 \sigma^{inc}}{dt dM^2} &= \frac{1}{s} \text{disc}_{M^2} T_{33} = \sum_i s \frac{d\sigma^{exc}}{dt} (ab \rightarrow ci) \delta(M^2 - m_i^2) \\ &+ (\text{Regge cut background}) \end{aligned} \quad (3)$$

where $d\sigma^{exc}/dt$ denotes the differential cross-section of the exclusive process $a + b \rightarrow c + i$. Following the proposals of Ref. 8), there are seven types of processes contributing in the planar dual resonance model to the cross-section of the reaction. The corresponding amplitudes, shown in Fig. 2, differ by the number of resonances out of which the anything X is composed.

The contributions of these processes to the inclusive cross-section should be calculated according to Eq. (3) by taking the discontinuity in the missing mass variable of the six-point amplitudes of Fig. 3. In Fig. 4, we have added, for completeness, also the graphs for diffraction dissociation, i.e., with vacuum quantum numbers in the $u(t)$ channels.

The contributions I_1 - I_3 of Fig. 3 have been extensively studied recently in the framework of the B_6 model ¹⁾. Therefore we restrict ourselves to the remaining graphs containing twisted loops. In the following we shall try to construct phenomenological six-point amplitudes which simulate essential features of the singularity structure of the loop amplitudes such as leading Regge behaviour in the corresponding Mandelstam channels. The main assumption of this proposal is, of course, that the loops have a duality structure similar to that of a tree amplitude, at least in some kinematical regions. Such a property has been shown explicitly, e.g., for the twisted box graph in the fixed angle limit ⁵⁾.

Non-planar loop amplitudes contain also Regge cuts whose branch point is given in the case of N intermediate Reggeons by ^{6) *)}

$$\alpha_c^N(v) = N(\alpha_0 - 1) + 1 + \frac{\alpha'}{N} v \quad . \quad (4)$$

Here $\alpha(s) = \alpha_0 + \alpha's$ is the parametrization of usual trajectories. In order to describe Regge cuts in the B_6 model we use the smoothing prescription of Ref. 7)

$$u_i^{-\alpha(s_i)} \rightarrow \int_0^{\gamma} d\lambda_i f_N(\lambda_i) u_i^{\lambda_i - \alpha_c^N(s_i)} \quad (5)$$

where u_i is the Chan variable of the corresponding channel and $\alpha_c^N(s_i)$ is the leading branch point introduced in Eq. (4). The prescription (5) yields also a series of cuts in the energy variable s_i with

$$\ell \leq \alpha_c^N(s_i) \leq \ell + \gamma \quad (\ell = 0, 1, 2, \dots) \quad (6)$$

as well as the Regge cut behaviour

$$\Gamma(-\alpha_c^N(s_i)) \frac{(-\alpha' s_k)^{\alpha_c^N(s_i)}}{\log^{N-1}(-\alpha' s_k)} \quad . \quad (7)$$

*) If the intermediate Reggeons can decay into a single Reggeon they lead to renormalization corrections instead of Regge cuts. These effects will not be considered here.

Here s_k are corresponding asymptotic energy variables and the power of the logarithm is connected with a behaviour

$$f_N(\lambda) \underset{\lambda \rightarrow 0}{\sim} \lambda^{N-2} c_1, \quad f_N(\lambda) \underset{\lambda \rightarrow \gamma}{\sim} (\lambda - \gamma)^{N-2} c_2$$

of the spectral function $f_N(\lambda)$. The latter one is assumed to be an analytic function of λ for $0 \leq \text{Re } \lambda \leq \gamma$. Finally, we use the following parametrization of the Pomeron trajectory

$$\alpha_P(v) = 1 + \frac{1}{2} \alpha' v. \quad (8)$$

The slope $\frac{1}{2} \alpha'$ is suggested by the dual loop approach ^{5),6)}. Its value is in agreement with measurements of the slope of the diffraction cone for pp scattering ^{6) *)}.

The phenomenological B_6 model for a loop amplitude with m Regge cuts as well as ordinary and Pomeron trajectories is now given by

$$B_6 = \int_0^\gamma d\lambda_1 f_{N_1}(\lambda_1) \dots \int_0^\gamma d\lambda_m f_{N_m}(\lambda_m) \int_0^1 \int_0^1 \frac{dx_1 dx_2 dy}{(1-x_1 y)(1-x_2 y)} x_1^{-\beta_{bc}(t)-1} y^{-\beta(M^2)-1} \\ \cdot x_2^{-\beta_{bc}(t)-1} \left(\frac{1-x_1}{1-x_1 y} \right)^{-\beta_{ac}(u)-1} \left(\frac{1-x_2}{1-x_2 y} \right)^{-\beta_{ac}(u)-1} \left(\frac{1-y}{1-x_1 y} \frac{1-x_1 x_2 y}{1-x_2 y} \right)^{-\beta_{a\bar{a}}(0)-1} \\ \cdot \left(\frac{1-x_1 y}{1-x_1 x_2 y} \right)^{-\beta_{a\bar{a}}(u_c^2)-1} \left(\frac{1-x_2 y}{1-x_1 x_2 y} \right)^{-\beta_{a\bar{a}}(u_c^2)-1} (1-x_1 x_2 y)^{-\beta_{bb}(0)-1} \quad (9)$$

where, depending on the graphs considered, one has to insert

$$\beta(v) = \left\{ \alpha(v); \alpha_c^N(v) - \lambda; \alpha_P(v) \right\}. \quad (10)$$

We emphasize that this phenomenological description does not include automatically all the singularity structures which are contained in a given loop due to duality transformations. In order to include all singularities properly we have to draw all configurations connected with a given loop by duality transformations, write down for each configuration a phenomenological

*) The dual loop approach yields for the "embryonic" Pomeron an intercept $\alpha_P(0) = \frac{1}{3}$. Hopefully, this intercept will be shifted to 1 when the number of degrees of freedom in the spectrum of states will be increased by including baryons and exotics or by using ghost free dual models. Differing from the loop calculations we describe the Pomeron as a Regge pole rather than a cut.

B_6 amplitude and to add them. We refer again to the example of the twisted loop graph ⁵⁾ where one had to add phenomenological (s,t) and (u,t) B_4 amplitudes in order to generate the signature of the Pomeron trajectory.

The Table gives a survey of the singularities inserted into the (asymptotic) t, u and M^2 channels of the loop configurations of Figs. 3 and 4.

For the following, it is also useful to quote the contribution of a given resonance $\alpha(M^2) = N$ to $\text{disc}_{M^2} B_6$. The corresponding residue R_6 is given by the product of two beta functions and a finite sum ¹⁰⁾

$$R_6 = \pi \int_0^x d\lambda_1 f_{N_1}(\lambda_1) \dots \int_0^x d\lambda_m f_{N_m}(\lambda_m) B(-\beta_{a\bar{c}}(u), -\beta_{b\bar{c}}(t)) \cdot B(-\beta_{a\bar{c}}(u), -\beta_{b\bar{c}}(t)) \left\{ \sum_{m=0}^N \sum_{n=0}^m \sum_{\ell=m-n}^m \frac{(1 + \beta_{a\bar{a}}(0))_{N-m}}{(N-m)!} \cdot \frac{(z_1)_{m-n}}{(m-n)!} \frac{(z_2)_{n+\ell-m}}{(n+\ell-m)!} \frac{(z_3)_{m-\ell}}{(m-\ell)!} \frac{(-\beta_{b\bar{c}}(t))_\ell}{(-\beta_{b\bar{c}}(t) - \beta_{a\bar{c}}(u))_\ell} \frac{(-\beta_{b\bar{c}}(t))_n}{(-\beta_{b\bar{c}}(t) - \beta_{a\bar{c}}(u))_n} \right\} \quad (11)$$

where

$$(x)_n \equiv \frac{\Gamma(x+n)}{\Gamma(x)}$$

and

$$\begin{aligned} z_1 &= \beta_{a\bar{a}\bar{c}}(m_c^2) - \beta_{a\bar{c}}(u) - \beta_{a\bar{a}}(0) \\ z_2 &= \beta_{a\bar{a}}(0) + \beta_{b\bar{b}}(0) - \beta_{a\bar{a}\bar{c}}(m_c^2) - \beta_{a\bar{a}\bar{c}}(m_c^2) \\ z_3 &= \beta_{a\bar{a}\bar{c}}(m_c^2) - \beta_{a\bar{c}}(u) - \beta_{a\bar{a}}(0) \end{aligned} \quad (12)$$

3. - TRANSVERSE MOMENTUM AND FIXED ANGLE BEHAVIOUR OF THE MODEL

In usual Regge phenomenology with fixed momentum transfers Regge cuts lead to non-leading contributions in comparison with Regge pole exchange, due to their lower intercept. When the momentum transfer becomes asymptotic, too, this argument may not hold any longer since now the Regge slope becomes also important. In order to determine the influence of Regge singularities with slopes smaller than those of ordinary trajectories we consider therefore the following two kinematical regions :

(i) region of double Regge expansion

$s, |t|, |u| \gg M^2(\text{fixed})$, but non asymptotic with $K \equiv (t \cdot u/s)$ fixed ^{*)} ;

(ii) fixed angle limit ($s \rightarrow \infty$)

(a) M^2 finite

$$t \rightarrow -\frac{s}{2}(1-z)$$

$$u \rightarrow -\frac{s}{2}(1+z)$$

$$z = \cos \theta ; \quad r = \frac{q}{p}$$

(b) $M^2 \rightarrow \infty$

$$t \rightarrow -\frac{s}{2}r(1-z)$$

$$u \rightarrow -\frac{s}{2}r(1+z)$$

$$M^2 \rightarrow s(1-r)$$

Region (i) has been discussed in Ref. 3), region (ii,b) in Ref. 4). Here θ denotes the angle between the outgoing and the incident particle in the centre-of-mass system and p, q are the incident and outgoing momenta.

Case (i)

Let us calculate the contribution of a given resonance at $M^2 = m_i^2$ of the diffraction dissociation graphs, Fig. 4, to this limit. Using Eq. (11) and Stirling's formula, we obtain

$$s^2 \frac{d^2 \sigma^{inc}}{d4 dM^2} \Big|_{M^2=m_i^2} = s^2 \frac{d\sigma^{exc}}{d4} (ab \rightarrow ci) = e^{-b \frac{t \cdot u}{s}} \cdot R_0 \quad (13)$$

^{*)} The variable $(t \cdot u/s)$ is the non-asymptotic version of the usual transverse momentum variable $q_{\perp}^2$.

where b takes the values 2, 3, 3 for the graphs IV_1 , IV_2 or IV_3 respectively, and R_0 is a function of s , m_i^2 and $(t \cdot u/s)$ that is bounded for fixed s by a power in $(t \cdot u/s)^*$. The slope of the leading exponential term $e^{-2(t \cdot u/s)^*}$ agrees well with the value found for the exclusive processes ³⁾ (see Fig. 5)

In particular, the above diffraction dissociation graphs dominate over the $e^{-4(t \cdot u/s)}$ fall-off of the tree B_6 model calculated by Pinsky ³⁾. As quoted there, the exponential of Eq. (13) yields a good fit in the interval $1 \leq (t \cdot u/s) \leq 5(\text{GeV})$. A slope of the exchanged singularity close to 0.5 has also been found for the inclusive process $p + p \rightarrow p + X$ measured near the kinematic boundary (missing masses from 2.5 to 4.0 GeV) ¹¹⁾. Besides the configurations shown in Fig. 4, one has to consider also twisted contributions where the particles $a, b, \bar{c}$ are interchanged on each side. Except the signature twist $(a\bar{c}b) \rightarrow (b\bar{c}a)$, the twist contributions are all suppressed asymptotically for sufficiently high s . Further, one has to add graphs with Pomerons in the t channels which give identical contributions.

Case (ii,a)

For brevity, we sketch here only the behaviour of the two-Pomeron graph IV_1 . Using again Stirling's formula, we obtain a universal behaviour

$$s^2 \frac{d^2 \sigma^{inc}}{dt dM^2} \Big|_{M^2=m_i^2} = s^2 \frac{d\sigma^{exc}}{dt} (ab \rightarrow ci) = e^{2 \frac{\alpha'}{2} s F(z)} \cdot R_0 \quad (14)$$

where R_0 is a power bounded function of s, z and

$$F(z) = \frac{1-z}{2} \ln \frac{1-z}{2} + \frac{1+z}{2} \ln \frac{1+z}{2} \quad (15)$$

is the fixed angle exponent of the Veneziano four-point function ¹²⁾.

*) There is also a dual configuration of the graph IV_1 where one Pomeron, say from the $\bar{a}c$ channel, is transformed into the $b\bar{b}$ channel. This configuration yields also a $e^{-3(t \cdot u/s)}$ behaviour.

We see again that the graph IV_1 should dominate the tree B_6 contribution given by $e^{2\alpha' s F(z)} R_0^{\text{tree}}$ for sufficiently large energies. As shown in Fig. 4, the two-loop graph IV_1 factorizes at the stable particle pole ($J = 0$) into two twisted one-loop graphs. We mention that using the explicit results of 5) for the twisted one-loop graphs one obtains the same exponent as in Eq. (14). It can easily be shown that twisted configurations yield the same universal function $F(z)$ as in Eq. (14). Another configuration of IV_1 with a Pomeron shifted into the $b\bar{b}$ channel (see footnote p. 7) yields a non-leading exponential behaviour $e^{\frac{5}{2}\alpha' s F(z)}$. One obtains the same behaviour for the configurations IV_2 and IV_3 .

Case (ii, b)

Starting from the integral representation Eq. (9) and using Laplace's method, we obtain

$$s^2 \frac{d^2 \sigma}{dt dM^2} \xrightarrow{\text{inc}} e^{\alpha' s G(r, z)} \cdot \widetilde{R}_0. \quad (15)$$

The function $G(r, z)$ is defined by

$$\begin{aligned} \alpha' G(r, z) = & [\alpha'_1 r(1-z) + \alpha'_3(1-r)] \ln [\alpha'_1 r(1-z) + \alpha'_3(1-r)] \\ & + [\alpha'_2 r(1+z) + \alpha'_3(1-r)] \ln [\alpha'_2 r(1+z) + \alpha'_3(1-r)] \\ & - [\alpha'_1 r(1-z) + \alpha'_2 r(1+z) + \alpha'_3(1-r)] \ln [\alpha'_1 r(1-z) + \alpha'_2 r(1+z) + \alpha'_3(1-r)] \\ & - [\alpha'_3(1-r)] \ln [\alpha'_3(1-r)] \end{aligned} \quad (17)$$

Here α'_1 , α'_2 and α'_3 denote the slopes of the Regge singularities in the t , u and M^2 channels. [For simplicity, we do not consider the non-symmetric graphs IV_2 , IV_3 which can be shown to be dominated by the graph IV_1 .] For $\alpha'_1 = \alpha'_2 = \alpha'_3 = \alpha'$ $G(r, z)$ reduces to the result of the tree B_6 model

$$\begin{aligned} G(r, z) = & (1+r z) \ln(1+r z) + (1-r z) \ln(1-r z) \\ & - (1+r) \ln(1+r) - (1-r) \ln(1-r) \end{aligned} \quad (18)$$

discussed in Ref. 4).

Let us compare in some detail how the "deep inelastic scaling law"

$$\ln \left(\frac{d^2 \sigma}{dt dM^2} \right)_{\text{c.m.}} / s G(r, z) \xrightarrow{s \rightarrow \infty} \alpha' \quad (19)$$

of Huang, Segrè⁴⁾ works for the loop graphs of Figs. 3 and 4. Using data points from 4) and references found therein for $p + p \rightarrow (p; \pi^\pm) + X$ at momenta of 10-30 GeV/c ($|z| < 0.7$) we obtain the curves of Fig. 6a-c. In Fig. 6a, we see that the loop contributions to the process $p + p \rightarrow p + X$ give higher lying scaling curves as the tree case. The diffraction dissociation graph IV₁ yields the leading contribution which can take the predicted value (19), although the variation of the curve in r is somehow unpleasant.

In Figs. 6b and 6c, the corresponding curves for the processes $p + p \rightarrow \pi^\mp + X$ with an exotic missing mass channel are shown. As one sees, the configurations II₁(a) and III lead to considerable higher values than the tree graph I. For r not too small these curves lie near the predicted value $\alpha' \approx 1$. The configuration II₁(b) can be calculated analogously. Its contribution can be shown to be non-leading in comparison with the one of II₁a. The calculations leading to Eq. (16) have been performed in the limit $s, |u|, |t|, M^2 \rightarrow \infty$ with $|z| < 1, 0 < r < 1$. If one goes to the phase space boundary one obtains

$$r \rightarrow 0 \text{ (pionization)}$$

$$\alpha' s G(r, z) \rightarrow -4 \frac{\alpha'_1 \alpha'_2}{\alpha'_3} q^2 \sin^2 \theta = -4 \frac{\alpha'_1 \alpha'_2}{\alpha'_3} q_\perp^2 \quad (20)$$

$$|z| \rightarrow 1 \text{ (fragmentation)}$$

$$\alpha' s G(r, z) \rightarrow 2 \alpha'_1 t \ln \frac{2 \alpha'_2 r + \alpha'_3 (1-r)}{\alpha'_3 (1-r)} \quad (21)$$

$$r \rightarrow 1 \text{ (quasielastic limit)}$$

$$\alpha' G(r, z) \rightarrow \alpha'_1 (1-z) \ln \frac{\alpha'_1 (1-z)}{\alpha'_1 (1-z) + \alpha'_2 (1+z)} + \alpha'_2 (1+z) \ln \frac{\alpha'_2 (1+z)}{\alpha'_1 (1-z) + \alpha'_2 (1+z)} \quad (22)$$

One can obtain the same results (20)-(22) without using the fixed angle limit as an intermediate step by calculating immediately the appropriate limits. Note that Eq. (22) gives for $\alpha'_1 = \alpha'_2 = \frac{1}{2}\alpha'$, $\alpha'_3 = \alpha'$ again the result (15) *).

Let us conclude by comparing the single particle distributions of Figs. I₃, II₁(a), II₂, II₃, (III) and IV₁ in the pionization limit. According to Eq. (20) the slopes of the exponentials are 4, 2, 4, 4, ($\frac{4}{3}$, 2, 3) and 1.

Since the single particle distribution $s(d^2\sigma^{\text{inc}}/dtdM^2)$ contains also a factor $s^{\beta_{a\bar{a}}(0)-1} x^{\beta_{a\bar{a}}(0)-\beta_{b\bar{b}}(0)}$, only the graph (III) with Pomerons in the $\bar{a}a$ and $b\bar{b}$ channels contributes to pionization with an asymptotic $q_{\perp}^2$ slope ($\frac{4}{3}$, 2, 3). Comparing with the experimental fit $e^{-2.7q_{\perp}^2}$ for π^+ distributions in the range $1 < q_{\perp}^2 < 4 \text{ GeV/c}^2$ ¹³⁾, the singularity structure leading to the slope 3 should be preferred.

Finally, we mention that Amati et al. ¹⁴⁾ calculated recently a graph of the type II₂ finding the same slope value 4 as obtained by the phenomenological approach.

4. - CONCLUDING REMARKS

In this note we have given a phenomenological description of the twisted loop graphs of Figs. 3 and 4 which contribute to the single particle cross-section. These graphs are of particular interest since they include the Pomeron singularity, too.

*) For twisted graphs, there can arise difficulties in the case of different slopes of the Regge singularities in the s and u (or t) channel. If one considers, for example, the twisted graph II₂ with $a \neq \bar{c}$, $\bar{a} \neq c$ one obtains a $G(z) > 0$ for $z > 0$ leading to an exponential increase in s . Hopefully, such configurations of the twisted graphs will be suppressed in the fixed angle or pionization region by an internal loop mechanism. These difficulties do not concern the diffraction dissociation graphs.

As has been shown the Pomeron graph IV_1 of Fig. 4 can explain the $e^{-2(t.u/s)}$ fit for the exclusive process $p + p \rightarrow p + N^*$. In order to investigate further the influence of the Pomeron near resonances in the missing mass it would be very interesting to compare inclusive processes where the particle c is identical with the projectile or target particles a, b (e.g., $\pi^\pm p \rightarrow \pi^\pm + X$) with processes where this is not the case (e.g., $K^- p \rightarrow \pi^- + X$). In the first processes we may have Pomeron exchange in the $a\bar{c}$ or $b\bar{c}$ channels, in the latter ones not.

Finally, we have calculated the fixed angle behaviour of some phenomenological amplitudes which is in a better agreement with the Huang-Segrè scaling law than in the tree B_6 case. Needless to say that all these results are, of course, rather heuristic. To derive (or disprove) them exactly, one has to show the duality structure of the corresponding one and two-loop graphs in the above kinematical regions.

ACKNOWLEDGEMENTS

I would like to thank R. Baier, P. Frampton and G. Veneziano for discussions and helpful comments. The author is grateful for the hospitality extended to him in the Theoretical Study Division of CERN where most of this work was done.

TABLE

β	$\beta_{a\bar{c}}(u)$	$\beta_{\bar{a}c}(u)$	$\beta_{b\bar{e}}(t)$	$\beta_{\bar{b}e}(t)$	$\beta(M^2)$
$\text{II}_1(a)$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}u$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}u$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}t$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}t$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}M^2$
II_2	$\alpha_0 + \alpha' u$	$\alpha_0 + \alpha' u$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}t$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}t$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}M^2$
II_3	$2(\alpha_0-1)+1+\frac{\alpha'}{2}u$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}u$	$\alpha_0 + \alpha' t$	$\alpha_0 + \alpha' t$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}M^2$
$\text{III}^{(*)}$	$3(\alpha_0-1)+1+\frac{\alpha'}{3}u$	$3(\alpha_0-1)+1+\frac{\alpha'}{3}u$	$3(\alpha_0-1)+1+\frac{\alpha'}{3}t$	$3(\alpha_0-1)+1+\frac{\alpha'}{3}t$	$3(\alpha_0-1)+1+\frac{\alpha'}{3}M^2$
	$3(\alpha_0-1)+1+\frac{\alpha'}{3}u$	$3(\alpha_0-1)+1+\frac{\alpha'}{3}u$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}t$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}t$	$3(\alpha_0-1)+1+\frac{\alpha'}{3}M^2$
	$2(\alpha_0-1)+1+\frac{\alpha'}{2}u$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}u$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}t$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}t$	$3(\alpha_0-1)+1+\frac{\alpha'}{3}M^2$
IV_1	$2(\alpha_0-1)+1+\frac{\alpha'}{2}u$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}u$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}t$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}t$	$\alpha_0 + \alpha' M^2$
IV_2	$2(\alpha_0-1)+1+\frac{\alpha'}{2}u$	$\alpha_0 + \alpha' u$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}t$	$\alpha_0 + \alpha' t$	$\alpha_0 + \alpha' M^2$
IV_3	$\alpha_0 + \alpha' u$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}u$	$\alpha_0 + \alpha' t$	$2(\alpha_0-1)+1+\frac{\alpha'}{2}t$	$\alpha_0 + \alpha' M^2$

*) There are singularities in the $u(t)$ channel with two and three intermediate Reggeons.
 One has also non-symmetric terms with different singularities in the $(\bar{a}c)$, $(\bar{a}e)$ channels, etc., which will not be considered here.

REFERENCES

- 1) M.A. Virasoro - Phys.Rev. D3, 2834 (1971) ;
D. Gordon and G. Veneziano - Phys.Rev. D3, 2116 (1971) ;
C.E. DeTar, K. Kang, C.J. Tan and J.H. Weis - Phys.Rev. D4, 425 (1971) ;
K.J. Biebl, D. Bebel and D. Ebert - Berlin-Zeuthen Preprint (1971), to
be published in Fortschr.Phys. ;
R.C. Brower and R.E. Waltz - CERN Preprint TH. 1335 (1971).
- 2) D. Bebel, K.J. Biebl, D. Ebert and H.J. Otto - Berlin-Zeuthen Preprint
(1971) ;
R.C. Arnold and S. Fenster - Argonne Preprint (1971) ;
G.H. Thomas - Argonne Preprint (1971).
- 3) J.S. Ball and S. Pinsky - Phys.Rev.Letters 27, 1820 (1971) ;
S. Pinsky - Utah Preprint (1971).
- 4) K. Huang and G. Segrè - Phys.Rev.Letters 27, 1095 (1971) ;
R. Baier - CERN Preprint TH. 1458 (1972).
- 5) V. Alessandrini, D. Amati and B. Morel - Nuovo Cimento 7A, 797 (1972).
- 6) C. Lovelace - Phys.Letters 34B, 500 (1971).
- 7) D. Atkinson and A.P. Contogouris - Bonn Preprint (1971) ;
G.C. Joshi - Nuovo Cimento Letters 3, 243 (1972) ;
For a first application to single particle distributions, see also :
K.J. Biebl et al. - Ref. 1).
- 8) See :
D. Gordon et al. - Ref. 1) ;
G. Veneziano - Nuovo Cimento Letters 1, 681 (1971).
- 9) A.H. Mueller - Phys.Rev. D2, 2963 (1970).
- 10) J.F. Hopkinson and E. Plahte - Phys.Letters 28B, 489 (1969).
- 11) Min-Shih Chen, Ling-Lie Wang and T.F. Wang - Brookhaven Preprint
No BNL 15834 (1971).
- 12) G. Veneziano - Nuovo Cimento 57A, 190 (1968).
- 13) D.G. Crabb et al. - Phys.Rev.Letters 21, 830 (1968).
- 14) D. Amati and V. Alessandrini - Private communication.

FIGURE CAPTIONS

- Figure 1 Graphical representation of the process $a + b \rightarrow c + X$ (anything).
- Figure 2 The seven processes contributing to the inclusive reaction
 $a + b \rightarrow c + X$.
- Figure 3 Twisted loop graphs resulting from the processes of Fig. 2.
- Figure 4 The graphs contributing to diffraction dissociation.
- Figure 5 This figure is taken from Ref. 3). Exponential slope parameter
of $e^{(t \cdot u / \nu_0(t+u))}$ vs. laboratory momentum ($t+u \rightarrow -s$).
- Figure 6 (a)-(c) Plot of experimental points of $s^{-1} \ln(d^2\sigma/dq d\Omega)_{c.m.}$
divided by the function $G(r, z)$ for different graphs of Figs. 3
and 4 vs. r for different values of z . The experimental dif-
ferential cross-section $d^2\sigma/dq d\Omega$ is in units of $mb/(sr \text{ GeV}/c)$,
and s is in GeV^2 . The curve III corresponds to the singularities
of the last row of III in the Table.

FIG.1

FIG.2

FIG. 3

FIG. 4

FIG. 5

FIG. 6a

FIG. 6b

FIG 6c