Deuteron photodisintegration with polarized photons at astrophysical energies

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Following precise experimental studies at the Duke Free-Electron Laser Laboratory, we discuss photodisintegration of deuterons with 100% linearly polarized photons using a model independent theoretical approach taking together $M1$ and $E1$ amplitudes simultaneously. The isoscalar $M1_s$ contribution is also taken exactly into account. From the existing experimental measurement on doubly polarized thermal neutron capture, it is seen that the isoscalar $M1_s$ contribution could be of the same order of magnitude as the experimentally measured cross sections at energies relevant to Big Bang Nucleosynthesis (BBN). Therefore appropriate measurements on deuteron photodisintegration are suggested to empirically determine the $M1_s$ contribution at astrophysical energies.

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The photodisintegration of the deuterium and its inverse reaction viz., $n$-$p$ fusion in the neutron energy range of order 10 to $10^2$ keV is of considerable interest to astrophysics. It is important to the nucleosynthesis scenarios \cite{1} from the Big Bang to stellar evolution under various conditions. The earliest estimates of the reaction rates by Fowler, Caughlan and Zimmerman (FCZ) \cite{2} used theoretical calculations \cite{3} of deuteron photodisintegration normalized to the then available thermal neutron radiative capture cross section measurements \cite{4}. In a comprehensive evaluation of the reaction rates and uncertainties in 1993, Smith, Kawano and Malaney \cite{5} have pointed out: "With a binding energy of 2.22 MeV, deuterium is the most fragile of the primordial isotopes: it is rapidly destroyed in stellar interiors... Given that significant quantities of deuterium can only be produced during primordial nuclesynthesis, detection of deuterium provides important evidence in favor of the big bang model... Given the range of D/H observed in the interstellar medium, it is difficult to directly determine a lower limit... a determination of the upper limit is plagued by uncertainties arising from chemical evolution effects. The ratio of the primordial abundance of deuterium to that observed today could be any where between 1 and 50". Laboratory measurements and decisive developments in astronomical observations go hand in hand to remove crucial ambiguities in nuclear physics input parameters and sharpen theoretical predictions in the astrophysical context. Although laboratory measurements with thermal neutrons date back to 1936 by Fermi and collaborators \cite{6}, it has not been possible for a long time to measure the cross section at astrophysical energies due to the tendency of the neutrons to thermalize at low energies. The first cross section measurements between 20 keV and 64 keV have been reported in 1995 by Suzuki et al. \cite{7} and subsequently, by Nagai et al. \cite{8} at 550 keV. Burles and Tytler \cite{9} measured the deuterium abundance in high-red-shift-hydrogen clouds, where (it may be expected that) almost none of the deuterium could have been destroyed subsequent to the primordial stage. However, in a re-examination of the estimates of the uncertainties in 1999 to sharpen the predictions of big bang nucleosynthesis (BBN), Burles, Nollett, Truran and Turner \cite{10} have observed: "Our method breaks down for the process $p+n \rightarrow d+\gamma$. This is because of a near-complete lack of data at the energies relevant for BBN. The approach used for this reaction is a constrained theoretical model that is normalized to high precision thermal neutron capture cross-section measurements". The measured cross section of $334.2 \pm 0.5$mb by Cox, Wynchank and Collie \cite{11} for thermal neutrons is considered as standard.

The thermal neutron cross section has traditionally been interpreted in terms of a dominant isovector $M1$ amplitude for radiative capture from the initial $^1S_0$ state of the n-$p$ system in the continuum. Theoretical calculations \cite{12} based on potential models led to a 10% discrepancy with the experimental measurements. Breit and Rustgi \cite{13} proposed a polarized target-beam-test to detect the possibility of radiative capture from the initial $^1S_0$ state as well, which can take place through isoscalar $M1$ and possibly also isoscalar $E2$ transitions. However, the surprising accuracy with which Riska and Brown \cite{14} explained the 10% discrepancy by including Meson Exchange Current (MEC) contributions, set the trend for theoretical discussion in later years. It has been noted by Nagai \textit{et al.}, \cite{8} that the measured cross section is in agreement with the theoretical calculations by Sato \textit{et al.}, \cite{15} including MEC’s, isobar currents and pair currents. They have also pointed out that "the theory is in good agreement with the cross section measured for neutrons above 14 MeV, but it deviates by about 15% from the measured cross section of the $d(\gamma,n)p$ reaction by using the $\gamma$ ray of between 2.5 and 2.75 MeV \cite{16}, corresponding to neutron energies of 550 and 1080 keV" \cite{8}. Experimental studies on photodisintegration of the deuteron for photon energies from 2.62 MeV and above is well documented \cite{17}. The cross section at 2.62 MeV is $1.30 \pm 0.029$mb which increases slowly to $2.430 \pm 0.17$mb at 4.45 MeV and starts slowly decreasing with energy there-
after. The disintegration process is dominantly through $E1$ transitions leading to final triplet $P$-states of the n-p system in the continuum. Apart from the 15% discrepancy with the measured cross section noted by Nagai et al., the measured angular distribution and neutron polarization at photon energy of 2.75 MeV and in the range 6 to 13 MeV were found to be in disagreement with theoretical predictions which included the meson exchange currents. Measurements of the analyzing power in $p(n, \gamma)d$ at neutron energies of 6.0 and 13.43 MeV were consistent with and theoretical calculations showed that meson exchange currents produce a significant change but the effect is to move the theoretical curve to more negative values, thus making the discrepancy between theory and experiment more pronounced. An observable which is sensitive to the presence of isoscalar $M1$ and $E2$ transitions from the triplet $S$-state is the circular polarization of the emitted radiation with initially polarized neutrons. The first measurement to detect the presence of isoscalar amplitudes was not quite encouraging but a subsequent measurement yielded a value $P_\gamma = -(2.29 \pm 0.9) \times 10^{-3}$. An attempt to explain the large measured value by introducing a six quark admixture in the deuteron wave function led however to a disagreement with the well-known deuteron magnetic moment. Later calculations in the zero range approximation and the wavefunction for a Reid soft core potential led to a theoretical prediction $P_\gamma$ of the order of $-1.1 \times 10^{-3}$ with an estimated accuracy of 25%. The measured value of $P_\gamma = -(1.5 \pm 0.3) \times 10^{-3}$ is in reasonable agreement with the theoretical calculation. The importance of measuring the photon polarization with initially polarized neutrons on a polarized proton target has been pointed out. When the initial preparation of the neutron and proton polarizations and $P(n)$ and $P(p)$ are such that they are either opposite to each other or orthogonal to each other, the interference of the small isoscalar amplitudes with the large isovector amplitude could substantially contribute to the observable photon polarization.

Anticipating the experimental results of polarized thermal neutron capture by polarized protons by Müller et al., the possibility of the initial $^3S_1$ state contributions at thermal neutron energies was discussed using two different versions of effective field theory. Although the measured value of $(1.0 \pm 2.5) \times 10^{-4}$ for the $\gamma$ anisotropy $\eta$ was not sufficiently sensitive to distinguish between the two theoretical predictions, we may use equation (2) of Müller et al., to estimate the ratio $R$ of the triplet to singlet capture cross sections to be $1.202 \times 10^{-3}$. If we multiply $R$ by the well-known cross section, we get an estimate of 401.7 $\mu$b for the $^3S_1$ contribution to the cross section at thermal neutron energies. Quite surprisingly, this number is of the same order as the measured cross sections for capture at astrophysical energies of 20, 40 and 64 keV. In fact, it is even larger by a factor of 10 than the measured cross section at 550 keV. This raises an open question as to what could possibly be the ratio $R$ at astrophysical energies relevant to BBN.

The influential paper of Burles, Nollett, Truran and Turner has inspired several theoretical studies as well as experimental studies. Since photodisintegration of the deuteron is well documented for photon energies of 2.62 MeV and above and is known to be dominated by $E1$ transitions leading to final triplet $P$-states in the n-p continuum, these studies were motivated towards the determination of the relative $M1$ and $E1$ contributions to the process at astrophysical energies. The experiment was concerned with the measurement of the near threshold beam analyzing power using for the first time a laser based $\gamma$-ray source at 3.58 MeV. This was followed by measurements at seven $\gamma$-ray energies between 2.39 and 4.05 MeV. These measurements with 100% linearly polarized photons have been analyzed, making several simplifying assumptions viz.,

a) only $l = 0, 1$ partial waves were considered in the final state due to the low energies involved,

b) of the allowed two $M1$ and four $E1$ transitions, the isoscalar $E1$ leading to $^1P_0$ is set to zero,

c) the isoscalar $M1$ term leading to $^3S_1$ is neglected, using the traditional arguments for its suppression,

d) the three isovector $E1$ terms were combined to form a single $P$-wave amplitude, using the theoretical formalism, where $M1$ and $E1$ contributions were calculated separately.

The purpose of the present paper is to study $d(\gamma, n)p$ theoretically, using a model independent formalism, without making any simplifying assumptions except that only the dipole transitions are considered with $l = 0, 1$ partial waves in the final state. Since the strength of the isovector $M1_\gamma$ amplitude which is dominant at thermal neutron energies is known to decrease by several orders of magnitude as energies relevant to BBN is approached and an estimate of 401.7 $\mu$b of the contribution of the isoscalar $M1_\gamma$ amplitude to the cross section at thermal neutron energies is seen to be of the same order of magnitude as the measured cross sections at energies relevant to BBN, it is not unreasonable to pay attention to the contribution of the isoscalar $M1_\gamma$ amplitude at the energies of astrophysical interest. Moreover, spin observables are generally sensitive to the interference of a leading amplitude with other amplitudes which are not expected to be large. It is therefore appropriate to study the sensitivity of the beam analyzing powers to the isoscalar $M1_\gamma$ amplitude leading to final $^3S_1$ state at astrophysical energies.

We choose the linearly polarized photon momentum $\mathbf{k}$ in c.m. frame to be along z-axis and the linear polarization to be along x-axis of a right handed cartesian coordinate system and the neutron momentum $\mathbf{p}$ in c.m. frame to have polar coordinates $(p, \theta, \phi)$, following If the left and right circular states of photon polarization are
defined following Rose [37] through $u_{\lambda} = -\mu \xi_{\mu}, \mu = \pm 1$, the above state of linear polarization may be represented by $\frac{1}{\sqrt{2}}(u_{1} + u_{-1})$. We use natural units, $\hbar = c = 1$.

The unpolarized differential cross section for the reaction $d(\gamma, n)p$, in c.m frame at energy $E$ is given by

$$
\frac{d\sigma_{0}}{d\Omega} = \frac{1}{6} \left( \frac{E_{n}E_{p}E_{d}}{(2\pi E)^{3}} \right) \sum_{\mu = -1, 1} Tr(T(\mu)T^\dagger(\mu))
$$

where $Tr$ denotes the trace or spur and $T(\mu)$ denotes the on-energy-shell matrix for $d(\gamma, n)p$ when photons are in the polarized state $u_{\mu}$. The c.m. energies of the neutron, proton and deuteron are denoted respectively by $E_{n}$, $E_{p}$ and $E_{d}$. Following [38], we express

$$
M(\mu) = \sum_{s=0}^{1} \sum_{\lambda=|s-1|} \left( S^\lambda(s, 1) \cdot F^\lambda(s, \mu) \right),
$$

in terms of irreducible tensor operators, $S^\lambda_0(s, 1)$ of rank $\lambda$ in hadron spin space [38] connecting the initial spin $1$ state of the deuteron with the final singlet and triplet states, $s = 0, 1$ of the $n - p$ system in the continuum. Making use of the multipole expansion for $u_{\mu}e^{ik \cdot r}$ [37] and expressing the continuum states of the $n-p$ system in terms of partial waves, the irreducible tensor amplitudes, $F_{\nu}^\lambda(s, \mu)$ of rank $\lambda$ are given, in general, by

$$
F_{\nu}^\lambda(s, \mu) = \frac{1}{2} \sum_{L=1}^{\infty} \sum_{l=0}^{L} \sum_{j=-|s-l|}^{j=|s-l|} (i)^{L-l} \left[ 1 - (-1)^{l+s+j}(j)^{L-l-1}[L][j][s]^{-1} \right] W(L1ls; j\lambda) F_{ls; l, \mu}^{i\lambda} f_{\nu}^{i\lambda}(l, L, \mu),
$$

where $l, I$ denote the orbital angular momentum and isospin in the final state, $j$ denotes the conserved total angular momentum, $L$ denotes the total angular momentum of the photon and the shorthand notation $[L]$ stands for $\sqrt{2L + 1}$. The partial wave multipole amplitudes $F_{ls; l, \mu}^{i\lambda}$ depend only on c.m. energy $E$, while the

$$
f_{\nu}^{i\lambda}(l, L, \mu) = 4\pi \sqrt{2}(i\mu)^{s} \times C(l, L; \lambda; m_l, -\mu, \nu) Y_{lm_l}(\theta, \phi),
$$

take care of the angular dependence and also the dependence on photon polarization. The projection operators

$$
\pi^{\pm} = \frac{1}{2} \{ 1 \pm (-1)^{L-l-1} \}
$$

assume either of the values 0, 1 such that, if $\pi^{+} = 1$ implies $\pi^{-} = 0$ and vice versa. The $F_{ls; L}^{i\lambda}$ denotes electric $2^{-}$-pole amplitudes, if $\pi^{+} = 1$ and magnetic $2^{-}$-pole amplitudes, if $\pi^{-} = 1$. It may be noted that the reaction is completely characterized at any energy by the set of four irreducible tensor amplitudes $F_{\nu}^\lambda(s, \mu)$, given by [33]. But the contributing partial wave multipole amplitudes $F_{ls; L}^{i\lambda}$ increase as the c.m. energy increases.

In the region of interest to BBN, we may restrict ourselves to only $L = 1$ and to $l = 0, 1$ partial waves as in [33]. Then, we clearly have two $M1$ amplitudes viz., the isovector $M1_{v}$ leading to the final $1S_{0}$ state, the isoscalar $M1_{s}$ leading to the final $3S_{1}$ state and four $E1$ amplitudes viz., three isovector $E1_{v}^{j=0,1,2}$ leading to the final $3P_{0}$ states and an isoscalar $E1_{s}$ leading to the final $1P_{0}$ state. In terms of these limited number of partial wave multipole amplitudes, the four irreducible tensor amplitudes $F_{\nu}^\lambda(s, \mu)$ may explicitly be written as

$$
F_{\nu}^0(0, \mu) = -iM_{1v} f_{\nu}^{0}(0, 1, \mu) - \sqrt{3}E_{1s} f_{\nu}^{0}(1, 1, \mu),
$$

$$
F_{\nu}^0(1, \mu) = \frac{1}{3} E_{1v}(0) f_{\nu}^{0}(1, 1, \mu),
$$

$$
F_{\nu}^1(1, \mu) = \frac{1}{6} E_{1v}(1) f_{\nu}^{1}(1, 1, \mu) + iM_{1s} f_{\nu}^{1}(0, 1, \mu),
$$

$$
F_{\nu}^1(1, \mu) = \frac{1}{6} E_{1v}(2) f_{\nu}^{2}(1, 1, \mu).
$$

The $E_{1v}(\lambda)$ amplitudes contributing to the triplet irreducible tensor amplitudes $F_{\nu}^\lambda(1, \mu)$ with $\lambda = 0, 1, 2$ are related to the $E_{1v}^{j}$ amplitudes with $j = 0, 1, 2$ through

$$
\begin{bmatrix}
E_{1v}(0) \\
E_{1v}(1) \\
E_{1v}(2)
\end{bmatrix}
= \begin{bmatrix}
1 & 3 & 5 \\
2 & 3 & -5 \\
2 & -3 & 1
\end{bmatrix}
\begin{bmatrix}
E_{1v}^{0} \\
E_{1v}^{1} \\
E_{1v}^{2}
\end{bmatrix}.
$$

(10)

The differential cross section relevant to [33, 34] for $d(\gamma, n)p$ with linearly polarized photons is given, in c.m. frame, by

$$
\frac{d\sigma}{d\Omega} = \frac{2\pi^{2}}{6} [a + b \sin^{2} \theta(1 + \cos 2\phi) - c \cos \theta],
$$

where

$$
M = M(+1) + M(-1).
$$

(12)

Using known properties [33] of the irreducible tensor operators and standard Racah algebra, we have

$$
\frac{d\sigma}{d\Omega} = 2\pi^{2} \right| \begin{array}{c}
a + b \sin^{2} \theta(1 + \cos 2\phi) - c \cos \theta,
\end{array}
$$

(13)

where

$$
a = [8|M_{1s}|^{2} + 24|M_{1s}|^{2} + 36|E1_{s}|^{2} + 8|E1_{v}^{0}|^{2} + 18|E1_{v}^{1=1}|^{2} + 26|E1_{v}^{1=2}|^{2} - 16Re(E1_{v}^{1=0}E1_{v}^{1=2*}) - 36Re(E1_{v}^{1=1}E1_{v}^{1=2*})],
$$

(14)

$$
b = [9|E1_{v}^{1=1}|^{2} + 21|E1_{v}^{1=2}|^{2} + 24Re(E1_{v}^{1=0}E1_{v}^{1=2*}) + 54Re(E1_{v}^{1=1}E1_{v}^{1=2*}) - 18|E1_{v}|^{2}],
$$

(15)
\[ c = 4\sqrt{6} \text{Re}(2E_1^{i=0} + 3E_1^{i=1} - 5E_1^{i=2})M_1^s]. \]  

It is readily seen from (10) that the third term \( c \cos \theta \) in (13) arises due to the interference of the \( M_1 \) amplitude with the \( E_1 \) amplitudes. This term does not find place in (8), since the calculations there have been carried out separately for the \( E_1 \) and \( M_1 \) transitions. If we identify \( 2\pi^2 F_{1s1}^{ij} \) with \( 2\pi^2 I_{lsb} \) of (5) where \( b \) denotes \( j \), there is complete agreement between our expressions given by (14) and (15) for \( a \) and \( b \) and the corresponding expressions in (20). If it is assumed that \( E_1 = 0 \) and

\[ E_1^{i=0} = E_1^{i=1} = E_1^{i=2} = E_1, \]  

it follows that \( a, b, c \) simplify to

\[ a = 8(|M_1|^2 + 3|M_1s|^2); \quad b = 108E_1v; \quad c = 0. \]

leading to the beam analyzing power \( \Sigma(\theta) \) defined by (2) of (33) which now assumes the form

\[ \Sigma(\theta) = \frac{27}{2} |E_1v|^2 \sin^2 \theta / D, \]

where the denominator

\[ D = |M_1v|^2 + 3|M_1s|^2 + \frac{27}{2} |E_1v|^2 \sin^2 \theta, \]

is proportional to the unpolarized differential cross section. The \( \Sigma(\theta) \) was determined experimentally at \( \theta = 150^\circ \) in (33) at \( \gamma \)-ray energy 3.58 MeV and at \( \theta = 90^\circ \) in (34) at seven \( \gamma \)-ray energies between 2.39 and 4.05 MeV. The measurements of \( \Sigma(\theta) \) in (33) have led to empirical estimates of

\[ X = |M_1|^2/|E_1v|^2 = (|M_1v|^2 + 3|M_1s|^2)/|E_1v|^2, \]  

(21)

if \( M_1s \) is not set equal to zero. Under the same simplifying assumptions, it is interesting to note that the tensor target analyzing power (33) is given by

\[ A_2^0 = \frac{1}{\sqrt{2}} |M_1s|^2 - \frac{3}{2} |M_1s|^2 / D. \]  

(22)

Thus experimental measurements of \( A_2^0 \) can lead to an empirical estimate of

\[ Y = (|M_1v|^2 - \frac{3}{2} |M_1s|^2)/|E_1v|^2 \]  

(23)

in the energy region of astrophysical interest. Since \( X \) and \( Y \) are known empirically as functions of energy, it is possible to estimate

\[ R = \frac{|M_1s|^2}{|M_1v|^2} = \frac{2 (X - Y)}{3 (X + 2Y)}. \]  

(24)

to study the energy dependence of \( R \) empirically in the energy region of interest to astrophysics.

Finally, we may point out that, when the above simplifying assumptions are not made, the unpolarized differential cross section (1) itself is given by

\[ \frac{d\sigma}{d\Omega} = \frac{2\pi^2}{6} [a + b \sin^2 \theta - c \cos \theta], \]

(25)

where the coefficient \( c \) in third term can be determined by taking the difference between measurements of \( \frac{d\sigma}{d\Omega} \) at two angles \( \theta(\neq \pi/2) \) and \( \pi - \theta \). It can also be determined in the same way from \( \frac{d\sigma}{d\Omega} \) given by (19). For eg., Schreiber et al., (32) have measured (19) at \( \theta = 150^\circ \) and \( \phi = 0 \) and \( 90^\circ \). Additional measurements at \( \theta = 30^\circ \) for the same angles \( \phi \) and at the same energy, could easily estimate \( c \) at 3.58 MeV. The measurements by Tornow et al., (34) at lower energies have been carried out at \( \theta = 90^\circ \) and therefore not suitable for this purpose. It would therefore be desirable to carryout measurements at \( \theta \neq 90^\circ \) and at \( \pi - \theta \) at lower energy, to determine \( c \). The coefficient \( b \) is readily determined by taking the difference between (13) and (25) at any angle \( \theta \neq 0 \) or \( \pi \) and for any value of \( \phi \neq \pi/4 \). Since \( b \) and \( c \) are thus known, one can determine \( a \) by measuring (19) or even (25). Thus \( a, b, c \) given by (19), (25) and (26) are determinable empirically without making simplifying assumptions as in (33). We may note from (19) that \( c \) goes to zero either if \( M_1s \) is zero or if (17) holds exactly. On the other hand, if an empirical determination leads to \( c \neq 0 \), it implies simultaneously that \( M_1s \neq 0 \) and the simplifying assumption (17) is invalid.

Therefore an empirical determination of \( c \) appears desirable before carrying out the more incisive analysis of the experimental data suggested above. If \( c \) is found to be zero experimentally and (17) is assumed to be valid, the measurements of (19) along with (13), (22) and (26) hold promise for the more incisive empirical analysis, wherein \( R \) given by (24) also gets determined as a function of energy along with (21), where \( |M_1|^2 \) represents \( |M_1v|^2 + 3|M_1s|^2 \). This will lead to a better understanding of the photodisintegration of deuterons at astrophysical energies of relevance for sharpening the predictions of BBN.

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