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SOME REMARKS ABOUT THE USEFULNESS OF BEAM POLARIZATION
AT e^+e^- STORAGE RINGS

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A B S T R A C T

We study e^+e^- annihilation into hadrons and show to what extent, in the simplest experimental situations, beam polarization might be useful to measure electromagnetic structure functions in the one-photon approximation to this process.

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I. INTRODUCTION

Theoretical estimates show that quasi-two-body hadronic final states will be produced in e^+e^- collisions

$$e^+ + e^- \rightarrow a + b \quad (1)$$

with sufficient cross-section to be measurable at high luminosity storage rings up to an energy $\sqrt{s} \simeq 36 \text{ GeV}$ ¹⁾⁻³⁾. Moreover, it has been put forward that, in particular, high spin high mass resonances, whose production in hadronic reactions is difficult to establish, have a good chance of being detected in e^+e^- annihilation ³⁾ within the one-photon approximation (OPA). We thus might hope that with the development of more powerful detection equipment storage ring experiments will reveal a wealth of information about electromagnetic form factors in the time-like region.

The number n of independent (complex) form factors increases, of course, with the spin of the outgoing particles ⁴⁾. The problem to disentangle, from the measurement, the $(2n-1)$ real independent functions involved [i.e., n moduli and $(n-1)$ relative phases of the form factors], might become quite difficult as soon as n differs appreciably from one. In general, this separation cannot be achieved without detection of final state polarization: at a given energy the angular distribution yields only two independent quantities, namely the longitudinal and transverse cross-sections. If the final state contains at least one unstable particle, the determination of spin density matrix elements from this decay can be used to increase the number of independent measurements to more than two. For not too high spin systems such as $\Delta \bar{N}$ (where $n=3$) this procedure yields all $(2n-1)$ independent reals, without recourse to spin correlation measurements. Fortunately, most of the more important cases are of this type, as has been demonstrated and worked out in detail in a recent work of Kramer and Walsh ⁵⁾. However, one need not go to too exotic final states (like $\Sigma^+ \Sigma^-, \Delta \bar{\Delta}$) to meet the situation where spin correlation measurements are necessary for a complete determination of the form factors.

On the other hand, there is increasing experimental evidence from ACO that the storage ring beams may get substantially polarized due to synchrotron radiation ⁶⁾, an effect which was predicted by Sokolov and Ternov in 1963 ⁷⁾. As decay correlation measurements seem prohibitively difficult, when counting rates and spectrometer acceptance are limited, we want to study here the question to what extent this beam polarization, if really established, might be useful experimentally to gain information on hadronic form factors, or, more generally, on the structure functions that describe inclusive type processes

$$e^+ + e^- \rightarrow a + X \quad (2)$$

in OPA, where a is a resonance. We shall not question the validity of C, P and T invariance in electromagnetic interactions.

Section II is devoted to the kinematics of the lepton vertex which is fully parameterized in the photon density matrix. Section III discusses the information content of differential cross-section and single polarization measurements for the various beam polarizations. Some applications to a number of final state constellations are given in Section IV.

II. THE PHOTON DENSITY MATRIX

Since we want to deal with polarized electron and positron beams we will start with a short reminder about the parameterization of this polarization. Following Bjorken and Drell ⁸⁾, we introduce a unit polarization three-vector: a pure (electron or positron) state is said to have the polarization $\hat{s}$, if it obeys

$$\begin{aligned} \hat{s} \cdot \vec{\sigma} \, u(\vec{p}=0, \hat{s}_-) &= u(\vec{p}=0, \hat{s}_-) & (\text{for electrons}) \\ -\hat{s}_+ \cdot \vec{\sigma} \, v(\vec{p}=0, \hat{s}_+) &= v(\vec{p}=0, \hat{s}_+) & (\text{for positrons}). \end{aligned} \quad (3)$$

A projection operator $\sum_{\underline{s}} (s)$ onto these states can be written in a covariant manner (with the Bjorken-Drell conventions)

$$\Sigma(\underline{s}) = \frac{1 + \gamma_5 \underline{s}}{2} \quad (4)$$

The polarization of the two incoming beams in reactions (1) and (2) can be characterized by two such polarization four-vectors, $\underline{s}_+$ and $\underline{s}_-$. These are now no longer unit vectors; their norms equal the degrees of polarization, $P_{\pm}$:

$$P_{\pm} = \left(-\underline{s}_{\pm}^2 \right)^{1/2} \quad (5)$$

$\Sigma(\underline{s}_{\pm})$ represents now the covariant form of the spin density matrix. Since we agreed to work in the one-photon approximation (see Fig. 1 which also explains our notation), the whole spin information carried by the initial state through annihilation into the final hadron state

$$e^+ + e^- \rightarrow \gamma_V \rightarrow a + b \quad (6a)$$

$$e^+ + e^- \rightarrow \gamma_V \rightarrow a + \pi \quad (6b)$$

is contained in the (unnormalized) spin density matrix of the virtual photon γ_V

$$\Gamma_{\mu\nu} = \left\{ \bar{v}(k_+) \gamma_{\mu} u(k_-) \right\}^* \left\{ \bar{v}(k_+) \gamma_{\nu} u(k_-) \right\}, \quad (7)$$

where we are supposed to inject properly the lepton polarizations. The easiest way to do this is to use the covariant spin density matrix introduced above, and to take the traces. The result is

$$\Gamma_{\mu\nu}(\underline{s}_+, \underline{s}_-) = \text{tr} \left\{ \Sigma(\underline{s}_-) \Lambda(k_-) \gamma_{\mu} \Sigma(\underline{s}_+) \Lambda(-k_+) \gamma_{\nu} \right\} \quad (8)$$

with

$$\Lambda(p) = \frac{\not{p} + m}{2m} \quad (9)$$

the usual Dirac projection operator for the beam particles. The

cross-section for reaction (6a) can be expressed in terms of $\Gamma_{\mu\nu}$ in the c.m.s.

$$\frac{d\sigma}{d\Omega} = (2\pi)^6 \alpha^2 \frac{2m^2 |\vec{p}_b|}{q_0^3} \sum_{\substack{\mu, \nu \\ \text{spins}}} \langle ab | j_\mu | 0 \rangle^* \langle ab | j_\nu | 0 \rangle \Gamma_{\mu\nu}. \quad (10)$$

where $\langle ab | j_\mu | 0 \rangle$ is the matrix element of the electromagnetic current operator between vacuum and the two-particle state $|ab\rangle$ ⁹⁾. These are essentially the form factors mentioned above and can, e.g., be classified in the helicity basis.

An important point to note is that, due to current conservation, $\partial^\mu j_\mu = 0$, we have

$$\langle \vec{p}_a, \vec{p}_b | j_0 | 0 \rangle = 0 \quad \text{in the c.m.s.} \quad (11)$$

Consequently, in the c.m.s., only the space-space components of $\Gamma_{\mu\nu}$ (which we call Γ_{mn}) are physically significant and, therefore, we need only consider these.

Another important remark is that the scalar products vanish:

$$\begin{aligned} \underline{S}_+ \cdot \underline{k}_+ &= 0 \\ \underline{S}_- \cdot \underline{k}_- &= 0. \end{aligned} \quad (12)$$

This can easily be verified in the respective rest systems.

The parameterization of the final state polarization is best achieved by the spin-space density matrix $S_{ab, a'b'}$ which reads ⁹⁾

$$S_{ab, a'b'} = (2\pi)^6 \alpha^2 \frac{2m |\vec{p}_b|}{q_0^3} \sum_{\mu, \nu} \langle a'b' | j_\mu | 0 \rangle^* \langle ab | j_\nu | 0 \rangle \Gamma_{\mu\nu}. \quad (13)$$

In the following we shall evaluate Eq. (8) for various cases of polarization in increasing order of complexity.

a) No initial polarization

In this case $\underline{s}_+^2 = \underline{s}_-^2 = 0$, and, therefore,

$$\Sigma(s_+) = \Sigma(s_-) = \hat{2}$$

Inserting this into Eq. (8), we retrieve the well-known result

$$\Gamma_{\mu\nu}^{\text{unpol}} = \frac{-1}{4m^2} \{ (k_\mu^+ k_\nu^- + k_\mu^- k_\nu^+) - \frac{s}{2} g_{\mu\nu} \} \quad (14)$$

where we used the notation $s = \underline{K}^{+2}$, $K^+ = k^+ + k^-$.

b) Only one beam polarized

Remembering the useful formula

$$\text{tr} \{ \gamma_5 \gamma_\alpha \gamma_\beta \gamma_\gamma \gamma_\delta \} = 4i \epsilon_{\alpha\beta\gamma\delta} \quad (15)$$

the trace can be performed easily. With the electron beam polarized, the result is

$$\Gamma_{\mu\nu}(s_-) = \Gamma_{\mu\nu}^{\text{unpol}} + \Gamma_{\mu\nu}^{e-}(s_-), \quad \Gamma_{\mu\nu}^{e-}(s_-) = \frac{-i}{4m} \epsilon_{\alpha\beta\mu\nu} k^{+\alpha} s_-^\beta \quad (16)$$

where repeated indices are being summed over. A corresponding formula holds for the case of a polarized positron beam.

c) Both beams polarized

We use the recursion formula

$$\text{tr} \{ \gamma_{\mu_1} \dots \gamma_{\mu_n} \} = \sum_i g_{\mu_1 \mu_i} (-1)^i \text{tr} \{ \gamma_{\mu_2} \dots \gamma_{\mu_{i-1}} \gamma_{\mu_{i+1}} \dots \gamma_{\mu_n} \} \quad (17)$$

to perform the traces in Eq. (8). The result is a lengthy expression

$$\Gamma_{\mu\nu}(s_+, s_-) = \Gamma_{\mu\nu}^{e+}(s_+) + \Gamma_{\mu\nu}^{e-}(s_-) + \Gamma_{\mu\nu}^{\text{unpol}} + B_{\mu\nu}(s_+, s_-) \quad (18)$$

with

$$\begin{aligned}
 B_{\mu\nu}(\underline{s}_+, \underline{s}_-) = \frac{1}{4m^2} \bigg\{ & m^2 [s_\mu^+ s_\nu^- + s_\mu^- s_\nu^+ - (\underline{s}^+ \cdot \underline{s}^-) g_{\mu\nu}] \\
 & - s_\mu^+ [(k_\mu^+ \underline{s}^-) k_\nu^- - (k_\mu^+ k_\nu^-) s_\nu^-] \\
 & - s_\nu^+ [(k_\mu^+ \underline{s}^-) k_\mu^- - (k_\mu^+ k_\mu^-) s_\mu^-] \\
 & + (\underline{s}^+ \cdot \underline{s}^-) [k_\mu^+ k_\nu^- + k_\mu^- k_\nu^+ - (\underline{k}^+ \cdot \underline{k}^-) g_{\mu\nu}] \\
 & - (\underline{s}^+ \cdot \underline{k}^-) [k_\mu^+ s_\nu^- + k_\nu^+ s_\mu^- - (\underline{k}^+ \cdot \underline{s}^-) g_{\mu\nu}] \bigg\}.
 \end{aligned} \tag{19}$$

Since we are seeking to gain additional information on the absorption factors by use of e^+ , e^- polarization, we want to concentrate on those parts of B , which contain correlations between $\underline{s}_+$, $\underline{s}_-$ and the final state quantities. For this reason, we decompose $B_{\mu\nu}$ into three terms, $B_{\mu\nu}^{(0)}$, $B_{\mu\nu}^{(1)}$ and $B_{\mu\nu}^{(2)}$ in which none, one or both spin-vectors are non-contracted:

$$\begin{aligned}
 B_{\mu\nu} &= B_{\mu\nu}^{(0)} + B_{\mu\nu}^{(1)} + B_{\mu\nu}^{(2)} \\
 B_{\mu\nu}^{(0)} &= \frac{1}{4m^2} \left\{ (\underline{s}^+ \cdot \underline{s}^-) [k_\mu^+ k_\nu^- + k_\mu^- k_\nu^+ - \frac{s}{2} g_{\mu\nu}] + (\underline{s}^+ \cdot \underline{k}^-) (\underline{s}^- \cdot \underline{k}^+) g_{\mu\nu} \right\} \\
 B_{\mu\nu}^{(1)} &= \frac{i}{4m^2} \left\{ (\underline{s}^+ \cdot \underline{k}^-) [k_\mu^+ s_\nu^- + k_\nu^+ s_\mu^-] + (\underline{s}^- \cdot \underline{k}^+) [k_\mu^- s_\nu^+ + k_\nu^- s_\mu^+] \right\} \\
 B_{\mu\nu}^{(2)} &= \frac{s}{8m^2} \{ s_\mu^+ s_\nu^- + s_\mu^- s_\nu^+ \}.
 \end{aligned} \tag{20}$$

In order to investigate the implication of the expressions (14), (16) and (20), we introduce a co-ordinate system, in the c.m.s., with z axis parallel to $\vec{p}_a$, and y axis perpendicular to the reaction plane:

$$\hat{e}_y = \frac{\vec{k}^- \times \vec{p}_a}{|\vec{k}^- \times \vec{p}_a|}. \tag{21}$$

Furthermore, for convenience, we go over to spherical co-ordinates by the unitary transformation U

$$\begin{pmatrix} a_- \\ a_0 \\ a_+ \end{pmatrix} = U \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}, \quad U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i & 0 \\ 0 & 0 & \sqrt{2} \\ -1 & -i & 0 \end{pmatrix}. \quad (22)$$

The easiest way to evaluate Eqs. (14), (16) and (20) which contain only initial state vectors, is to insert these first in a system with z axis parallel to $\vec{k}_-$ and then do a rotation acting only on the indices μ, ν by the scattering angle θ (see Fig. 1).

To construct the polarization vector $\underline{s}_-$, we start from the electron rest system, where $\vec{s}^-$ is parameterized by the two angles α^- , ϕ^- and the degree of polarization P_- :

$$(s_x^-, s_y^-, s_z^-) = P_- (0, \sin \alpha^- \cos \phi^-, \sin \alpha^- \sin \phi^-, \cos \alpha^-) \quad (23)$$

in cartesian co-ordinates. After a Lorentz boost $L(-\beta)$ in the negative z direction to the centre-of-mass system this transforms to

$$\underline{s}_- = P_- (\beta \gamma \cos \alpha^-, \sin \alpha^- \cos \phi^-, \sin \alpha^- \sin \phi^-, \gamma \cos \alpha^-) \quad (24)$$

with

$$\beta = \left(1 - \frac{4m^2}{s}\right)^{1/2}, \quad \gamma = (1 - \beta^2)^{-1/2}$$

A corresponding expression holds for $\underline{s}_+$:

$$\underline{s}_+ = P_+ (-\beta \gamma \cos \alpha^+, \sin \alpha^+ \cos \phi^+, \sin \alpha^+ \sin \phi^+, \gamma \cos \alpha^+) \quad (24')$$

Note that $\alpha = \pi/2$ amounts to transverse and $\alpha = 0$ to longitudinal polarization.

We are now in the position to evaluate the photon density matrix, which we denote in the spherical basis (now with z axis along $\vec{p}_a$) by P . The formula is

$$P_{\lambda\lambda'} = d_{\mu\lambda}^{\dagger}(\theta) U_{\mu m} \Gamma_{mn} U_{n\nu}^{-1} d_{\nu\lambda'}^{\dagger}(\theta). \quad (25)$$

Here and in the following repeated indices are summed over. The d 's are the rotation functions in the notation of Rose¹⁰⁾. In particular, we obtain

$$P_{\lambda\lambda'}^{\text{unpol}}(\theta) = \frac{1}{8m^2} \{ -(s-4m^2) d_{0\lambda}^{\dagger}(\theta) d_{0\lambda'}^{\dagger}(\theta) + s \delta_{\lambda\lambda'} \}. \quad (26)$$

In the approximation $m^2/s=0$, this agrees with the result of Ref. 5).

Turning to the case of a polarized electron beam, unpolarized positron beam, we find from Eqs. (16) and (25):

$$P_{\lambda\lambda'}(s, \theta) = P_{\lambda\lambda'}^{\text{unpol}}(\theta) + P_{\lambda\lambda'}^{e-}(s, \theta) \quad (27)$$

with

$$P_{\lambda\lambda'}^{e-}(s, \theta) = \frac{\sqrt{s}}{4m} s_b^{-1} E_{b\mu\mu'} d_{\mu\lambda}^{\dagger}(\theta) d_{\mu'\lambda'}^{\dagger}(\theta), \quad (28)$$

where

$$E_{b\mu\mu'} = i U_{\mu'm'} E_{bm'm} U_{mp}^{-1}$$

For further reference, we list $E^e(s)_{\mu'\mu} \equiv s_b^{-1} E_{b\mu\mu'}$, which is a 3×3 matrix in the spherical base

$$E^e(s) = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sqrt{2}s_z, & s_{x-iy}, & 0 \\ s_{x+iy}, & 0, & s_{x-iy} \\ 0, & s_{x+iy}, & \sqrt{2}s_z \end{pmatrix}, \quad (29)$$

where we used the shorthand notation $s_{\pm x \pm iy} \equiv \pm s_x \pm i s_y$.

When both beams are polarized the various pieces of the photon density matrix read in spherical co-ordinates, according to Eqs. (18) and (20):

$$P_{\lambda\lambda}(s^+, s^-, \theta) = P_{\lambda\lambda}^{\text{unpol}}(\theta) + P_{\lambda\lambda}^{e^-}(s^-, \theta) + P_{\lambda\lambda}^{e^+}(s^+, \theta) + \sum_{i=0}^2 P_{\lambda\lambda}^{(i)}(s^+, s^-, \theta) \quad (30)$$

where

$$P_{\lambda\lambda}^{(0)} = -(s^+ s^-) P_{\lambda\lambda}^{\text{unpol}} - \frac{1}{4m^2} (s^+ k^-)(s^- k^+) \delta_{\lambda\lambda} \quad (31)$$

$$P_{\lambda\lambda}^{(1)} = -\frac{\sqrt{s-4m^2}}{8m^2} \left\{ (s^- k^+) E_{\mu\mu}^{(1)}(s^+) - (s^+ k^-) E_{\mu\mu}^{(1)}(s^-) \right\} d_{\mu\lambda}^{\dagger} d_{\mu\lambda} \quad (32)$$

with

$$E^{(1)}(s) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & , & s_x - iy & , & 0 \\ s_x + iy & , & \sqrt{2} s_z & , & s_{-x} + iy \\ 0 & , & s_{-x} - iy & , & 0 \end{pmatrix} \quad (33)$$

and

$$P_{\lambda\lambda}^{(2)} = \frac{s}{8m^2} \left(E_{\mu\mu}^{(2)}(s^+, s^-) + E_{\mu\mu}^{(2)}(s^-, s^+) \right) d_{\mu\lambda}^{\dagger} d_{\mu\lambda} \quad (34)$$

with

$$E^{(2)}(s^+, s^-) = \frac{1}{2} \begin{pmatrix} s_{x-iy}^+ s_{x+iy}^- & , & \sqrt{2} s_{x-iy}^+ s_z^- & , & s_{x-iy}^+ s_{-x+iy}^- \\ \sqrt{2} s_z^+ s_{-x+iy}^- & , & 2 s_z^+ s_z^- & , & \sqrt{2} s_z^+ s_{-x+iy}^- \\ s_{-x-iy}^+ s_{-x+iy}^- & , & \sqrt{2} s_{-x-iy}^+ s_z^- & , & s_{-x-iy}^+ s_{-x+iy}^- \end{pmatrix} \quad (35)$$

We now want to discuss the symmetries of P . As one verifies from Eq. (26)

$$\begin{aligned} \text{Re } P_{-\lambda' - \lambda}^{\text{unpol}}(\theta) &= (-)^{\lambda' - \lambda} \text{Re } P_{\lambda' \lambda}^{\text{unpol}}(\theta) \\ \text{Im } P_{\lambda' \lambda}^{\text{unpol}} &= 0. \end{aligned} \quad (36)$$

This reflection symmetry can easily be generalized to the case of linear beam polarization, as can be seen from Eqs. (28) and (29) together with the well-known symmetry $d_{-\lambda' - \lambda}^1(\theta) = (-)^{\lambda' - \lambda} d_{\lambda' \lambda}^1(\theta)$. The result is

$$\text{Im } \left\{ P_{-\lambda' - \lambda}^{e\pm}(s^\pm, \theta) \right\} = -\text{Re } \left\{ (-)^{\lambda' - \lambda} P_{\lambda' \lambda}^{e\pm}(s^\pm, \theta) \right\}. \quad (37)$$

Since this symmetry will be quite important in the following, we want to give it a name: we call a matrix $B_{\lambda\lambda'}$ symmetric of Class I, if $B_{-\lambda - \lambda'} = +(-)^{\lambda' - \lambda} B_{\lambda\lambda'}$ and symmetric of Class II, if $B_{-\lambda - \lambda'} = -(-)^{\lambda' - \lambda} B_{\lambda\lambda'}$. Note that the symmetry of $P^{e\pm}$ depends on the direction of $\vec{s}$: $P^{e\pm}$ is symmetric of Class I for $\vec{s}$ parallel to the normal of the reaction plane (y axis) and symmetric of Class II for $\vec{s}$ lying in the reaction plane.

Similarly, from Eq. (35), $P^{(1)}$ and $P^{(2)}$ obey

$$\text{Re } \left\{ P_{-\lambda' - \lambda}^{(i)}(s^\pm, \theta) \right\} = +\text{Re } \left\{ (-)^{\lambda' - \lambda} P_{\lambda' \lambda}^{(i)}(s^\pm, \theta) \right\} \quad (38)$$

To put this into physical terms: $P^{(1)}$ and $P^{(2)}$ obey Class II reflection symmetry, if one and only one of the two polarization vectors $\vec{s}^\pm$ lies parallel to the normal to the reaction plane, and Class I for all other cases. There is no such symmetry for circular polarization.

For completeness we close this section by adding the expressions for the joint density matrix elements in terms of P , using spherical components of j [Eq. (22) sandwiched between vacuum and the $J=1$ state of particles a and b with helicity λ_a and λ_b]

$$S_{\lambda_a \lambda_b, \lambda'_a \lambda'_b} = (2\pi)^6 d^3 \frac{2m\vec{p}_0}{s^{7/2}} \sum_{\lambda \lambda'} \langle p_z, \lambda'_a \lambda'_b | j_{\lambda' 10} \rangle^* \langle p_z, \lambda_a \lambda_b | j_{\lambda 10} \rangle P_{\lambda \lambda} \quad (39)$$

$$\frac{d\sigma}{d\Omega} = \text{tr } S \quad (40)$$

III. INFORMATION CONTENT OF VARIOUS MEASUREMENTS

In this Chapter we will look for the maximum amount of information that can be gained from inclusive type experiments, in which only particle a and its decay distribution is detected, i.e., nothing shall be assumed about the object b . Can beam polarization be of any use to gain more knowledge from this type of experiment? And which are the useful polarization conditions?

Differential Cross-Section

Let us assume first the simplest situation, namely that only the differential cross-section is measured. Since, by rotational invariance

$$\langle p_z, \lambda_a \lambda_b | j_{\lambda 10} \rangle = \left(\frac{3}{4\pi}\right)^{1/2} \delta_{\lambda, \lambda_a - \lambda_b} \langle j=1, \lambda_a \lambda_b | j_{\lambda 10} \rangle, \quad (41)$$

the cross-section has essentially the structure $\sum_{\lambda} a_{\lambda} P_{\lambda \lambda}$, where the a 's are bilinear in the form factors

$$a_{\lambda} = \sum_{\lambda_a - \lambda_b = \lambda} |\langle j=1, \lambda_a \lambda_b | j_{\lambda 10} \rangle|^2 \quad (42)$$

i.e., independently of $P_{\lambda\lambda}$, it contains only moduli of, not phases between, the form factors. From parity conservation

$$\langle J=1, \lambda_a \lambda_b | j \lambda | 0 \rangle = n_a n_b \langle J=1, -\lambda_a, -\lambda_b | j -\lambda | 0 \rangle \quad (43)$$

(where n_i stands for normality of particle i), we have $a_+ = a_-$. So there are only two terms to separate, a_0 and a_1 , corresponding to longitudinal and transverse cross-sections, which can already be performed without beam polarization from the θ dependence, as is well known.

Furthermore, we note that the differential cross-section with one beam linearly polarized equals the unpolarized cross-section and hence shows no azimuthal asymmetry. This follows from $a_+ = a_-$ together with the reflection symmetry and hermiticity of $P_{\lambda'\lambda}^{e\pm} : P_{++}^{e\pm} = -P_{--}^{e\pm}$, $P_{00}^{e\pm} = 0$.

Density Matrix of Produced Resonances

So let us consider the case where the decay of one outgoing particle is detected, say a , i.e., some of the spin density matrix elements

$$S_{\lambda'_a \lambda_a} = \sum_{\lambda_b} S_{\lambda'_a \lambda_b, \lambda_a \lambda_b} \quad (44)$$

are measured. From Eq. (39) we learn that $S_{\lambda'_a \lambda_a}$ has the structure

$$S_{\lambda'_a \lambda_a} = \sum_{\lambda\lambda'} b_{\lambda'\lambda}^{\lambda'_a \lambda_a} P_{\lambda\lambda} \quad (45)$$

with

$$b_{\lambda'\lambda}^{\lambda'_a \lambda_a} = (2\pi)^6 d^2 \frac{2m|\vec{p}_0|}{s^{7/2}} \sum_{\lambda_b} \langle p_e, \lambda'_a \lambda_b | j \lambda' | 0 \rangle^* \langle p_e \lambda_a \lambda_b | j \lambda | 0 \rangle \quad (46)$$

Because of Eqs. (41) and (43), we have

$$b_{\lambda'\lambda}^{\lambda_a\lambda_a} \sim \delta_{\lambda'-\lambda, \lambda_a-\lambda_a} \quad (47)$$

and

$$b_{-\lambda'-\lambda}^{-\lambda_a-\lambda_a} = b_{\lambda'\lambda}^{\lambda_a\lambda_a}. \quad (48)$$

From the last two relations, it follows immediately that P and S have the same reflection symmetry. In particular, if one defines $S^{\text{unpol.}}$, $S^{e\pm}$, $S^{(1)}$, $S^{(2)}$ through Eq. (45) by inserting on the right-hand side, $P^{\text{unpol.}}$, $P^{e\pm}$, $P^{(1)}$, $P^{(2)}$ respectively, all the statements made in the last Chapter about Class I and Class II symmetries of the P 's hold as well for the respective S 's.

Before we come to discuss the information content of density matrix elements $S_{\lambda'a\lambda_a}$ let us first repeat a few well-known facts about the observables from the decay of a resonance a . For most of this paper we shall restrict ourselves to the usual situation where the decay is parity conserving and determined by one coupling (such as $N^* \rightarrow N\pi$, $S \rightarrow 2\pi$, $\omega \rightarrow 3\pi$) and no polarization of the decay products is detected; the decay angular distribution has the structure

$$W(\Omega) \sim \sum_{\lambda'\lambda m} D_{\lambda'm}^{sa*}(\Omega) S_{\lambda'\lambda} D_{\lambda m}^{sa}(\Omega) \quad (49)$$

$$= \frac{1}{2} \sum_{\lambda'\lambda m} d_{\lambda'm}^{sa} d_{\lambda m}^{sa} \left\{ e^{i(\lambda-\lambda')\varphi} S_{\lambda'\lambda} + (-)^{\lambda'-\lambda} e^{i(\lambda'-\lambda)\varphi} S_{-\lambda'-\lambda} \right\}.$$

Here we have used only $d_{mn}^s(\theta) = (-)^{m-n} d_{-m-n}^s(\theta)$. As a result, one can measure only the combinations $\text{Re} (S_{\lambda'\lambda} + (-)^{\lambda'-\lambda} S_{-\lambda'-\lambda})$ and $\text{Im} (S_{\lambda'\lambda} - (-)^{\lambda'-\lambda} S_{-\lambda'-\lambda})$. Consequently, this "usual" decay reveals only the real part of Class I symmetric and only the imaginary part of Class II symmetric density matrix elements. There are, of course, additional constraints from hermiticity, like $\text{Im } S_{\lambda\lambda} = 0$.

What is the lesson to learn from $S_{\lambda_a' \lambda_a}$? We realize immediately from Eq. (41) that $S_{\lambda_a' \lambda_a}$ yields at most the bilinears b . With the help of Eq. (37) we can classify the latter for a given λ_a :

$$\lambda_a' = \lambda_a : \quad b_{+1, +1}^{\lambda_a \lambda_a}, \quad b_{00}^{\lambda_a \lambda_a}, \quad b_{-1, -1}^{\lambda_a \lambda_a} \quad (3 \text{ real numbers})$$

$$\lambda_a' = \lambda_a \pm 2 : \quad b_{\pm 1, \mp 1}^{\lambda_a \pm 2, \lambda_a} \quad (1 \text{ complex number})$$

$$\lambda_a' = \lambda_a \pm 1 : \quad b_{\pm 1, 0}^{\lambda_a \pm 1, \lambda_a}, \quad b_{0, \mp 1}^{\lambda_a \pm 1, \lambda_a} \quad (2 \text{ complex numbers})$$

These need not all be different from each other, or from zero, of course, for b obeys Eq. (48) and we must always satisfy $\lambda_a' \leq s_a$, $\lambda_b = \lambda_a - \lambda \leq s_b$, $\lambda_b' = \lambda_a' - \lambda \leq s_b$. In order to discuss the usefulness of beam polarization, in general, let us assume that the values of the λ 's and s ' are such that all b 's are different from each other and non-zero.

We start with the consideration of the diagonal density matrix elements. Since these are real by hermiticity, they can only be observed in Class I symmetric situations, where $P_{++} = P_{--}$. Therefore, the observable $S_{\lambda_a \lambda_a}$ has the structure

$$S_{\lambda_a \lambda_a} = (b_{+1, +1}^{\lambda_a \lambda_a} + b_{-1, -1}^{\lambda_a \lambda_a}) P_{++} + b_{00}^{\lambda_a \lambda_a} P_{00} \quad (50)$$

and, hence, irrespective of P_{++} , i.e., beam polarization, does not allow separation of the two transverse contributions.

Let us now turn to $S_{\lambda_{a+2}, \lambda_a}$ which reads

$$S_{\lambda_{a+2}, \lambda_a} = b_{+1, -1}^{\lambda_{a+2}, \lambda_a} P_{+-} \quad (51)$$

By hermiticity, P_{+-} has to be real for the Class I symmetry and imaginary for the Class II symmetry situation. So, regardless of whether one observes $\text{Re } S_{\lambda_{a+2}, \lambda_a}$ or $\text{Im } S_{\lambda_{a+2}, \lambda_a}$, one always measures $\text{Re } b_{+, -}^{\lambda_{a+2}, \lambda_a}$, as in the unpolarized case.

The result so far is that we learn nothing from beam polarization that we could not learn as well with unpolarized beams. But let us look at $S_{\lambda_{a+1}, \lambda_a}$:

$$S_{\lambda_{a+1}, \lambda_a} = b_{+, 0}^{\lambda_{a+1}, \lambda_a} P_{+0} + b_{0, -}^{\lambda_{a+1}, \lambda_a} P_{0-}. \quad (52)$$

In case of Class I symmetry, we observe

$$\text{Re } S_{\lambda_{a+1}, \lambda_a} = \text{Re} \left\{ b_{+, 0}^{\lambda_{a+1}, \lambda_a} P_{+0} - b_{0, -}^{\lambda_{a+1}, \lambda_a} P_{+0}^* \right\}. \quad (53)$$

Depending on whether P_{+0} is real or imaginary, $\text{Re } S_{\lambda_{a+1}, \lambda_a}$ therefore measures either $\text{Re} \left\{ b_{+, 0}^{\lambda_{a+1}, \lambda_a} - b_{0, -}^{\lambda_{a+1}, \lambda_a} \right\} = r_{\lambda_a}$ or $\text{Im} \left\{ b_{+, 0}^{\lambda_{a+1}, \lambda_a} + b_{0, -}^{\lambda_{a+1}, \lambda_a} \right\} = i_{\lambda_a}$. Vice versa, if we consider Class II symmetry cases, we can observe

$$\text{Im } S_{\lambda_{a+1}, \lambda_a} = \text{Im} \left\{ b_{+, 0}^{\lambda_{a+1}, \lambda_a} P_{+0} + b_{0, -}^{\lambda_{a+1}, \lambda_a} P_{+0}^* \right\} \quad (54)$$

which yields again i_{λ_a} and r_{λ_a} .

Since the unpolarized beam case is of Class I symmetry with $P_{\lambda, \lambda}^{\text{unpol.}}$ real [see Eq. (26)], r_{λ_a} can be measured without beam polarization. On the other hand, i_{λ_a} can be measured with one beam polarized. As $S^{e\pm} / S^{\text{unpol.}} = 0(m/\sqrt{s}) \sim 10^{-3}$ for transverse beam polarization [see Eqs. (26) and (28)], we need longitudinal polarization to obtain appreciable effects¹²⁾.

According to Eqs. (28) and (29), we then measure

$$\text{Im } S_{\lambda_{a+1}, \lambda_a}^{e\pm}(s_{\pm}^{\pm}, \theta) = \frac{\sqrt{s}}{4m} s_{\pm}^{\pm} d_{\lambda_0}^{\pm} (d_{\lambda_1}^{\pm} + d_{-\lambda_1}^{\pm}) i_{\lambda_a} \quad (55)$$

Nothing in addition can be learned from the polarization of both beams.

IV. DISCUSSION OF A FEW EXAMPLES

The analysis so far has been rather general in order to establish that beam polarization, in principle, is capable of providing additional information from one-arm type experiments within the OPA. To what extent this is true for a concrete final state is another issue that we shall be concerned with in the present Chapter.

The reader will have noticed that all our arguments so far hold irrespective of the quantum numbers of the object b produced along with a : although the symmetry, Eq. (43), is lost if the object b has no definite quantum numbers, the bilinears b obey the relations (47) and (48) even in the case of genuine inclusive reactions $e^+e^- \rightarrow aX$. So we might treat inclusive hadron production and two-body final states in parallel.

Let us assume that a is a resonance of spin $\frac{1}{2}$. There are four independent structure functions, $b_{++}^{\frac{1}{2}\frac{1}{2}}$, $b_{--}^{\frac{1}{2}\frac{1}{2}}$, $b_{00}^{\frac{1}{2}\frac{1}{2}}$, $b_{+0}^{\frac{1}{2}-\frac{1}{2}}$, the first three of which are real by hermiticity. If X has definite spin-parity, we have $|b_{+0}^{\frac{1}{2}-\frac{1}{2}}|^2 = |b_{++}^{\frac{1}{2}\frac{1}{2}}| |b_{00}^{\frac{1}{2}\frac{1}{2}}|$, and, therefore, at most four real functions instead of five to determine. One can only determine $\{b_{++}^{\frac{1}{2}\frac{1}{2}} + b_{--}^{\frac{1}{2}\frac{1}{2}}\}$ and $b_{00}^{\frac{1}{2}\frac{1}{2}}$ in case of parity conserving decay (which does not suffice to determine the density matrix). From a parity violating decay, such as $\Lambda \rightarrow N\bar{\nu}$, however, one can also determine $\text{Im } b_{+0}^{\frac{1}{2}-\frac{1}{2}}$ and $\text{Re } b_{+0}^{\frac{1}{2}-\frac{1}{2}}$ from Class I and Class II symmetry situations, respectively. If, in addition, X has spin $\frac{1}{2}$, as in $e^+e^- \rightarrow \Lambda\bar{\Lambda}$, one has $b_{--}^{\frac{1}{2}\frac{1}{2}} = 0$, and thus no beam polarization is necessary for a complete determination of the form factors. In conclusion, beam polarization is only of interest for $e^+e^- \rightarrow \Lambda + X$.

If a is a spin $\frac{3}{2}$ resonance, the number of structure functions increases to 14 reals, six of which can be measured without beam polarization. Longitudinal polarization of one beam allows additional measurement of $\text{Im } \{b_{+0}^{\frac{3}{2}\frac{1}{2}} + b_{0-}^{\frac{3}{2}\frac{1}{2}}\}$, which makes seven reals. Again, if X is of definite spin-parity, the number of structure functions reduces to at most 10 reals. In the case of $e^+e^- \rightarrow \Lambda\bar{\Lambda}$,

due to charge conjugation symmetry, one is left with four complex form factors ⁴⁾ $\langle \frac{3}{2}\frac{3}{2} | j_0 | 0 \rangle$, $\langle \frac{3}{2}, \frac{1}{2} | j_+ | 0 \rangle$, $\langle \frac{1}{2}-\frac{1}{2} | j_+ | 0 \rangle$ and $\langle \frac{1}{2}\frac{1}{2} | j_0 | 0 \rangle$, which makes seven real functions that can all be determined from Δ decay with beam polarization. For $e^+e^- \rightarrow \Delta \bar{N}$, of course, beam polarization is not needed to get the three complex form factors.

Turning now to boson resonances, we consider first vector meson production, $e^+e^- \rightarrow 1^\pm + X$. The number of real structure functions is ten: b_{++}^{++} , b_{--}^{++} , b_{00}^{++} , b_{++}^{00} , b_{00}^{00} , b_{+-}^{+-} , $b_{+0}^{+0} \pm b_{+0}^{+0*}$, $b_{0-}^{+0} \pm b_{0-}^{+0*}$, six (seven) of which can be determined without (with) beam polarization. If X has definite spin-parity, $b_{+-}^{+-} = n_a n_b b_{++}^{++}$ and $|b_{+0}^{+0}|^2 = |b_{++}^{++}| |b_{00}^{00}|$, $|b_{0-}^{+0}|^2 = |b_{00}^{00}| |b_{++}^{++}|$, yielding at most seven independent real structure functions. Without (with) beam polarization, five (six) out of these can be measured. If X has spin one, $b_{--}^{++} = 0$, and, therefore, beam polarization allows a complete measurement of $e^+e^- \rightarrow \underline{3}^+ \underline{3}^-$ from the decay distribution of one of the $\underline{3}$'s.

Similarly, if a has spin 2, there are 19 real structure functions, out of which 10 (12) are measurable without (with) beam polarization. The number of independent real structure functions is reduced to ≤ 13 if X has definite spin-parity. If X has spin 2, there are seven complex form factors (13 real functions). Evidently, if the spin of a becomes high enough, one has to measure decay correlations in order to obtain all form factors from "usual" parity conserving decays.

V. SUMMARY AND CONCLUSIONS

Motivated by the observed beam polarization in e^+e^- storage rings, we have looked into the question to what extent this polarization might be a useful experimental tool to investigate the hadronic bubble in e^+e^- annihilation within one photon exchange approximation. Since cross-section measurements (single particle distributions) do not allow more than a separation of longitudinal and transverse photon contributions, we envisaged an inclusive type situation, where a produced resonance a is detected through its two-particle decay. We found that the observed spin density matrix element $\underline{3}_{\lambda+1, \lambda}$ of this resonance, indeed carries additional

information due to beam polarization; one has to choose one beam longitudinally polarized to separate the interesting term. The case of two polarized beams is of no practical interest. For the processes $e^+e^- \rightarrow \Delta \bar{\Delta}$ and $e^+e^- \rightarrow \Xi^+ \Xi^-$, longitudinal polarization of one beam would allow extraction of all form factors from the observation of one resonance decay. For simpler final states such as Σ^0 , $\Delta \bar{N}$, beam polarization is not required and for more complicated ones such as ΞX , ΔX it is not quite sufficient to determine the full structure of the hadronic bubble.

On the whole, the advantage of using beam polarization in OPA processes seems only of marginal interest. In practice, beam polarization might be more useful to single out annihilation via two photon exchange ^{13),14)} at intermediate energies where this mechanism starts competing against the OPA contribution. The method would be to look for polarization dependence of the production cross-sections, with only one beam polarized. Since the OPA cross-section is independent of polarization, this procedure should reveal the two-photon exchange contributions.

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is linear in the pseudovector s_μ and gauge invariant is
 $\Gamma_{\mu\nu} \sim \epsilon_{\mu\nu\alpha\beta} s^\alpha K^{+\beta}$. In the expression for the cross-
section, this is to be contracted with any of the final state
momenta, $(\underline{p}_a \pm \underline{p}_b)$. Since $K^+ = \underline{p}_a + \underline{p}_b$ and $\Gamma_{\mu\nu}$ are anti-
symmetric, the contribution to the cross-section is zero.

- 12) For this reason, an asymmetry in cross-section or final state density matrix can only show up in case of two polarized beams and is proportional to $P_+ \cdot P_-$, i.e., hard to detect for $P_{\pm} \lesssim 0.4$. See also the discussion after the talk of Lefrançois in the Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies, Cornell 1971.
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FIGURE CAPTION

One photon exchange diagram explaining our notations for the quasi-two-body process $e^+e^- \rightarrow ab$. Particle b may be replaced by X (= anything).

FIG.1