Heavy Quarkonia and Quark Drip Lines in Quark-Gluon Plasma

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Abstract. Using the potential model and thermodynamical quantities obtained in lattice gauge calculations, we determine the spontaneous dissociation temperatures of color-singlet quarkonia and the 'quark drip lines' which separate the region of bound $\bar{Q}Q$ states from the unbound region. The dissociation temperatures of $J/\psi$ and $\chi_b$ in quenched QCD are found to be $1.62T_c$ and $1.18T_c$ respectively, in good agreement with spectral function analyses. The dissociation temperature of $J/\psi$ in full QCD with 2 flavors is found to be $1.42T_c$. For possible bound quarkonium states with light quarks, the characteristics of the quark drip lines severely limit the stable region close to the phase transition temperature. Bound color-singlet quarkonia with light quarks may exist very near the phase transition temperature if their effective quark mass is of the order of 300-400 MeV and higher.

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1. Introduction

The degree to which the constituents of a quark-gluon plasma can combine to form composite entities is an important property of the plasma. It has significant implications on the plasma equation of state, the probability of recombination of plasma constituents prior to the phase transition, and the chemical yields of the observed hadrons. Recent spectral analyses of quarkonium correlators indicated that $J/\psi$ may be bound up to $1.6T_c$ where $T_c$ is the phase transition temperature [1,2]. Subsequently, there has been renewed interest in quarkonium states in quark-gluon plasma as Zahed and Shuryak suggested that $Q\bar{Q}$ states with light quarks may be bound up to a few $T_c$ [3]. Quarkonium bound states and instanton molecules in the quark-gluon plasma have been considered by Brown, Lee, Rho, and Shuryak [4]. As heavy quarkonia may be used as diagnostic tools [5], there have been many recent investigations on the stability of heavy quarkonia in the plasma [6,7,8,9,10,11,12,13,14,15].

Previously, DeTar [17], Hansson, Lee, and Zahed [18], and Simonov [19] observed that the range of the strong interaction is not likely to change drastically across the phase transition and suggested the possible existence of relatively narrow low-lying $Q\bar{Q}$ states in the plasma. On the other hand, Hatsuda and Kunihiro [20] considered the persistence of soft modes in the plasma which may manifest themselves as pion-like
and sigma-like states. The use of the baryon-strangeness correlation and the charge fluctuation to study the abundance of light quarkonium states in the plasma have been suggested recently [21, 22].

We would like to use the potential model to investigate the composite properties of the plasma and to determine its ‘quark drip lines’. Here we follow Werner and Wheeler [23] and use the term ‘drip line’ to separate the region of bound color-singlet $Q\bar{Q}$ states from the unbound region of spontaneous quarkonium dissociation. It should be emphasized that a quarkonium can be dissociated by collision with constituent particles to lead to the corresponding ‘thermal-dissociation line’ and ‘particle-dissociation line’, which can be interesting subjects for future investigations.

2. The color-singlet $Q\bar{Q}$ potential

The most important physical quantity in the potential model is the $Q\bar{Q}$ potential between the quark $Q$ and the antiquark $\bar{Q}$ in a color-singlet state. Previous works in the potential model use the color-singlet free energy $F_1$ [8, 9, 16] or the color-singlet internal energy $U_1$ [10, 3, 15] obtained in lattice gauge calculations as the color-singlet $Q\bar{Q}$ potential without rigorous theoretical justifications. The internal energy $U_1$ is significantly deeper and spatially more extended than the free energy $F_1$. Treating the internal energy $U_1$ as the $Q\bar{Q}$ potential led Shuryak and Zahed to suggest the possibility of bound color-singlet quarkonium states with light quarks in the plasma [3]. The conclusions will be quite different if one uses the free energy $F_1$ as the $Q\bar{Q}$ potential.

While $F_1$ or $U_1$ can both be used as the $Q\bar{Q}$ potential at $T = 0$ (at which $F_1 = U_1$), the situation is not so clear in a thermalized quark-gluon plasma. In Ref. [6], we show that the equation of motion for the $Q\bar{Q}$ state can be obtained in two steps. First, one considers a lattice gauge calculation for a static pair of color-singlet $Q$ and $\bar{Q}$ at a fixed separation $R$ at the temperature $T$ and obtains the free energy $F_1(R)$, the internal energy $U_1(R)$, and their difference $TS_1(r) = U_1 - F_1$. In the second step, one considers a $Q$ and a $\bar{Q}$ in dynamical motion in the quark-gluon plasma. The dynamical degrees of freedom can be taken to be the set of quarkonium and plasma constituent wave functions and their corresponding state occupation numbers. The equilibrium of such a thermalized system at a fixed temperature and volume occurs when the grand potential is a minimum. One explicitly writes out the grand potential, which is the sum of the free energy and appropriate Lagrange multiplier terms, in terms of the quarkonium and plasma constituent wave functions, the state occupation numbers, and the mutual $Q\bar{Q}$, $Q$-(constituent), $\bar{Q}$-(constituent), and (constituent)-(constituent) interactions. The minimization of the grand potential with respect to a color-singlet quarkonium wave function leads to the Schrödinger equation for the quarkonium state with a color-singlet potential $U^{(1)}_{Q\bar{Q}}$ containing only $Q\bar{Q}$, $Q$-(constituent), and $\bar{Q}$-(constituent) interactions. On the other hand, the internal energy $U_1$ in the lattice gauge calculation contains not only contributions from these $Q\bar{Q}$, $Q$-(constituent), and
Dissociation temperatures for $\chi$ from 1.62 $T_{J/\psi}$ of the spectral functions, the temperature at which the bound at 0.78 $T_{J/\psi}$ becomes already unbound at 1.70 $T_{J/\psi}$, then it would be reasonable to infer that $J/\psi$ is stable and bound at 0.78 $T_{c}$, then it would be reasonable to infer that $J/\psi$ is also bound and stable at 2.33 $T_{c}$ [1]. These results are further supported by the presence of a similar relationship between the total internal energy and the $Q\bar{Q}$ potential in the analogous case of Debye screening, for which analytical expressions for various thermodynamical quantities can be readily obtained and compared [23].

3. The color-singlet $Q\bar{Q}$ potential in quark-gluon plasma

In order to subtract out the $R$-dependent internal energy of the quark-gluon plasma from the total internal energy to obtain the color-singlet $Q\bar{Q}$ potential in the plasma, additional lattice gauge calculations may be needed to evaluate the quark-gluon plasma internal energy in the presence of a color-singlet $Q$ and $\bar{Q}$ pair. It is nonetheless useful at this stage to suggest approximate ways to evaluate this quark-gluon plasma internal energy. The subtraction can be carried out by noting that the quark-gluon plasma internal energy density $\epsilon$ is related to its pressure $p$ and entropy density $\sigma$ by the First Law of Thermodynamics, $\epsilon = T\sigma - p$, and the quark-gluon plasma pressure $p$ is also related to the plasma energy density $\epsilon$ by the equation of state $p(\epsilon)$ that is presumed known by another independent lattice gauge calculation. Thus, by expressing $p$ as $(3\epsilon/\epsilon)(\epsilon/3)$ with the ratio $a(T) = 3p/\epsilon$ given by the known equation of state, the plasma internal energy density $\epsilon$ is related to the entropy density $T\sigma$ by $\epsilon = [3/(3 + a(T))]T\sigma$, and the plasma internal energy integrated over the volume is given by $[3/(3 + a(T))]|TS|_1$ where $TS_1$ has already been obtained as $U_1 - F_1$. The proper $Q\bar{Q}$ potential, $U_{Q\bar{Q}}^{(1)}$, as determined from $U_1$ by subtracting the plasma internal energy (which is equal to $[3/(3 + a(T))](U_1 - F_1)$), is then a linear combination of $F_1$ and $U_1$ given by [3],

$$U_{Q\bar{Q}}^{(1)}(R, T) = \frac{3}{3 + a(T)}F_1(R, T) + \frac{a(T)}{3 + a(T)}U_1(R, T).$$

The potential $U_{Q\bar{Q}}^{(1)}$ is approximately $F_1$ near $T_c$ and $3F_1/4 + U_1/4$ for $T > 1.5T_c$ [3].

In the spectral function analyses, the widths of many color-singlet heavy quarkonium broaden suddenly at various temperatures [1, 2, 13]. In the most precise calculations for $J/\psi$ in quenched QCD using up to 128 time-like lattice slices, the spectrum has a sharp peak for $0.78T_{c} \leq T \leq 1.62T_{c}$ and a broad structure with no sharp peak for $1.70T_{c} \leq T \leq 2.33T_{c}$ [1]. The spectral peak at the bound state has the same structure and shape at $0.78T_{c}$ as it is at $1.62T_{c}$. If one can infer that $J/\psi$ is stable and bound at $0.78T_{c}$, then it would be reasonable to infer that $J/\psi$ is also bound and stable at $1.62T_{c}$. The spectral function at $1.70T_{c}$ has the same structure and shape as the spectral function at $2.33T_{c}$. If one can infer that $J/\psi$ is unbound at $2.33T_{c}$, then it would be reasonable to infer that $J/\psi$ become already unbound at $1.70T_{c}$. Thus, from the shape of the spectral functions, the temperature at which the $J/\psi$ width broadens suddenly from $1.62T_{c}$ to $1.70T_{c}$ corresponds to the $J/\psi$ spontaneous dissociation temperature. Dissociation temperatures for $\chi_{c}$ and $\chi_{b}$ in spectral analyses in quenched QCD have...
also been obtained \[2\] \[13\]. We list the heavy quarkonium spontaneous dissociation


temperatures obtained from spectral analyses in quenched QCD in Table I. They can

be used to test the potential model of \(U^{(1)}_{QQ}\), the linear combination of \(U_1\) and \(F_1\) proposed

in Eq. (4), as well as \(F_1\) and \(U_1\).

Table I. Spontaneous dissociation temperatures obtained from different potentials.

<table>
<thead>
<tr>
<th>States</th>
<th>Spectral Analysis</th>
<th>Quenched QCD</th>
<th>Full QCD (2 flavors)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(J/\psi, \eta_c)</td>
<td>1.62-1.70(T_c)</td>
<td>1.62(T_c)</td>
<td>1.40(T_c)</td>
</tr>
<tr>
<td>(\chi_c)</td>
<td>below 1.1(T_c)</td>
<td>unbound</td>
<td>unbound</td>
</tr>
<tr>
<td>(\psi', \eta_c)</td>
<td>(\psi', \eta_c)</td>
<td>unbound</td>
<td>unbound</td>
</tr>
<tr>
<td>(\Upsilon, \chi_b)</td>
<td>4.1(T_c)</td>
<td>3.5(T_c)</td>
<td>(\sim 5.0)</td>
</tr>
<tr>
<td>(\chi_c)</td>
<td>1.15-1.54(T_c)</td>
<td>1.18(T_c)</td>
<td>1.10(T_c)</td>
</tr>
<tr>
<td>(\Upsilon', \chi_b)</td>
<td>1.38(T_c)</td>
<td>1.19(T_c)</td>
<td>2.28(T_c)</td>
</tr>
</tbody>
</table>

\[1\text{Ref.}\ [1], \ 2\text{Ref.}\ [2], \ 3\text{Ref.}\ [13]\]

4. Quark drip-lines in quark-gluon plasma

To evaluate the \(Q-\bar{Q}\) potential, we use the free energy \(F_1\) and the internal energy \(U_1\)

obtained by Kaczmarek et al. in both quenched QCD \[11\] and full QCD with 2 flavors \[12\]. In quenched QCD, \(F_1\) and \(U_1\) can be parametrized in terms of a screened Coulomb

potential with parameters shown in Figs. 1 and 2 of Ref. \[3\]. In full QCD with 2 flavors, \(F_1\) and \(U_1\) can be represented by a color-Coulomb interaction at short distances and a

completely screened, constant, potential at large distances as given in Ref. \[7\], although

other alternative representations have also been presented \[15\] \[14\]. To determine the

\(U^{(1)}_{QQ}\) potential as given by Eq. (4), we also need the ratio \(a(T) = 3p/\epsilon\) from the plasma
equation of state. We use the quenched equation of state of Boyd et al. \[25\] for quenched

QCD, and the equation of state of Karsch et al. \[26\] for full QCD with 2 flavors. Using

quark masses \(m_c = 1.4\) GeV and \(m_b = 4.3\) GeV, we can calculate the binding energies

color-singlet heavy quarkonia and their spontaneous dissociation temperatures. We

list in Table I the heavy quarkonium spontaneous dissociation temperatures calculated

with the \(U^{(1)}_{QQ}\) potential, the \(F_1\) potential, and the \(U_1\) potential, in both quenched QCD

and full QCD.

The spontaneous dissociation temperatures of \(J/\psi\) and \(\chi_b\) obtained with the \(U^{(1)}_{QQ}\)

potential in quenched QCD are found to be 1.62\(T_c\) and 1.18\(T_c\) respectively. On the other

hand, spectral analyses in quenched QCD give the spontaneous dissociation temperature

of 1.62-1.70\(T_c\) for \(J/\psi\) \[1\] and 1.15-1.54\(T_c\) for \(\chi_b\) \[13\]. Thus, the \(U^{(1)}_{QQ}\) potential of Eq. (1)
gives spontaneous dissociation temperatures that agree with those from spectral function

analyses. This indicates that the \(U^{(1)}_{QQ}\) potential, defined as the linear combination of

\(U_1\) and \(F_1\) in Eq. (1), may be an appropriate \(Q-\bar{Q}\) potential for studying the stability of
heavy quarkonia in quark-gluon plasma.

Figure 1. The drip lines in quenched QCD calculated with the $U^{(1)}_{Q\bar{Q}}$, $F_1$, and $U_1$ potentials. The solid-circle symbols represent results from lattice gauge spectral function analyses.

5. The stability of a $Q-\bar{Q}$ pair

To examine the stability of a $Q-\bar{Q}$ pair, we consider the quark mass $m_Q$ as a variable and evaluate the spontaneous dissociation temperature as a function of the reduced mass $\mu_{\text{red}} = m_Q m_{\bar{Q}} / (m_Q + m_{\bar{Q}})$. The results for quenched QCD are shown in Fig. 1. A state is bound in the $(T/T_c, \mu_{\text{red}})$ space above a drip line and is unbound below the drip line.

The drip lines for quenched QCD in Fig. 1 are useful to provide a comparison of the potential model results with those from spectral analyses in the same quenched approximation. However, as dynamical quarks may provide additional screening, it is necessary to include dynamical quarks to assess their effects on the stability of quarkonia. Accordingly, we use the $U^{(1)}_{Q\bar{Q}}$ potential defined by Eq. (1) with $F_1$, $U_1$, and $a(T)$ evaluated in full QCD with 2 flavors [12, 26] to determine the drip lines shown as solid curves in Fig. 2. In comparison with quenched QCD results, the 1s drip line in full QCD is shifted to lower temperatures for $T > 1.2 T_c$ while the 1p drip line in full QCD is only slightly modified.
For heavy quarkonia, the results in Table I and Fig. 2 indicate that for the $U_{Q\bar{Q}}^{(1)}$ potential $J/\psi$, $\Upsilon$, and $\chi_b$ may be bound in the plasma up to $1.42T_c$, $3.40T_c$, and $1.22T_c$ respectively. As light quarks have current-quark masses of a few MeV, we may be advised that non-relativistic potential models should not be used to describe light quarkonia. However, due to its strong interaction with other constituents, a light quark becomes a dressed quasiparticle and acquires a large quasiparticle mass. In the low temperature region when spontaneous chiral symmetry breaking occurs with $\langle \bar{\psi}\psi \rangle \neq 0$, the quasiparticle mass is $m_q \sim |g\langle \bar{\psi}\psi \rangle|+(\text{current quark mass})$, where $g$ is the strong coupling constant and $\langle \bar{\psi}\psi \rangle$ the quark condensate [20, 27, 28]. This quasiparticle mass is the origin of the constituent-quark mass in non-relativistic constituent quark models [20, 28]. In the high temperature perturbative QCD region, the quasiparticle mass is $m_q \sim gT/\sqrt{6}$, which is of the order of a few hundred MeV [29].

As the restoration of chiral symmetry is a second order transition, $\langle \bar{\psi}\psi \rangle$ decreases gradually as the temperature increases beyond $T_c$. The light quark quasiparticle mass associated with $\langle \bar{\psi}\psi \rangle$ will likewise decrease gradually from the constituent-quark mass to the current-quark mass when the temperature increases beyond $T_c$. This tendency for the quasiparticle mass to decrease will be counterbalanced by the opposite tendency for the quasiparticle ‘thermal mass’ to increase with increasing temperature. In the region of our interest, $T_c < T < 2T_c$, various estimates give the light quark quasiparticle
masses from 0.3 GeV to 1.2 GeV [30, 31, 32, 33]. As in the case of $T = 0$, where light quarks with a constituent-quark mass of about 350 MeV mimic the effects of chiral symmetry breaking and non-relativistic constituent quark models have been successfully used for light hadron spectroscopy [28, 34], so the large estimated quasiparticle mass (from 0.3 to 1.2 GeV) may allow the use of a non-relativistic potential model as an effective tool to estimate the stability of light quarkonia at $T_c < T < 2T_c$. At this stage when the uncertainties in the quasiparticle mass are much greater than the effects due to relativistic kinematics, an estimate based on a non-relativistic model suffices. It will be of interest to investigate the relativistic effects in the future, when the quasiparticle masses are more definitively determined.

By examining the effects of the light quark quasiparticle masses on the quark-gluon plasma equation of state, Levai et al. [30], Szabo et al. [31], and Ivanov et al. [32] estimate that $m_q$ is about 0.3 to 0.4 GeV at $T_c < T < 2T_c$. From the results in Fig. 2 for the $U^{(1)}_{Q\bar{Q}}$ potential, we can estimate that as a quarkonium with light quarks has a reduced mass of 0.15-0.2 GeV, it may be bound at temperatures below $(1.05 - 1.07)T_c$. An open heavy quarkonium with a light quark and a heavy antiquark has a reduced mass of about 0.3-0.4 GeV and may be bound at temperatures below $(1.11 - 1.19)T_c$.

Another lattice gauge calculation gives $m_q/T = 3.9 \pm 0.2$ at $1.5T_c$ [33], which implies that at $T=1.5T_c$ (or about 0.3 GeV), the quark mass will be $\sim 1.2$ GeV for $(u, d, s)$ quarks. Such a ‘light’ quark quasiparticle mass appears to be quite large and may be uncertain, as the plasma will have a relatively large abundance of charm quarks and antiquarks, and there may be difficulties in reproducing the plasma equation of state. With this mass, a ‘light’ quarkonium will have a reduced mass of 0.6 GeV, and the quarkonium may be bound at temperatures below $\sim 1.31T_c$.

In either case, the drip lines of Fig. 2 calculated with the $U^{(1)}_{Q\bar{Q}}$ potential for full QCD with 2 flavors do not support bound $Q\bar{Q}$ states with light quarks beyond $1.5T_c$. A recent study of baryon-strangeness correlations suggests that the quark-gluon plasma contains essentially no bound $Q\bar{Q}$ component at $1.5T_c$ [21].

In conclusion, we have used the thermodynamical quantities obtained in lattice gauge calculations to determine the quark drip lines in quark-gluon plasma for full QCD with two flavors. The characteristics of the quark drip lines severely limit the region of possible quarkonium states with light quarks to temperatures close to the phase transition temperature. If the light quark quasiparticle mass is 0.3-0.4 GeV as estimated from [30, 31, 32], a quarkonium with light quarks may be bound below $(1.05-1.07)T_c$ and an open heavy quarkonium may be bound below $(1.11-1.19)T_c$.

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References


