Neutrino physics at large colliders

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Abstract

Large colliders are not sensitive to light neutrino masses and character, but they can produce new heavy neutrinos, allowing also for the determination of their Dirac or Majorana nature. We review the discovery limits at the next generation of large colliders.

1 Introduction

Future colliders will probe Nature up to TeV scales with high precision, discovering new heavy particles with sizeable couplings or setting stringent limits on their existence. Thus, we expect that the Higgs will be copiously produced at the LHC, as may also be the lightest superpartners [1, 2]. At any rate, these facilities will be a window to any new physics near the electroweak scale which couples to the Standard Model (SM). This could be the case, for example, of new heavy neutrinos. In the following we review what can be learned about them at the next generation of large colliders.

SM extensions with heavy neutrinos at the TeV scale or below usually include new interactions and extra matter. If the latter are also in this mass range, they give new signals and further contributions to heavy neutrino production processes. We neglect them here, assuming that they do not affect the significance of the specific processes discussed. (In this sense the resulting limits are conservative, because we do not consider other possible effects eventually larger.) Hence, all the additional parameters involved in heavy neutrino production at colliders are the heavy neutrino masses and their mixings with SM fermions. Let $N$ be the lightest heavy neutrino, which can be of Dirac (D) or Majorana (M) nature. In the former case two extra bispinors $N_{L,R}$ are added and lepton number is conserved, whereas in the latter the SM is enlarged with only one bispinor satisfying $(N_L)^c \equiv C \bar{N}_L^T = N_R$, and thus lepton number is violated [3, 4]. It should be stressed that:
The effects of light neutrino masses at large colliders are suppressed by factors $\sim m_\nu/\sqrt{s}$, and they can be completely neglected. Hence, we can assume without loss of generality that the SM neutrinos are strictly massless, and lepton number is preserved in the light sector. In other words, it does not matter in this context whether the light neutrinos are of Dirac or Majorana nature [5].

The heavy Dirac neutrino case is a particular limit of two heavy Majorana neutrinos, when both are degenerate and their mixings with the SM leptons are lepton number conserving (LNC). On the other hand, one extra heavy Majorana neutrino corresponds to the limit in which the second Majorana neutrino is much heavier.

In both situations, the relevant additional terms of the Lagrangian are the heavy neutrino mass term

$$\mathcal{L}_{\text{mass}} = -m_N \bar{N}_L N_R + \text{H.c.}, \quad (D)$$
$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} m_N \bar{N}_L N_R + \text{H.c.}, \quad (M)$$

with $N_{L,R} = P_{L,R} N$), and its interactions with the light fermions [8]. The charged current vertex with a charged lepton $\ell$ reads

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} \left( \ell \gamma^\mu V_{\ell N} P_L N W_\mu + \bar{N} \gamma^\mu V_{\ell N}^* P_L \ell W^\dagger_\mu \right). \quad (2)$$

The neutral current gauge couplings with a light neutrino $\nu_\ell$ are

$$\mathcal{L}_Z = -\frac{g}{2 c_W} \left( \bar{\nu}_\ell \gamma^\mu V_{\ell N} P_L N + \bar{N} \gamma^\mu V_{\ell N}^* P_L \nu_\ell \right) Z_\mu, \quad (3)$$

and the scalar interactions

$$\mathcal{L}_H = -\frac{g m_N}{2 M_W} \left( \bar{\nu}_\ell V_{\ell N} P_R N + \bar{N} V_{\ell N}^* P_L \nu_\ell \right) H. \quad (4)$$

Thus, all the necessary parameters for the evaluation of the $N$ production cross sections are its mass $m_N$ and its couplings to the different lepton flavours $V_{\ell N}$, $\ell = e, \mu, \tau$. The main difference between a heavy Dirac or Majorana neutrino is the non-zero lepton number violating (LNV) propagator for two $N$ fields $\langle N N \rangle$ in the Majorana case, besides the common LNC one $\langle N \bar{N} \rangle$ [9]. Dirac and Majorana heavy neutrinos have

\footnote{In definite models with extra interactions and matter content light neutrino masses and mixing parameters may be related to the new sectors and, through them, to collider observables (see for examples Refs. [6,7]). Then, the latter can give indirect information about the former, which are otherwise unmeasurable at high energies.}
the same couplings but for the Majorana case the second term in Eqs. (3) and (4) can be rewritten using the condition $N = N^c$, giving \(^2\)

\[
\mathcal{L}_Z = -\frac{g}{2c_W} \bar{\nu}_{\ell} \gamma^\mu (V_{\ell N} P_L - V^*_{\ell N} P_R) N Z^\mu, \quad (M)
\]

\[
\mathcal{L}_H = -\frac{g m_N}{2M_W} \bar{\nu}_{\ell} (V_{\ell N} P_R + V^*_{\ell N} P_L) N H. \quad (M)
\]

For equal values of the couplings, production cross sections are equal in general for a Dirac and Majorana heavy neutrino, as we will explicitly see in section 5 (in some processes, this requires summing $N$ and $\bar{N}$ production for Dirac neutrinos). For the analysis of the different heavy neutrino signals it is necessary to know its decay modes as well. $N$ can decay in the channels $N \to W^+ \ell^-$ (if $N$ is a Majorana fermion $N \to W^- \ell^+$ is also allowed), $N \to Z\nu_\ell$ and $N \to H\nu_\ell$. The partial widths for these decays are \([8, 10, 11]\)

\[
\Gamma(N \to W^+ \ell^-) = \Gamma(N \to W^- \ell^+) = \frac{g^2}{64\pi} |V_{\ell N}|^2 \frac{m_N^3}{M_W^2} \left(1 - \frac{M_W^2}{m_N^2} \right)^2 \left(1 + \frac{M_W^2}{m_N^2} - 2 \frac{M_W^4}{m_N^4} \right),
\]

\[
\Gamma_D(N \to Z\nu_\ell) = \frac{g^2}{128\pi c_W^2} |V_{\ell N}|^2 \frac{m_N^3}{M_Z^2} \left(1 - \frac{M_Z^2}{m_N^2} \right)^2 \left(1 + \frac{M_Z^2}{m_N^2} - 2 \frac{M_Z^4}{m_N^4} \right),
\]

\[
\Gamma_M(N \to Z\nu_\ell) = 2\Gamma_D(N \to Z\nu_\ell),
\]

\[
\Gamma_D(N \to H\nu_\ell) = \frac{g^2}{128\pi} |V_{\ell N}|^2 \frac{m_N^3}{M_H^2} \left(1 - \frac{M_H^2}{m_N^2} \right)^2,
\]

\[
\Gamma_M(N \to H\nu_\ell) = 2\Gamma_D(N \to H\nu_\ell),
\]

with $\Gamma_D$ and $\Gamma_M$ standing for the widths of a Dirac and Majorana neutrino. The factors of two in the partial widths of $N \to Z\nu_\ell$, $N \to H\nu_\ell$ for a Majorana neutrino are the consequence of the extra $V^*_{\ell N}$ couplings in Eqs. (5) resulting from the Majorana condition, and which are not present for a Dirac neutrino. From Eqs. (6) it follows that for equal values of the mixing angles $V_{\ell N}$ the width of a heavy Majorana neutrino is twice as large as for a Dirac neutrino. Another straightforward consequence of the previous expressions is that the partial widths for $W$, $Z$ and Higgs decays are in the ratios $2 : 1 : 1$ (assuming $m_N \gg M_W, M_Z, M_H$).

The reader mainly interested in the reach of the different experiments can go directly to section 5, where we review the results for heavy neutrino production at future colliders. In next section we discuss the motivation for having heavy neutrinos at the

\(^2\)This also assumes that $\nu = \nu^c$, otherwise both fields $\nu$ and $\nu^c$ would appear. As it has been emphasised, results are independent of this assumption.
electroweak scale, and in section 3 we provide more details on the neutrino mass matrices we are interested in, for easier comparison with other analyses. Present indirect constraints on heavy neutrino mixing are summarised in section 4. Section 6 is devoted to our conclusions.

2 Motivation

New fermions are present in many extensions of the SM. If their masses are well above the electroweak scale they must be vector-like, that is, with their left- and right-handed parts transforming in the same representation of the SM gauge group. This guarantees that their mass terms preserve the electroweak gauge symmetry, but by the same token the natural size of their masses is of the order of the largest mass scale allowed by the symmetries of the model [12]. In grand unified theories this is usually the grand unified scale, near the Planck mass. This is for example the case of Majorana right-handed neutrinos [13–16]. However, in the presence of extra symmetries these vector-like masses can be much smaller [17–19], and exotic fermions can also manifest at lower energies (for reviews see Refs. [20, 21]).

This scenario has become more interesting lately because the possible existence of low extra dimensions near the TeV scale [22, 23], with SM fermions in the bulk, implies infinite towers of vector-like fermions, known as Kaluza-Klein modes [24–26], whose lightest states could be also eventually observable [27, 28]. Fermions in five dimensions have both chiralities and, although the zero modes can be chiral in appropriate backgrounds, the Kaluza-Klein modes are in general Dirac particles. If neutral, they can have Majorana mass terms as well [29, 30]. Moreover, as an alternative to supersymmetry at the TeV scale, SM extensions with a larger global symmetry have been proposed to cancel the undesired large quantum corrections to the Higgs mass [31–33]. In these models the SM Higgs is a pseudo-Goldstone boson, what justifies their name of Little Higgs models. They are characterised by a new scale near the TeV and include extra matter content to realise the larger symmetry. In this context the existence of new fermions near the electroweak scale is not a possibility but a requirement. In general they mix with the SM fermions and can be eventually produced at large colliders [34–36]. In particular, if the model includes extra neutrinos transforming as singlets under the SM group, it seems natural that they have TeV masses and relatively large mixings with the SM fermions. For example, in the simplest Little Higgs models [37],

\footnote{In particular, fermions transforming trivially under the SM group can have Majorana nature. In this case particles and antiparticles coincide, and their (Majorana) masses violate fermion number.}
the matter content belongs to SU(3) multiplets, and the SM lepton doublets must be enlarged with one extra neutrino $N'_{\ell L}$ per family. When a combination of Higgs triplets acquires a large vacuum expectation value (VEV) $f$ of few TeV, reducing the SU(3) symmetry down to SU(2)$_L$, the extra neutrinos can get a large Dirac mass of the order of the new scale, provided the model also includes the necessary right-handed neutrinos transforming as SU(3) singlets [38]. This mechanism provides a natural way of giving masses to the SM neutrinos while explaining the absence of exotic fermions below the electroweak scale. In this framework the mixing between the light leptons and the heavy neutrinos is of order $v/\sqrt{2f}$, with $v = 246$ GeV the electroweak VEV. If this is the case, it is precisely the non-observation of these fermions at future colliders what will set very stringent limits on the model.

Besides their appearance in specific models, heavy Majorana neutrinos are introduced in seesaw models [13–16]. They give contributions to light neutrino masses $m_\nu$ of the order $Y^2v^2/2m_N$, where $Y$ is a Yukawa coupling. In the minimal seesaw realisations this is the only source for light neutrino masses, and the Yukawas are assumed of order unity without any particular symmetry. Therefore, having $m_\nu \sim Y^2v^2/2m_N$ requires heavy masses $m_N \sim 10^{13}$ GeV to reproduce the observed light neutrino spectrum. Additionally, the light-heavy mixing is predicted to be $V_{\ell N} \sim \sqrt{m_\nu/m_N}$. These ultra-heavy particles are unobservable, and thus the seesaw mechanism is not directly testable. Nevertheless, non-minimal seesaw models can be built, with $m_N \sim 1$ TeV or smaller, if some approximate flavour symmetry suppresses the $\sim Y^2v^2/2m_N$ contribution from seesaw [39–41]. Moreover, the light-heavy mixing can be decoupled from the mass ratio $\sqrt{m_\nu/m_N}$ [42]. In this situation, the heavy states could be observable in future collider experiments.

In addition to providing a mechanism for neutrino mass generation, heavy neutrino decays can give a succesful leptogenesis. A beautiful feature of the minimal seesaw is that the same heavy neutrino scale $m_N \sim 10^{13}$ GeV which reproduces light neutrino masses predicts a lepton asymmetry large enough to account for the observed baryon asymmetry through $B + L$ violating sphaleron interactions [43, 44]. The heavy neutrino scale can be lowered to the TeV, if two neutrinos are nearly degenerate and the CP asymmetry is resonantly enhanced [45–47], or for other SM extensions [48–50]. It seems to be a general property that the heavy neutrino needed to generate the CP asymmetry must couple very weakly to the SM fields, and then it cannot be produced at existing or planned colliders through the usual mechanisms. But this does not preclude the existence of other heavy neutrinos with larger couplings which, although not participating actively in leptogenesis, may be observable at colliders. In the example of Refs. [51, 52], with 3 heavy neutrinos with masses $m_{N_i} \sim 250$ GeV, the observed
light neutrino spectrum is reproduced and a baryon excess is generated through a τ lepton asymmetry. The heavy neutrino $N_1$ actively involved in τ leptogenesis couples very weakly to the SM leptons, $V_{eN_1} \sim 10^{-6}$, and cannot be produced at existing or planned colliders. The other two heavy neutrinos $N_{2,3}$ form a quasi-Dirac neutrino $N$ mainly coupling to the first two families, $V_{eN}, V_{\mu N} \sim 10^{-2}$, and with a small mixing with the third one, $V_{\tau N} \sim 10^{-6}$, to avoid washing-out the τ lepton asymmetry. This heavy state could be observable at $e^+e^-$ colliders.

3 Beyond the Neutrino Standard Model

In this section we consider a SM extension with heavy neutrino singlets, deriving their interactions with the light leptons [53]. The general situation is that $n$ additional singlets $N'_{iL}$ ($i = 1, ..., n$), which can be taken to be left-handed, are introduced. We will assume $n = 3$ for definiteness and easier comparison with the heavy neutrino production analysis we are interested in, but the formalism is general for any number of singlets. The neutrino weak isospin $T_3 = 1/2$ eigenstates $\nu'_{iL}$ are the same as in the SM, and the extended mass term reads

$$L_{\text{mass}} = -\frac{1}{2} (\nu'_{iL} N'_{iL}^c) \begin{pmatrix} M_L & v \sqrt{2} Y \\ v^T \sqrt{2} M_R \end{pmatrix} \begin{pmatrix} \nu'_{jR} \\ N'_{jR} \end{pmatrix} + \text{H.c.},$$

where $\nu'_{iR} \equiv (\nu'_{iL})^c$, $N'_{jR} \equiv (N'_{jL})^c$, and the blocks $M_L$, $Y$ and $M_R$ stand for $3 \times 3$ matrices. $M_L$ is a lepton number violating mass matrix for the light neutrinos, $Y$ stands for the Yukawa interactions and $M_R$, which can be assumed diagonal, real and positive without loss of generality, corresponds to bare masses which also violate lepton number. The mass eigenstates are obtained diagonalising the complete $6 \times 6$ mass matrix $\mathcal{M}$ with the unitary transformation $U_L$,

$$\begin{pmatrix} \nu_L \\ N_L \end{pmatrix} = U_L^\dagger \begin{pmatrix} \nu'_{iL} \\ N'_{iL} \end{pmatrix}, \quad \begin{pmatrix} \nu_R \\ N_R \end{pmatrix} = U_L^T \begin{pmatrix} \nu'_{iR} \\ N'_{iR} \end{pmatrix},$$

In the case of light Dirac neutrinos we must introduce 3 additional SU(2)$_L$ singlet fields to allow for Dirac neutrino masses. However, as it has already been emphasised, light neutrino masses are not relevant at large colliders, so these fermions can be considered massless and their right-handed parts ignored. Any other more general scenario can be brought into this form in this limit. In particular, this is consistent with whatever specific model of neutrino masses one may advocate. Here we do not make any attempt to accommodate them, neither claim that the heavy neutrino masses and mixings we consider are natural in any given model of neutrino mass generation one can think of. But heavy neutrinos can be introduced, as long as they fulfill the experimental constraints discussed in the next section and can reproduce the pattern of neutrino masses and mixings implied by neutrino oscillations.
with $U_L^H \mathcal{M} U_L^* = \mathcal{M}_{\text{diag}}$. The weak interaction Lagrangian in the weak eigenstate basis is

$$
\mathcal{L}_W = -\frac{g}{\sqrt{2}} \bar{l}_L \gamma^\mu \nu_L^i W_\mu + \text{H.c.}, \\
\mathcal{L}_Z = -\frac{g}{2 c_W} \bar{\nu}_L^i \gamma^\mu \nu_L^i Z_\mu, \\
\mathcal{L}_H = -\frac{1}{\sqrt{2}} \bar{\nu}_L^i Y \nu_R^i H + \text{H.c.},
$$

(9)

with $l_L^i$ the charged lepton weak eigenstates. Using Eqs. (8), these terms read in the mass eigenstate basis

$$
\mathcal{L}_W = -\frac{g}{\sqrt{2}} \bar{l}_L \gamma^\mu U_L^H U_L^U L \left( \nu_L \right) W_\mu + \text{H.c.}, \\
\mathcal{L}_Z = -\frac{g}{2 c_W} (\bar{\nu}_L \bar{N}_L) \gamma^\mu U_L^H U_L^U L \left( \nu_L \right) Z_\mu, \\
\mathcal{L}_H = -\frac{1}{\sqrt{2}} (\bar{\nu}_L \bar{N}_L) U_L^H Y U_L^U L \left( \nu_R \right) H + \text{H.c.},
$$

(10)

where the $3 \times 6$ matrices $U_L$ and $U_L^U$ are the upper and lower blocks of $U_L$, 

$$
U_L = \begin{pmatrix} U_L \\ U_L^U \end{pmatrix},
$$

(11)

and $U_L$ is a $3 \times 3$ unitary matrix resulting from the diagonalisation of the charged lepton mass matrix. In the weak basis where the latter is diagonal $U_L$ is the identity, so we can assume without loss of generality that the extended $3 \times 6$ Maki-Nakagawa-Sakata (MNS) matrix [54,55]

$$
V = U_L^H U_L
$$

(12)

equals $U_L$. Hence, from Eqs. (10) we see that this matrix completely fixes the couplings of light leptons to heavy neutrinos, including their scalar interactions, although in this case the couplings are also proportional to the heavy neutrino masses $m_{N_i}$. The charged current matrix can be further decomposed into $3 \times 3$ blocks for convenience,

$$
V = \begin{pmatrix} V^{(\nu)} & V^{(N)} \end{pmatrix},
$$

(13)

where $V^{(\nu)}$ and $V^{(N)}$ mix the charged leptons with light and heavy neutrinos, respectively. Its approximate form can be obtained observing that the $3 \times 3$ blocks of the mass matrix $\mathcal{M}$ exhibit a strong hierarchy $M_L \ll v Y \ll M_R$, where here $M_L$, $Y$ and $M_R$ stand for generic values of their entries. Then, to first order in small ratios,

$$
U_L = \begin{pmatrix} U_L \\ U_L^U \end{pmatrix} \approx \begin{pmatrix} \mathbb{I} & V^{(N)} \\ -V^{(N)*} & \mathbb{I} \end{pmatrix},
$$

(14)
where $V^{(N)}$ has matrix elements

$$V_{\ell N_i} = \frac{v}{\sqrt{2} M_i} Y_{\ell i}, \quad (15)$$

being $M_i, i = 1, \ldots, 3$ the eigenvalues of the diagonal matrix $M_R$. Heavy neutrino masses are $m_{N_i} \simeq M_i$. Their couplings $V_{\ell N_i}$ are very small and it makes sense to keep only the first order terms. (Experimental data constrain these quantities to be $O(10^{-2})$ at most, see next section.) $V^{(\nu)}$ is unitary up to small corrections $O(V^2_{\ell N_i})$ and has been taken as the $3 \times 3$ identity matrix, because light neutrino masses can be safely neglected at high energies. The interactions for heavy Majorana neutrinos presented in the introduction can be easily obtained substituting the expressions for $U_L$ and $V$ into Eqs. (10). We note that while the charged current interaction has the form shown in Eq. (2) by definition, the neutral current and Higgs terms correspond to the leading term for small $V^{(N)}$. In the general case of three heavy Majorana neutrinos the vertices in Eqs. (2)–(4) correspond to any of them.

For a heavy Dirac neutrino the interactions are the same but some redefinitions are necessary. In this case there are two degenerate eigenstates, say $N_1$ and $N_2$, and their Yukawa couplings (the first two columns) are proportional,

$$m_{N_2} = m_{N_1}, \quad Y_{12} = i Y_{11}. \quad (16)$$

From the latter equality, $V_{\ell N_2} = i V_{\ell N_1}$. These conditions realise the lepton number symmetry $L$ inherent to this case. Defining

$$N_L \equiv \frac{1}{\sqrt{2}} (N_{1L} + i N_{2L}),$$
$$\hspace{1cm} (N_R)^c \equiv \frac{1}{\sqrt{2}} (N_{1L} - i N_{2L}), \quad (17)$$

it is easily found that $N_L$ couples to the charged leptons with

$$V_{\ell N} = \sqrt{2} V_{\ell N_1}, \quad \hspace{1cm} (18)$$

while $(N_R)^c$ decouples. Lepton numbers $L = 1, L = -1$ can be assigned to $N_L, (N_R)^c$, respectively. Rewriting the Majorana mass term for $N_1$ and $N_2$

$$\mathcal{L}_{\text{mass}} = -\frac{m_{N_1}}{2} \left( \bar{N}_{1L} (N_{1L})^c + \bar{N}_{2L} (N_{2L})^c \right) + \text{H.c.} \quad (19)$$

gives the usual LNC mass term of a Dirac neutrino with mass $m_N \equiv m_{N_1}$ in Eq. (1),

$$\mathcal{L}_{\text{mass}} = -m_N \bar{N}_L N_R + \text{H.c.} \quad (20)$$
4 Experimental constraints

The mixing of neutrinos heavier than the $Z$ boson with charged leptons is limited by two sets of processes [19,42,56–60]: (i) $\pi \rightarrow \ell \nu$, $Z \rightarrow \nu \bar{\nu}$ and other tree-level processes involving light neutrinos in the final state; (ii) $\mu \rightarrow e\gamma$, $Z \rightarrow \ell^+\ell^-$ and other flavour violating (LFV) processes to which heavy neutrinos can contribute at one loop level. These limits are independent of the Dirac or Majorana nature of the neutrinos, and constrain the quantities

$$\Omega_{\ell\ell} \equiv \delta_{\ell\ell} - \sum_{i=1}^{3} V_{\ell \nu_i} V_{\ell \nu_i}^* = \sum_{i=1}^{3} V_{\ell N_i} V_{\ell N_i}^*, \quad (21)$$

where the equality is a consequence of the unitarity of $U_L$. A global fit to the processes in the first group gives the bounds [59]

$$\Omega_{ee} \leq 0.0054, \quad \Omega_{\mu\mu} \leq 0.0096, \quad \Omega_{\tau\tau} \leq 0.016, \quad (22)$$

with a 90% confidence level (CL). In the limit of heavy neutrino masses in the TeV range, LFV processes in the second group require [42]

$$|\Omega_{e\mu}| \leq 0.0001, \quad |\Omega_{e\tau}| \leq 0.01, \quad |\Omega_{\mu\tau}| \leq 0.01. \quad (23)$$

The bounds in Eqs. (22) are model-independent to a large extent, and independent of heavy neutrino masses as well. They imply that the mixing of the heavy eigenstates with the charged leptons is very small, $\sum_i |V_{\ell N_i}|^2 \leq 0.0054, 0.0096, 0.016$ for $\ell = e, \mu, \tau$, respectively. On the other hand, in general the bounds in Eqs. (23) do not directly constrain the products $V_{\ell N_i} V_{\ell' N_i}^*$, but their sums, and cancellations might occur between two or more terms and also with other new physics contributions. These cancellations may be more or less natural, but in any case such possibility makes the limits in Eqs. (23) relatively weak if more than one heavy neutrino exists [10,61]. When discussing the future collider limits in next section we will take into account only the bounds in Eqs. (22), eventually using those in Eqs. (23) for comparison.

Specific neutrino mass models must reproduce the observed light neutrino spectrum. In the block-diagonal basis defined by Eq. (14) the neutrino mass matrix in Eq. (7) reads (neglecting small corrections)

$$U_L^\dagger \mathcal{M} U_L \simeq \begin{pmatrix} M_L - \frac{\nu^2}{2} Y M_R^{-1} Y^T & 0 \\ 0 & M_R \end{pmatrix}. \quad (24)$$

The light neutrino masses and mixings result from the diagonalisation of the light neutrino $3 \times 3$ mass matrix

$$M_\nu \simeq M_L - \frac{\nu^2}{2} Y M_R^{-1} Y^T. \quad (25)$$
The first term $M_L$ vanishes unless a Higgs triplet is included in the theory, in which case it must be justified why its matrix elements are small. For $V_{LN} \sim 0.01$ and $m_N \sim 1$ TeV, the typical size of the second term is 0.1 GeV, 8 orders of magnitude larger than the light neutrino mass scale $m_\nu \sim 1$ eV. Then, one has to arrange either (i) some suppression mechanism, or (ii) a fine-tuned cancellation with $M_L$. In any of these cases one expects that some symmetry is at work. A particular example of the former appears when two degenerate Majorana neutrinos form a (quasi) Dirac neutrino. In such case, their contributions cancel due to the conditions in Eqs. (16). (If all heavy neutrinos are Dirac particles the light neutrinos can be also Dirac fermions, what requires the addition of three extra SM singlets.) This possibility has been explored in definite models [40, 51, 52]. Indeed, if we write

$$Y = Y^{(0)} + \varepsilon Y^{(1)},$$

with $\varepsilon$ small and $Y^{(0)}$ taking the form

$$Y^{(0)} = \begin{pmatrix} y_1 & y_2 & y_3 \\ \alpha y_1 & \alpha y_2 & \alpha y_3 \\ \beta y_1 & \beta y_2 & \beta y_3 \end{pmatrix},$$

where

$$\frac{y_1^2}{M_1} + \frac{y_2^2}{M_2} + \frac{y_3^2}{M_3} = 0,$$

the term $Y^{(0)} M_R^{-1} Y^{(0)T}$ identically vanishes, and the light neutrino mass matrix reduces to

$$M_\nu \simeq \varepsilon \left(Y^{(0)} M_R^{-1} Y^{(1)T} + Y^{(1)} M_R^{-1} Y^{(0)T}\right) + O(\varepsilon^2).$$

In this case it is also possible to cancel the contributions to neutrinoless double $\beta$ decay as well, which otherwise would require in general much heavier neutrino masses $m_{N_i} \geq 1$ TeV [62, 63]. Moreover, this framework can accommodate leptogenesis by making $\beta \ll \alpha$ [51, 52]. At any rate, constructing models where this structure and the size of $\alpha, \beta$ result from symmetries with a natural breaking does not seem straightforward.

5 Heavy neutrino signals at large colliders

Heavy neutrino signals can conserve or violate lepton number. In the LNC case final state angular distributions must be used in order to determine the Dirac or Majorana nature of the heavy neutrino. On the contrary, LNV signals certify its Majorana character. (Although lepton number can be violated by light Majorana neutrino masses, they have no relevance at large colliders because their effects are suppressed by powers
of \( m_\nu / \sqrt{s} \).) Lepton flavour is also conserved within the SM in the limit of vanishing neutrino masses. This makes LFV signals interesting as well. SM backgrounds to LNV or LFV processes involve the production of extra neutrinos in the final state (this is because both lepton number and flavour are preserved within the minimal SM, in the limit of vanishing light neutrino masses). Then, in addition to being higher order processes, they can be greatly reduced in general requiring the absence of significant missing energy. Consequently, SM backgrounds to these kind of signals are much smaller than for the ones conserving lepton number and flavour. This usually translates in more stringent constraints on heavy Majorana neutrinos, and justifies concentrating on LNV processes in some cases. For each accelerator it is convenient to classify the possible signals according to their LNC and lepton flavour conserving (LFC) character in order to address the discovery potential in each case.

We review the estimates for heavy neutrino production at \( e^- p, \bar{p}p, e^+ e^-, \) and \( e^- \gamma \) colliders in turn, restricting ourselves to masses \( m_N > M_Z \). Since final state neutrinos are undetected, the observation of LNV signals requires a change of two units in the charge of the leptons involved. This makes \( e^- p \) and \( e^- \gamma \), a priori, more adequate to search for Majorana neutrinos, because the initial state has a single charged lepton. The most significant processes are \( e^- p \rightarrow N j \) [39, 40, 64] and \( e^- \gamma \rightarrow NW^- \) [65]. For \( pp \) collisions the most interesting process is \( q \bar{q}' \rightarrow \ell^+ N \) (and its charge conjugate). We extend the analysis in Refs. [66–68] to Dirac neutrinos, for which backgrounds are much larger. Finally, the neutral character of the initial state in \( e^+ e^- \) colliders makes this machine equally sensitive to Dirac and Majorana neutrinos in the process \( e^+ e^- \rightarrow N \nu \) [8].

It is also important to realise that discovering a heavy neutrino seems to require its on-shell production. This is so because its mixing with the SM particles is rather small, what makes necessary the pole enhancement factor to observe the heavy neutrino signal over the background. Additionally, the production of an on-shell heavy neutrino allows to reconstruct its mass and reduce the backgrounds further.

### 5.1 \( e^- p \) scattering

Heavy neutrinos can be produced in the processes \( e^- q \rightarrow N q', e^- \bar{q}' \rightarrow N \bar{q} \), being \( q = u, c \), \( q' = d, s \). These processes take place with \( t \) channel exchange, and hence their cross section, equal for Dirac and Majorana \( N \), is not suppressed for large \( m_N \) by \( s \)-channel propagators. On the other hand, \( N \) is produced through its mixing with the electron \( V_{eN} \), and heavy neutrinos mixing significantly with the muon or tau but with \( V_{eN} \approx 0 \) are not observable. Depending on the mixing and character of \( N \), we
can have the following signals:

(i) For Dirac $N$ coupling only to the electron, the decay $N \to e^- W^+$ gives an SM-like
final state $e^- W^+ j$ with a huge background.

(ii) For Dirac $N$ coupling also with the muon or tau, the decays $N \to \mu^- /\tau^- W^+$
give clean LFV signals $e^- p \to \mu^- /\tau^- W^+ j$.

(iii) For Majorana $N$, apart from the previous modes we have $N \to \ell^+ W^-$, yielding
a clean LNV signal $e^- p \to \ell^+ W^- j$, as depicted in Fig. 1.

Figure 1: Feynman diagram for the LNV process $e^- q \to \ell^+ W^- q'$. An additional process
$e^- q' \to N q$, obtained interchanging the quark lines, contributes to $e^- p \to \ell^+ W^- j$ as well.

The $W^\pm$ bosons in the final state can be taken to decay hadronically, what, in
addition to the larger branching ratio, avoids the complication of additional final state
leptons which may hide the non-conservation of lepton number and/or flavour. The
LNV signal $e^- p \to e^- jjj$ of a heavy Majorana neutrino has been studied in the lit-
erature [39, 40, 64]. At the centre of mass (CM) energy $\sqrt{s} = 314$ GeV available at
HERA heavy neutrino production cross sections are very small, due to the suppression
of the parton density functions (PDFs) at high $x$. We have rescaled the limits obtained
in Ref. [40], saturating the improved upper bound $|V_{eN}|^2 \leq 0.0054$. For a luminosity
of 200 pb$^{-1}$, assuming no backgrounds and a perfect detection efficiency, HERA could
give 2$\sigma$ evidence (conventionally taken as 3 signal events in the absence of background)
for a 100 GeV neutrino coupling only to the electron. This sensitivity is similar to the
one achieved at LEP [69].

A hypothetical LEP $\otimes$ LHC $ep$ machine, with $\sqrt{s} = 1.3$ TeV and a luminosity
of 2 fb$^{-1}$ per year, could extend the LHC sensitivity for heavy Majorana neutrinos.
Taking as statistical criterion for 5$\sigma$ discovery the observation of 10 signal events, this
significance would be achieved for heavy neutrino masses up to $\sim 550$ GeV, doubling
the LHC reach (see next subsection). If no signal is found, the bounds $|V_{eN}| \lesssim 0.02$ at
90% CL could be set for $m_N \simeq 300$ GeV, and useful constraints would be obtained up
to $m_N \sim 700$ GeV.
5.2 \(^{(5)}\) \(pp\) collisions

Hadron colliders can produce heavy neutrinos in association with a charged lepton in \(q\bar{q}' \rightarrow \ell^+ N\), as shown in Fig. 2 (plus its charge conjugate \(\bar{q}q' \rightarrow \ell^- N\)). This process is relevant for moderate \(N\) masses. It is mediated by \(s\)-channel \(W\) exchange, and then for large \(m_N\) it is suppressed not only by PDFs but also by the \(W\) propagator (in contrast to \(ep \rightarrow Nj\) scattering previously discussed). However, one important advantage compared to other colliders is that \(\ell^+ N\) production at LHC does not require a sizeable coupling to the electron, as it can be observed from Fig. 2. It can lead to the following signals:

(i) For a Dirac \(N\) mixing with only one lepton flavour, the decay \(N \rightarrow \ell^- W^+\) yields a \(\ell^+ \ell^- W^+\) final state, with a huge SM background.

(ii) For a Dirac \(N\) coupled to more than one charged lepton we can also have \(N \rightarrow \ell' - W^+\) with \(\ell' \neq \ell\), giving the LFV signal \(\ell^+ \ell' - W^+\) shown in Fig. 2(a), which has much smaller backgrounds.

(iii) For a Majorana \(N\), in addition to LNC signals we have LNV ones arising from the decay \(N \rightarrow \ell'(\pm) W^-\) in Fig. 2(b), which have small backgrounds too.

![Figure 2: Feynman diagrams for the process \(q\bar{q}' \rightarrow \ell^+ N\), followed by LNC decay \(N \rightarrow \ell'(\pm) W^+\) (a) and LNV decay \(N \rightarrow \ell'(\pm) W^-\) (b). Additional diagrams with off-shell \(N\) contributing to the same final state are not shown. For the charge conjugate processes the diagrams are analogous.](image)

For the same values of the couplings, cross sections for on-shell \(N\) production are the same for heavy Dirac and Majorana neutrinos (for the latter, additional diagrams with off-shell \(N\) can mediate the final states considered, but their contribution is very small). Therefore, since a Majorana \(N\) has LNV decays as well as LNC ones, the relation

\[
\sigma_D (pp \rightarrow \ell^\pm \ell'^\mp W^\pm) \simeq 2\sigma_M (pp \rightarrow \ell^\pm \ell'^\mp W^\pm) \simeq 2\sigma_M (pp \rightarrow \ell^\pm \ell'^\pm W^\mp) \quad (30)
\]

13
holds, although \( \sigma_M (pp \rightarrow \ell^+ \ell^- W^-) \neq \sigma_M (pp \rightarrow \ell^- \ell^+ W^+) \) in \( pp \) collisions due to the different PDFs involved.

Apart from angular distributions, the obvious difference between the processes (i)–(iii) above is the SM background, huge for the first case and small for the other two. In order to estimate the LHC discovery potential we have implemented in ALPGEN [70] the relevant vertices presented in section 4, finding results consistent with previous analyses for Majorana neutrinos [66]. We will present a detailed study of signals and backgrounds with a proper simulation of the experimental detection elsewhere. For our numerical estimates we assume a heavy neutrino \( N \) coupling only to muons and saturating the present limit \( |V_{\mu N}|^2 \leq 0.0096 \), which yields \( \mu^\pm \mu^\mp W^\pm \) final states and, provided \( N \) is a Majorana fermion, \( \mu^\pm \mu^\pm W^\pm \) as well. This is the most interesting situation for LHC, where it can outperform other planned \( e^+e^- \) and \( e^-\gamma \) colliders, as it will be shown later. The analysis if \( N \) couples only to the electron is completely analogous, while if it couples to the tau the decays and tagging efficiency of this lepton must be considered in the analysis. We select \( W^\pm \) hadronic decays, and fix the Higgs mass \( M_H = 120 \) GeV. The processes considered are

\[
\begin{align*}
pp \rightarrow \mu^+ \mu^- jj, & \quad \text{(LNC)} \\
pp \rightarrow \mu^\pm \mu^\pm jj, & \quad \text{(LNV)}
\end{align*}
\]

Among the most relevant SM backgrounds we select as example

\[
pp \rightarrow Z/\gamma^* jj \rightarrow \mu^+ \mu^- jj
\]

in the LNC case, and

\[
pp \rightarrow W^\pm W^\pm W^\mp \rightarrow \mu^\pm \mu^\pm jj + \text{missing energy}
\]

in the LNV one, computing them with ALPGEN. In the former case (LNC) the background is huge. For instance, the signal cross section without cuts for \( pp \rightarrow \mu^\pm N \rightarrow \mu^\pm \mu^\mp W^\pm \rightarrow \mu^\pm \mu^\pm jj \) at LHC is 86 fb for \( m_N = 100 \) GeV, while the \( Zjj \) background is around 400 times larger. The cuts adopted to simulate the detector coverage and particle / jet isolation are

\[
\begin{align*}
p_T^\mu & > 20 \text{ GeV} , \quad p_T^l > 30 \text{ GeV} , \\
|\eta_{\mu,j}| & < 2.5 , \quad \Delta R_{jj,j\mu,\mu\mu} > 0.4 ,
\end{align*}
\]

with \( p_T \) the transverse momentum, \( \eta \) the pseudorapidity and \( \Delta R \) the lego-plot distance, in standard notation. Besides these minimal “pre-selection” criteria we impose cuts on
various final state invariant masses $M$ to suppress backgrounds. In the LNC case we require

$$
60 \text{ GeV} < M_{jj} < 100 \text{ GeV},
40 \text{ GeV} < M_{\mu\mu} < 70 \text{ GeV} \quad \text{or} \quad M_{\mu\mu} > 110 \text{ GeV},
$$

(35)
in order to reconstruct the $W$ boson from $N$ decay and reduce the $Z/\gamma^*$ contributions. In the LNV case we ask

$$
60 \text{ GeV} < M_{jj} < 100 \text{ GeV},
\not{p}_t < 20 \text{ GeV},
$$

(36)
so as to reconstruct the $W$ boson and take advantage of the fact that the signal has no significant missing momentum $\not{p}_t$. Moreover, as the observation of a heavy neutrino requires its mass reconstruction, we also impose in both cases

$$
0.9 \, m_N < M_{jj\mu} < 1.1 \, m_N,
$$

(37)
for at least one of the two $\mu$ assignments. The corresponding cross sections are given in Tables 1 and 2 for LHC and Tevatron, respectively. For the LNC signal the heavy neutrino is assumed to have Dirac nature (for a Majorana neutrino this cross section would be roughly one half). We have concentrated on the mass region $m_N > M_Z$ where the signal cross section gets smaller (the case $m_N < m_Z$ has been also considered for $p\bar{p}\to \mu^+\mu^-jj$ in Ref. [66]). As we stressed at the beginning of this subsection, cross sections are suppressed for larger $m_N$ values. We observe for example that changing $m_N = 100 \text{ GeV}$ to $m_N = 500 \text{ GeV}$ implies a signal reduction of almost 2 orders of magnitude at LHC.

<table>
<thead>
<tr>
<th>$m_N$</th>
<th>$\mu^+\mu^-jj$ signal</th>
<th>$Z_{jj}$ background</th>
<th>$\mu^\pm\mu^\pm jj$ signal</th>
<th>$W^\pm W^\mp W^\pm$ background</th>
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<td>44</td>
<td>0.16</td>
<td>0.0023</td>
</tr>
<tr>
<td>400</td>
<td>0.13</td>
<td>22</td>
<td>0.068</td>
<td>0.0012</td>
</tr>
<tr>
<td>500</td>
<td>0.066</td>
<td>11</td>
<td>0.034</td>
<td>0.0007</td>
</tr>
</tbody>
</table>

Table 1: Signal and background cross sections (in fb) as a function of the heavy neutrino mass (in GeV) at LHC ($\sqrt{s} = 14 \text{ TeV}$) for the cuts given in the text.

Tevatron does not seem to have a chance to observe a new heavy neutrino in this mass range with a luminosity of 2 fb$^{-1}$. For LHC, although the comparison between the
first two columns of Table 1 seems to indicate that it may be difficult to observe a Dirac heavy neutrino, one could still carefully refine the cut selection to improve the signal to background significance. In this way neutrino masses $\sim 100$ GeV might be observable. The LNV process clearly offers better prospects to detect a heavy Majorana neutrino. In this case one expects to discover with $5\sigma$ significance (10 signal events without background) a Majorana neutrino coupling only to the muon with a mass up to 350 GeV in this channel, assuming a luminosity of $100 \text{ fb}^{-1}$. This result is in agreement with previous ones [66]. Conversely, if no signal is observed present bounds on the mixing angles are improved. For instance, for $m_N = 200$ GeV we would obtain the 90% CL upper limit $|V_{\mu N}| \leq 0.022$, improving the present bound by a factor of four. (This limit is obtained assuming no observed events, what yields an upper limit of 2.44 for the signal in the absence of background [71].) Obviously, further backgrounds must be taken into account and a realistic detector simulation performed, especially regarding the background suppression with the requirement $p_t < 20$ GeV, which seems more delicate. On the other hand, the kinematical cuts used to enhance the signal significance can still be improved.

For a heavy Majorana neutrino simultaneously coupling to the electron and muon the results (summing all relevant signals) are expected to be similar, since the backgrounds involving electrons and muons have the same size. In case that $N$ couples significantly to the tau lepton, results will be worse because decays involving taus are more difficult to tag. An interesting possibility is that of a heavy Dirac neutrino coupling to more than one charged lepton. In this case, LFV signals as described in point (ii) above have much smaller SM backgrounds than LFC ones discussed here, and better limits could be obtained.

<table>
<thead>
<tr>
<th>$m_N$</th>
<th>$\mu^+\mu^-jj$ signal</th>
<th>$Zjj$ background</th>
<th>$\mu^\pm\mu^\pm jj$ signal</th>
<th>$W^\pm W^\pm W^\mp$ background</th>
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<tr>
<td>100</td>
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<td>500</td>
<td>0.0015</td>
<td>0.24</td>
<td>0.0008</td>
<td>0.00001</td>
</tr>
</tbody>
</table>

Table 2: The same as in Table 1 but at Tevatron ($\sqrt{s} = 1.96 \text{ TeV}$).
5.3 $e^+e^-$ annihilation

The process $e^+e^- \rightarrow N\nu$ can produce heavy neutrinos which couple to the electron, for masses up to nearly the kinematical limit imposed by CM energy [72–75]. In the Dirac case both the neutrino $N$ and antineutrino $\bar{N}$ are produced (with equal cross sections), and a Majorana $N$ is produced with a cross section two times larger. The subsequent decays $N \rightarrow \ell^-W^+$, $\bar{N} \rightarrow \ell^+W^-$ for a Dirac neutrino and $N \rightarrow \ell^-W^+$, $N \rightarrow \ell^+W^-$ for a Majorana one yield $\ell^\pm W^\mp$ final states, with total cross sections $\sigma(e^+e^- \rightarrow \ell^-W^+\nu) = \sigma(e^+e^- \rightarrow \ell^+W^-\nu)$. Additionally, these two cross sections are almost independent of the Dirac or Majorana nature of the produced neutrino. The diagrams for $e^+e^- \rightarrow \ell^-W^+\nu$ mediated by on-shell $N$ exchange are shown in Fig. 3. The SM background is given by $e^+e^- \rightarrow \ell^-W^+\nu$, including resonant $W^+W^-$ production and several other Feynman diagrams, which dominate at high energies. We observe that the only indication of lepton number violation in diagram 3 (b) is the helicity of the final state neutrino, which remains undetected. Therefore, there is no advantage for LNV processes in what respects to background reduction, and limits obtained are the same for Dirac and Majorana fermions.

![Feynman diagrams](image)

Figure 3: Feynman diagrams for the process $e^+e^- \rightarrow \ell^-W^+\nu$ involving on-shell Majorana $N$ exchange. Diagram (b) is only present if $N$ is a Majorana fermion. Additional diagrams with off-shell $N$ are not shown.

We consider $e^+e^-$ annihilation at a CM energy of 500 GeV, as proposed for an international linear collider (ILC) and 3 TeV, as might be the case of a future compact linear collider (CLIC), examining their discovery potential for heavy neutrinos [8]. Since they are produced through its mixing with the electron, two cases are worth discussing for this process: (a) $N$ only couples to the electron; (b) $N$ also mixes with the muon or tau lepton. $W$ hadronic decays are selected, and the luminosities for ILC and CLIC are of 345 fb$^{-1}$ and 1000 fb$^{-1}$, respectively, corresponding to one year of

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5If $N$ does not couple to the electron it can still be produced through the diagram (c), but this contribution is very suppressed by the $s$-channel $Z$ propagator, and does not lead to an observable signal [61].
running. Beam polarisations $P_{e^+} = 0.6$, $P_{e^-} = -0.8$ are also used. The discovery potential for both machines is summarised in Fig. 4. (Notice that the mass range in the second plot has been enlarged with respect to Ref. [8].)

Figure 4: Dependence of the discovery and upper limits on $V_{eN}$ on the heavy neutrino mass, for ILC (a) and CLIC (b). Both plots assume mixing only with the electron.

For ILC the sensitivity is nearly the same for masses between 100 and 400 GeV, because the reduction of the background at larger transverse momenta makes up for the decrease in signal cross sections for larger $m_N$. A heavy neutrino with a coupling $|V_{eN}| \geq 0.01$ could be discovered with 5σ significance, and if no signal is seen the limit $|V_{eN}| \leq 0.006$ could be set at 90% CL, improving the eventual LHC bounds by a factor of 4. CLIC would be able to explore $N$ masses in the TeV range, as it can be observed in Fig. 4 (b), and provide very stringent limits $|V_{eN}| \leq 0.001 - 0.002$ for $m_N = 400 - 1000$ GeV. These estimates have been obtained at the partonic level and with a simple simulation of the detector resolution. Therefore, it is expected that with a detailed simulation the results will be a little worse. We also remark that the Dirac or Majorana nature of a heavy neutrino eventually discovered could be unambiguously established with the analysis of its opening angle distribution [8].

For heavy neutrinos mixing also with the muon or the tau, additional signals $\mu^\pm W^\mp \nu$ or $\tau^\pm W^\mp \nu$ are present. The discovery potential for ILC in this case is shown in Fig. 5 considering mixing either with the muon (a) or with the $\tau$ (b). In the first case the combined limits do not depend on the mixing with the muon, essentially due to the fact that backgrounds involving electrons and muons have the same size. In the second case the limits are worse for larger $|V_{\tau N}|$ because in this situation the heavy neutrino has a larger branching fraction to $\tau^\pm W^\mp$, and $\tau$ detection is experimentally more difficult. We have also included the constraints from low-energy LFV processes.
for comparison, which are relevant only for $N$ coupling simultaneously to $e$ and $\mu$. The direct limit from $N$ production complements the indirect ones for small $|V_{\mu N}|$.

Figure 5: Combined limits obtained at ILC on: $V_{eN}$ and $V_{\mu N}$, for $V_{\tau N} = 0$ (a); $V_{eN}$ and $V_{\tau N}$, for $V_{\mu N} = 0$ (b). The red areas represent the 90% CL limits if no signal is observed. The white areas extend up to present bounds $V_{eN} \leq 0.073$, $V_{\mu N} \leq 0.098$, $V_{eN} \leq 0.13$, and correspond to the region where a combined statistical significance of 5$\sigma$ or larger is achieved. The indirect limit from $\mu - e$ LFV processes is also shown. We take $m_N = 300$ GeV.

Figure 6: The same as in Fig. 5 but for CLIC, taking $m_N = 1500$ GeV.

At CLIC energies the behaviour is completely different. Since backgrounds involving muons or taus are much smaller [8], mixing with a second charged lepton increases the observability of the heavy neutrino. The combined limits are shown in Fig. 6 for mixing with the muon (a) or the tau (b). In the first case the limits are greatly
improved for $|V_{\mu N}| \gtrsim 0.005$, while in the second case the effect is partially compensated by the worse detection of $\tau$ leptons. We also point out that the direct limit eventually obtained from $N$ production is far better than the indirect one from present low-energy LFV processes, and would remain competitive with future improvements of the upper bounds on $\text{Br}(\mu \rightarrow e\gamma)$ [76] and $\mu - e$ conversion in nuclei [77].

A heavy neutrino $N$ with $V_{eN} \simeq 0$ can still be produced in $e^+e^-$ annihilation through the higher-order process $e^+e^- \rightarrow \ell^-NW^+$ (and its charge conjugate). Its cross section is much smaller than for $e^+e^- \rightarrow N\nu$, however. Depending on the nature and mixing of $N$, we can have the following final states:

(i) For Dirac $N$ coupling only to the muon or tau, $N \rightarrow \ell^+W^- (\ell = \mu, \tau)$ gives an SM-like $\ell^+\ell^-W^+W^-$ signal with a large background.

(ii) A Dirac $N$ coupling to $\mu$ and $\tau$ also yields the LFV final state $\mu^\pm\tau^\mp W^+W^-$, for which the background is much smaller.

(iii) For a Majorana $N$, the decay $N \rightarrow \ell^-W^+$ leads to a LNV signal $\ell^-\ell^-W^+W^+$, as shown in Fig. 7. SM background is very small as well in this case.

![Feynman diagrams](image)

Figure 7: Feynman diagrams for the process $e^+e^- \rightarrow \ell^-\ell^-W^+W^+$ involving on-shell Majorana $N$ exchange. Additional diagrams with off-shell $N$ are not shown.

We have estimated the ILC discovery potential in the latter case (iii), for $m_N = 200$ GeV, $V_{eN} = V_{\tau N} = 0$, $|V_{\mu N}|^2 = 0.0096$. Assuming perfect detection efficiency, 8 $\mu^\pm\mu^\pm jjjj$ events could be obtained within one year of running. The SM background is given by $e^+e^- \rightarrow W^-W^-W^+W^- \rightarrow \mu^\pm\mu^\pm\nu\nu jjjj$, and it is assumed that it can be reduced to negligible levels (what must be confirmed with a detailed simulation) requiring the absence of significant missing momentum and the reconstruction of the $N$ invariant mass. Hence, the heavy neutrino could be clearly observed but with a significance smaller than $5\sigma$. If no signal is found, the limit $|V_{\mu N}| \leq 0.055$ could be set at 90% CL, a factor of two lower than the one obtained at LHC.
5.4 $e^-\gamma$ collisions

A proposed option for ILC is to have $e^-\gamma$ collisions at CM energies of several hundreds of GeV. From the point of view of heavy neutrino physics, this would be a very interesting possibility, complementing the capabilities of $e^+e^-$ annihilation. Heavy neutrinos can be produced in the process $e^-\gamma \rightarrow NW^-$. The cross section is the same for Dirac and Majorana $N$, but depending on its character and mixing with the charged leptons we can have the following final states:

(i) For a Dirac $N$ coupling only to the electron, $N \rightarrow e^-W^+$ gives a SM-like signal $e^-\gamma \rightarrow e^-W^+W^-$ with a large background.

(ii) For a Dirac $N$ coupling also to the muon or tau, we can have $N \rightarrow \mu^-/\tau^-W^+$. The resulting LFV signals $e^-\gamma \rightarrow \mu^-/\tau^-W^+W^-$ are not present in the SM, hence their backgrounds are small.

(iii) For a Majorana $N$, apart from these final states we have $N \rightarrow \ell^+W^-$, giving a clean LNV signal $e^-\gamma \rightarrow \ell^+W^-W^-$, shown in Fig. 8.

![Feynman diagrams](image)

Figure 8: Feynman diagrams for the LNV process $e^-\gamma \rightarrow \ell^+W^-W^-$. Additional diagrams with off-shell $N$ are not shown.

The $W$ bosons can be taken to decay hadronically, what in addition to the larger branching ratio avoids the complication of additional final state leptons. SM backgrounds to the LFV and LNV signals involve additional final state neutrinos, and they can be reduced by requiring the absence of significant missing transverse momentum. This is however a delicate issue and requires detailed simulations to confirm the parton-level expectations.

The process (iii), for a Majorana $N$ coupling mainly with the electron, has been studied in Ref. [65]. The SM background is given by $e^-\gamma \rightarrow W^+W^-\nu_e \rightarrow e^+\nu_eW^-W^-\nu_e$. For a CM energy of 500 GeV and a luminosity of 100 fb$^{-1}$, 5$\sigma$ evidence (taking as criterion the production of 10 signal events) for a 200 GeV Majorana neutrino could be achieved for mixings $|V_{eN}| \geq 0.0046$. Conversely, if no signal is observed the bound
$|V_{eN}| \leq 2.3 \times 10^{-3}$ can be set at 90% CL. These limits assume that SM background can be essentially eliminated without affecting the signals, what may be too optimistic. Thus $e^{-}\gamma$ collisions improve the ILC limit by a factor of two for $m_N$ around this value. For heavier $N$ the production cross section decreases quickly, and for $m_N = 400$ GeV the limits for $e^+e^-$ and $e^-\gamma$ collisions are similar. On the other hand, limits for heavy Dirac neutrinos are much better at ILC.

As in $e^+e^-$ annihilation, heavy neutrinos which do not couple to the electron can be produced in a sub-leading process, in this case $e^{-}\gamma \to N\ell^-\nu$, $\ell = \mu, \tau$. The following final states are possible:

(i) For Dirac $N$ coupling only to $\ell = \mu$ or $\ell = \tau$, $N \to \ell^+W^-$ gives a signal $\ell^+\ell^-W^-\nu$ which has a large SM background

(ii) A Dirac $N$ coupling to both can give a LFV signal $\mu^\pm\tau^\mp W^-\nu$ which is easier to detect

(iii) For a Majorana $N$, the decay $N \to \ell^-W^+$ leads to a LNV signal $\ell^-\ell^-W^+\nu$, as shown in Fig. 9. SM background is very small in this case.

Figure 9: Feynman diagrams for the process $e^{-}\gamma \to \ell^-\ell^-W^+\nu$ involving on-shell Majorana $N$ exchange. Additional diagrams with off-shell $N$ are not shown.

The third case has been studied in Ref. [65]. Assuming that the background can be essentially eliminated with kinematical cuts (a fact which must be confirmed with a detailed simulation), a 200 GeV neutrino coupling only to the muon can be discovered at 5$\sigma$ level for $|V_{\mu N}| \geq 0.09$. If no signal is observed, the bound $|V_{\mu N}| \leq 0.045$ can be set. These figures improve slightly the ones from $e^+e^-$ annihilation at 500 GeV, shown in the previous subsection, but are still a factor of two worse than the ones achievable at LHC.
6 Conclusions

At present, neutrinos are known to be massive. The minimal SM extension necessary to account for this experimental fact includes three ultra-heavy eigenstates with masses of the order of $10^{13}$ GeV, which lead to light neutrino masses via the seesaw mechanism. These states are directly unobservable. Moreover, at low energies the number of parameters in the neutrino sector (when heavy states are integrated out) is smaller than at the high scale, and this minimal seesaw mechanism is untestable (for a discussion see Ref. [78]). This has given extra motivation for models lowering to the TeV the scale of new physics which originates light neutrino masses. (Other possibility to obtain predictive models is to introduce extra symmetries.) The new heavy neutrinos appearing at this scale could be directly observed in future colliders, provided their mixing with the charged leptons is $O(10^{-2})$ or larger. With a completely different motivation in mind, extra-dimensional and Little Higgs models are also proposed, in the first case aiming to reduce the huge hierarchy between the electroweak and Planck scales, and in the second to cancel large corrections to the Higgs boson mass. Both classes of models can have new heavy neutrinos in their additional particle spectrum, light enough to be observable.

Independently of their origin, the mixing of heavy neutrinos is constrained by low-energy data, including lepton universality, LFV processes and neutrinoless double $\beta$ decay. Additionally, their seesaw-like contributions to light neutrino masses must be kept under control. Model-independent bounds from universality restrict their mixing with a given charged lepton, while LFV processes constrain the simultaneous mixing to more than one charged lepton. Neutrinoless double $\beta$ decay imposes a strong constraint on $V_{eN}$ if only one heavy Majorana neutrino $N$ is introduced, but when more than one exist cancellations are possible, including the natural case in which two (nearly) degenerate Majorana neutrinos with opposite CP parities form a (quasi) Dirac fermion. This is a general property: heavy (quasi) Dirac neutrinos at the TeV scale or below are less constrained and can reproduce light neutrino masses with less fine tuning.

We have reviewed the potential of various colliders to discover heavy neutrinos. Their relative sensitivities strongly depend on the mass, mixing and character of $N$. Some general statements can be made, however:

(i) Lepton colliders have a better discovery potential for heavy neutrinos with a significant coupling to the electron. They can be produced, up to high masses, via $t$-channel diagrams.

(ii) LHC has the best discovery potential for a heavy Majorana neutrino with $V_{eN} =$
0. The sensitivity of $e^-\gamma$ and $e^+e^-$ colliders is similar in this situation, but a factor of two smaller.

(iii) Neutrinos which couple to more than one charged lepton are easier to detect, because: (a) LFV backgrounds are smaller in all cases; (b) in $e^+e^-, e^-p$ and $e^-\gamma$ collisions backgrounds involving $\mu$ or $\tau$ leptons are also smaller. However, the better observability at lepton colliders does not translate into better bounds on $V_{\mu N}$ or $V_{\tau N}$. Production cross sections are independent of these couplings and, when $V_{\mu N}$ or $V_{\tau N}$ are sufficiently large so that $\mu$ or $\tau$ final states dominate, decay branching ratios are independent as well. This fact can be clearly observed in Figs. [3] [4]

(iv) Dirac neutrinos are best studied in $e^+e^-$ collisions, where they are copiously produced and the environment is sufficiently clean.

<table>
<thead>
<tr>
<th></th>
<th>Majorana</th>
<th></th>
<th>Dirac</th>
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<td>LHC</td>
</tr>
<tr>
<td>$e, \mu$</td>
<td>$e^-\gamma$</td>
<td>CLIC</td>
<td>CLIC</td>
</tr>
<tr>
<td></td>
<td>ILC</td>
<td>LEP $\otimes$ LHC</td>
<td>LEP $\otimes$ LHC</td>
</tr>
<tr>
<td></td>
<td>LHC</td>
<td>LHC</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Summary of the relative discovery potential for heavy Majorana and Dirac neutrinos, in the low ($100 - 400$ GeV), intermediate ($400 - 1000$ GeV) and large ($1 - 2.5$ TeV) mass regions, and coupling to $e$, $\mu$ or both. In each cell, better to worse discovery potentials are ordered from top to bottom. Dashes are shown when there is no significant sensitivity. Results for mixing with the $\tau$ are analogous than for the muon.
Table 3 summarises the relative sensitivities of the different colliders studied. It is useful to consider three approximate mass ranges for heavy neutrinos: “low”, from 100 to 400 GeV, “intermediate” from 400 GeV to 1 TeV and “large”, from 1 to 2.5 TeV, which is close to the maximum $m_N$ which can be probed at CLIC (see Fig. 4). It is also convenient to take several limits for the mixing of $N$: when it only couples to the electron, muon or tau, and when it couples significantly to the electron and either muon or tau lepton. In this latter case, we assume a significant coupling to the electron but with decays dominated by muon / tau final states. We observe that for heavy Dirac neutrinos, which naturally appear in some Little Higgs and extra-dimensional models, $e^+e^-$ collisions at CLIC and ILC provide the best limits.

<table>
<thead>
<tr>
<th>Neutrino Type</th>
<th>Majorana</th>
<th>Dirac</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Low</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e$</td>
<td>$</td>
<td>V_{eN}</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$</td>
<td>V_{\mu N}</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$</td>
<td>V_{\tau N}</td>
</tr>
<tr>
<td>$e, \mu$</td>
<td>$</td>
<td>V_{eN}</td>
</tr>
<tr>
<td>$e, \tau$</td>
<td>$</td>
<td>V_{eN}</td>
</tr>
<tr>
<td><strong>Intermediate</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e$</td>
<td>$</td>
<td>V_{eN}</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$\sim$</td>
<td>$\sim$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$\sim$</td>
<td>$\sim$</td>
</tr>
<tr>
<td>$e, \mu$</td>
<td>$</td>
<td>V_{eN}</td>
</tr>
<tr>
<td>$e, \tau$</td>
<td>$</td>
<td>V_{eN}</td>
</tr>
<tr>
<td><strong>Large</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e$</td>
<td>$</td>
<td>V_{eN}</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$\sim$</td>
<td>$\sim$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>$\sim$</td>
<td>$\sim$</td>
</tr>
<tr>
<td>$e, \mu$</td>
<td>$</td>
<td>V_{eN}</td>
</tr>
<tr>
<td>$e, \tau$</td>
<td>$</td>
<td>V_{eN}</td>
</tr>
</tbody>
</table>

Table 4: Estimated 90% CL upper bounds on the mixing of heavy Majorana and Dirac neutrinos which couple to the charged leptons in the left column, for the low (100 – 400 GeV), intermediate (400 – 1000 GeV) and large (1 – 2.5 TeV) mass regions. Dashes are shown when there is no significant sensitivity.

We collect in Table 4 the approximate 90% CL bounds expected if no heavy neutrino
signal is found at any of the colliders discussed. They have been obtained from the various limits and plots presented in section 5. Limits on $V_{\mu N}$ and $V_{\tau N}$ originate from $N$ production at LHC, and so they are relevant only for low $m_N$ values. They are shown for the Majorana case. Limits for a Dirac neutrino are not still available nor can be safely estimated with present analyses. $5\sigma$ discovery limits can be obtained from the figures presented by multiplying by a factor $\sim 1.7 - 2$.

To conclude, we stress the importance of searches for heavy neutrinos at future colliders. Light neutrino masses are at present the only piece of evidence for physics beyond the SM, but their source is (and may remain forever) unknown. A possible discovery in this direction would clarify the situation, confirming and discarding possible scenarios for neutrino mass generation and possibly leptogenesis.

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**References**


For a recent review see C. Aalseth et al., *Neutrinoless double beta decay and direct searches for neutrino mass*, hep-ph/0412300


