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UNDERSTANDING AMPLITUDES FROM AN s CHANNEL APPROACHB. Schrempp ^{*)} and F. Schrempp ^{**)}

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A B S T R A C T

The systematics of inelastic two-body reactions at high energies is discussed on the basis of a geometrical s channel approach which we proposed recently. The amplitudes are given in terms of an asymptotically universal s (and u) channel trajectory representing the peripheral resonances. Its imaginary part is responsible for the amount of shrinkage, its real part for the structure in differential cross-sections, polarizations and density matrix elements. An approximation, valid for $|t| \gtrsim 0.25$ (GeV/c)², allows for a very simple, qualitative understanding of these structures in a large sample of reactions. The predictions are confronted with many experimental examples, among these the amplitude analysis for $\pi^- p \rightarrow \pi^0 n$ at 6 GeV/c. The agreement is good in all cases. As characteristic features for $|t| \gtrsim 0.25$ (GeV/c)² emerge a certain universality of inelastic reactions and the regularity of all structures with $\sqrt{-t}$ instead of $-t$.

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1. INTRODUCTION

In a recent paper ¹⁾ we presented an s channel picture for inelastic high energy scattering (s large and t fixed, say). The t dependence and also to a considerable extent the s dependence of the data are predicted in terms of a single peripheral resonance trajectory $b_0(s)$: at high energies the meaning of $\text{Re } b_0(s)$ is the interaction radius and $\text{Im } b_0(s)$ is connected to the interaction time and controls shrinkage. Since there is only one trajectory $b_0(s)$ in the model, its predictions appear to be very restrictive.

In Ref. 1) the predictions were already quantitatively tested in a sample of suitable meson baryon reactions. The results were very encouraging.

In this paper we shall adopt a more global point of view: we establish general qualitative rules on how to look at experimental data of inelastic two-body reactions from the s channel point of view. In more detail: without numerical calculations we describe the expected structure (dips, bumps, their positions and depths) and shrinkage of differential cross-sections, the structure of polarizations and density matrix elements. We illustrate the agreement of these qualitative rules with experiment by representative examples.

The paper consists of three main parts. The first section gives a short summary of the ideas which led to the s channel picture and the results which emerged from it ¹⁾.

The second main section deals with the most transparent predictions, those for exotic reactions, i.e., for reactions which do not have resonances in the s or in the u channel. We illustrate these predictions by the experimental examples $K^-p \rightarrow \bar{K}^0 n$, $K^\pm p \rightarrow K^*(890)N$, $K^*(1420)N$, $K \Delta(1236)$ and $np \rightarrow pn$, $pp \rightarrow \Delta(1236)N$.

In the third part the predictions are extended to non-exotic reactions. An important test for these predictions is the amplitude analysis ^{2),3)} of $\pi^-p \rightarrow \pi^0 n$. Further illustration is given by the reactions $\pi^-p \rightarrow \eta n$, $\eta^0 n$, $\pi^+p \rightarrow \eta \Delta^{++}(1236)$ and many others.

2. s CHANNEL PICTURE

As is well known, experimentally the imaginary parts of inelastic impact parameter amplitudes seem to be peaked ⁴⁾ at a common value of the impact parameter $b \approx 1$ fermi at high energies. Harari ⁵⁾ successfully explained this feature by the dominance of peripheral s channel resonances being excited mainly at the edge of an interaction sphere with radius $R \approx 1$ fermi.

Our aim was to pursue the idea of peripheral resonance dominance further. The basic question was the following: to which extent is the high energy amplitude dominated by one effective, peripheral Regge pole $\alpha_{\text{eff}}(s)$ in the s channel? What do we need in addition?

Therefore, we started from an effective, complex s channel Regge pole $\alpha(s)$, to be associated with the peripheral s channel resonances. Its contribution to the partial wave amplitudes is:

$$f_{\frac{1}{2}}^{\text{s-pole}}(s) \propto \frac{f(s)}{(j - \alpha(s))(j + 1 + \alpha(s))} \quad (1)$$

[the factor $1/(j+1+\alpha(s))$ is due to a reggeization of resonance poles]. We feel that at high energies the impact parameter plane is much more adequate for the implementation of geometrical ideas: the typical low energy notion of spins and widths of single resonances is replaced by the more relevant notion of an interaction radius ^{*)} and an interaction time. So, at high energies we preferred to "translate" the contribution (1) of the peripheral resonances into the b language

$$h^{\text{s-pole}}(b, s) \propto \frac{R(s)}{b^2 - b_0^2(s)} \quad (2)$$

by making use of the well-known substitution law, valid for large s

$$j + \frac{1}{2} \rightarrow k \cdot b, \alpha(s) + \frac{1}{2} \rightarrow k \cdot b_0(s), f_{\frac{1}{2}}(s) \rightarrow h(b, s) \quad (3)$$

with $k = \text{c.m.s. momentum}$.

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*) This also obviously induced Harari ⁵⁾, to formulate his ideas in the impact parameter language.

In the Appendix we show in detail that asymptotically for $s \rightarrow \infty$, t fixed, the two formulations (1) and (2) lead to equivalent results for the scattering amplitude $T(s, t)$

$$\left[- \frac{P_{\alpha}(s) (-\cos \Theta_s)}{\sin \pi \alpha(s)} \longrightarrow i H_0^{(1)}(b_0(s) \sqrt{-t}) \right]$$

Moreover, we show there that the results from ansatz (1) and ansatz (2) are practically equal for $\sqrt{s} \gtrsim 3$ GeV and $|t| < s/2$, i.e., at rather low energies already.

Thus, in assuming ansatz (2) for large energies, we rather considered the peripheral s channel resonances to be represented by a moving pole $b_0(s)$ in the complex b plane of the impact parameter amplitude $h(b, s)$.

In order to identify that part of the high energy amplitude, which is dominated by the pole contribution

$$T^{s\text{-pole}}(s, t) = \int_0^{\infty} b \, db \, f_0(b \sqrt{-t}) h^{s\text{-pole}}(b, s), \quad (4)$$

we took the duality content of the Veneziano formula⁶⁾ as a guide. We found the following correspondence

$$\begin{aligned} T^{s\text{-pole}}(s, t) &\longleftrightarrow V(s, t) \text{ in the Veneziano formula} \\ &\longleftrightarrow T^{\text{rot.}} = \text{"rotating term" in the } t \text{ channel} \\ &\hspace{10em} \text{Regge language} \\ &\longleftrightarrow T^{\text{r.h.c.}} = \text{right-hand cut term in a fixed } t \text{ dispersion relation} \end{aligned} \quad (5)$$

$$[V(s, u) \rightarrow 0 \text{ for } s \rightarrow \infty, t \text{ fixed}]$$

and the necessity for an additional real background term of leading order corresponding to

$$V(u, t) \longleftrightarrow T^{\text{real}} = \text{"real term"} \longleftrightarrow T^{\text{l.h.c.}} = \text{left hand cut term} \quad (6)$$

respectively. Thus, from the u - t Veneziano term we abstract that the real background term is the analytic continuation of the contribution of the u channel peripheral resonances.

Finally, in our s channel approach for large s and t fixed the amplitude may be written as a sum of two terms

$$T(s, t) = T^{s\text{-pole}}(s, t) + T^{u\text{-pole}}(s, t) \quad (7)$$

$T^{s\text{-pole}}$ is dominated in its real and imaginary parts by the peripheral s channel resonances, $T^{u\text{-pole}}$ is real and represents the effect of the peripheral u channel resonances in the s channel. This may be schematically represented as in Fig. 1a.

The impact parameter projection of $T(s, t)$ looks typically as represented in Fig. 1b: $T^{s\text{-pole}}$ yields a Breit-Wigner type distribution and $T^{u\text{-pole}}$ a real central contribution.

Finally the analytic form of the full amplitude

$$T(s, t) = \int_0^\infty b \, db \, \gamma_0(b \sqrt{-t}) h(b, s) \quad (8)$$

may be found from Eqs. (2), (7). In a discussion ¹⁾ including also spin the general result reads for the s channel helicity amplitudes

$$T_{\Delta\lambda=n}(s, t) \stackrel{s \text{ large, } t \text{ fixed} \neq 0}{=} \underbrace{R_n(s) i H_n^{(1)}(b_0(s) \sqrt{-t})}_{T^{s\text{-pole}}} + \underbrace{\bar{R}_n(u) i H_n^{(1)}(b_0(u) \sqrt{-t})}_{T^{u\text{-pole}}} \quad (9)$$

(taking into account that the s - u crossing matrix for helicity amplitudes is diagonal for large s). $\Delta\lambda = n$ is the total helicity flip in the s channel. $H_n^{(1)}(z)$ are the Hankel functions ^{*} of order n defined by

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^{*}) We ignore the log-singularity of the Hankel functions ⁷⁾ [cf., Ref. 1)] at $t=0$ in this paper, since all our investigations will be confined to $-t \gtrsim 0.25(\text{GeV}/c)^2$.

$$i H_n^{(1)}(z) = -Y_n(z) + i J_n(z) \quad (10)$$

(Y=Neumann functions, J=Bessel functions).

Let us emphasize that the Hankel functions provide the amplitudes (9) with physical s, t, u, cuts where the t cut starts at t=0, however.

The residue functions R_n and $\bar{R}_n$ have the following properties:

- i) the behaviour under s-u crossing is made explicit by the decomposition

$$R_n(s) = R_n^{(+)}(s) + R_n^{(-)}(s), \quad (11)$$

$$\bar{R}_n(u) = R_n^{(+)}(u) - R_n^{(-)}(u) \quad (12)$$

with $R_n^{(\pm)}(s) \equiv 0$ for the pure s-u crossing (odd) case;
even

- ii) hermitian analyticity requires ¹⁾ for $u < 0$

$$\bar{R}_n(u) \text{ to be real for even } n \text{ and imaginary for odd } n \quad (13)$$

- iii) in order to build in the Harari limit ⁵⁾

$$" \text{Im } T_n \propto_{\text{Re } b_0} \gg \text{Im } b_0 \cdot J_n(\text{Re } b_0(s) \sqrt{-t}) "$$

(at least for $p_{\text{lab}} \lesssim 20 \text{ GeV/c}$) we assumed

$$R_n(s) \text{ to be (at least approximately) real in this energy range.} \quad (14)$$

A second ingredient in our s channel picture is a plausible ansatz for the analytic form of $b_0(s)$ for large values of $|s|$. Actually, hermitian analyticity, power law behaviour for $s \rightarrow \infty$, t fixed and a detailed comparison with the data led us to a form ¹⁾:

$$b_0(s) \underset{|s| \text{ large}}{=} i a \log \frac{-s}{s_0} \quad (15)$$

with $a \approx 1.6(\text{GeV}/c)^{-1}$, as determined from the amount of shrinkage in high energy data, such that

$$b_0(s) \underset{|s| \text{ large}}{=} \begin{cases} a\pi + ia \log \frac{s}{s_0} & \text{for } s > 0 \\ ia \log \frac{|s|}{s_0} & \text{for } s < 0. \end{cases} \quad (16)$$

This trajectory very naturally

- i) produces the expected constant behaviour and the correct magnitude of

$$\text{Re } b_0(s) \approx a\pi \approx 5 (\text{GeV}/c)^{-1} \approx 1 \text{ fermi}$$

[Notice the crucial interlocking of $\text{Re } b_0(s)$ and $\text{Im } b_0(s)$ by the common parameter a .]

- ii) induces the u term $T^{u\text{-pole}}$ in Eq. (9) to be real [see also the property (13)].

Translated into the J language according to the substitution (3) we obtain an effective J plane trajectory

$$\alpha(s) \underset{|s| \text{ large}}{=} -\frac{a}{2} \sqrt{-s} \log \frac{-s}{s_0} \quad (17)$$

which is negative for negative s and behaves like

$$\alpha(s) \underset{s \text{ large}}{=} \frac{a}{2} \pi \sqrt{s} + i \frac{a}{2} \sqrt{s} \log \frac{s}{s_0} \quad (18)$$

for large positive s .

In Ref. 1) we made sure by checking against experiment in case of meson-baryon s channels that the baryon trajectory $b_0(s)$ is essentially universal ($s_0 \simeq 4.3 \text{ GeV}^2$) at high energies and that it extrapolates down smoothly to the peripheral resonances.

3. APPROXIMATION FOR THE SCATTERING AMPLITUDES

In the following sections we shall derive extremely simple qualitative rules for the behaviour of measurable quantities in two-particle inelastic reactions. The main point of interest will be the t dependence. The key to these rules is a simple approximation of the Hankel functions for large values of their argument [see Ref. 1)]:

$$i H_n^{(1)}(b_0(s)\sqrt{-t}) \propto \frac{1}{\sqrt{b_0(s)}} \frac{1}{\sqrt{-t}} e^{-\text{Im} b_0(s)\sqrt{-t}} e^{i(\text{Re} b_0(s)\sqrt{-t} + \frac{\pi}{4})} e^{-i\frac{\pi}{2}n} \times \left(1 + O\left(\frac{1}{b_0(s)\sqrt{-t}}\right)\right) \quad (19)$$

which appears to be quantitatively good for $-t \gtrsim 0.25 \text{ (GeV/c)}^2$ for $n \lesssim 2$.
Accordingly all following considerations will be confined to $|t| \gtrsim 0.25 \text{ (GeV/c)}^2$.

The nicest property of the approximation (19) is that it factorizes into a term

$$e^{-\text{Im} b_0(s)\sqrt{-t}} \approx \left(\frac{|s|}{s_0}\right)^{-a\sqrt{-t}} \text{ for large } |s| \quad (20)$$

depending only on the imaginary part of the trajectory and a term

$$e^{i \text{Re} b_0(s)\sqrt{-t}} \approx \begin{cases} e^{i\pi a\sqrt{-t}} & \text{for large positive } s \\ 1 & \text{for large negative } s \end{cases} \quad (21)$$

being determined by its real part only. Both terms simulate, in a dual manner, typical features of a t channel Regge behaviour [see Ref. 1]. Obviously the term (20) is responsible for shrinkage and the term (21) for the s independent structure in differential cross-sections, polarizations, etc.

In the following we shall consistently use the high energy approximation $u \simeq -s$ [applied already in Eqs. (20) and (21)] valid for fixed t .

4. PREDICTIONS FOR REACTIONS WITH EXOTIC s OR u CHANNEL

Let us start with the simplest and most transparent predictions of our model: those for reactions which do not exhibit ^{*}) any resonances (i.e., are exotic) in the s or in the u channel. An exotic s (u) channel is described by

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^{*}) According to this definition of exotic channels reactions like hypercharge exchange reactions (which are exotic only from duality diagram arguments) are excluded from this discussion.

$$R_n(s) \equiv 0 \quad \text{and} \quad \bar{R}_n(u) \equiv 0 \quad \text{respectively} \quad (22)$$

in Eq. (9). As usually done we shall ignore in the following predictions the experimental evidences for resonances with exotic quantum numbers. Any possible failure of these predictions has to be attributed first to the presence of such exotic resonances.

From the approximation (19) we deduce for both types of reactions the common property

$$\sqrt{-t} \frac{d\sigma}{dt} \approx e^{-2 \operatorname{Im} b_0(s) \sqrt{-t}} \times \text{some function of energy} \quad (23)$$

For reactions with exotic u channel we have

$$T_{\Delta\lambda=n} \approx \frac{R_n(s)}{R_0(s)} e^{-i \frac{\pi}{2} n} T_{\Delta\lambda=0} \quad (24)$$

with real, t independent (!) coefficients $R_n(s)/R_0(s)$, the phase differences being given by $e^{-i(\pi/2)n}$.

Analogously we derive for reactions with exotic s channel

$$T_{\Delta\lambda=n} \approx \frac{\bar{R}_n(-s) (-i)^n}{\bar{R}_0(-s)} T_{\Delta\lambda=0} \quad (25)$$

with the t independent coefficients

$$\bar{R}_n(-s) (-i)^n / \bar{R}_0(-s)$$

being real according to the result (13).

From Eqs. (23)-(25) we derive the following predictions for the t dependence of measurable quantities, valid for $|t| \gtrsim 0.25(\text{GeV}/c)^2$:

- i) $\log(\sqrt{-t}(d\sigma/dt))$ plotted versus $\sqrt{-t}$ is a straight line with slope = $-2 \operatorname{Im} b_0(s)$. All reactions with the same initial particles have the same type of intermediate resonances; correspondingly we expect them to have the same slope at a given energy. [To the (qualitative) extent that $b_0(s)$ is universal this is in fact correct for all reactions.]

- ii) Polarization (confined to $0\frac{1}{2} \rightarrow 0\frac{1}{2}$ reactions): $P = -(2 \operatorname{Im} T_0 T_1^*) / (|T_0|^2 + |T_1|^2)$
exotic u channel

$$P \approx \text{constant} = - \frac{2 R_0 R_1}{R_0^2 + R_1^2}. \quad (26)$$

This allows for a determination of the ratio of the non-flip and flip residues.

exotic s channel

$$P \equiv 0. \quad (27)$$

- iii) The density matrix elements $\rho_{\lambda\mu}^H$ of any produced resonance in the helicity system have the following properties:

- a) exotic s or u channel: in both cases:

$$\rho_{\lambda\mu}^H \approx \text{constant} \quad (28)$$

- b) exotic u channel

$$\begin{array}{ll} \rho_{\lambda\mu}^H & \text{is real} \quad \text{if } \lambda - \mu \text{ is even} \\ & \text{is imaginary} \quad \text{if } \lambda - \mu \text{ is odd} \end{array} \quad (29)$$

Particular examples (being relevant if the decay angular distribution of the resonance is measured only):

$$\begin{aligned} \operatorname{Re} \rho_{10}^H &\approx 0 \quad (\text{e.g. for } \bar{K}^*(890) \text{ in } K^- p \rightarrow \bar{K}^* N) \\ \operatorname{Re} \rho_{\frac{1}{2}\frac{1}{2}}^H &\approx 0 \quad (\text{e.g. for } \Delta(1236) \text{ in } K^- p \rightarrow \bar{K} \Delta) \\ \operatorname{Re} \rho_{10}^H &\approx \operatorname{Re} \rho_{21}^H \approx \operatorname{Re} \rho_{2-1}^H \approx 0 \\ &(\text{e.g. for } \bar{K}^*(1420) \text{ in } K^- p \rightarrow \bar{K}^* N) \end{aligned} \quad (30)$$

- c) exotic s channel: all elements

$$\rho_{\lambda\mu}^H \text{ are real} \quad (31)$$

Notice: the predictions (26), (28) of constancy are rather basic predictions of our model, while the phase predictions (27), (29)-(31) furthermore involve the reality assumption (14). All predictions, however, crucially depend on the unimportance of exotic resonances.

5. EXAMPLES FOR REACTIONS WITH EXOTIC s OR u CHANNEL

The selection of examples for illustration of the predictions of Section 4 has been performed according to the following criteria:

- a) sufficiently good statistics;
- b) data available up to $-t \simeq 1(\text{GeV}/c)^2$ at least.

Differential cross-section [ad. Section 4 - i)]

$\sqrt{-t}(dG/dt)$ was plotted logarithmically versus $\sqrt{-t}$ for the following reactions

$$pp \rightarrow \left. \begin{array}{l} \Delta^{++}(1236) n \\ \Delta^+(1236) p \end{array} \right\} \begin{array}{l} \text{between 6.6 and 16.9 GeV/c} \\ \text{(see Fig. 2)} \end{array} \quad (32)$$

$$np \rightarrow pn \quad \begin{array}{l} \text{between 15 and 27.3 GeV/c} \\ \text{(see Fig. 3)} \end{array} \quad \text{and} \quad (33)$$

the set of charge-exchange reactions

$$K^- p \rightarrow \left. \begin{array}{l} \bar{K}^0 n \\ \bar{K}^{*0}(890) n \\ \bar{K}^{*0}(1420) n \end{array} \right\} \quad (34)$$

at 3.9 and 10 GeV/c (see Fig. 4).

We reach the following conclusions.

- i) A nice straight line behaviour is seen in case of all reactions (32)-(34).
- ii) Independence of the slope $-2 \text{Im} b_0$ from the final states is verified by comparing the three $K^- p$ charge exchange final states (34) with each other at the same energy. Thus the same simple s channel content [equal peripheral baryon resonance trajectory $b_0(s)$] connects reactions having a different and partly rather involved t channel content.

- iii) In the cases (32) and (33) we consider for the first time in our model reactions which involve a meson trajectory: they are exotic in the s channel and have meson resonances in the u channel, which enter our formula (9) via their effective peripheral trajectory

$$b_0^{\text{mes.}}(u) \approx b_0^{\text{mes.}}(-s). \quad (35)$$

We determined the slopes $[-2 \text{Im} b_0(s)]$ of the reactions (32), (33) at various energies. We found first of all shrinkage in agreement with a behaviour (15). Moreover all values of $\text{Im} b_0(s)$ nicely fit onto the trajectory of the πN initiated baryonic resonances as determined in Ref. 1). This is shown in Fig. 5. Thus, at least for sufficiently high energies ($p_{\text{lab}} \gtrsim 7 \text{ GeV/c}$) our conjecture of one universal peripheral trajectory $b_0(s)$ seems to have found new support.

Polarization [ad. Section 4 - ii)]

A measurement of the K^-p CEX polarization has been performed ⁸⁾ at 8 GeV/c which is still being analyzed. The deviation of the polarization in np charge exchange ⁹⁾ from our prediction $P \equiv 0$ is a measure for the existence of exotic resonances in the s channel. This is a problem shared with all dual resonance models neglecting the existence of exotic resonances.

Density matrix [ad. Section 4 - iii)]

We display data for the density matrix elements of $K^*(890)$ at $p_{\text{lab}} = 3.9-5.5 \text{ GeV/c}$ for the two reactions with exotic u channel

They are compatible with a constant behaviour for $|t| \gtrsim 0.25 (\text{GeV/c})^2$. The prediction $\text{Re } g_{10}^H \equiv 0$ is confirmed experimentally very well in case of both reactions.

As examples for an exotic s channel we chose the density matrix elements of $K^*(890)$ in

$$\begin{aligned}
 & K^+ p \rightarrow K^{*+} (890) p \\
 & \text{and those of } \Delta(1236) \text{ in} \\
 & K^+ p \rightarrow K^0 \Delta^{++} (1236)
 \end{aligned} \tag{37}$$

at two energies 1.4-1.7 GeV/c and 4.3 GeV/c, displayed in Figs. 7a, b, respectively. These data show a nice constant behaviour as predicted. Note that the data in Fig. 7a extend over the whole angular range.

6. AMPLITUDES IN THE GENERAL CASE

Let us now insert the approximation (19) of the Hankel functions into the general form (9) of the s channel helicity amplitudes $T_{\Delta\lambda=n}$:

$$\begin{aligned}
 T_{\Delta\lambda=n} \approx & \frac{1}{\sqrt{-t}} \left(\frac{s}{s_0} \right)^{-a\sqrt{-t}} \frac{1}{\sqrt{a \log \frac{s}{s_0}}} u_n(s) \times \\
 & \times \left[1 + v_n(s) \sqrt{\frac{\log s/s_0}{\pi + i \log s/s_0}} e^{-i\pi \left(\frac{n}{2} - \frac{1}{4} - a\sqrt{-t} \right)} \right]
 \end{aligned} \tag{38}$$

where

$$\sqrt{\frac{\log s/s_0}{\pi + i \log s/s_0}} \approx \text{real}, > 0 \quad \text{for } p_{\text{lab}} \lesssim 20 \text{ GeV/c},$$

$u_n(s)$ is real according to (13) and

$$v_n(s) = \frac{R_n(s)}{\bar{R}_n(-s) e^{-i\pi n/2}} \approx \text{real according to (13) and the Harari limit (14)}$$

Obviously the moduli of the amplitudes (38) show a dip or a bump at positions where the imaginary part vanishes, i.e., at

$$\sin \pi \left(a\sqrt{-t} + \frac{1}{4} - \frac{n}{2} \right) \approx 0 \tag{39}$$

This is already known since Harari⁵⁾. Notice: these positions are independent of $v_n(s)$. The question of whether there is either a dip or a bump at a given position involves the knowledge of the sign of $v_n(s)$ only. It is then finally the depth of the dip or the height of the bump which actually depends on the magnitude of $v_n(s)$.

What can be said about the signs of $v_n(s)$ for different reactions? The answer requires some more discussion and assumptions about the residues $R_n(s)$. Let us confine to $\Delta\lambda = n = 0, 1$, here.

To the extent that $R_0(s)$ is real for positive s [Harari's limit (14)] and for negative s [real analyticity (13)] $R_0(s)$ is a polynomial ^{*)} in s , since it, moreover, has to be bounded by a power of s ; so we write (Froissart bound!)

$$R_0(s) = -\alpha_0 s + \beta_0 = -\alpha_0^{(+)} s + \beta_0^{(+)} - \alpha_0^{(-)} s + \beta_0^{(-)},$$

hence

$$\overline{R_0}(-s) = \alpha_0^{(+)} s + \beta_0^{(+)} - \alpha_0^{(-)} s - \beta_0^{(-)}, \quad (40)$$

where the crossing $\begin{Bmatrix} \text{even} \\ \text{odd} \end{Bmatrix}$ decomposition $\{\pm\}$, Eqs. (11), (12) have been inserted. The real analyticity constraint (13) tells us that in contrast with $R_0(s)$, $R_1(s)$ has to have a branch point which is common to all reactions. In order to treat $R_1(s)$ on the same footing as $R_0(s)$, we plug this common singularity into a function $\vartheta(s)$ and write

$$R_1(s) = \vartheta(s) \tau_1(s) \quad (41)$$

with

$$\frac{\vartheta(s)}{\overline{\vartheta}(-s) e^{-i\frac{\pi}{2}}} = C(s), \quad (42)$$

where $C(s)$ is essentially real and

$$\tau_1(s) = -\alpha_1 s + \beta_1 = -\alpha_1^{(+)} s + \beta_1^{(+)} - \alpha_1^{(-)} s + \beta_1^{(-)}, \quad (43)$$

$$\text{hence} \quad \overline{\tau_1}(-s) = \alpha_1^{(+)} s + \beta_1^{(+)} - \alpha_1^{(-)} s - \beta_1^{(-)}.$$

The unknown sign of $C(s)$, defined by Eq. (42), which is common to all reactions, is normalized, once for all reactions, by the dip structure in $\pi^-_p \rightarrow \pi^0_n$ to be

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*) Notice that the physical s and u cuts are already contained in the Hankel functions.

$$C(s) > 0 \quad (44)$$

In Table 1 we present $v_{n=0,1}(s)$ for different physical constellations. The expressions "dominantly leading" and "dominantly non-leading" refer to the dominance of the leading $(-\alpha_n)$ and non-leading (β_n) s powers respectively in the polynomials (40) and (43) at a given energy.

	dominantly leading	dominantly non-leading
$v_n = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} (s) \approx$	$-\begin{Bmatrix} 1 \\ C(s) \end{Bmatrix} \times \frac{\alpha_n^{(+)} + \alpha_n^{(-)}}{\alpha_n^{(+)} - \alpha_n^{(-)}}$	$\begin{Bmatrix} 1 \\ C(s) \end{Bmatrix} \times \frac{\beta_n^{(+)} + \beta_n^{(-)}}{\beta_n^{(+)} - \beta_n^{(-)}}$
crossing even > crossing odd	< 0	> 0
crossing odd > crossing even	> 0	< 0
pure crossing even	$-\begin{Bmatrix} 1 \\ C(s) \end{Bmatrix}$	$+\begin{Bmatrix} 1 \\ C(s) \end{Bmatrix}$
pure crossing odd	$+\begin{Bmatrix} 1 \\ C(s) \end{Bmatrix}$	$-\begin{Bmatrix} 1 \\ C(s) \end{Bmatrix}$

TABLE 1

To sum up polynomial behaviour of the residues $R_0(s)$ and $r_1(s)$ together with the sign normalization $C(s) > 0$ led to a complete classification of the structure in the amplitudes.

7. APPLICATION TO THE $\pi^-_p \rightarrow \pi^0_n$ AMPLITUDE ANALYSIS

One should certainly test the form of our amplitudes (38) in the most clear-cut case of a crossing symmetric or antisymmetric reaction. there the interference between the real and rotating terms is maximal. We choose the crossing odd reaction $\pi^-_p \rightarrow \pi^0_n$, where model independent amplitude analyses ^{2),3)} at 6 GeV/c are available.

In order to make the discussion most transparent let us fix the pole trajectory from the very beginning to be

$$b_0(s) = 1 \text{ fm} + i \frac{1}{\pi} \log \frac{s}{4.7} \text{ fm}, \quad (45)$$

i.e. $a = 1.59 (\text{GeV}/c)^{-1}$, $s_0 = 4.7 \text{ GeV}^2$

as taken from Ref. 1).

Let us first list our predictions for $|t| \gtrsim 0.25 (\text{GeV}/c)^2$, $u \simeq -s$:

Amplitudes

$$T_{\Delta\lambda=0} \simeq \frac{1}{\sqrt[4]{-t}} \left(\frac{s}{s_0} \right)^{-a\sqrt{-t}} \frac{1}{\sqrt{a \log s/s_0}} u_0(s) \times$$

$$\times \left[1 - w_0(s) g(s) e^{-i\pi(\frac{3}{4} - a\sqrt{-t})} \right], \quad (46)$$

$$T_{\Delta\lambda=1} \simeq \frac{1}{\sqrt[4]{-t}} \left(\frac{s}{s_0} \right)^{-a\sqrt{-t}} \frac{1}{\sqrt{a \log s/s_0}} u_1(s) \times$$

$$\times \left[1 - w_1(s) q(s) e^{-i\pi(\frac{5}{4} - a\sqrt{-t})} \right] \quad (47)$$

with $g(s) = \sqrt{\frac{\log s/s_0}{\pi + i \log s/s_0}} \simeq |g(s)| \left[= 0.54 \text{ at } 6 \text{ GeV}/c \right],$

$$w_0(s) = \frac{\alpha_0^{(-)} - \beta_0^{(-)}/s}{\alpha_0^{(-)} + \beta_0^{(-)}/s} \quad \text{expected to be near } 1$$

("dominantly leading"),

$$w_1(s) = \frac{\alpha_1^{(-)} - \beta_1^{(-)}/s}{\alpha_1^{(-)} + \beta_1^{(-)}/s} \quad C(s) > 0 \quad \text{"dominantly leading"}.$$

t structure of the amplitudes

$\text{Im } T_{\Delta\lambda=0}$ has a zero at $\frac{3}{4} - a\sqrt{-t} = 0$, i.e. at $-t \simeq 0.22 (\text{GeV}/c)^2$
 while $|T_{\Delta\lambda=0}|$ has a dip there.
 $\text{Im } T_{\Delta\lambda=1}$ has a zero at $\frac{5}{4} - a\sqrt{-t} = 0$, i.e. at $-t \simeq 0.62 (\text{GeV}/c)^2$
 while $|T_{\Delta\lambda=1}|$ has a dip there.

Zeros in the polarization

The position of zeros in the polarization

$$P = - \frac{2 \text{Im}(T_0 T_1^*)}{|T_0|^2 + |T_1|^2}$$

is given by

$$-w_0 \sin \varphi + w_1 \cos \varphi - w_0 w_1 q = 0 ,$$

$$\text{with } \varphi = \pi (3/4 - a\sqrt{-t}) . \quad (48)$$

A simple estimate with $w_0 \simeq w_1 \simeq 1$ leads to a zero at $t \simeq -0.75 (\text{GeV}/c)^2$.

Confrontation with experiment

In order to be able to quantitatively compare the predictions (46), (47) for the amplitudes with the result of the amplitude analyses ^{2),3)} at 6 GeV/c we performed a fit to that of Ref. 3) for $0.25 \leq |t| \leq 1.0 (\text{GeV}/c)^2$. The trajectory $b_0(s)$ (i.e., a and s_0) was fixed as given in Eq. (45). This leaves us with the four free parameters u_0, u_1, w_0, w_1 altogether.

The amplitude analyses ^{2),3)} provide the moduli of the two helicity amplitudes and their phase difference. Figs. 8a-d show the result of the fit. We find a good agreement in case of the moduli (Fig. 8a). The structure of the phase difference (see Fig. 8b) is well reproduced, even though it seems to be systematically a little bit too small. Figs. 8c, d show how the observables $d\sigma/dt$ and P are described by the fit solution. The parameters chosen by the fit are:

$$u_0 = 0.29 \sqrt{mb}/\text{GeV}/c \quad u_1 = 0.41 \sqrt{mb}/\text{GeV}/c$$

$$w_0 = 1.14 \quad w_1 = 1.74 . \quad (49)$$

Note that w_0 actually is near to 1 as expected (for a "dominantly leading" solution). The agreement is altogether very satisfactory; especially if we remember the various approximations we introduced: i) the approximation for the Hankel functions, ii) $u \simeq -s$ up to $t = -1(\text{GeV}/c)^2$ and iii) a diagonal s - u crossing matrix, which is violated to order $\mathcal{O}(1/s)$.

Consequences for $\pi^- p \rightarrow \eta n$

From the previous analysis we also learn something about a "dominantly leading", crossing symmetric reaction, where $w_0(s) \rightarrow \simeq -w_0(s)$, $w_1(s) \rightarrow \simeq -w_1(s)$. We then expect

- i) a bump ^{*)} in $|T_{\Delta\lambda=0}|$ at $t \simeq -0.22(\text{GeV}/c)^2$;
- ii) a bump ^{*)} in $|T_{\Delta\lambda=1}|$ at $t \simeq -0.62(\text{GeV}/c)^2$
- iii) a zero in the polarization, where

$$w_0 \sin \varphi - w_1 \cos \varphi - w_0 w_1 \cdot g = 0$$

with

$$\varphi = \pi (3/4 - a\sqrt{-t})$$

i.e., at $t \simeq -1 (\text{GeV}/c)^2$ for the values of w_0 , w_1 given in (49) and no zero in $0.25 \leq |t| < 1(\text{GeV}/c)^2$.

8. STRUCTURE OF DIFFERENTIAL CROSS-SECTIONS IN NON-EXOTIC REACTIONS

The relative sign between the real and rotating terms in the amplitudes, as displayed in Table 1, leads to the natural classification of the dip and bump structure in differential cross-sections presented in Table 2. There we also gave a connection to the t channel quantum numbers exchanged.

- 1) The crossing even (odd) part of the amplitude corresponds to signature $\zeta_t = +1$ (-1) exchange in the t channel.
- 2) We very tentatively suggest that the leading (non-leading) s channel contribution corresponds asymptotically to the t channel naturality $\eta_t = +1$ (-1). The motivation was mainly the empirical fact that $\eta_t = -1$ trajectories seem to lie systematically lower than $\eta_t = +1$ trajectories.

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^{*)} If typically plotted $\sqrt[4]{-t} |T|$ versus $\sqrt{-t}$.

Table 2 applies to either of the two cases: flip dominance or non-flip dominance. The case where both of these amplitudes are of comparable size leads also to a transparent result: the positions of the extrema (dips and bumps) of the differential cross-section - or to be exact of $s^{2a\sqrt{-t}}$ $\sqrt{-t}(d\sigma/dt)$ - may be calculated from the general amplitudes (38) to be given by

$$\operatorname{tg}(\pi a \sqrt{-t}) = \frac{\gamma - 1}{\gamma + 1}$$

with the mixing ratio

$$\gamma = \frac{v_1(s) u_1^2(s)}{v_0(s) u_0^2(s)} = \frac{R_1(s) \bar{R}_1(-s) e^{-i\frac{\pi}{2}}}{R_0(s) \bar{R}_0(-s)} \quad (50)$$

Take as an example the leading, crossing odd case. While non-flip dominance leads to a dip at $t \simeq -0.22$, flip dominance to one at $t \simeq -0.62$, an equal mixture of both amplitudes ($\gamma \simeq 1$)

$$\text{shifts the dip to } t \simeq -0.4 \text{ (GeV/c)}^2 \quad (51)$$

Thus in principle one can tell from the position of the extrema of $d\sigma/dt$ the rough mixing ratio of the two amplitudes.

Of course also the depths of the dip (or the heights of the bump) depend on the mixing ratio. It is intuitively clear that in case of comparable contributions from both amplitudes the dips and bumps are less pronounced. So far we discussed the helicity non-flip and flip amplitudes $T_{\Delta\lambda=0,1}$ only. As soon as double helicity flip (or higher flips) give considerable contributions, the discussion is complicated very much. One of the complications lies in the identity of the Hankel functions ⁷⁾

$$H_0^{(1)}(z) + H_2^{(1)}(z) = \frac{2}{z} H_1^{(1)}(z) \quad (52)$$

i.e., a suitable combination of non-flip and double flip amplitudes might simulate the behaviour of a flip amplitude in certain density matrix elements.

As a test for our classification given in Table 2 [supplemented by the content of Eq. (50)] we collected the meson-baryon reactions with prominent dip and bump structures in Table 3. They give nice support for our classification.

9. TYPICAL PATTERN FOR DIFFERENTIAL CROSS-SECTIONS

Let us study the case where all quantities (energy, non-flip/flip, leading/non-leading) are fixed and only the ratio of the crossing even to the crossing odd contribution is varied.

Figure 9a shows the typically expected pattern for $\sqrt{-t}(d\sigma/dt)$ in the case of flip dominance. Of course the various cross-sections have to be suitably normalized. The tangents to the extrema have the slope

$$-2 \operatorname{Im} b_0(s) = -2a \log s/s_0.$$

In order to get rid of the distortion of a log-plot we also show the corresponding pattern for $s^{2a\sqrt{-t}}\sqrt{-t}(d\sigma/dt)$ in Fig. 9b (onions on a string). The two outmost curves with the "wildest" structure correspond to crossing symmetric and antisymmetric reactions. In the case "crossing even = crossing odd" (exchange degeneracy in the t channel language) the curve reduces to a straight line. This special case was discussed already in Sections 4 and 5.

The Figures 10a, b show experimental examples for such a pattern at two fixed energies for the flip dominated case. As shown in Fig. 10a at $\langle p_{\text{lab}} \rangle \simeq 3.5$ GeV/c there are data for $\pi^- p \rightarrow \pi^0 n$ (crossing odd) available for one and a half periods thus fixing the tangents to the maxima and minima. The data for $\pi^+ p \rightarrow \eta \Delta^{++}(1236)$ ($\simeq$ crossing even), suitably normalized, just fit into the strip between the two tangents. Moreover they touch the tangents at t values coincident with those of $\pi^- p \rightarrow \pi^0 n$, but with minima and maxima interchanged. These features as well as the regularity of the pattern as a function of $\sqrt{-t}$ (in contrast to $-t$!) support well our predictions. The only drawback is that the oscillation period requires $\operatorname{Re} b_0 \simeq 6$ rather than $5(\text{GeV}/c)^{-1}$. This may be due to the rather low energy ($\simeq 3.5$ GeV/c), where the backward direction is not too far.

In Figure 10b a similar situation at $\simeq 18$ GeV/c is presented. $\pi^- p \rightarrow \pi^0 n$ (crossing odd) is compared to $\pi^- p \rightarrow \eta n$ and $\pi^- p \rightarrow \rho^0 n$ (both $\simeq$ crossing even). In agreement with our prediction the two latter flip dominated reactions lie on top of each other for $|t| \gtrsim 0.25 (\text{GeV}/c)^2$,

if suitably normalized. This time $\pi^- p \rightarrow \pi^0 n$ is normalized relative to $\pi^- p \rightarrow \eta n$ according to SU(3) (i.e., divided by 3)! Moreover, the data are in agreement with the expected value $\text{Re } b_0 \simeq 5 (\text{GeV}/c)^{-1} \simeq 1 \text{ fm}$.

Let us add one remark: the pattern discussed above is in its general formulation quantitatively correct only to the extent that $b_0(s)$ is universal (i.e., only for sufficiently high energies). It should, however, hold best for sets of reactions of the type "fixed initial state $\rightarrow$ different final states". The examples chosen in Figs. 10a, b, are exactly of this type: they all have a πN initial state.

10. DISCUSSION

As was pointed out already in Section 3 our s channel picture leads to high energy amplitudes (at fixed t) which bear, in a dual manner, some essential features of a t channel Regge pole term. They contain a factor

$$\left(\frac{s}{s_0} \right)^{\alpha_1(t)} \quad (53)$$

and a rotating and real term proportional to

$$e^{-i\pi\alpha_2(t)} \quad \text{and} \quad 1 \quad (54)$$

respectively.

But there are major differences to a simple minded t channel Regge pole picture. These differences seem to remedy the Regge pole deficiencies since they actually cope very well with the experimental situation. From the large amount of experimental material which we successfully understand (see Figs. 2-8, 10) the following conclusions seem to be well supported.

a) Universality

For t not too close to the forward direction, i.e., $|t| \gtrsim 0.25(\text{GeV}/c)^2$, all inelastic two-body reactions seem to bear very similar qualitative features at high energies. This was also observed empirically by Fox¹⁰⁾. In our model the common features are due to the asymptotically universal peripheral trajectory

$$b_0(s) \underset{|s| \text{ large}}{\approx} i a \log \frac{-s}{s_0} \quad \text{with} \quad a \approx 1.6 (\text{GeV}/c)^{-1} \\ s_0 \approx 4 - 4.7 \text{ GeV}^2 \\ = a\pi + i a \log \frac{s}{s_0} \quad \text{for } s > 0.$$

In Ref. 1) it was already verified that there is essentially one single baryonic trajectory; that the meson trajectory joins it (for $p_{\text{lab}} \gtrsim 7 \text{ GeV}/c$) was made sure in Section 5, Fig. 5.

The consequences are:

- i) shrinkage (in our sense) is asymptotically universal;
- ii) the structure of $d\sigma/dt$, P , $\rho_{ik}^* d\sigma/dt$ is universal in the sense that it only depends on (t - and probably also s -independent) coefficients, which determine the relative strength of the rotating and real terms in the amplitudes and the relative strength of the different helicity amplitudes. This is in detail contained in Table 2 and Fig. 9 and successfully compared with experiment in Table 3.

These universal properties have to hold even quantitatively in case of reactions with common initial states. In Fig. 4 some reactions with common K^-p initial state and in Figs. 10a, b, some with common πN initial state were compared and the expected universal features were found.

(In contrast to the region, where we expect universality to hold, the nearforward direction is strongly influenced by t channel unitarity which in general induces non-universal structure there.)

b) Different phases for different helicity amplitudes

In our model (for $|t| \gtrsim 0.25 \text{ GeV}/c$) the phase difference between the rotating part of the s channel helicity n -flip amplitude and that of the non-flip amplitude is (at least for $p_{\text{lab}} \lesssim 20 \text{ GeV}/c$)

$$\varphi_n^{\text{rot}} - \varphi_0^{\text{rot}} \approx -n \frac{\pi}{2}, \quad (55)$$

which allows for non-vanishing polarizations. This phase difference was tested in $K^-p \rightarrow \bar{K}^*(890)N$ by $\text{Re } \mathcal{P}_{10} \equiv 0$ and partly also by the constancy of the density matrix elements of K^-p initiated processes (see Fig. 6). An important test was the phase difference in $\pi^-p \rightarrow \pi^0n$ (see Fig. 8b). All these results suggest the behaviour (55).

c) Regularity of all structures in $\sqrt{-t}$ instead of $-t$

I.e., $\alpha_1(t)$, $\alpha_2(t)$ in (53) and (54) are predicted to be linear functions of $\sqrt{-t}$ for $|t| \gtrsim 0.25(\text{GeV}/c)^2$.

The linearity of $\alpha_1(\sqrt{-t})$ was demonstrated in Figs. 2, 3, 4 where $\log(\sqrt{-t}(d\mathcal{G}/dt))$ becomes a straight line if plotted versus $\sqrt{-t}$ as well as in Figs. 10a, b, where the tangents to the extrema and the line connecting the intersection points are straight lines in a similar plot.

The linearity of $\alpha_2(\sqrt{-t})$ is shown best in Fig. 10a, where the positions of the extrema appear to be equally spaced in $\sqrt{-t}$.

d) Line reversal breaking

In our model the rotating term " $e^{-i\pi\alpha_2(t)}$ " has a coefficient roughly given by

$$\frac{1}{\sqrt{\pi + i \log s/s_0}},$$

while the real term "1" has a coefficient roughly given by

$$\frac{1}{\sqrt{\log s/s_0}}.$$

For non-asymptotic energies this makes the modulus of the real term larger than that of the rotating term, especially so at small energies (2-3 GeV/c). This provides an explanation for

i) line reversal breaking, e.g., in KN CEX, where

$$\frac{d\mathcal{G}}{dt} (K^+n \rightarrow K^0p) > \frac{d\mathcal{G}}{dt} (K^-p \rightarrow \bar{K}^0n)$$

at lower and intermediate energies;

ii) that in general the moduli of the amplitudes have dips rather than the zeros of a single Regge pole expression.

Asymptotic conditions for phases and ratios rotating term/real term are then reached only for $\log s/s_0 \rightarrow \infty$ (as one would also expect in a t channel Regge pole + cut model).

Let us finally emphasize that we consider the prediction of the t dependence to be the strongest point of our approach; the s dependence and **phase** of the residues is based on somewhat more input (which might require modifications at very high energies, $p_{lab} \gtrsim 20$ GeV). In this sense the forementioned results a) and c) are more basic than b) and d).

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A P P E N D I X

In this Appendix we shall establish the approximate equivalence of an (effective) pole description in the J and b planes respectively:

$$\begin{aligned} T_J^{s\text{-pole}}(s,t) &= -\beta(s) \frac{P_{\alpha(s)}(-\cos \theta_s)}{\sin \pi \alpha(s)} \longrightarrow \\ T_b^{s\text{-pole}}(s,t) &= \beta(s) i H_0^{(1)}(b_0(s)\sqrt{-t}) \end{aligned} \quad (56)$$

$$b_0(s) = \frac{\alpha(s) + 1/2}{k}$$

in the following limit:

$$\begin{aligned} s \rightarrow \infty, \text{ i.e. } \alpha(s) \rightarrow \infty \text{ such that } \operatorname{Re} \alpha(s) \rightarrow \infty \text{ and } \arg \alpha(s) > \epsilon > 0, \\ t < 0, \text{ fixed} \end{aligned} \quad (57)$$

We take first $4k^2(\approx s) > -2t > 0$ (i.e., $1 > \cos \theta_s > 0$) and rewrite $T_J^{s\text{-pole}}$ in Eq. (56) by using the reflection formula ¹¹⁾ $(-\cos \theta \leftrightarrow \cos \theta)$ for $P_{\alpha}(-\cos \theta)$

$$T_J^{s\text{-pole}} = i\beta(s) \left\{ i \operatorname{ctg} \pi \alpha(s) P_{\alpha(s)}(\cos \theta_s) - \frac{2i}{\pi} Q_{\alpha(s)}(\cos \theta_s) \right\}. \quad (58)$$

Next we observe that in the limit (57)

$$i \operatorname{ctg} \pi \alpha(s) \rightarrow 1. \quad (59)$$

Finally, with the well-known limiting formulas ⁷⁾ ($t < 0$)

$$\begin{aligned} P_{\alpha(s)}\left(1 + \frac{t}{2k^2}\right) &\underset{\text{limit (57)}}{\sim} Y_0(b_0(s)\sqrt{-t}), \\ Q_{\alpha(s)}\left(1 + \frac{t}{2k^2}\right) &\underset{\text{limit (57)}}{\sim} -\frac{\pi}{2} Y_0(b_0(s)\sqrt{-t}) \end{aligned} \quad (60)$$

and

$$Y_0 + i Y_0 = H_0^{(1)} \quad (61)$$

we arrive immediately at Eq. (56). From Eqs. (58), (60), we, moreover, see that the $t \leftrightarrow u$ exchange term in the J plane, $T_J^{s\text{-pole}}(s,u)$ is unimportant in the limit (57):

$$T_J^{s\text{-pole}}(s,u) \underset{\text{limit (57)}}{\approx} O(e^{-Jm\alpha(s)\pi}) \times T_J^{s\text{-pole}}(s,t). \quad (62)$$

Analogously it can be shown that our conclusions remain unaltered if we start in the J plane with a Regge pole form à la Mandelstam ¹²⁾

$$T_J^{s\text{-pole}}(s,t) = \frac{2}{\pi} \beta(s) Q_{-\alpha(s)-1}(\cos \theta_s) \xrightarrow{\text{limit (57)}} T_b^{s\text{-pole}}(s,t). \quad (63)$$

The Mandelstam form [in contrast to the one in Eq. (56) provides also a suitable J plane representation of the second leading contribution: the peripheral u channel pole which gives the real non-peripheral background in the s channel (cf., Section 2). The analytic continuation into the s channel is easily done, with the result:

$$T_J^{u\text{-pole}}(s,t) = \frac{2}{\pi} \bar{\beta}(u) Q_{-\alpha(u)-1}(\cos \theta_u) \xrightarrow{\text{limit (57)}} T_b^{u\text{-pole}}(s,t) = \bar{\beta}(u) i H_0^{(1)}(b_0(u) \sqrt{-t}) \quad (64)$$

Notice that in the limit (57), $\alpha(u)$ is real, $-(\alpha(u)+1) \rightarrow +\infty$ and $\cos \theta_u > 1$.

In order to have a quantitative impression of how fast the convergence of the J and b plane descriptions actually is, we have made a numerical comparison. The result is illustrated in Fig. 11. Already for $\sqrt{s} \approx 3$ GeV we have excellent agreement of the J and b pole descriptions for $|t| \lesssim s/2$.

TABLE 2 Structure in $d\sigma/dt$ or in $\rho_{ik}^H \sqrt{s} d\sigma/dt$ [if $\sqrt{-t}(d\sigma/dt)$ plotted versus $\sqrt{-t}$]

s channel language	structure	for non-flip for flip dominance dominance	possibly to be identified with the t channel quantum numbers
a) dominantly leading crossing odd > crossing even	dip at $t \simeq -0.22$ $t \simeq -0.6$ **) at $t \simeq -1.2$ $t \simeq -2.0$ bump	$\eta_t = +1, \sigma_t = -1 > \sigma_t = +1$ b) $\eta_t = -1, \sigma_t = +1 > \sigma_t = -1$ next dip $t \simeq -3.0$ $t \simeq -4.1$	
c) dominantly leading crossing even > crossing odd	bump at $t \simeq -0.22$ $t \simeq -0.6$ **) at $t \simeq -1.2$ $t \simeq -2.0$ dip	$\eta_t = +1, \sigma_t = +1 > \sigma_t = -1$ d) $\eta_t = -1, \sigma_t = -1 > \sigma_t = +1$ next bump $t \simeq -3.0$ $t \simeq -4.1$	

Explanations: "dominantly leading (non-leading)": the term $-\alpha s$ (β) dominates the residues $-\alpha s + \beta$ at a given energy

$\sigma_t = t$ channel signature $\eta_t = t$ channel naturality

Notice: The position of the "dip" or "bump" has to be interpreted from Eqs. (38) as follows: it is not the position of an extremum in $d\sigma/dt$ in a log-plot, but the position of an extremum in $(s/s_0)^{2\alpha} \sqrt{-t} (d\sigma/dt)$. This is in other words the position, where the tangent with slope $-2 \operatorname{Im} b_0(s) = -2\alpha \log s/s_0$ touches the extrema of $\sqrt{-t}(d\sigma/dt)$ in a log-plot! For higher energies these positions differ already considerably from the simple dip and bump positions

*) ρ_{ik} in the helicity system

**) This is only to be expected for sufficiently high energies

TABLE 3 :

Reaction	Structure	s channel interpretation	Predictions for $M_{\text{target}}^2 \text{lab} > t $	Dominant t channel quantum numbers
$\left(\frac{d\sigma}{dt} \right) \begin{cases} \pi^- p \rightarrow \pi^0 n \\ \pi^- p \rightarrow \pi^- \Delta^{++} \\ \pi^+ p \rightarrow \pi^0 \Delta^{++} \\ \pi^+ p \rightarrow A_2^0 \Delta^{++} \end{cases}$ $\left(\rho_{11}^H + \rho_{1-1}^H \right) \frac{d\sigma}{dt} (\pi^+ p \rightarrow \omega \Delta^{++})$ $d\sigma/dt (\gamma N \rightarrow \pi^0 N)$	Dip at $-t_1 \simeq 0.5-0.6$	crossing odd $\gg$ crossing even leading $\gg$ non-leading flip dominated	next dip at $\sqrt{ t_2 } \simeq \sqrt{ t_1 } + 2\pi/\text{Re } b_0$ i.e., $t_2 \simeq 3.5-4.0$	$\begin{cases} \sigma_t = - \begin{pmatrix} (9) \\ (9) \\ (9) \\ (9) \end{pmatrix} \\ \eta_t = + \begin{pmatrix} (9) \\ (9) \end{pmatrix} \end{cases}$
$d\sigma/dt (\pi N \rightarrow \rho N)_{I_t=0}$ $\left(\rho_{11}^H + \rho_{1-1}^H \right) \frac{d\sigma}{dt} (\pi N \rightarrow \rho N)_{I_t=0}$ $d\sigma/dt (K^- p \rightarrow \eta \Lambda)$	Dip at $-t_1 \simeq 0.3-0.4$	crossing odd $\gg$ crossing even leading $\gg$ non-leading flip $\approx$ non-flip	next dip at $t_2 \simeq 3.0-3.5$	$\begin{cases} \sigma_t = - \begin{pmatrix} (\omega) \\ (\omega) \end{pmatrix} \\ \eta_t = + \begin{pmatrix} (\omega) \\ (K^*) \end{pmatrix} \end{cases}$
$\rho_{00}^H \frac{d\sigma}{dt} (\pi^- p \rightarrow \rho^0 n)$ $\rho_{00}^H \frac{d\sigma}{dt} (\pi^+ p \rightarrow \rho^0 \Delta^{++})$	Dip (?) at $-t_1 \simeq 0.6-0.75$	crossing even $\gg$ crossing odd non-leading $\gg$ leading flip dominated	next dip at $t_2 \simeq 3.5-4.5$	$\begin{cases} \sigma_t = + \begin{pmatrix} (\pi) \end{pmatrix} \\ \eta_t = - \begin{pmatrix} (\pi) \end{pmatrix} \end{cases}$
$\sqrt{-t} (d\sigma/dt) \begin{cases} \pi^- p \rightarrow \eta n \\ \pi^+ p \rightarrow \eta \Delta^{++} \\ \pi^- p \rightarrow \rho_{0n}^0 \\ \pi N \rightarrow A_2 N \end{cases}$	Bump at $\sqrt{-t_1} \simeq 0.78$ i.e., $-t_1 \simeq 0.6$	crossing even $\gg$ crossing odd leading $\gg$ non-leading flip dominated	next dip at $\sqrt{ t_2 } \simeq \sqrt{ t_1 } + \pi/\text{Re } b_0$ i.e., $t_2 \simeq 1.5-2.0$	$\begin{cases} \sigma_t = + \begin{pmatrix} (A_2) \\ (A_2) \\ (A_2) \end{pmatrix} \\ \eta_t = + \begin{pmatrix} (f) \end{pmatrix} \end{cases}$

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FIGURE CAPTIONS

Figure 1 :

- a) Schematical graphs representing the contribution $T^{s\text{-pole}}$ of the peripheral s channel resonances ($\sim$) and of the crossed term $T^{u\text{-pole}}$ (for large s and fixed t).
- b) Typical impact parameter distribution at $p_{\text{lab}} \simeq 6$ GeV/c. $h^{s\text{-pole}}(b,s) \propto 1/(b^2 - b_0^2(s))$ [with $b_0(s) \simeq 5 + 1.5i$] represents the peripheral s channel resonances and yields a Breit-Wigner type distribution. The contribution $h^{u\text{-pole}}(b,s)$ from the peripheral u channel resonances is large, real and central.

Figure 2 :

$\sqrt{-t} (dG/dt)$ of $pp \rightarrow \Delta^{++}(1236)n$ and $\Delta^+(1236)p$ plotted versus $\sqrt{-t}$ at energies between 6.6 and 16.9 GeV/c. The full data points [$|t| \gtrsim 0.25(\text{GeV}/c)^2$] were very well fitted by an exponential $\exp(-2 \text{Im } b_0(s) \sqrt{-t})$ leading to the straight lines in the plots. The increase of the imaginary part of the trajectory $b_0(s)$ indicates shrinkage. The data were taken from Refs. 10), 13)-16). The shaded barrier signals the lower limit $|t| \simeq 0.25(\text{GeV}/c)^2$ of the range of validity of the basic approximation (19). Data points left of this value have been drawn with open symbols.

Figure 3 :

The same plots, fits and conclusions as in Fig. 2 for $np \rightarrow pn$ at energies from 15 to 27.3 GeV/c. The data were taken from Ref. 17).

Figure 4 :

The same plots, fits and conclusions as in Fig. 2 for $K^-p \rightarrow \bar{K}^0 n$ at 3.9 and 10 GeV/c. The data of the two reactions $K^-p \rightarrow \bar{K}^{*0}(890)n$ and $K^-p \rightarrow \bar{K}^{*0}(1420)n$, having the same initial state as $K^-p \rightarrow \bar{K}^0 n$, appear to be well consistent with the slope determined from $K^-p \rightarrow \bar{K}^0 n$ at the same energy. The data were taken from Refs. 18)-22).

Figure 5 :

The imaginary part of the "universal" trajectory $b_0(s)$. The open squares represent the values from πN initiated reactions as determined in Ref. 1) $[\text{Im } b_0(s) \simeq 1.6 \log(s/4.7)]$. The full symbols represent the values determined from $pp \rightarrow \Delta N$ and $np \rightarrow pn$ in Figs. 2 and 3. They are due to a mesonic trajectory $b_0(u)$ and apparently fall nicely onto the baryonic πN line, i.e., show the same amount of shrinkage at high energies.

Figure 6 :

Density matrix elements of $K^*(890)$ in the helicity system for the reactions $K^-p \rightarrow K^{*-}(890)p$ and $K^-p \rightarrow \bar{K}^{*0}(890)n$, between 3.9 and 5.5 GeV/c. In agreement with our prediction the density matrix elements are reasonably constant for $|t| \gtrsim 0.25(\text{GeV}/c)^2$, i.e., to the right of the shaded barrier, and $\text{Re } \rho_{10}^H \simeq 0$ there. The broken line was inserted in order to guide the eye. The data were taken from Refs. 21), 23), 24).

Figure 7 :

The same as in Fig. 6 for $K^{*+}(890)$ produced in $K^+p \rightarrow K^{*+}p$ and for $\Delta^{++}(1236)$ produced in $K^+p \rightarrow K^0 \Delta^{++}$ at two energies, 1.4-1.7 GeV/c (Fig. 7a) and 4.3 GeV/c (Fig. 7b). [Second shaded barrier in Fig. 7a: backward direction minus $0.25(\text{GeV}/c)^2$]. All density matrix elements are nicely constant as predicted. The data were taken from Refs. 25), 35).

Figure 8 :

A fit of the approximative amplitudes (46), valid for $|t| \gtrsim 0.25(\text{GeV}/c)^2$ to the amplitude analysis of Ref. 3) of $\pi^-p \rightarrow \pi^0 n$ at 6 GeV/c was performed for $0.25 \leq |t| \leq 1.0(\text{GeV}/c)^2$. The data for $|F_{++}^1|$ were drawn with the symbols $\bullet$ [Ref. 3)], $\square$ [Ref. 2)], those for $|F_{+-}^1|$ with $*$ [Ref. 3)], Δ [Ref. 2)]. The data for the flip-non-flip phase difference, $\delta_{+-} - \delta_{++}$, are drawn with symbols $\circ$ [Ref. 3)], $\square$ [Ref. 2)]. The result for $|F_{++}^1|$ is displayed in a), for the phase difference in b) for $d\sigma/dt$ in c) and for the polarization in d). The fit result is given by solid lines, the broken lines are their extrapolations to lower values of $-t$. The data for $d\sigma/dt$ and P were taken from Refs. 26)-29).

Figure 9 :

The typical pattern (onions on a string) for suitably normalized flip dominated reactions at a fixed energy with varying ratio "crossing even/crossing odd contributions" ($G^t = +1/G^t = -1$).

- a) $\sqrt{-t} (dG/dt)$ versus $\sqrt{-t}$ in a log-plot, the two solid lines being the tangents to the outmost extrema, the broken line connecting the points of intersection [all having the common slope $-2 \operatorname{Im} b_0(s)$]. The solid arrows denote the positions where the tangent touches the extrema, the broken arrows the position of intersection points.
- b) The analogous plot for $e^{2 \operatorname{Im} b_0(s) \sqrt{-t}} \sqrt{-t} (dG/dt)$ versus $\sqrt{-t}$ in a linear scale.

Figure 10 :

- a) As a nice example for the pattern displayed in Fig. 9, $\sqrt{-t} (dG/dt)$ of the two reactions $\pi^- p \rightarrow \pi^0 n$ (crossing odd) and $\pi^+ p \rightarrow \eta \Delta^{++}(1236)$ ($\simeq$ crossing even) was plotted at $\simeq 3.5$ GeV/c with suitable normalizations. The solid curves were drawn in order to guide the eye. The data were taken from Refs. 30)-32).
- b) As a further example $\pi^- p \rightarrow \pi^0 n$ and $\pi^- p \rightarrow \eta n$, $\pi^- p \rightarrow \rho^0 n$ ($\simeq$ crossing even) are shown at $\simeq 18$ GeV/c; $\pi^- p \rightarrow \rho^0 n$ was normalized suitably to $\pi^- p \rightarrow \eta n$, $\pi^- p \rightarrow \pi^0 n$ according to the SU(3) prescription. The solid lines were drawn in order to guide the eye; for $\sqrt{-t} \gtrsim 1.3$ (GeV/c)² they are rather predictions (applying at least to $\pi^- p \rightarrow \pi^0 n$ and $\pi^- p \rightarrow \eta n$). Data are from Refs. 26), 27), 33), 34).

Figure 11 :

Comparison of the results from an effective, complex b plane pole $b_0(s)$ with those from an effective J plane Regge pole $\alpha(s)$ [$\alpha(s) + \frac{1}{2} = b_0(s) \cdot k$]. It is shown how well the relation

$$i H_0^{(1)}(b_0(s) \sqrt{-t}) \simeq - P_{\alpha(s)}(-\cos \theta_s) / \sin \pi \alpha(s),$$

valid for $s \rightarrow \infty$, t fixed, is satisfied already for the rather low energy (resonance mass) $\sqrt{s} \simeq 3$ GeV over a large range of t values.

FIG. 1a

FIG. 1b

FIG.2

FIG. 3

FIG. 4

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FIG. 5

FIG. 6

FIG.7a

$K^+ p \rightarrow K^{*+}(890) p$ 4.27 GeV/c

$K^+ p \rightarrow K^0 \Delta^{++} (1236)$

$-t \text{ [(GeV/c)}^2]$

FIG.7b

FIG. 8a

FIG. 8b

FIG. 8 c

FIG. 8 d

FIG.10a

FIG.10 b

$\sqrt{s} = 3.16 \text{ GeV}$

FIG.11

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