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SCALING AND LIGHT CONE DOMINANCE IN HADRONIC INDUCED
REACTIONS FROM SPECTRAL REPRESENTATIONS

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A B S T R A C T

A generalized Deser-Gilbert-Sudarshan representation for the product of two currents, which has been recently proposed by Cornwall, is used to study the processes $pp \rightarrow \mu^+ \mu^- X$ and $pp \rightarrow \pi X$ in the limit of large c.m. energy s with fixed ratios of all kinematic invariants, and the possible relevance of the leading light cone singularity for this scaling limit. We conclude that the existence of the scaling limit does not necessitate light cone dominance. The requirement of scaling amounts to assuming suitable smoothness for the spectral weight occurring in the representation. However, such smoothness assumptions must not be pushed too far in order to avoid unphysical results. This is demonstrated with the help of the limiting case of finite transverse momenta and its connection to ordinary hadronic scaling.

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Spectral representations such as the Jost-Lehmann-Dyson or the Deser-Gilbert-Sudarshan (DGS) representation have been used to some extent for the study of matrix elements describing deep inelastic processes. Their extension to matrix elements governing hadronic induced inclusive reactions (e.g., $pp \rightarrow \mu^+ \mu^- X$ or $pp \rightarrow \pi X$) is however not easy because one can no longer use causality. A generalized DGS representation for the product of two currents, taken between two particle states, has recently been made plausible by Cornwall ¹⁾, based mainly on perturbation theoretical arguments. Some applications of that representation to inclusive processes have already been made ²⁾. Its use in scaling limits is complicated by the fact that the spectral function also depends on the total energy s of the initial state which due to kinematics cannot be held fixed in the scaling region. Keeping these difficulties in mind we use the representation as proposed by Cornwall ¹⁾. Assuming scaling behaviour of the amplitudes for $pp \rightarrow \mu^+ \mu^- X$ and $pp \rightarrow \pi X$ we investigate the consequences for the spectral function and the question of light cone (LC) dominance. The main conclusion is that one can have perfect scaling (in the sense defined in the text) without any LC dominance. In fact, a very simple scaling form for the spectral function in the limit of large energy s yields a scaling behaviour of the entire amplitude but the leading LC singularity does not dominate. Other choices for the spectral function are also possible which do give LC dominance but there is no way to discriminate between the different possibilities. This lack of LC dominance has already been noticed within the parton model ³⁾ and is shown here on more general grounds. It is to be contrasted with the situation in deep inelastic scattering, where scaling and light cone dominance are intimately connected.

The use of spectral representations is however not without pitfalls. They must be handled with care in order to avoid incorrect conclusions. The reason is that the weight function occurring in these representations is smooth only up to a certain set of points which become decisive in particular physical limits. Going uncritically to the limit at such a dangerous point one may for instance lose the dependence on a kinematic variable. Then one arrives at results which are contradicted by commonly accepted physical principles. This, however, does not constitute a counter-example against the spectral representations which are general enough if only applied with caution. As an illustration for this phenomenon we discuss the case of finite transverse momenta $q_{\perp}$ of the produced particle. We observe that the statement made in Ref. 1) concerning the relative insensitivity of the cross-section for hadronic induced reactions to $q_{\perp}$ at large q^2 does not necessarily follow from the spectral representation. Other

examples for dangerous limits within this context can be found in the literature ⁴⁾.

The amplitude for the process $P(p) + P(p') \rightarrow \gamma(q) + X \rightarrow \mu^+ \mu^- + X$ is determined by the current correlation function:

$$\tilde{W}(x) = \langle p p' \text{ in } | j_\mu(x) j^\mu(0) | p p' \text{ in} \rangle \quad (1)$$

For this matrix element we assume a spectral representation which is, so to speak, "half of a DGS representation" and was first suggested by Cornwall ¹⁾:

$$\tilde{W}(x) = \int_0^\infty ds \int_{-1}^1 d\beta \int_{-1}^1 d\beta' h(s, \beta, \beta'; s, m^2) e^{i(\beta p + \beta' p')x} \Delta^+(x; s) \quad (2)$$

The distinctive features of this more generalized DGS representation are first, the additional dependence of the spectral function h on the variable $s = (p + p')^2$ ($p^2 = p'^2 = m^2$), characterizing the internal degree of freedom of the two-particle state $|pp'\rangle$, and second, the appearance of the positive frequency part $\Delta^+(x)$ instead of the full causal function $\Delta(x)$ found in the usual DGS expression for the commutator. It is this separation of $\Delta(x)$ into $\Delta^+(x)$ and $\Delta^-(x)$ corresponding to the two orderings of the commutator which cannot be established on the same footing as the ordinary DGS representation ^{*)} (which by itself is not completely rigorous). Nevertheless, the form (2) seems to be sufficiently general ^{**) to deserve some further attention. For some supporting arguments based on perturbation theory see Cornwall's work ¹⁾.}

^{*)} Since due to the spectrum conditions the overlap of the supports in momentum space of the two orderings of the commutator is compact, one might be tempted to argue that $W(q)$ has to look as in Eq. (3) apart from a function with compact support which can be neglected for asymptotic considerations. However, due to kinematics the physical region lies entirely in the overlap region whose boundaries also grow with s !

^{**) A possible subtraction in (3) would have the form of a polynomial in q^2 , ν , ν' with s dependent coefficients. Such a term can obviously be written as a function of s times the same polynomial in the three ratios α , α' , γ and its support in configuration space is concentrated at the tip of the light cone. Therefore, our main conclusions would remain unaltered.}

We shall mainly use the Fourier transform of $\tilde{W}(x)$:

$$W(q) = (2\pi)^{-4} \int d^4x e^{-iqx} \tilde{W}(x) \\ = \int_0^\infty da \int_{-1}^1 d\beta \int_{-1}^1 d\beta' h(a, \beta, \beta'; s, m^2) \delta[(p\beta + p'\beta' - q)^2 - a] \theta(p_0\beta + p'_0\beta' - q_0) \quad (3)$$

For the process under consideration, $s > 4m^2$. Then the integration path determined by the δ function is a hyperbola in the $\beta\beta'$ plane. The step function, which after going to the c.m. system $\vec{p} + \vec{p}' = 0$ can be written as $\theta(\beta + \beta' - 2((p+p')q)/s)$, selects the branch of the hyperbola lying entirely in the first quadrant. Suppressing from now on the θ function we write $W(q)$ as:

$$W(q) = \int_0^\infty da \int_0^1 d\beta \int_0^1 d\beta' h(a, \beta, \beta'; s, m^2) \cdot \\ \cdot \delta[m^2(\beta^2 + \beta'^2) + (s - 2m^2)\beta\beta' - 2\nu\beta - 2\nu'\beta' + q^2 - a] \quad (4)$$

where $\nu = pq$, $\nu' = p'q$.

Introducing the ratios $\alpha = \nu/s$, $\alpha' = \nu'/s$, $\gamma = q^2/s$, which in the physical region all run between zero and one, $W(q)$ becomes

$$W(q) = \int_0^\infty da \int_0^1 d\beta \int_0^1 d\beta' \frac{1}{s} h(a, \beta, \beta'; s, m^2) \cdot \\ \cdot \delta\left[\frac{m^2}{s}(\beta^2 + \beta'^2) + (1 - \frac{2m^2}{s})\beta\beta' - 2\alpha\beta - 2\alpha'\beta' + \gamma - \frac{a}{s}\right] \quad (5) \\ = F(s, m^2; \alpha, \alpha', \gamma)$$

Our basic assumption now is that the limit $q^2 \rightarrow \infty$ (and hence by kinematics also $s \rightarrow \infty$) with fixed ratios α , α' , γ exists. However, because of the additional s dependence of the spectral function h we consider the more general "scaling" form:

$$F(s, m^2; \alpha, \alpha', \gamma) \xrightarrow[s \rightarrow \infty]{} \frac{1}{m^2} \left(\frac{s}{m^2}\right)^n f(\alpha, \alpha', \gamma) \quad , \quad (6)$$

ratios fixed

where f is a dimensionless scaling function of the three ratios, the factor $1/m^2$ is inserted because $W(q)$ has dimension -2 and the constant n is left arbitrary. The power $(s/m^2)^n$ is included to account for a possible "hadronic" (i.e., Regge-like) behaviour of the spectral function h . It should be noted that we do not speak of a "Regge region" $s \gg q^2$ nor of any interchange of Regge and A limits. (This is done in the LC treatment of the same process by Brandt and Preparata⁵⁾: they consider the A limit, which is identical to our above scaling limit, and the Regge limit, where s, ν and/or ν' are much larger than q^2 . Furthermore, they assume commutativity of these two limits.)

Under the stated assumption the m^2/s terms in the δ function in (5) can be neglected which does not change the degree of the integration curve, contrary to the corresponding case in deep inelastic scattering. Nevertheless, for this neglect under the integral sign to be allowed the spectral weight $h(a, \beta, \beta'; s, m^2)$ must be sufficiently well behaved. Thus, our scaling requirement is equivalent to a certain smoothness assumption for h . We shall see below that it can be dangerous to push such smoothness conditions too far.

One may now ask whether the assumed scaling behaviour has anything to do with light-cone dominance. First, we note the well-known form^{*)} of the function $\Delta^+(x; a)$ appearing in $\tilde{W}(x)$:

$$\begin{aligned} \Delta^+(x; a) = & \frac{i}{4\pi^2} \frac{1}{x^2 - i\epsilon x_0} - \frac{ia}{16\pi^2} \ln \frac{a\gamma^2}{4e} (-x^2 + i\epsilon x_0) + \frac{a}{8\pi} \epsilon(x_0) \theta(x^2) \cdot \\ & \cdot \left[\frac{J_1(\sqrt{ax^2})}{\sqrt{ax^2}} - \frac{1}{2} \right] - \theta(x^2) \frac{ia}{8\pi} \left[\frac{N_1(\sqrt{ax^2})}{\sqrt{ax^2}} + \frac{2}{\pi ax^2} - \frac{1}{\pi} \ln \frac{\gamma \sqrt{ax^2}}{2} + \frac{1}{2\pi} \right] \\ & - \theta(-x^2) \frac{ia}{4\pi^2} \left[\frac{K_1(\sqrt{-ax^2})}{\sqrt{-ax^2}} + \frac{1}{ax^2} - \frac{1}{2} \ln \frac{\gamma \sqrt{-ax^2}}{2} + \frac{1}{4} \right] \end{aligned} \quad (7)$$

$$(\gamma = e^c = 1.781..., \quad c = 0.577... = \text{Euler's constant})$$

*) In the textbook by N.N. Bogoliubov and D.V. Shirkov, Introduction to the Theory of Quantized Fields, Interscience (New York), there is an error in Eqs. (15.16) and (15.18) because the constant $\ln \gamma^{-\frac{1}{2}}$ is missing there.

In (7) the LC singularities are exhibited explicitly, since the square brackets vanish for $ax^2 \rightarrow 0$.

Since the leading LC singularity of $\Delta^+(x;a)$ is a -independent we take out this part by rewriting $W(q)$ with the help of the mean value theorem as: $(v = p\beta + p'\beta' - q)$

$$\begin{aligned} W(q) &= \int da d\beta d\beta' h(a, \beta, \beta'; s, m^2) \delta(v^2 - a) \theta(v_0) \\ &= \int da d\beta d\beta' h(a, \beta, \beta'; s, m^2) \delta(v^2) \theta(v_0) \\ &\quad - \int da d\beta d\beta' h(a, \beta, \beta'; s, m^2) a \delta'(v^2 - \eta a) \theta(v_0) \end{aligned} \quad (8)$$

$$0 < \eta < 1$$

This decomposition only makes sense if both integrals exist separately. Clearly this is connected with the asymptotic behaviour of h for large a and is true for sufficiently decreasing spectral functions ^{*}). But then the first term in (8) will also be the leading LC singularity of $\tilde{W}(x)$ [$\delta(v^2)\theta(v_0)$ is the Fourier transform of $1/x^2 - i\epsilon x_0$] and LC dominance holds if:

$$\left| \int da d\beta d\beta' h(a, \beta, \beta'; s, m^2) \delta(v^2) \theta(v_0) \right| \gg \left| \int da d\beta d\beta' h(a, \beta, \beta'; s, m^2) a \delta'(v^2 - \eta a) \theta(v_0) \right| \quad (9)$$

Note that both sides of (9) can still grow with s !

As a side remark we mention that it is not straightforward to carry the expansion in a [Eq. (8)], one step further because $(\partial/\partial a)\Delta^+(x;a)$ does not exist at $a=0$ due to a logarithmic divergence, as can be seen from (7). Alternatively speaking, $\delta'(v^2)\theta(v_0)$ does not have a Fourier transform ⁷⁾. However, the condition (9) is sufficient for our purposes.

^{*}) For deep inelastic scattering, the relation between LC dominance, scaling and asymptotic fall-off of the spectral function has already been studied very intensively, see e.g., Ref. 6).

Together with our scaling assumption and carrying out the β' integration, say, with the help of the δ function, the "LC condition" Eq. (9) can be transcribed into:

$$\langle a \rangle \equiv \frac{\left| \int_0^\infty da a \int_0^1 \frac{d\beta}{(\beta-2\alpha')^2} \frac{\partial}{\partial \beta'} h(a, \beta, \beta'; s, m^2)_{\beta' = \hat{\beta} + \eta \frac{a}{s} \frac{1}{\beta-2\alpha'}} \right|}{\left| \int_0^\infty da \int_0^1 \frac{d\beta}{(\beta-2\alpha')} h(a, \beta, \hat{\beta}; s, m^2) \right|} \ll s \quad (10)$$

($\hat{\beta} = 2\alpha\beta - \gamma/\beta - 2\alpha'$), where we have introduced a sort of "mean value" $\langle a \rangle(s)$ which still depends on s and on the ratios α , α' , γ . Whether condition (10) can be satisfied or not in the limit $s \rightarrow \infty$ now depends on the behaviour of $h(a, \beta, \beta'; s, m^2)$ for large s and the consequent rate of growth of $\langle a \rangle$ with s .

For instance, one may consider the possibility:

$$h(a, \beta, \beta'; s, m^2) \xrightarrow{s \rightarrow \infty} \left(\frac{s}{m^2}\right)^n \hat{h}\left(\frac{a}{s}, \beta, \beta'; m^2\right) \quad (11)$$

This looks even like a very natural choice for h to be inserted in (5) in order to satisfy the scaling form (6) [substitute $t = a/s$ in (5) and remember that m^2/s is neglected inside the δ function]. But for this choice $\langle a \rangle$ is seen to grow linearly with s [again $t = a/s$ can be substituted everywhere in (10) leaving one extra power of s] with a numerical coefficient which is not quite the ratio of the first to the zeroth moment (in t) of h because of the additional t dependence in $\beta' = \hat{\beta} + \eta t(1/\beta - 2\alpha')$. Unless this number happens to be very small, LC dominance is excluded, but scaling is perfectly all right.

One can certainly invent other examples for h yielding a growth of $\langle a \rangle$ with s which is weaker than linear thereby enabling the LC to dominate. But since one has no further information on the spectral function one cannot decide between the different possibilities. Therefore for hadronic induced reactions "scaling" as formulated above does by no means necessitate LC dominance ³⁾.

Returning to our basic scaling hypothesis (6) we note the ensuing form of the cross-section. The differential cross-section for production of a μ pair with invariant mass $\sqrt{q^2}$ is given by:

$$\frac{d\sigma(q^2, s)}{dq^2} = - \frac{2\alpha^2\pi}{3q^2 [s(s-4m^2)]^{1/2}} \int \frac{d^3q}{q_0} W(q) \quad (12)$$

In the c.m. system

$$\frac{d^3q}{q_0} = 2\pi \frac{q_{\perp} dq_{\perp} dq_{\parallel}}{q_0} = 4\pi s d\alpha d\alpha',$$

where

$$\nu + \nu' = \sqrt{s} q_0, \quad q_{\parallel} = \frac{\nu' - \nu}{\sqrt{s}}, \quad q_{\perp}^2 = \frac{4\nu\nu'}{s} - q^2$$

has been used. Inserting the above scaling form for $W(q)$ into (12) the cross-section becomes for large q^2 , γ fixed:

$$\frac{d\sigma(q^2, s)}{dq^2} = - \frac{8\pi^2 \alpha^2}{m^4} \left(\frac{s}{m^2}\right)^{n-1} G(\gamma) \quad (13)$$

The dimensionless function $G(\gamma)$ is defined by:

$$G(\gamma) = \frac{1}{\gamma} \int d\alpha d\alpha' f(\alpha, \alpha', \gamma) \quad (14)$$

It is instructive to apply a similar analysis to single particle inclusive distributions, say $pp \rightarrow \pi X$ [see also Ref. 1]. The relevant matrix element is analogous to $W(q)$ with J_{μ} replaced by the pion current J_{π} and $q^2 = m_{\pi}^2$. The kinematics is very similar to before and the scaling limit is defined as; $s \rightarrow \infty$, α and α' fixed, the third ratio γ now being zero. Again a representation like Eq. (3) holds.

The condition for light cone dominance is the same as Eq. (10) and the subsequent remarks apply also here. In any case, one can have a scaling behaviour for $W_\pi(q)$ of the form:

$$W_\pi(q) \xrightarrow{s \rightarrow \infty} \frac{1}{m^2} \left(\frac{s}{m^2}\right)^n f(\alpha, \alpha') \quad (15)$$

The scaling considered here is of a more general nature. In order to make the connection to ordinary hadronic scaling, we investigate the case of finite transverse momentum $q_\perp$. Using $q_\perp^2 = (4 \nu \nu' / s) - m_\pi^2$ (in the c.m. or lab. system) we can rewrite the δ function in the DGS representation:

$$\delta\left(\beta\beta' - 2\alpha\beta - 2\alpha'\beta' - \frac{q_\perp^2}{s}\right) \quad \text{as} \quad \delta\left[(\beta - 2\alpha')(\beta' - 2\alpha) - \frac{q_\perp^2}{s} - \frac{a}{s}\right] \quad (16)$$

Finite $q_\perp$ (as $s \rightarrow \infty$) means $\alpha = 0$ or $\alpha' = 0$. Take for instance the longitudinal c.m. momentum $q_\perp > 0$, then $\alpha = (q_\perp^2 + m_\pi^2 / 2xs) \rightarrow 0$, $\alpha' \rightarrow \frac{1}{2}x$, where $x = 2(q_\perp / \sqrt{s})$ is the Feynman variable.

If we neglected uncritically $q_\perp^2/s$ in the δ function, we would obtain:

$$W_\pi(q) \rightarrow \frac{1}{m^2} \left(\frac{s}{m^2}\right)^n f(0, \frac{x}{2}) \quad (17)$$

The free parameter n can be adjusted at will, but still we do not get the complete hadronic scaling because the $q_\perp$ dependence has disappeared completely. Since we know from physical reasons that the result should depend on $q_\perp$, the limit $q_\perp^2/s = 0$ inside the integral is not allowed. The explanation is that the spectral function $h(a, \beta, \beta'; s, m^2)$ can (and on physical grounds must) have a singularity which invalidates the exchange of the above limit with the integration.

As an example, consider a special spectral function:

$$h_1(a, \beta, \beta'; s, m^2) = \frac{1}{\beta'} \bar{h}_1(a, \beta; s, m^2) \quad (18)$$

Integrating over β with the help of the δ function we get as the contribution of h_1 to W_π :

$$W_1(q) = \int_0^\infty d\alpha \int_0^1 d\beta \frac{\bar{h}_1(\alpha, \beta; s, m^2)}{q_\perp^2 + \alpha + (q_\perp^2 + m_\pi^2) \frac{\beta - x}{x}} \quad (19)$$

which contains the desired $q_\perp$ dependence. We do not attribute any physical significance to this particular example. It serves merely to illustrate the mechanism. The point is that due to the functional form of the spectral function an integration region of the size $1/s$ can give a contribution proportional to s thus yielding a finite result. Therefore one must always be careful in performing such limits in spectral representations in order not to get unphysical results ⁴⁾.

Going back to the previous process we can again look at the case of finite $q_\perp^2$. There, $q_\perp^2 = (4\nu\nu'/s) - q^2$, and putting $q_\perp^2/s = 0$ amounts to a relation between the three ratios: $4\alpha\alpha' = \gamma$. Hence, one would conclude that for finite $q_\perp^2$ the scaling function $f(\alpha, \alpha', \gamma)$ becomes a function of only two variables. However, from the lesson learned before one may expect a similar mechanism to work, such that for finite $q_\perp$ the scaling function $f(\alpha, \alpha', \gamma)$ is converted into a new function $\tilde{f}(\alpha, \alpha', q_\perp^2)$ corresponding to an incomplete scaling behaviour. A few numbers may illustrate the present situation concerning $q_\perp$ in μ pair production: the BNL-Columbia experiment ⁸⁾ was done at energies s ranging from 46 to 61 GeV², a momentum cut $12 \text{ GeV}/c < |\vec{q}|_{\text{lab}} < 29 \text{ GeV}/c$ and an angular cut $\cos \theta_{\text{lab}} \geq 0.998$. The corresponding maximal transverse momenta $q_\perp^2$ range from 0.57 to 3.33 (GeV/c)², indeed quite small compared to s . In this kinematical domain it may well be that the mechanism discussed before applies and a complete scaling behaviour is not yet reached.

Finally, we observe that at large q^2 the spectral representation used by us does allow for large sensitivity (for instance in the sense of a cut-off) with respect to variations in $q_\perp^2$ and no sensitivity in q^2 , contrary to statements in the literature ¹⁾. Our incomplete scaling function $\tilde{f}(\alpha, \alpha', q_\perp^2)$ can serve as a candidate for such a behaviour since the $q_\perp$ dependence is not specified.

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