Mean excitation energies in the nuclear muon capture*

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The parametrization of the *total* muon capture rates in terms of a mean nuclear excitation energy $\bar{E}'$, proposed in a recent letter of Christillin, Dellafiore, and Rosa-Clot (CDR), is examined. We show that this parametrization ignores the importance of the allowed transitions in light nuclei and fails to reproduce the experimentally observed trend of the T_1 giant resonance energy as a function of nuclear mass number. The removal of contributions due to higher multipoles in heavier nuclei aggravates this discrepancy. The failure of $\bar{E}'$, extracted by CDR, to correspond to physically meaningful nuclear excitation energies in heavier nuclei is attributable to the intrinsic arbitrariness in its definition, and to the oversimplifying assumption of the nuclear Hamiltonian to be a sum of kinetic energy and Wigner potential alone. A first-order improvement of the Primakoff approximation is discussed.

[NUCLEAR REACTIONS $\mu^- + A(N, Z) \rightarrow \nu + A(N+1, Z-1)$; muon capture; closure approximation; mean nuclear excitation energies.]

Studies¹ of muon capture reactions leading to definite final nuclear states have provided valuable knowledge of nuclear weak hadron form factors and have underscored the complexities of the underlying nuclear structure problem. Attempts to interpret the *total* muon capture rates in varied nuclear targets, on the other hand, have always been beset with ambiguities, despite the outstanding early successes of Primakoff² in understanding the role of nuclear correlations in muon capture. Thus, the recent attempt³ of Christillin, Dellafiore, and Rosa-Clot (CDR) in understanding the total muon capture rates in terms of a mean nuclear excitation energy (denoted as $\bar{E}'$ by these authors) merits a careful scrutiny, in view of CDR's claim that this parametrization is model-independent, and appears to be the "most natural one" in interpreting the total muon capture rate.

In this comment we examine the procedure adopted by CDR in extracting $\bar{E}'$ and the validity of the underlying approximations. We find that $\bar{E}'$ is arbitrarily defined and it is without any compelling physical significance. Thus, in the light nuclei, the agreement of $\bar{E}'$ with experimental⁴ giant dipole resonance⁵ energies are fortuitous and do not reflect the importance⁶ of the excitation of the giant magnetic dipole resonance analog states in light nuclei. In heavy nuclei, the lack of physical significance of $\bar{E}'$ becomes more apparent—thus, for ²⁰⁸Pb, if one isolates⁷ the giant dipole resonance contribution, the mean energy $\bar{E}'$, calculated by the CDR approach, is about 70 MeV,

while the experimental giant dipole resonance energy is only 13.4 MeV.

We recapitulate here the essential steps in the CDR approach. It starts with an expression for the total capture rate $\Lambda_{\mu c}$ that ignores the relativistic effects and the effects of SU(4) breaking:

$$\Lambda_{\mu c} = \frac{G_{\mu c}^2}{2\pi} \times \sum_b \nu_{ab}^2 \int \frac{d\hat{\nu}}{4\pi} |\langle b | \sum_{i=1}^A \tau_i^- \exp(-i\vec{\nu} \cdot \vec{x}_i) \phi(x_i) | a \rangle|^2. \quad (1)$$

Then the definition for the mean energy $\bar{E}$ is introduced by means of the equation

$$\bar{E} = \frac{G_{\mu c}^2}{2\pi} \sum \frac{\bar{\nu}^2 (E_b - E_a)}{\Lambda_{\mu c}} \times \int \frac{d\hat{\nu}}{4\pi} |\langle b | \sum_{i=1}^A \tau_i^- \exp(-i\vec{\nu} \cdot \vec{x}_i) \phi(x_i) | a \rangle|^2, \quad (2)$$

in which the mean neutrino momentum $\vec{\nu}$ is evaluated in terms of the mean nuclear excitation energy $\bar{E}$. Finally, the capture rate is expressed in terms of a double commutator:

$$\Lambda_{\mu c} = -\frac{1}{2} \frac{G_{\mu c}^2}{2\pi} \frac{\bar{\nu}^2}{\bar{E}} \langle a | [H, \sum_{i=1}^Z \exp(-i\vec{\nu} \cdot \vec{x}_i), \phi(x_i)] , \sum_{j=1}^Z \exp(i\vec{\nu} \cdot \vec{x}_j) \phi(x_j)] | a \rangle, \quad (3)$$

where $\bar{E}' = \bar{E} + E_c$. The double commutator is evaluated in the approximation $H = T + V^{\text{Wigner}}$, providing CDR with a parametrization of the capture rate in terms of $\bar{\nu}$ and $\bar{E}'$.

The definition of the mean energy $\bar{E}$ adopted in Eq. (2) is by no means unique. Thus, one could have started with the definition of the mean energy in the following way, after Luyten, Rood, and Tolhoek (LRT)⁸:

$$\bar{E} = \langle E_b - E_a \rangle_{av} = \frac{\sum_b \Lambda_{\mu c}(a-b)(E_b - E_a)}{\sum_b \Lambda_{\mu c}(a-b)}. \quad (4)$$

Indeed, the definition of mean energy in Eq. (4) is better than that of Eq. (2) because, in this definition, the neutrino momentum is not averaged out, but taken appropriately for each partial transition. The difficulties of using this preferable definition have been pointed out by LRT, the primary one being the dependence of the value of $\bar{E}$ on the choice of the ground state nuclear wave function. Clearly, the merit of the definition of Eq. (2) has to be justified *a posteriori*, one of the most compelling tests coming from the consistency of this definition with known physical situations. Thus, if the giant dipole resonance contribution to the muon capture process in a given nucleus is *known* (e.g., by means of other reliable theoretical or experimental analyses), the calculated value of $\bar{E}'$ must correspond to the *observed* giant resonance energy in the target. Equation (2) fails in this crucial test in heavy nuclei, as we shall see later.

We now will examine the physical significance of the parameter $\bar{E}'$ obtained by CDR for targets throughout the Periodic Table. We first consider the light nuclei in the *p*- and *s-d* shell. Allowed transitions can be very important in light nuclei⁶; thus, in ¹⁰B, allowed transitions are responsible for more than 30% of the *total* muon capture strength. In this context, SU(4) invariance of the nuclear Hamiltonian, assumed by CDR, may yield very erroneous results in certain uses.⁹ Thus, in ¹²C, SU(4) wave functions imply no allowed strength in that target, while, actually, allowed capture is responsible for about $\frac{1}{6}$ of the total capture rate. In some targets such as ⁶Li, even in the SU(4) limit, one obtains a sizeable allowed capture rate. In ⁶Li, the ground-state to ground-state transition is superallowed and the rate for this alone is about a third of the total capture rate.¹⁰

Despite this demonstrated importance of the allowed transitions in light nuclei, the CDR parametrization is not very sensitive to their importance. Thus, calculating the values of $\bar{\nu}$ and $\bar{E}$ using the CDR prescription before and after

subtraction of the allowed capture rates from the total rates does not result in large differences in the values of $\bar{E}$ and $\bar{\nu}$, even in cases in which the giant dipole dominance is not overwhelming.¹¹ Thus the agreement of $\bar{E}'$ with giant dipole resonance energies in light nuclei, obtained by CDR, is not compelling in physical significance. Indeed, in some targets, significant contributions from the broken channels,¹² neglected by CDR, make this agreement look even more accidental.

To examine the correlation of $\bar{E}'$ obtained by CDR with giant dipole resonance energies even further, we come to the heavier target nuclei. Thus, for $Z > 20$, $\bar{E}'$ obtained by the CDR procedure follows roughly the following empirical relation

$$\bar{E}' \sim [25 + (Z - 20)0.3] \text{ MeV}, \quad (5)$$

while the giant dipole resonance energies (E_{GDR}) go like^{13,14}

$$E_{\text{GDR}} \sim 80A^{-1/3}, \quad (6)$$

reflecting no direct correlation between $\bar{E}'$ and E_{GDR} . Thus, in ²⁰⁸Pb, CDR obtain $\bar{E}'$ to be ~45 MeV, while E_{GDR} is only 13.4 MeV. This lack of agreement, contrary to the speculations of CDR, cannot be attributed to the contributions of the multipoles of order higher than that of the electric dipole operator. Thus, it is easy to demonstrate that subtracting the contributions of higher multipoles from the total rate and recalculating $\bar{E}'$, after CDR, from this corrected rate do not improve the agreement between $\bar{E}'$ and E_{GDR} , they only make the agreement worse. This demonstration will be given below.

Specializing to the giant dipole excitation, we replace $e^{i\vec{\nu} \cdot \vec{x}}$ by $i\vec{\nu} \cdot \vec{x}$, and evaluate the commutator¹⁵

$$\sum_{i=1}^Z [[x_i \nu, [H, x_i \nu]]] = \nu^2 Z / M. \quad (7)$$

This then leads to the capture rate for the giant dipole excitation, $\Lambda_{\mu c}$ (dipole):

$$\Lambda_{\mu c} (\text{dipole}) = \frac{G_{\mu e}^2}{2\pi} \frac{1}{2M} \frac{\bar{\nu}^4}{\bar{E}'} \times \left\langle a \left| \left\{ \sum_{i=1}^Z \phi^2(x_i) + \sum_{i=1}^Z x_i^2 [\nabla_i \phi(x_i)]^2 \right\} \right| a \right\rangle. \quad (8)$$

Defining

$$\frac{\langle a | \sum_{i=1}^Z x_i^2 [\nabla_i \phi(x_i)]^2 | a \rangle}{\langle a | \sum_{i=1}^Z \phi^2(x_i) | a \rangle} = \Delta, \quad (9)$$

we obtain

$$\Lambda_{\mu c}(\text{dipole}) = \frac{G_{\mu c}^2 (m_{\mu} \alpha)^3}{(2\pi)^2 M} \frac{\bar{\nu}^4}{(m_{\mu} - \epsilon_{\alpha} + E_c - \bar{\nu})} \times Z_{\text{eff}}^4 (1 + \Delta), \quad (10)$$

using

$$\langle a | \sum_{i=1}^Z \phi^2(x_i) | a \rangle = \frac{(m_{\mu} \alpha)^3}{\pi} Z_{\text{eff}}^4. \quad (11)$$

Equation (10) is to be compared with Eq. (7) of CDR for total capture rate. If the parametrization of the rate in terms of $\bar{\nu}$ or $\bar{E}'$ makes sense, the value of $\bar{E}'$, obtained by using Eq. (10) and a reasonable estimate of $\Lambda_{\mu c}(\text{dipole})$, should compare favorably with the giant dipole resonance energy in ^{208}Pb .

An estimate of the dipole contribution to the rate is obtainable from the Migdal-theoretic analysis.⁷ For ^{208}Pb , various multipole decompositions of the total capture rate are as follows:

$$\Lambda(l=0) = 31 \times 10^5 \text{ s}^{-1}, \quad \Lambda(l=1) = 17.3 \times 10^5 \text{ s}^{-1}, \quad (12)$$

$$\Lambda(l=2) = 57.0 \times 10^5 \text{ s}^{-1}, \quad \Lambda(l=3) = 14.8 \times 10^5 \text{ s}^{-1},$$

and the other multipole contributions are negligible. Using the $\Lambda(l=1)$ value from Eq. (12) as an estimate for $\Lambda_{\mu c}(\text{dipole})$, we obtain a lower limit of $\bar{E}'$ for $\Delta=0$. This estimate is ~ 70 MeV, which is further off the experimental giant dipole resonance energy than the estimate obtained from the total rate in CDR. The energy $\bar{E}'$ has, therefore, no physical significance as far as the giant resonance excitation by muon capture in ^{208}Pb is concerned. It should be kept in mind that the accuracy of the various multipole contributions in Eq. (12) is not crucial to this argument. Any *nonzero* contribution of the other multipoles will make $\bar{E}'$ further off the experimental giant resonance energy.

We now return to some of the basic assumptions of the CDR calculation, besides the definition of $\bar{E}'$ discussed above, since they have wider implications in nuclear physics beyond the interpretation of total muon capture rates in nuclei. Thus, the assumption that the nuclear Hamiltonian H is the sum $(T + V^{\text{Wigner}})$ is untenable even at the end of the $1p$ shell^{9,16} because of the importance of Majorana force and increasingly important roles of the spin-orbit force and those of the SU(4)-breaking two-body interactions. The reason, advanced by CDR, for neglecting both the relativistic and the SU(4)-breaking effects is also not persuasive for high-mass nuclei, since with the increase of the mass number A , SU(4)-breaking effects^{9,16} generally increase, while the relativ-

istic effects¹⁷ drop off as $A^{-2/3}$, thereby making the partial compensation of those two effects a poorer possibility.

A dramatic manifestation of the SU(4) breaking is obtained in the possible role of the isovector Lane potential, defined by the term involving V_1 in the following more exact nuclear Hamiltonian, considered by Bressani and Molinari¹⁸:

$$H = \sum_{i=1}^A [p_i^2/2M + V_0(r_i) + V_{1s}(r_i)\hat{1}_i \cdot \hat{s}_i + (V_1/A)\hat{t}_i \cdot \hat{T}_0 + (\frac{1}{2} - t_i^Z)\phi_{\text{Coulomb}}(r_i)] + \sum_{i < j} V_{ij}. \quad (13)$$

The effect of this one-body force is the "blocking" or strong suppression of the T_0 giant dipole strength,¹⁸ e.g., in the muon capture in ^{208}Pb . Thus, the definition of the mean energy $\bar{E}$ by an equation of the type

$$\bar{E} \langle ^{208}\text{Tl}(\text{GDR}) | D^+ | ^{208}\text{Pb}_{\text{g.s.}} \rangle^2 \approx \frac{1}{2} [D^+, [H, D^+]], \quad (14)$$

with D^+ as the dipole operator $\sum_{i=1}^A x_i \tau_i^+$, is no longer meaningful, since the matrix element

$$\langle ^{208}\text{Tl}(\text{GDR}) | D^+ | ^{208}\text{Pb}_{\text{g.s.}} \rangle$$

nearly vanishes, while $\bar{E} \neq 0$. Thus, the CDR approach runs into serious difficulty.

Any improvement of the interpretation of the total capture rate in terms of a mean neutrino momentum $\bar{\nu}$ has to begin with the fact that the usual closure approximation of Primakoff² is a zeroth order reduction of a complicated energy-weighted sum rule. A first-order improvement of the Primakoff approximation is possible by expanding in a Taylor series a partial capture rate Λ_{ab} in the neighborhood of $\bar{\nu}$ ¹⁹:

$$\Lambda_{ab} = [\Lambda_{ab}]_{\nu_{ab}=\bar{\nu}} + \left[\frac{d\Lambda_{ab}}{d\nu_{ab}} \right]_{\nu_{ab}=\bar{\nu}} (\nu_{ab} - \bar{\nu}); \quad (15)$$

summing over all the final states b , one obtains¹⁹

$$\Lambda_a = \left\{ 1 + (m_{\mu} - \bar{\nu}) \frac{d}{d\bar{\nu}} \right\} \sum_b [\Lambda_{ab}]_{\nu_{ab}=\bar{\nu}} - \frac{d}{d\bar{\nu}} \left\{ \sum_b (E_b - E_a) [\Lambda_{ab}]_{\nu_{ab}=\bar{\nu}} \right\}. \quad (16)$$

The first term in the right-hand side of Eq. (16) is the result of the Primakoff approximation. For realistic ground-state wave functions of the target, this procedure [Eq. (16)] yields a wide region of value of $\bar{\nu}$ for light nuclei.

To summarize, the mean excitation energy $\bar{E}'$ obtained by the CDR approach does not appear to be the "most natural" parametrization of the total muon capture rate. For light nuclei, it ignores the importance of the allowed strength, which, in several targets, can account for a third of the total rate. For heavy nuclei, $\bar{E}'$ is in complete disagreement with the observed giant resonance energies; isolation of the giant resonance contribution of the total capture rate makes this agreement even worse. Finally, the assumption that the nuclear Hamiltonian is just the sum of kinetic energy and Wigner potential is exceedingly inaccurate, even in the light nuclei, and profound consequences, such as the blocking of the $T_>$ giant resonance excitation, as a result of improvement

of this approximation, are ignored in the CDR approach. The importance of corrections to the simple nuclear Hamiltonian $H = (T + V^{\text{Wigner}})$ in the interpretation of the total muon capture rate does not decrease in heavy nuclei. Neglect of these corrections cannot be taken into account by a simple scaling of the rate, e.g., by a factor ~ 1.3 in Eq. (7) of CDR.

Thus the problem of correctly interpreting the experimentally measured total muon capture rate, in our opinion, remains an open one.

Note added in proof: A more complete discussion of the recent theoretical progress in interpreting the observed total muon capture rates will be given in a forthcoming article by Mukhopadhyay.²⁰ See also Cannata, Graves, and Überall.²¹

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allowed rate differs only by 2 MeV in ^{12}C , whereas the observed difference in centroid energies of giant magnetic and electric dipole resonance is ~ 8 MeV.

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