NNLO ANALYSIS OF UNPOLARIZED DIS STRUCTURE FUNCTIONS

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We present the results of a NNLO QCD analysis of the World data on unpolarized DIS Non-Singlet Structure functions.

1. Introduction

The increasing accuracy of DIS experiments will further reduce the experimental errors on the determination of the strong coupling constant calling for an improvement on the theoretical errors, which are by now the dominant ones. One way to achieve it is to include NNLO QCD effects in the analysis.

Our goal is to perform a NNLO QCD analysis of World data on unpolarized DIS structure functions to determine $\alpha_s$ with an accuracy of $O(2\%)$ along with a parametrization of the parton distribution functions with fully correlated errors. As a first step in this direction we concentrate on the non-singlet (NS) sector.

We presented the first results of our analysis in [1]. Here we give an update of the main results and refer the interested reader to [2] for all the details.

2. Theoretical Framework

We carried out our analysis in Mellin–$N$ space [3], where the non-singlet part of the electromagnetic DIS structure function $F_2(N,Q^2)$ is written in terms of the non-singlet quark combinations $q_{\pm,v}(N,Q^2)$ and the corresponding Wilson coefficients $C_2^{(k)}(N)$ as

$$F_2^{\pm,v}(N,Q^2) = \{1 + a_s(Q^2)C_2^{(1)}(N) + a_s^2(Q^2)C_2^{(2)}(N)\}q_{\pm,v}(N,Q^2), \quad (1)$$
with $a_s(Q^2) \equiv \alpha_s(Q^2)/4\pi$, the normalised coupling constant.

In the region $x > 0.3$ we adopt the quark valence dominance hypothesis under which the proton and deuteron structure functions are given by the following quark distribution combinations

$$F_2^p = \frac{4}{9}x u_v + \frac{1}{9}x d_v, \quad F_2^d = \frac{5}{18}x(u_v + d_v).$$

(2)

For $x < 0.3$ we analyse the NS combination

$$F_2^{NS} \equiv 2(F_2^p - F_2^d) = \frac{1}{3}x(u_v - d_v) - \frac{2}{3}x(d - \bar{u}).$$

(3)

The valence parton distribution functions are parametrised at the reference scale $Q_0^2 = 4 \text{ GeV}^2$ with the functional form

$$x u_v(Q_0^2, x) = A_u x^{\alpha_u} (1 - x)^{b_u} (1 + \rho_u \sqrt{x} + \gamma_u x),$$

(4)

and

$$x d_v(Q_0^2, x) = A_d x^{\alpha_d} (1 - x)^{b_d} (1 + \rho_d \sqrt{x} + \gamma_d x),$$

(5)

where the normalization constants $A_u$ and $A_d$ are not free parameters of the fit but are determined to satisfy the valence quark counting: $\int_0^1 u_v(x)dx = 2$ and $\int_0^1 d_v(x)dx = 1$.

The remaining non-singlet parton density, $(\bar{d} - \bar{u})$, is not constrained by the electromagnetic structure function data and we adopted the form given in [11], which provides a good description of the Drell-Yan dimuon production data from the E866 experiment. The heavy flavor corrections were accounted for as described in [11].

3. Data

The results we present are based on 551 data points for the structure function $F_2(x, Q^2)$ measured on proton and deuteron targets. The experiments contributing to the statistics are: BCDMS[4], SLAC[7], NMC[8], H1[9], and ZEUS[10].

The BCDMS data were recalculated replacing $R_{QCD}$ with $R_{1998}^{[11]}$. All deuteron data were corrected for Fermi motion and off-shell effects [11].

We used the measured structure functions $F_2^p$ and $F_2^d$ in the region $x > 0.3$ which is expected to valence dominated, while in the region $x < 0.3$ we construct the non-singlet structure function $F_2^{NS} \equiv 2(F_2^p - F_2^d)$ from proton and deuteron data measured at the same $x$ and $Q^2$.

We imposed different cuts on the data. Only data points with $Q^2 > 4$ GeV$^2$ were included in the analysis and a cut on the hadronic mass of
$W^2 > 12.5 \text{ GeV}^2$ was imposed in order to reduce higher twist effects on the determination of $\Lambda_{QCD}$ and the PDF parameters. The latter cut was then relaxed in the extraction of higher twist effects. Moreover we imposed additional cuts on BCDMS ($y_H > 0.3$) and NMC ($Q^2 > 8 \text{ GeV}^2$) data in order to exclude regions with potentially significant correlated systematic errors.

In the fitting procedure we allowed for a relative normalization shift between the different data sets within the systematic uncertainties quoted by the single experiments. These normalization shifts were fitted once and then kept fixed.

4. Results

The results we obtain for the fit parameters are collected in Table 1. We note that the fit doesn’t constrain the $\rho_i$ and $\gamma_i$ parameters, which have therefore been kept fixed after the first minimization and their value is quoted without errors.

The remaining parameters to be determined in the fit are the low- and high-$x$ ones ($a_i$ and $b_i$) alongside with $\Lambda_{QCD}$. From the value for $\Lambda_{QCD}^{(4)}$

\begin{table}[h]
\begin{tabular}{|c|c|}
\hline
Parameter & Value \\
\hline
$u_0$ & $a$ \hspace{1cm} $0.291 \pm 0.008$ \\
 & $b$ \hspace{1cm} $4.013 \pm 0.037$ \\
 & $\rho$ \hspace{1cm} $6.227$ \\
 & $\gamma$ \hspace{1cm} $35.629$ \\
\hline
$d_0$ & $a$ \hspace{1cm} $0.488 \pm 0.033$ \\
 & $b$ \hspace{1cm} $5.878 \pm 0.239$ \\
 & $\rho$ \hspace{1cm} $-3.639$ \\
 & $\gamma$ \hspace{1cm} $16.445$ \\
\hline
$\Lambda_{QCD}^{(4)}$, MeV & $226 \pm 25$ \\
\hline
$\chi^2$/ndf & $472/546 = 0.86$ \\
\hline
\end{tabular}
\end{table}

obtained in the fit we extract the following value for the strong coupling constant

$$\alpha_s(M_Z^2) = 0.1134^{+0.0019}_{-0.0021} \text{ (expt).} \quad (6)$$

We note that this value is in agreement within the errors with results obtained from other NNLO QCD analyses\cite{13,14} and with the the world average $0.1182 \pm 0.0027$\cite{15} within 2$\sigma$.

In Figure 1 we compare the parton distribution functions $xu_v(x)$ and
$xd_v(x)$ at the reference scale $Q_0^2 = 4 \text{ GeV}^2$ as extracted from our fit with the results obtained in other NNLO QCD fits.

Figure 1. The parton densities $xu_v$ (left) and $xd_v$ (right) at the input scale $Q_0^2 = 4 \text{ GeV}^2$ compared to results obtained by MRST$^{13}$ and Alekhin$^{11}$. The shaded areas represent the fully correlated $1\sigma$ error bands.

References